ESTIMATION OF HEATING IN THICK TARGETS COMPOSED OF SEVERAL ELEMENTS

Accurate calculation of the heat deposited in each volume increment of thick targets is a complex problem. When the target is composed of laminations of several elements, application of theory becomes impracticable. For engineering applications, a method of estimation has been devised which provides the heat deposited by an electron beam and the pulse temperature rise at any depth in a laminated target.

Mathematical Model

The method assumes a mathematical model that matches several known properties of shower production in thick targets:

1. First interception of the beam by a thin target deposits energy at a rate about $dW/dx = 1.52$ Mev/(g/cm^2).[1]

2. Until other reactions become sufficiently probable to be controlling, both photon and pair production depend upon the mass of material traversed. Initial energy deposition therefore rises quadratically with depth.[2]

3. Maximum heat deposition occurs at the depth corresponding to peak showering, about 5.4 radiation lengths.[3]

4. Energy deposition declines exponentially after the peak. Consider the function

$$y = \frac{e^2 - 3c}{4} u^2 e^{-u} + cue^{-u} + ce^{-u}$$

where $u = \frac{2x}{5.4}$ and x is the depth into the target in radiation lengths. y is the fraction of peak showering energy deposition. $c \sim 0.02$ is a small fraction which will be assigned a value to yield 1.52 Mev/(g/cm) initial deposition. Thus, for $u = 0$, $y' = 0$ and $y = c$.

At small u, since $c \ll e^2$, y increases quadratically:

$$y \sim \frac{e^2}{4} u^2$$

For peak showering, $x = 5.4$, $u = 2$, and then $y = 1$ as required. The actual maximum, $y' = 0$, occurs at

$$u = 2 - \frac{4c}{e^2 - 3c}$$

rather than at the exact point $u = 2$. However, for $c \ll e^2$ the difference is negligible and the complexity of the exact function may be avoided.

As u increases, the curve decreases exponentially, for $u \to \infty$ $y \to y' \to 0$. A typical curve for y is shown in Fig. 1.

Heat Deposition

At peak showering, the energy deposition is given by

$$\left(\frac{dW}{dx}\right)_{max} = W'_m = \frac{0.31}{\sqrt{\mathrm{Ln}\left(0.689 \frac{E_o}{\epsilon_1}\right)}} \cdot \frac{E_o}{\epsilon_1} \cdot \frac{\epsilon_2}{x_2} \qquad (4)$$

where E_o is the initial beam energy, ϵ_1 is the critical energy of the material in which the shower is generated, ϵ_2 is the critical energy of the material in which energy deposition is desired, and x_2 is the radiation length of the latter. When W'_m has been found, c may be evaluated:

$$c = \frac{1.52}{W'_m}$$

The target depth x in radiation lengths determines u, which can then be substituted with c to evaluate y. The heat deposited is then

$$H = y\, W'_m\, \rho_2\, t_2\, I$$

where ρ_2 is the density and t_2 the thickness of the material, and I is the beam current. By considering several thin sections at various depths, the variation in H with depth may be determined.

Weighting of Several Materials

Where more than one material has contributed to the shower, the radiation lengths are added to find the depth x. But the critical energy, ϵ_1, must reflect the contributions of the several elements. The function y could be used for this purpose but it is more convenient to use a simpler form:

$$z = u^2 e^{-u}$$

where again $u = \frac{2x}{5.4}$ so that the maximum, $z' = 0$, occurs at $x = 5.4$ and $u = 2$. The function z appears in Figure 2. Since only ratios will be used, normalizing coefficients are unnecessary.

Use of this weighting function is based on the following argument: In a target stack consisting of, say, tungsten, copper, and aluminum, the radiation lengths of each may be added as shown by the successive vertical lines in Figure 2. The height of the curve is a measure of the number of particles developed at any point of the shower process. Of course, a heavy metal will develop more particles in one radiation length than a light metal, so that the same height of z for, say, tungsten and aluminum does not mean an equal number of actual particles in each. The actual number of particles is introduced into W_m' by the factor $1/\epsilon_1$ and it is this factor, therefore, that must be adjusted to reflect the contribution of each target element. Function z must be regarded therefore as a fictitious particle count which will be used to define areas over x. These areas as shown in Figure 2 are taken to represent the contribution of each target element to an effective value of ϵ_1.

The area between $x = m$ and $x = n$ will be

$$\int_m^n z\, dx = 2.7 \int_a^b u^2 e^{-u}\, du$$

if $m = 2.7\,a$ and $n = 2.7\,b$. Then

$$\int_a^b u^2 e^{-u}\,du = (a^2 + 2a + 2)\,e^{-a} - (b^2 + 2b + 2)\,e^{-b}$$

Assuming 9.24 mm of tungsten (2.7 radiation lengths, u = 1), 19.8 mm of copper (1.35 rl, u = 0.5), and 24.0 mm of aluminum (0.27 rl, u = 0.1), the effective critical energy would be

$$\frac{1}{\epsilon_1} = \frac{(2 - 5e^{-1})\frac{1}{\epsilon_W} + (5e^{-1} - 7.25\,e^{-1.5})\frac{1}{\epsilon_{Cu}} + (7.25\,e^{-1.5} - 7.76\,e^{-1.6})\frac{1}{\epsilon_{Al}}}{(2 - 7.76\,e^{-1.6})}$$

Pulse Temperature Rise

The pulse temperature rise is the heat deposited in the beam spot on a thin foil divided by the heat capacity of that volume of the foil under the beam spot. In practice, the density and thickness of the foil drop out:

$$\Delta T = \frac{y\,W'_m\,\rho_2\,t_2\,I}{\pi\,r^2\,t_2\,\rho_2\,c_p}$$

$$\Delta T = \frac{y\,W'_m\,I}{\pi\,r^2\,c_p}$$

where r is the radius of the beam spot and c_p is the heat capacity of the material. The result is very sensitive to the estimate of r that is used. Further, the spot size will increase due to scattering. With high energy beams and targets too thin to reach peak showering, this broadening is small and was omitted from consideration, along with any energy variation across the beam spot. Estimating the original beam spot size may contribute the largest uncertainty to calculation of temperature rise.

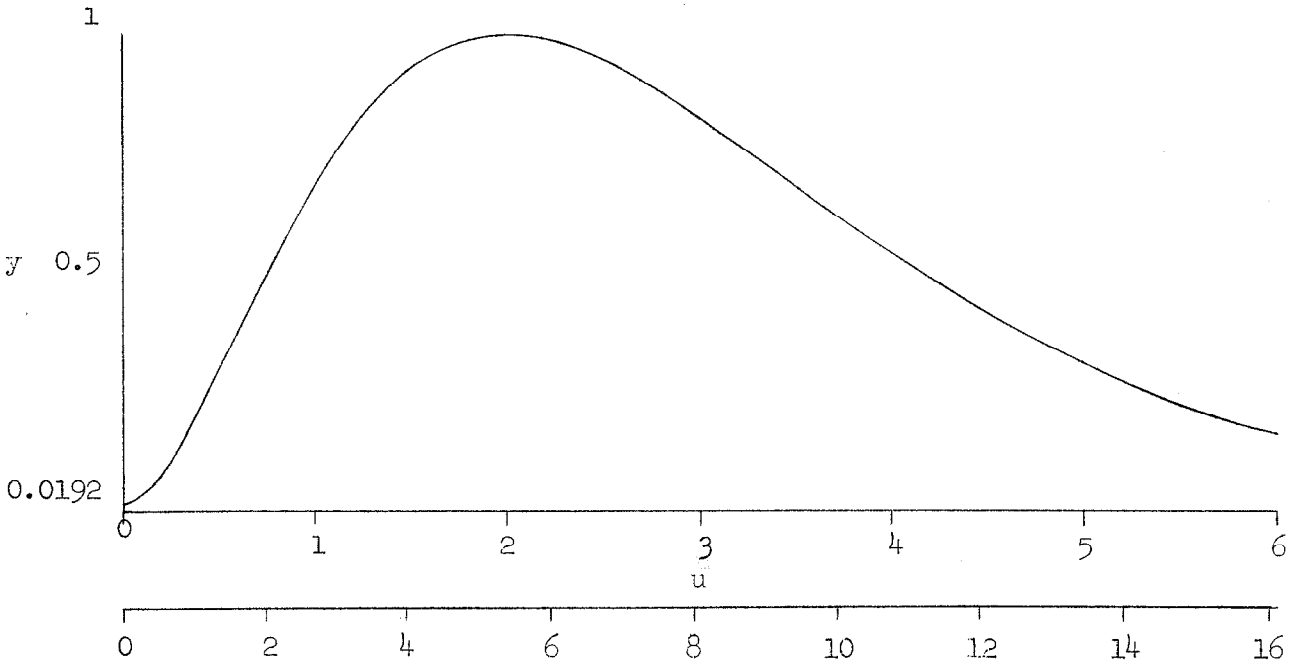
Computer Program

Although the calculations are simple they are time-consuming. A computer program in B5500 Algol has therefore been assembled to provide y, W'_m, and ΔT for target combinations of the following elements: Be, C, Mg, Al, Ti, Cr, Fe, Ni, Cu, Zr, Nb, Mo, Ag, In, Sn, W, Au, Pb. The program listing is given in Appendix A. Sample data cards for three problems, based on the example of Figure 2, are shown in Appendix B. Computer output for the three problems is reproduced in Appendix C.

Any number of problems may be stacked and run at the same time.

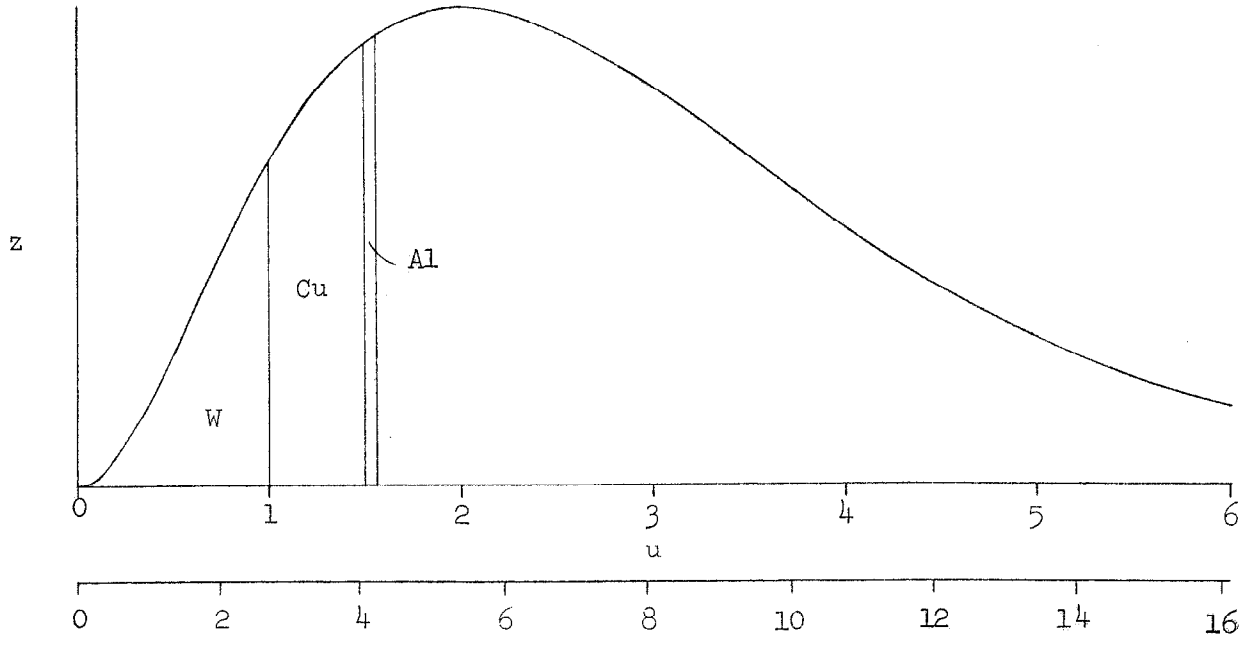
REFERENCES

1. H. DeStaebler, personal communication.
2. J. Pine, personal communication.
3. Z. Guiragossian, "Longitudinal and Radial Distribution of Shower Development in Cu, H_2O, and Al," TN-63-85, Oct. 1963.
4. H. DeStaebler, "Energy Density in the Positron Radiator," SLAC DeS-20, Dec. 1964.



x - Radiation Lengths

Fig. 1--Shower model for copper



x - Radiation Lengths

Fig. 2--Weighting functions for critical energy

APPENDIX A

PROGRAM LISTING

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                                                                BEGIN COMMENT
HEATING IN THICK TARGETS OF MULTIPLE ELEMENTS;

REAL A,AREA,B,BEV,C,CP,DA,DT,DWDX,EC,H,MA,R,Y;   INTEGER J,K,N;
ARRAY EO,RHO,TM,X,XO[0:40], CA,EO,RHO,XO[0:18];
ALPHA ARRAY ID[0:12], EL[0:40], L[0:18];
LABEL  START, QUIT, SKIP;
FORMAT  F1(12A6), F2("PARAMETERS ARE NOT AVAILABLE FOR ELEMENT ",A2),
   F3(/"BEAM ENERGY =",F5.1," BEV,  PULSE CURRENT =",F5.1,
   " MA,  RADIUS =",F4.1," MM."//,X6,"EO",X7,
   "XO     DENSITY   THICKNESS    X"/"MATL   MEV/RL   G/CM2/RL   G/CM3",
   X6,"MM",X7,"RL"/),  F4(X1,A2,5F9.2),
   F5(//"TOTAL RADIATION LENGTHS =",F7.3,",  FUNCTION Y =",F7.4,","//
   "EFFECTIVE CRITICAL ENERGY =",F6.2," MEV/RL,  HEAT CAPACITY =",
   F6.3," CAL/G/K"//"(DW/DX)MAX =",F7.2," MEV/(G/CM2),"//
   "HEAT DEPOSITED =",F6.1," WATTS/MM OF DEPTH,"//
   "PULSE TEMPERATURE RISE =",F7.2," K.");
DEFINE  THRU = ←1 STEP 1 UNTIL #,  FORK = FOR K THRU N DO #,
   TITLE = F1, FOR K THRU 12 DO ID[K] #;

FILL L[*] WITH "BE","C","MG","AL","TI","CR","FE","NI","CU","ZR",
   "NB","MO","AG","IN","SN","W","AU","PB";
FILL EO[*] WITH 112,76,41.7,38.9,24.4,22.5,21,19.6,19,14.3,14,13.7,
   12.5,12.1,11.8,8.35,7.86,7.63;   COMMENT MEV/RL;
FILL XO[*] WITH 63.5,44.8,25.5,24,16,15,14.1,13.4,13.1,10.3,10.1,
   9.9,9.1,8.8,8.7,6.6,6.35,6.2;   COMMENT G/CM2/RL;
FILL RHO[*] WITH 1.86,2.25,1.74,2.7,4.5,7.1,7.86,8.9,8.94,6.4,8.4,
   10.2,10.5,7.28,7.28,19.3,19.32,11.35;   COMMENT G/CM3;
FILL CA[*] WITH .4849,.194,.2386,.2184,.1266,.1091,.1096,.1091,
   .0928,.0667,.0642,.0616,.0568,.0577,.0544,.0326,.0308,.0308;
                                                COMMENT CAL/G/K;
START:
READ (TITLE)[QUIT];   READ (N,BEV,MA,R);   FORK READ (EL[K],TM[K]);
WRITE ([PAGE]);   WRITE (TITLE);
                                                                COMMENT
LIST PARAMETERS FOR ELEMENTS;
FORK                                                    BEGIN
   FOR J THRU 18 DO
      IF L[J] = EL[K] THEN                       BEGIN
         EO[K] ← EO[J];   XO[K] ← XO[J];   RHO[K] ← RHO[J];  CP ← CA[J];
         GO TO SKIP;                             END;
   WRITE (F2, EL[K]);
   SKIP:   X[K] ← .1 × TM[K] / XO[K] × RHO[K];          END;

WRITE(F3,BEV,MA,R);  FORK WRITE(F4,EL[K],EO[K],XO[K],RHO[K],TM[K],X[K]);
                                                                COMMENT
ESTIMATE EFFECTIVE CRITICAL ENERGY, EC;
A ← AREA ← 0;      EC ← 1;
FORK                                                    BEGIN
B ← A + X[K]/2.7;
DA ← (A*2 + 2×A + 2) × EXP(-A) - (B*2 + 2×B + 2) × EXP(-B);


EC ← (AREA + DA) / (AREA/EC + DA/EO[K]);
AREA ← AREA + DA;      A ← B;                           END;

DWDX ← 310 / SQRT(LN(689.7×BEV/EC)) × BEV/EC × EO[N]/XO[N];      COMMENT
CONSTANTS:  .31 × 1000 MEV/BEV,  .6897 × 1000 MEV/BEV;
C ← 1.52 / DWDX;    Y ← ((EXP(2)-3×C)/4 × B*2 + C×B + C) × EXP(-B);
H ← .072 × Y × DWDX × RHO[N] × MA;                              COMMENT
CONSTANT:  .1 CM/MM × 1000 MUA/MA × 360 PULSE/SEC × 2@-6 SEC/PULSE;
DT ← .01521 × Y × DWDX × MA / R*2 / CP;                         COMMENT
CONSTANT:  1000 MUA/MA × .23889 CAL/JOULE / PI × 100 MM2/CM2 × 2@-6;

WRITE (F5, 2.7×B,Y,EC,CP,DWDX,H,DT);
GO TO START;                                    QUIT:          END.
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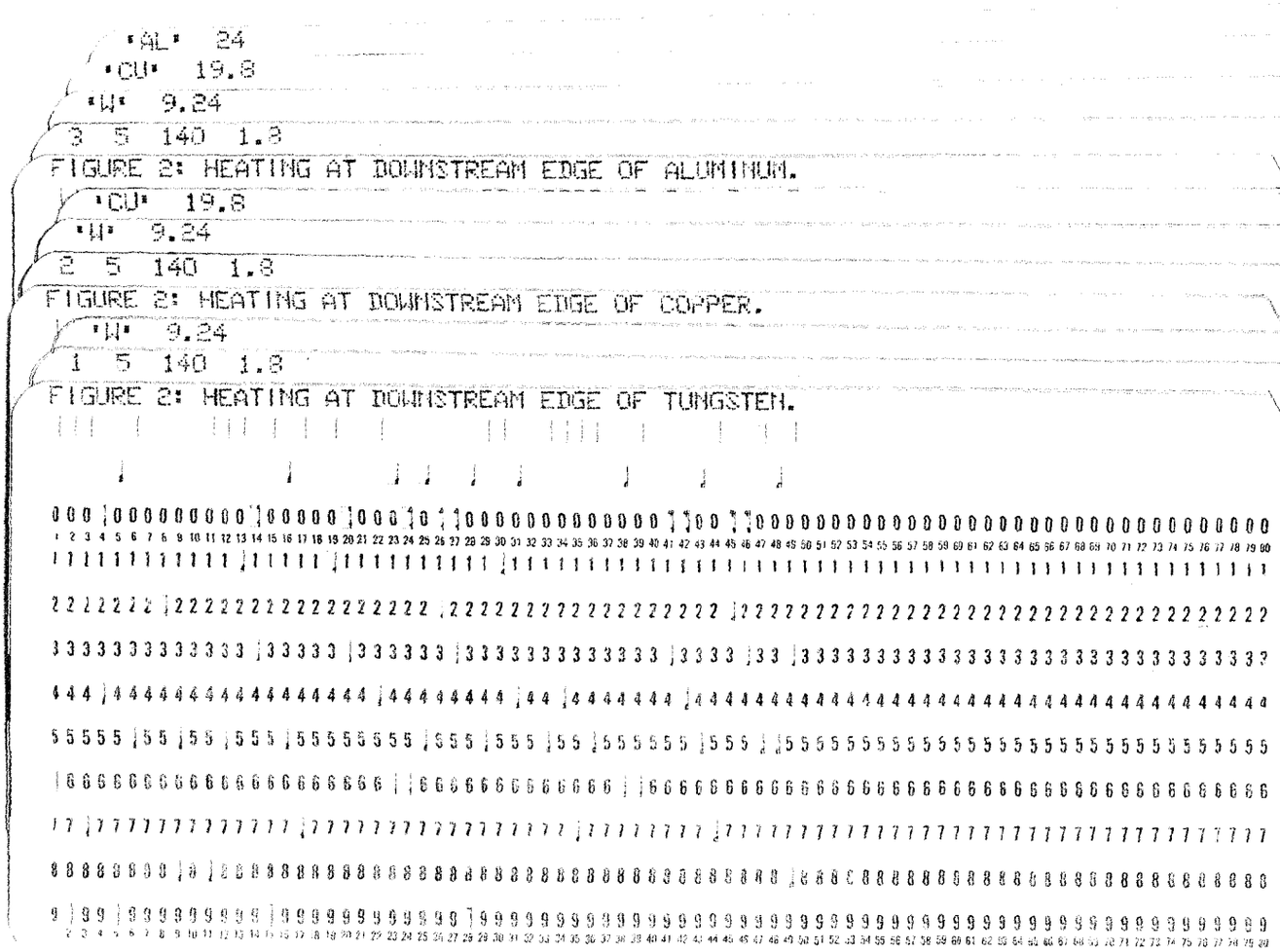
APPENDIX B

EXAMPLE OF DATA DECK



APPENDIX C

PROGRAM OUTPUT

FIGURE 2: HEATING AT DOWNSTREAM EDGE OF TUNGSTEN.

BEAM ENERGY = 5.0 BEV, PULSE CURRENT =140.0 MA, RADIUS = 1.8 MM.

MATL	EO MEV/RL	XO G/CM2/RL	DENSITY G/CM3	THICKNESS MM	X RL
OW	8.35	6.60	19.30	9.24	2.70

TOTAL RADIATION LENGTHS = 2.702, FUNCTION Y = 0.6874,

EFFECTIVE CRITICAL ENERGY = 8.35 MEV/RL, HEAT CAPACITY = 0.033 CAL/G/K

(DW/DX)MAX = 95.69 MEV/(G/CM2),

HEAT DEPOSITED = 12796.0 WATTS/MM OF DEPTH,

PULSE TEMPERATURE RISE =1326.02 K.

FIGURE 2: HEATING AT DOWNSTREAM EDGE OF ALUMINUM.

BEAM ENERGY = 5.0 BEV, PULSE CURRENT =140.0 MA, RADIUS = 1.8 MM.

MATL	EO MEV/RL	XO G/CM2/RL	DENSITY G/CM3	THICKNESS MM	X RL
OW	8.35	6.60	19.30	9.24	2.70
CU	19.00	13.10	8.94	19.80	1.35
AL	38.90	24.00	2.70	24.00	0.27

TOTAL RADIATION LENGTHS = 4.323, FUNCTION Y = 0.9577,

EFFECTIVE CRITICAL ENERGY = 13.45 MEV/RL, HEAT CAPACITY = 0.218 CAL/G/K

(DW/DX)MAX = 79.32 MEV/(G/CM2),

HEAT DEPOSITED = 2067.3 WATTS/MM OF DEPTH,

PULSE TEMPERATURE RISE = 228.58 K.

FIGURE 2: HEATING AT DOWNSTREAM EDGE OF COPPER.

BEAM ENERGY = 5.0 BEV, PULSE CURRENT =140.0 MA, RADIUS = 1.8 MM.

MATL	EO MEV/RL	XO G/CM2/RL	DENSITY G/CM3	THICKNESS MM	X RL
OW	8.35	6.60	19.30	9.24	2.70
CU	19.00	13.10	8.94	19.80	1.35

TOTAL RADIATION LENGTHS = 4.053, FUNCTION Y = 0.9314,

EFFECTIVE CRITICAL ENERGY = 12.37 MEV/RL, HEAT CAPACITY = 0.093 CAL/G/K

(DW/DX)MAX = 76.59 MEV/(G/CM2),

HEAT DEPOSITED = 6427.8 WATTS/MM OF DEPTH,

PULSE TEMPERATURE RISE = 505.16 K.