

# Degenerate Higher-Order Scalar-Tensor (DHOST) theories

David Langlois

*APC (Astroparticules et Cosmologie), CNRS-Université Paris Diderot,  
10, rue Alice Domon et Léonie Duquet, 75013 Paris, France*

This contribution reviews scalar-tensor theories whose Lagrangian contains second-order derivatives of a scalar field but nevertheless propagate only one scalar mode (in addition to the usual two tensor modes), and are thus not plagued with the Ostrogradsky instability. These theories, which encompass the so-called Horndeski and Beyond Horndeski theories, have recently been fully classified up to cubic order in second-order derivatives. After introducing these theories, I present a few phenomenological aspects. In cosmology, these theories can be included in the unified effective description of dark energy and modified gravity. Finally, neutron star solutions in some specific models are discussed.

## 1 Introduction

There have been numerous attempts to modify or extend general relativity, with either the motivation to account for dark energy (and sometimes dark matter) or, more modestly, to construct benchmark models that are useful to test general relativity quantitatively. Scalar-tensor theories have often played a prominent role in these attempts and, lately, special attention has been devoted to scalar-tensor theories whose Lagrangians contain second-order derivatives of a scalar field.

Lagrangians of this type, which contain “accelerations”, are generically plagued by an instability due to the presence, in addition to the usual scalar mode and tensor modes, of an extra scalar degree of freedom (unless the higher order terms can be treated as perturbative terms in the sense of low energy effective theories). Until recently, it was believed that only theories that yield second-order Euler-Lagrange equations were free of this dangerous extra degree of freedom. In the last couple of years, it has been realized that there in fact exists a much larger class of theories that satisfy this property.

## 2 From Horndeski to DHOST theories

### 2.1 Higher-Order Scalar-Tensor theories

In this section, we introduce scalar-tensor theories whose action is a functional of a metric  $g_{\mu\nu}$  and of a scalar field  $\phi$ , allowing for a dependence not only on  $\phi$  and its gradient  $\phi_\mu \equiv \nabla_\mu \phi$  as usual, but also on its second derivatives  $\phi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi$ . Restricting our investigation to Lagrangians that depend on  $\phi_{\mu\nu}$  up to cubic order, we are interested by actions of the form

$$\begin{aligned} S[g, \phi] = \int d^4x \sqrt{-g} & \left[ f_0(X, \phi) + f_1(X, \phi) \square \phi + f_2(X, \phi) R + C_{(2)}^{\mu\nu\rho\sigma} \phi_{\mu\nu} \phi_{\rho\sigma} + \right. \\ & \left. + f_3(X, \phi) G_{\mu\nu} \phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \phi_{\mu\nu} \phi_{\rho\sigma} \phi_{\alpha\beta} \right], \end{aligned} \quad (1)$$

where the functions  $f_i$  depend only on  $\phi$  and  $X \equiv \phi_\mu \phi^\mu$ , while  $R$  and  $G_{\mu\nu}$  denote, respectively, the usual Ricci scalar and Einstein tensor associated with the four-dimensional metric  $g_{\mu\nu}$ . The tensors  $\mathbf{C}_{(2)}$  and  $\mathbf{C}_{(3)}$  are the most general tensors constructed with the metric  $g_{\mu\nu}$  and the scalar field gradient  $\phi_\mu$ .

It is easy to see that the terms quadratic in  $\phi_{\mu\nu}$  can be rewritten as

$$C_{(2)}^{\mu\nu\rho\sigma} \phi_{\mu\nu} \phi_{\rho\sigma} = \sum_{A=1}^5 a_A(X, \phi) L_A^{(2)}, \quad (2)$$

with

$$\begin{aligned} L_1^{(2)} &= \phi_{\mu\nu} \phi^{\mu\nu}, & L_2^{(2)} &= (\Box\phi)^2, & L_3^{(2)} &= (\Box\phi) \phi^\mu \phi_{\mu\nu} \phi^\nu, \\ L_4^{(2)} &= \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu, & L_5^{(2)} &= (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2, \end{aligned} \quad (3)$$

where the  $a_A$  are five arbitrary functions of  $X$  and  $\phi$ . Similarly, the cubic terms can be written in terms of ten arbitrary functions  $b_A$ , as

$$C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \phi_{\mu\nu} \phi_{\rho\sigma} \phi_{\alpha\beta} = \sum_{A=1}^{10} b_A(X, \phi) L_A^{(3)}, \quad (4)$$

where

$$\begin{aligned} L_1^{(3)} &= (\Box\phi)^3, & L_2^{(3)} &= (\Box\phi) \phi_{\mu\nu} \phi^{\mu\nu}, & L_3^{(3)} &= \phi_{\mu\nu} \phi^{\nu\rho} \phi_\rho^\mu, & L_4^{(3)} &= (\Box\phi)^2 \phi_\mu \phi^{\mu\nu} \phi_\nu, \\ L_5^{(3)} &= \Box\phi \phi_\mu \phi^{\mu\nu} \phi_{\nu\rho} \phi^\rho, & L_6^{(3)} &= \phi_{\mu\nu} \phi^{\mu\nu} \phi_\rho \phi^{\rho\sigma} \phi_\sigma, & L_7^{(3)} &= \phi_\mu \phi^{\mu\nu} \phi_{\nu\rho} \phi^{\rho\sigma} \phi_\sigma, \\ L_8^{(3)} &= \phi_\mu \phi^{\mu\nu} \phi_{\nu\rho} \phi^\rho \phi_\sigma \phi^{\sigma\lambda} \phi_\lambda, & L_9^{(3)} &= \Box\phi (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2, & L_{10}^{(3)} &= (\phi_\mu \phi^{\mu\nu} \phi_\nu)^3. \end{aligned} \quad (5)$$

Theories described by an action of the form (1) in general contain, in addition to the usual scalar mode and two tensor modes, an extra scalar mode leading to the so-called Ostrogradsky instability [1]. However, by imposing some restrictions on the functions  $f_2$ ,  $f_3$  and  $a_A$  and  $b_A$ , it is possible to find theories that only one propagating scalar mode.

Historically, theories of this type were found in several steps. The starting point was the construction of higher-order theories leading to at most second-order Euler-Lagrange equations for the metric and for the scalar field, due to Horndeski [2]. Until recently, it was (wrongly) believed that requiring at most second-order Euler-Lagrange equations was necessary to get rid of the extra scalar degree of freedom and, as a consequence, Horndeski theories were considered to be the most general theories without Ostrogradsky instability. This belief was challenged by a new class of theories, now often called Beyond Horndeski (or GLPV), proposed in [3], extending Horndeski's theories and leading to higher-order equations of motion<sup>a</sup>. Beyond Horndeski theories were finally superseded by a larger class of theories, the DHOST theories, once it was understood that the crucial element that characterizes higher-order theories propagating a single scalar degree of freedom is the degeneracy of their Lagrangian [5], rather than the order of their equations of motion<sup>b</sup>. By using the degeneracy criterium, the quadratic DHOST theories were first identified in [5], extending both quadratic Horndeski and Beyond Horndeski Lagrangians<sup>c</sup>. More recently, all DHOST theories up to cubic order have been systematically classified [9].

<sup>a</sup>It is also worth noting that [4] had already pointed out the possibility to construct theories “beyond Horndeski” by applying disformal transformations of the metric to the Einstein-Hilbert action.

<sup>b</sup>In [6], after showing that the equations of motion of Beyond Horndeski theories, written in an arbitrary gauge, can be reformulated as a system of equations with at most second-order time derivatives, it was suggested that this property could provide a characterization of theories with a single scalar DOF. However, one can notice that this reformulation also applies to theories that are not degenerate (and thus propagate an extra scalar DOF), therefore invalidating this diagnostic. Note that, for scalar-tensor theories, the presence of redundant variables, due to the diffeomorphism invariance, makes it difficult to count the number of degrees of freedom just from the order of the equations of motion; instead, one needs to identify the number of independent initial conditions that are physically relevant.

<sup>c</sup>The name DHOST was not coined in the original paper but later in [7]. Note that the very same (quadratic) theories were also dubbed “Extended Scalar-Tensor” in [8].

## 2.2 Degeneracy of the Lagrangian

As mentioned above, the crucial ingredient that singles out higher-order theories with a single scalar degree of freedom is the degeneracy of their Lagrangian [5]. To better understand this notion of degeneracy, it is instructive to present a very simple toy model based on classical point dynamics. Let us consider the Lagrangian

$$L = \frac{1}{2}a\ddot{\phi}^2 + b\ddot{\phi}\dot{q} + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}c\dot{q}^2 - V(\phi, q), \quad (6)$$

where  $a$ ,  $b$  and  $c$  are constant coefficients and  $V(\phi, q)$  is some potential. This Lagrangian involves the acceleration of  $\phi$  but not that of  $q$ . If  $a \neq 0$ , one gets fourth-order equations of motion, whereas, if  $a = 0$  but  $b \neq 0$ , one obtains third-order equations of motion.

In order to work with a more familiar Lagrangian containing only velocities, let us introduce, following [5], the auxiliary variable

$$Q \equiv \dot{\phi}, \quad (7)$$

leading to the new (and equivalent) Lagrangian

$$L = \frac{1}{2}a\dot{Q}^2 + b\dot{Q}\dot{q} + \frac{1}{2}c\dot{q}^2 + \frac{1}{2}Q^2 - V(\phi, q) - \lambda(Q - \dot{\phi}), \quad (8)$$

which does not include any acceleration.

Let us now try to identify the number of independent physical degrees of freedom in the system. Equivalently, one can count the number of initial conditions that are needed to fully determine the system at some initial time. From the equations of motion, it is easy to see that two cases arise, depending on the nature of the Hessian matrix, defined by

$$M \equiv \left( \frac{\partial^2 L}{\partial v^a \partial v^a} \right) = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \quad (9)$$

where the symbol  $v^a$  denotes the velocities, i.e.  $v^a \equiv \{\dot{Q}, \dot{q}\}$ .

In the generic case where  $M$  is invertible, one finds that six initial conditions are needed, which corresponds to the existence of three degrees of freedom. While the variable  $q$  describes as usual one degree of freedom, the variable  $\phi$  is associated with two degrees of freedom. By contrast, in the particular cases where  $M$  is degenerate, i.e.

$$\det M = ac - b^2 = 0, \quad (10)$$

only four initial conditions are necessary, which means that only two degrees of freedom are present. The extra mode associated with  $\phi$  is eliminated when  $M$  is degenerate. By extension, it can be said that the initial Lagrangian (6) is degenerate in this situation.

The number of degrees of freedom can also be determined by resorting to a Hamiltonian analysis. When the Lagrangian is degenerate, the conjugate momenta satisfy a (primary) constraint. By writing down the time evolution of this constraint, one finds that it leads to a secondary constraint in phase space. These two constraints eliminate one degree of freedom, in agreement with the analysis based on the equations of motion.

The above discussion can be generalised to the case of  $n$  variables similar to  $\phi$ . In order to get rid of all of the  $n$  extra degrees of freedom that arise in general, one must not only impose a degeneracy of order  $n$  of the Hessian matrix, which guarantees the existence of  $n$  primary constraints, but also require additional constraints to ensure the presence of  $n$  secondary constraints [10, 11].

### 2.3 Horndeski and Beyond Horndeski theories

Well-known particular examples of DHOST theories are Horndeski's theories. They are characterized by four arbitrary functions of  $\phi$  and  $X$ , corresponding to the four functions  $f_A$  that appear in the general action (1). The other functions  $a_A$  and  $b_A$  are then completely determined in terms of  $f_2$  and  $f_3$ , respectively. The quadratic part of the Horndeski action, which it is convenient to denote  $L_{(2)}^H[f_2]$ , is thus fully determined by the function  $f_2$ , with the quadratic coefficient  $a_A$  given by

$$a_1 = -a_2 = 2f_{2,X}, \quad a_3 = a_4 = a_5 = 0. \quad (11)$$

Similarly, the cubic part of Horndeski theories,  $L_{(3)}^H[f_3]$ , depends only on the functions  $f_3$ , while

$$3b_1 = -b_2 = \frac{3}{2}b_3 = f_{3,X}, \quad b_A = 0 \quad (A = 4, \dots, 10). \quad (12)$$

The so-called Beyond Horndeski (or GLPV) theories, introduced in [3], extend Horndeski theories by including two additional Lagrangians, each characterized by a single arbitrary function. The first of these Lagrangians, which can be written as  $L_{(2)}^{\text{bH}}[g_2]$ , is quadratic and characterized by the coefficients

$$a_1 = -a_2 = Xg_2, \quad a_3 = -a_4 = 2g_2, \quad a_5 = 0. \quad (13)$$

The second new Lagrangian, which is cubic and will be denoted  $L_{(3)}^{\text{bH}}[g_3]$ , depends on a single arbitrary function  $g_3$  and its non vanishing coefficients  $b_A$  are given by

$$\frac{b_1}{X} = -\frac{b_2}{3X} = \frac{b_3}{2X} = -\frac{b_4}{3} = \frac{b_5}{6} = \frac{b_6}{3} = -\frac{b_7}{6} = g_3. \quad (14)$$

In the original paper [3], it was not yet fully clear whether arbitrary sums of the four Horndeski Lagrangians and of the two new Lagrangians  $L_{(2)}^{\text{bH}}[g_2]$  and  $L_{(3)}^{\text{bH}}[g_3]$  were viable. It was first pointed out in [12] that some Beyond Horndeski theories could be related to Horndeski theories via (invertible) conformal-disformal transformations, in which case they should have the same number of degrees of freedom as their Horndeski counterparts<sup>d</sup>. But it was only with the concept of degeneracy that this question was finally settled, with the results of [5] and [13].

Let us briefly summarize these results, by stressing that the sum of two degenerate Lagrangians is not necessarily degenerate. Moreover the terms  $f_0$  and  $f_1 \square \phi$  can always be added in the action without modifying the degeneracy of the total Lagrangian, so we do not need to worry about these terms any further. For the remaining terms, the following combinations involving Beyond Horndeski terms are degenerate:  $L_{(2)}^H + L_{(2)}^{\text{bH}}$ ,  $L_{(3)}^H + L_{(3)}^{\text{bH}}$  and  $L_{(2)}^{\text{bH}} + L_{(3)}^{\text{bH}}$ . By contrast, the following combinations are not degenerate:  $L_{(2)}^H + L_{(2)}^{\text{bH}} + L_{(3)}^H$ ,  $L_{(2)}^H + L_{(2)}^{\text{bH}} + L_{(3)}^{\text{bH}}$ .

### 2.4 DHOST theories

As discussed above, the crucial element that characterizes higher-order theories with a single scalar degree of freedom is the degeneracy of their Lagrangian, hence their name DHOST<sup>e</sup>.

DHOST theories were originally identified at quadratic order in  $\phi_{\mu\nu}$  (i.e. with the functions  $f_2$  and  $a_A$  only) in [5] and a complete Hamiltonian analysis in [14] soon confirmed that they indeed contained only one scalar degree of freedom. Quadratic DHOST theories were further studied in [7, 8, 15]. More recently, the identification of DHOST theories has been extended up to cubic order, i.e. by including the second line of (1), in [9] and the interested reader will find the full classification there.

<sup>d</sup>Note that the calculation in the final part of [12], directly inspired by a similar calculation in [4], does not lead to a manifestly second-order system, as originally claimed. But the main point of the paper, based on disformal transformations, remains valid.

<sup>e</sup>Amusingly, this acronym can be obtained by substituting the initial of 'ghost' with 'D' of 'degeneracy'.

In summary, DHOST theories include seven subclasses of quadratic theories (four classes with  $f_2 \neq 0$  and three classes with  $f_2 = 0$ ) and nine subclasses of cubic theories (two with  $f_3 \neq 0$  and seven with  $f_3 = 0$ ). These quadratic and cubic subclasses can be combined to yield degenerate hybrid theories, involving both quadratic and cubic terms, but all combinations are not possible: only 25 combinations (out of 63) lead to degenerate theories, often with extra conditions on the functions  $a_A$  and  $b_A$  in the Lagrangian (see [9] for details and for the explicit form of the functions in each subclass).

## 2.5 Disformal transformations

A legitimate question about this classification is whether seemingly different DHOST theories could correspond the same theory in different guises, in other words whether some theories could be identified via field redefinitions<sup>f</sup>. Since the Lagrangian depends on a metric and on a scalar field, natural field redefinitions of the metric involve disformal transformations [16]

$$\tilde{g}_{\mu\nu} = C(X, \phi)g_{\mu\nu} + D(X, \phi)\phi_\mu\phi_\nu. \quad (15)$$

Via this transformation, any action  $\tilde{S}$  given as a functional of  $\tilde{g}_{\mu\nu}$  and  $\phi$  induces a new action  $S$  for  $g_{\mu\nu}$  and  $\phi$ , when one substitutes the above expression for  $\tilde{g}_{\mu\nu}$  in  $\tilde{S}$ :

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}[\phi, \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \phi_\mu\phi_\nu]. \quad (16)$$

The actions  $S$  and  $\tilde{S}$  are then said to be related by the disformal transformation (15). The disformal transformations of all quadratic DHOST theories have been investigated in [7], where it was shown that all seven subclasses are stable under the action of disformal transformations.

Interestingly, there is a nice correspondence between the type of disformal transformations and the extent of the corresponding stable class of theories:

- Horndeski theories are stable under disformal transformations characterized by  $C(\phi)$  and  $D(\phi)$ , i.e. conformal and disformal factors that depend only on  $\phi$ , but not on  $X$  [17].
- Beyond Horndeski theories are stable under disformal transformations characterized by  $C(\phi)$  and  $D(\phi, X)$  [12].
- Finally, DHOST theories are stable under the most general disformal transformations where  $C$  and  $D$  depend on both  $\phi$  and  $X$  [7].

## 3 Cosmology and astrophysics

After the short introduction to DHOST theories given in the previous section, let us now discuss briefly some phenomenological consequences of these theories in the context of cosmology and of astrophysics.

### 3.1 Cosmology

In order to study the cosmology of DHOST theories, it is very convenient to resort to the unified formalism that has been developed for an effective description of Dark Energy and Modified Gravity (see e.g. [18] for a review).

This approach is based on a  $3 + 1$  decomposition of spacetime, in which the spatial slices coincide with uniform scalar field hypersurfaces. In this particular gauge, sometimes called unitary gauge, the action of DHOST theories is of the form

$$S = \int d^3x dt N \sqrt{h} L[N, K_{ij}, {}^3R_{ij}; t], \quad (17)$$

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<sup>f</sup>The coupling to matter is ignored here. If, after a redefinition of the metric, two related theories are minimally coupled to matter, then they are physically distinct.

where  $N$  is the lapse function [which appears in the  $3+1$  form of the spacetime metric  $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$ ,  $N^i$  being the shift vector,  $h_{ij}$  the spatial metric];  $K_{ij}$  is the extrinsic curvature tensor and  ${}^3R_{ij}$  the intrinsic curvature tensor.

The Friedmann equations associated with a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime  $ds^2 = -\bar{N}^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ , are then simply derived from the homogeneous action

$$S_{\text{homog}} = \int dt N a^3 L[N = \bar{N}(t), K_j^i = \frac{\dot{a}}{\bar{N}a}\delta_j^i, {}^3R_{ij} = 0; t]. \quad (18)$$

To study the dynamics of linear perturbations, one needs to write down the action at quadratic order in perturbations. These perturbations are associated with the three basic ingredients of the action:

$$\delta N \equiv N - \bar{N}, \quad \delta K_j^i = K_j^i - H\delta^i_j, \quad \delta {}^3R_j^i = {}^3R_j^i, \quad (19)$$

where  $H = \dot{a}/\bar{N}a$  is the Hubble parameter, and  ${}^3R_j^i$  is already a perturbation since it vanishes in the background. The Lagrangian at quadratic order is then obtained via a Taylor expansion, which is formally written as

$$L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A} \delta q^A + \frac{1}{2} \frac{\partial^2 L}{\partial q_A \partial q_B} \delta q^A \delta q^B + \dots \quad (20)$$

where  $q^A = \{N, K_j^i, {}^3R_j^i\}$ .

All (quadratic and cubic) DHOST theories lead to a Lagrangian quadratic in linear perturbations of the form [19]

$$S_{\text{quad}} = \int d^3x dt a^3 \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left(1 + \frac{2}{3}\alpha_L\right) \delta K^2 + (1 + \alpha_T) \left( {}^3R \frac{\delta\sqrt{h}}{a^3} + \delta_2 {}^3R \right) + H^2 \alpha_K \delta N^2 \right. \\ \left. + 4H\alpha_B \delta K \delta N + (1 + \alpha_H) {}^3R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\}, \quad (21)$$

where  $\delta_2 {}^3R$  denotes the second order term in the perturbative expansion of  ${}^3R$ , where the parameters  $M$ ,  $\alpha_L$ ,  $\alpha_T$ ,  $\alpha_K$ ,  $\alpha_B$ ,  $\alpha_H$ ,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  are time-dependent functions. Moreover, one finds that these parameters, for DHOST theories, are restricted to satisfy either one of the following sets of conditions:

$$\mathcal{C}_I : \quad \alpha_L = 0, \quad \beta_2 = -6\beta_1^2, \quad \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1(1 + \alpha_T)], \quad (22)$$

or

$$\mathcal{C}_{II} : \quad \beta_1 = -(1 + \alpha_L) \frac{1 + \alpha_H}{1 + \alpha_T}, \quad \beta_2 = -6(1 + \alpha_L) \frac{(1 + \alpha_H)^2}{(1 + \alpha_T)^2}, \quad \beta_3 = 2 \frac{(1 + \alpha_H)^2}{1 + \alpha_T}. \quad (23)$$

The category  $\mathcal{C}_I$  contains the subclass of Horndeski theories and of those related to Horndeski via disformal transformations.

In the static linear regime around Minkowski, one can define an effective Newton's constant given by [19]

$$8\pi G_N = M^{-2} \left[ \frac{(1 + \alpha_H)^2}{1 + \alpha_T} - \frac{\beta_3}{2} \right]^{-1}, \quad (24)$$

which diverges for theories satisfying  $\mathcal{C}_{II}$ .

The coupling to matter can be minimal or nonminimal. One can derive the transformation of the parameters that describe the matter coupling at linear level under disformal transformations (15). Similar transformations exist for the parameters of (21). See [19] for details.

### 3.2 Stars in Beyond Horndeski theories

Even if the main motivation for modified gravity arises from the observed acceleration of the cosmological expansion, it is indispensable to verify that any viable theory remains compatible with astrophysical observations and solar system constraints.

As part of this programme, let us concentrate on Beyond Horndeski theories. In these models, it has been noticed that the Vainshtein mechanism is partially broken inside matter [20]. For spherical bodies, a new term appears in the gravitational law,

$$\frac{d\Phi}{dr} = \frac{G_N M(r)}{r^2} + \frac{\Upsilon}{4} G_N \frac{d^2 M(r)}{dr^2}, \quad (25)$$

where  $\Phi$  is the gravitational potential and  $M(r)$  is the mass inside a sphere of radius  $r$ . This leads to a modified profile of Newtonian stars [21, 22].

Following the recent works [23] and [24], let us discuss neutron stars in a specific model described by the action

$$S = \int d^4x \sqrt{-g} \left[ M_{\text{P}}^2 \left( \frac{R}{2} - k_0 \Lambda \right) - k_2 X - \frac{\zeta}{2} X^2 + \mathcal{L}_{(2),\text{bH}}[g_2] \right], \quad (26)$$

where  $k_0$ ,  $k_2$ ,  $\zeta$  and  $g_2$  are assumed to be constant. By writing the Friedmann equations for this model, one can easily find de Sitter solutions where the Hubble parameter  $H$  is constant. In order to embed a spherical object within such cosmological spacetime, it is useful to rewrite the de Sitter solution in Schwarzschild-like coordinates,

$$ds^2 = -(1 - H^2 r^2) dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (27)$$

$$\phi(r, t) = v_0 t + \frac{v_0}{2H} \ln(1 - H^2 r^2). \quad (28)$$

One can then insert a spherical symmetric object in this cosmological solution by trying to solve the Einstein equations for a metric of the form

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (29)$$

going asymptotically to (27). The energy-momentum tensor receives a contribution from the scalar field as well as a contribution from a perfect fluid which is assumed to model the neutron star's matter. Remarkably, one can find an exact solution outside the star, corresponding to a Schwarzschild-de Sitter geometry. Inside the star, the equations of motion can be solved numerically, assuming some equation of state, in order to determine the matter density profile and the internal geometry.

When  $\Upsilon < 0$ , one finds that stars with fixed mass have a larger radius than their GR counterparts. Moreover, for the same equation of state, the maximum mass can increase significantly with respect to GR [23]. Modified gravity could thus provide a solution to the hyperon puzzle. Moreover, one can derive a relation between the dimensionless moment of inertia  $I c^2 / G^2 M^3$  and the compactness  $GM/Rc^2$ , which is robust in the sense that it weakly depends on the equation of state and which can discriminate between modified gravity and GR [24].

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