QCD and Deep Inelastic Scattering*

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Abstract

A number of current issues in QCD related to deep inelastic scattering are reviewed. After some general comments the question of measuring, and calculating, high energy scattering in the perturbative QCD domain is discussed. A brief account of issues and progress in partonic interactions with nuclear matter is given. A brief review of the status and issues in spin-dependent deep inelastic scattering is given. Finally, the relationship between the divergence of the perturbation series at the leading twist level and higher twist terms is discussed.

Résumé

Dans cet article, quelques uns des problèmes actuels en CDQ reliés à la diffusion profondément inélastique sont passés en revue.

1. General Comments

In some ways QCD suffers from too much success. (i) Consistent α-values are found using widely varying hard processes[1]. (ii) Extremely precise calculations are now available for a number of interesting, and measurable, processes. (iii) The essential correctness of perturbative QCD has been tested out to jet energies of 450 GeV setting the composite scale for quarks at greater than 1.5 TeV[2]. (iv) New calculational techniques are being developed for many-variable processes[3].

(i) The fundamental QCD coupling, α, has been determined at relatively low scales from deep inelastic lepton scattering, τ-decay, J/ψ and Υ-decays, and $e^+e^- \rightarrow jets$ and has been determined at the Z^0-mass from $\Gamma_{Z^0\rightarrow hadrons}$ and $Z^0 \rightarrow jets$. The α-values determined from these various process are consistent with $\alpha(M_Z) = 0.117 \pm 0.005$[1]. However, except for τ-decay, low energy determinations tend to give smaller values while high energy determinations, from Z^0-decay, tend to give somewhat higher values. Thus all is consistent, but there is perhaps the *hint* of something not quite right. We shall just have to wait and see.

(ii) There are now very precise calculations for a wide variety of processes. For example, for the Bjorken sum rule[4]

$$\Gamma_1^P - \Gamma_1^N = \int_0^1 dx[g_1^P(x,Q^2) - g_1^N(x,Q^2)]$$

$$\cdot = \frac{g_A}{6}[1 - \frac{\alpha}{\pi} - 3.58(\frac{\alpha}{\pi})^2 - 20.2(\frac{\alpha}{\pi})^3 + \cdots]$$

for three flavors in a $\overline{MS}$ renormalization scheme. Order α^3 corrections have been done for many "1-variable" quantities leading to an accuracy of 1-2% in the determination of such quantities.

In deep inelastic scattering one classifies the level of calculational precision according to the order of the formalism used. In a first order formalism the anomalous dimensions are determined to order α and the coefficient functions to order α^0. In a second order

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formalism the second order anomalous dimensions and first order coefficient functions are used to describe data. This is the present state of the art. Recently, the second order coefficient functions have been calculated[5] and as soon as third order, α^3, terms in the anomalous dimensions are known it will be possible to use a third order formalism to describe deep inelastic scattering which should raise the level of precision to a few percent.

Complete next-to-leading order calculations have been done, and are being used[6,7], for jet production in P-$\bar{\text{P}}$ collisions leading to about a 10% precision for very high-$p_\perp$ processes.

In $e^+e^- \rightarrow jets$ next-to-leading order calculations for three-jet production have long been used. The precision of the present data warrants a next-to-leading formalism for four-jet events and a next-to-next-to-leading order calculation of three-jet events.

(iii) The reliability and precision in predicting high $p_\perp jets$ in P-$\bar{\text{P}}$ collisions gives the best window for observing a possible composite structure of quarks. Present Fermilab data set a composite scale at greater than $1.5TeV$[3].

(iv) New calculational techniques[3], originally connected to string theory have recently been developed. One-loop amplitudes for processes involving many external variables have been constructed from general principles avoiding much of the tedious and painful work involved in doing the corresponding Feynman diagram calculations. While these one-loop calculations are important and useful phenomenologically, the real payoff would be to generalize the procedure to two and more loops. It is not clear whether this will be possible or not.

2. High Energy Scattering

2.1. Soft High Energy Scattering[8]

In hadronic collisions, total cross sections rise slowly with the center of mass energy, $\sqrt{s}$, roughly as $s^{0.08}$. The physics governing this energy dependence is nonperturbative QCD with the typical hadronic size of 1 fm $\sim 1/\Lambda$ determining the essential scale. There is a well developed phenomenology which successfully describes many high energy soft reactions. However, so far it has been very difficult to make a connection between this phenomenology and QCD. This difficulty is increased by the fact that lattice gauge theory, the only systematic approach to nonperturbative QCD, cannot be used to describe high energy soft scattering because of the intrinsic Minkowski space nature of such scattering.

2.2. Perturbative High Energy Scattering

Is there such a thing as high energy scattering in the perturbative domain? The answer is yes. Conceptually, high energy heavy onium-heavy onium forward scattering is an example[9]. For heavy enough quarks the onium radius, R, is much less than $1/\Lambda$ and perturbation theory applies to the forward scattering amplitude. In the two gluon exchange approximation

$$\sigma = c\pi R^2 \alpha^2(R) \tag{1}$$

is the total onium-onium cross section with the constant c a pure number.

Including extra gluons in a leading logarithmic approximation where all terms of the form $(\alpha Y)^n$, with $Y=\ell ns/M^2$ where M is the onium mass, are kept one finds

$$\sigma = \frac{16\pi R^2 \alpha^2 e^{(\alpha_P - 1)Y}}{\sqrt{7/2\ \alpha N_c \zeta(3) Y}}. \tag{2}$$

In (2) α_P is the Balitsky, Fadin, Kuraev and Lipatov (BFKL)[10,11] pomeron intercept with $\alpha_P - 1 = \frac{12\alpha}{\pi}\ell n2$, a number which is about 0.5 when $\alpha = 0.2$. If high energy heavy onium-heavy onium scattering data were available one could study[9]:

(i) How unitarity is restored at high energy.

(ii) The generalization of the Froissart bound for massless exchanges.

(iii) The connection between soft and hard high energy scattering.

(iv) Partonic saturation and high field strength QCD.

Of course we shall never have experimental data for onium-onium scattering. Nevertheless, onium-onium collisions furnish the simplest theoretical structure for studying questions of unitarity and parton saturation in high energy hard collisions. I believe we are quite close to a rather complete understanding of the energy domain where (2) holds and what energies are necessary to see unitarity corrections. We can also expect reasonably accurate calculations of unitarity corrections in the near future[12].

2.3. The Real World

What about the small-x region of deep inelastic scattering? Is it determined by high energy perturbative scattering? The answer to this question is not clear. High energy virtual Compton scattering on a proton, the process determining νW_2, is part way between high energy proton-antiproton scattering and high energy onium-onium scattering. All scales between Λ, the inverse proton radius, and Q, the photon virtuality can in principle be important. It may be that ordinary QCD evolution (the DGLAP equation)[13-15] gives a good description of the rapid rise of νW_2 at small x values. If such is the case, that the BFKL contribution to the small x behavior of νW_2 is small, then one will have to look elsewhere to study high energy perturbative scattering. If, on the other hand, the BFKL contribution to νW_2 is significant then one may be able to use νW_2 measurements to study high energy hard scattering. It is clearly very important to decide whether or not a second order DGLAP formalism is giving the correct behavior of νW_2 at small x.

There are processes which pick out BFKL evolution and strongly suppress DGLAP evolution and hence measure exactly the same physics as would be given by high energy onium-onium scattering. These processes occur both in deep inelastic scattering and in high energy proton-antiproton collisions. Let me here describe a measurement in proton-antiproton collisions which triggers on a BFKL mechanism of particle production[16]. Suppose the proton and antiproton have momenta P_1 and P_2 respectively in the center of mass of the collision. Consider events where two jets are measured inclusively (There may also be other jets in the event.) and where the longitudinal momenta of the two jets are $k_{1z} = x_1 P_1$ and $k_{2z} = x_2 P_2$, and where the two jets have $k_{1\perp}, k_{2\perp} > Q$ with Q setting the hard scale. Then a straightforward calculation gives, for large Y,

$$\frac{x_1 x_2 d\sigma}{dx_1 dx_2} = \frac{x_1 x_2 d\sigma^{Born}}{dx_1 dx_2} \cdot \frac{e^{(\alpha_P - 1)Y}}{\sqrt{\frac{7}{2}\ \alpha N_c \zeta(3) Y}} \qquad (3)$$

with $Y = \ell n \frac{x_1 x_2 s}{Q^2}$. $\frac{x_1 x_2 d\sigma^{Born}}{dx_1 dx_2}$ is the two-jet inclusive cross section when the hard scattering part of the cross section is given in the one gluon exchange approximation. If one had data at different values of s[17] for x_1 and x_2 fixed then the s-dependence of the cross section would be determined completely by the last factor in (3) and a comparison between theory and experiment could determine $\alpha_P - 1$. It would be very interesting to see this measurement made at Fermilab in the next few years. A similar measurement can be done at HERA[18-20] where one measures a single jet inclusively with $k_z = x_1 P_1$ and $k_\perp \approx Q$, where P_1 is the proton's momentum and Q the photon virtuality. A formula identical to (3) holds where now $Y = \ell n\ x_1/x$ with x the usual Bjorken variable. The advantage at HERA is that x can be varied, at a fixed x_1 within a single beam setting. The experiment is, however, difficult to perform because large values of Y requires values of x_1 not too small and such jets are difficult to measure.

Another process which may be useful in determining BFKL dynamics is high $p_\perp$ diffractive vector meson production at HERA. If the momentum transfer between the incoming virtual photon and the outgoing diffractively produced vector meson is large then the process will be a hard process. If there is a large reapidity gap between the vector meson and the recoil jet, then the process will be a high energy process and one can expect the BFKL pomeron to describe the rapidity gap dependence[21]. Indeed,

$$\frac{d\sigma}{dtdY} \alpha e^{2(\alpha_P - 1)Y},$$

where Y is the rapidity gap between the vector meson and the recoil jet. In principle, this process is a good place to measure α_P. However, detailed calculations[21] suggest that the behavior given in (4) may set in slowly. Nevertheless, it would be interesting to see what the data show. Clearly there is a pressing need for more processes where BFKL dynamics can be separated from the more prosiac DGLAP dynamics.

3. Partonic Interactions with Nuclear Matter

3.1. Color Neutral Systems

Hadrons are built of quarks and gluons. In the interaction picture the hadron's wavefunction fluctuates in time. Hard exclusive processes trigger on almost pointlike parts of the hadron's wavefunction. If such processes take place in a nucleus one can expect reduced initial and final state interactions as the hadron passes through the nucleus. This is the phenomenon of *color transparency*. The key ingredients for the existence of color transparency[22] are (i) a hard exclusive reaction, and (ii) a high momentum for all initial and final state particles entering or leaving the nucleus, a momentum high enough so that Lorentz time dilatation keeps the pointlike configuration small during its passage through the nucleus. Three major experiments have run which have results on color transparency.

The pioneering BNL experiment[23] $proton(p) + A \rightarrow proton(p_1) + proton(p_2) + (A-1)^*$, with $(p-p_1)^2$ the large momentum transfer and with $(A-1)^*$ a, possibly, excited nuclear state, showed a surprising result. Defining transparency by

$$T = \frac{\sigma(p + A \rightarrow p_1 + p_2 + (A-1)^*)}{\sigma(p + p_0 \rightarrow p_1 + p_2)}, \qquad (4)$$

with p_0 a proton at rest, the BNL experiment found an increasing T as p increased from about 4 GeV to about 9 GeV. This seemed to be a confirmation of theoretical expectations. However, as p further increased from 9 to 13 GeV T decreased. This result has now been confirmed by the EVA experiment and it will be very interesting to see what happens in EVA as p is further increased to about 22 GeV. There has been much discussion as to what causes the fall of T between 9 and 13 GeV, and so far no consensus has been reached. I personally find attractive the Pire and Ralston[24] explanation that the variation of T is due to the denominator in (4), that is that in the quasi-elastic nuclear reaction small hadron sizes are indeed being probed but that in the elastic proton-proton scattering there is an energy dependent destructive interference between large and small scales causing the proton-proton cross section to oscillate with energy. If this is indeed the case then T should follow the proton-proton oscillation as the beam energy is increased in the EVA experiment.

A precision experiment of quasi-elastic electron scattering[25], $\gamma^*(Q) + A \rightarrow proton + (A-1)^*$, was recently completed at SLAC. No evidence for color transparency was found. This might be due to the small values of Q^2 measured, $1 \leq Q^2 \leq 7GeV^2$, but more likely is due to the small Lorentz factor of the outgoing proton.

The most exciting recent news in this field has been the result from Fermilab[26,27] on diffractive ρ-production, $\gamma^*(Q) + A \rightarrow \rho + A^*$. If one parameterizes the cross section as $\sigma = \sigma_0 A^\alpha$, then E665 found a sharp increase with α as Q^2 was increased from 1 to 5 GeV^2. If the interpretation of this result, as due to an increasing color transparency of the q$\bar{\text{q}}$ pair making up the ρ as Q^2 increases, holds up it suggests that in certain circumstances color transparency effects can be seen at moderate values of Q^2.

3.2. Isolated Partons in a Nuclear Medium

The energy loss of fast electrons in matter is a classic problem in QED. Recently the Landau, Pomeranchuk, Migdal effect[28,29] has been measured at SLAC. The energy loss of quarks and gluons in both hot and cold nuclear matter is a problem of crucial importance for heavy ion physics. It is also an intriguing problem in its own right and one that has many implications for the interpretation of hard collisions in nuclei.

There has recently been rather striking progress made in the QCD problem and a new, much simpler, discussion of the classic LPM problem for electrons has been given[30]. In the QCD problem Baier, Dokshitzer, Peigné and Shiff[31], following a model of Gyulassy and Wang[32], have found a very unexpected result for energy loss of quarks and gluons in hot QCD matter. BDPS find

$$\frac{\omega dI}{d\omega dz} = \frac{3\alpha C_F}{4\lambda_g}\sqrt{\frac{\lambda_g \mu^2}{\pi\omega}}\ell n\frac{\omega}{\lambda_g\mu^2} \tag{5}$$

so long as $\lambda_g\mu^2 << \omega << \frac{L^2\mu^2}{\lambda_g}$, and where L is the length of the medium, ω is the energy of the radiated gluon contributing the main source of energy loss, μ is the screening mass in the hot medium and λ_g is the gluon mean free path. I believe the most striking way to represent this result is to give the energy loss off an extremely high energy quark as it passes through a medium of fixed length. Using (5), one finds

$$E_{loss} = \int_0^L dz\ d\omega\frac{\omega dI}{d\omega dz} = \frac{3\alpha C_F}{2\lambda_g}\frac{L\mu^2}{\sqrt{\pi}}\ell n\ L^2/\lambda_g^2. \tag{6}$$

Thus the energy loss varies quadratically with the length of the medium. This means that

$$< \frac{dE_{loss}}{dz} > \alpha\ z \tag{7}$$

a most unexpected result. The results of BDPS should also hold, with slight modificaitons, for cold matter. It will be very exciting to try and confirm the quadratic dependence on distance of the energy loss which seems to follow solidly from QCD.

4. Spin-Dependent Deep Inelastic Scattering

In the most straightforward parton-model interpretation one can represent the g_1 structure function of spin-dependent deep inelastic scattering as

$$g_1(x,Q^2) = \frac{1}{2}\sum_f e_f^2 \Delta q_f(x,Q^2) \tag{8}$$

where

$$\Delta q_f(x,Q^2) = q_{f\uparrow}(x,Q^2) - q_{f\downarrow}(x,Q^2) \tag{9}$$

where $\uparrow$ means positive helicity and $\downarrow$ means negative helicity. Integrating over all x gives

$$\int_0^1 dx g_1(x,Q^2) = \frac{1}{2}\sum_f \Delta q_f(Q^2) \tag{10}$$

where Δq_f has a *naive* interpretation as the fraction of the proton's spin being carried by quarks of flavor f. From the operator product expansion it follows also that[33]

$$\int_0^1 dx g_1(x,Q^2) = \frac{1}{2}\sum_f e_f^2 (p\lambda = \frac{1}{2}|j_{50}^f|p\lambda = \frac{1}{2}) \tag{11}$$

where $j^f_{5\mu} = \bar{q}_f\gamma_\mu\gamma_5 q_f$.

There is much good data recently from EMC[34] (proton), SMC[35] (deuteron), E142[36] (helium), SMC[37] (proton) and E143[38] (proton). Ellis and Karliner[39] combine all data, using higher order corrections to connect different Q^2. The find

$$\Delta u = 0.83 \pm 0.03$$

$$\Delta d = -0.43 \pm 0.03 \tag{12}$$

$$\Delta s = -0.10 \pm 0.03$$

giving

$$\Delta\Sigma = 0.33 \pm 0.04$$

Constituent quark models typically give

$$\Delta u \approx 1$$

$$\Delta d \approx -\frac{1}{4}$$

$$\Delta s \approx 0$$

$$\Delta\Sigma \approx \frac{3}{4}. \tag{13}$$

At stake here, is the validity of the constituent quark model which identifies

$$(p|j_\mu|p) \approx (P|J_\mu|P) \tag{14}$$

where j_μ is a fundamental QCD current and |p) is an eigenstate (the proton) of QCD while | P) is the proton's wavefunction as built of three constituent quarks, and J_μ is a current (corresponding to j_μ) built out of constituent quark fields. The quark model gives $g_A =$ Δu - Δd perfectly but $g_A^{singlet} = \Delta\Sigma = \Delta$ u + Δ d + Δs seems off by about a factor of 2. The problem seems to be only in the singlet channel. If one were to add 0.1-0.15 to Δ u, Δd and Δs in (12) we would get very close to the Δu, Δd and Δs of (13).

The two main attempts to explain the problems with the constituent quark model use a Skyrme picture of the nucleon[40] and the axial anomaly, respectively[41-44]. In the Skyrme model the proton is made of the octet of pseudoscalar mesons but not the η'. Because the η' does not occur in the proton $g_A^{singlet}$ is zero, as follows from the corresponding Goldberger-Treiman relation. The quark model simple does not work. Proponents of the axial anomaly suggest that the anomaly causes $j^f_{5\mu}$ not to be a proper measurer of the spin carried by quarks of flavor f, but that the identification should be

$$\Delta q_f = (p\lambda = \frac{1}{2}|j^f_{50}|p\lambda = \frac{1}{2}) + \frac{\alpha}{2\pi}\Delta g \tag{15}$$

where Δg is the spin carried by gluons in the proton.

Let me add a few comments and questions.

(i) It is important to check, experimentally, the Q^2-dependence of $\Gamma = \int_0^1 dx\ g_1(x, Q^2)$.

(ii) The relationship between the Skyrme model and large N_c QCD needs to be further clarified in order to understand how unique the Skyrme model predictions really are.

(iii) How big is Δg? In order for the anomaly to play a significant role it will be necessary that Δg be at least as big as 1.5 to 2. How well can Δg be measured at RHIC or HERA?

5. Higher Orders of Perturbation Theory and Higher Twist

5.1. An Example(The Bjorken Sum Rule)

The Bjorken sum rule is reasonably well tested experimentally and is a relation where where higher order corrections have been calculated[4] through order α^3.

$$\int_0^1 [g_1^P(x,Q^2) - g_1^N(x,Q^2)]dx = \frac{g_A}{6}[1 - \frac{\alpha}{\pi} - 3.58(\frac{\alpha}{\pi})^2$$

$$-20.2(\frac{\alpha}{\pi})^3 + \cdots] + c\frac{\Lambda^2}{Q^2} + \cdots. \tag{16}$$

The higher twist term involves a single unknown constant which is related to

$$(ps|\theta_\mu|ps) \tag{17}$$

where

$$\theta_\mu = \frac{1}{2}g[\bar{u}\gamma_\nu\frac{\lambda^a}{2}u - \bar{d}\gamma_\nu\frac{\lambda^a}{2}d]\epsilon_{\mu\nu\rho\sigma}F^a_{\rho\sigma}. \tag{18}$$

At large Q^2, say $Q^2 \geq 5GeV^2$, the higher twist contribution is likely very small and perturbation theory, through order α^3, should be a very good representation of the left-hand side of (16).

However, in general, the separation between higher order perturbation theory corrections to the leading twist term and higher twist terms is not unique[45,46]. Thus, the value of c in (16) is ambiguous with the ambiguities being compensated by ambiguities inherent in defining the divergent perturbation series for the leading twist term in (16). Thus, for example, lattice theorists cannot calculate c in an unambiguous fashion.

At $Q^2 = 2 - 3GeV^2$ the confusion between higher orders and higher twist may be significant at order α^3 or

α^4. This can be seen from noting that the perturbation theory for the leading twist term in (16) behaves like

$$c' n! \beta_2^n n^\gamma \alpha^{n+1} \quad (19)$$

for large n where $\beta_2 = \frac{33-2N_f}{12\pi}$, $\gamma = \frac{1}{\beta_2}(\frac{\beta_3}{\beta_2} - \gamma_2)$ and with γ_2 the anomalous dimension of θ_μ. The series (19) begins to diverge at $n \approx 1/\beta_2\alpha$ which is about 3 for $\alpha = 0.4$. The constant c' in (19) is very difficult to calculate in any standard scheme of renormalization.

5.2. Jet Event Shapes[47-51].

For many quantities determining jet event shapes $(2\beta_2)^n$ rather than β_2^n occurs in the perturbation series leading to $1/Q$ ambiguities in resumming the perturbation series and corresponding $1/Q$ higher twist contributions which are not negligible even when $Q = M_{Z^0}$. There has recently been a lot of work done on trying to understand the nature of the 1/Q corrections, in particular whether or not they are factorizable. The conclusion seems to be that $1/Q$ corrections are factorizable in perturbation theory, however, no good argument has been given as to their factorizability in general.

5.3. Brodsky, Lepage,Mackenzie[52](BLM) Scale Fixing and Commensurate Scale Relations[53].

With each QCD observable one can associate an effective charge[54]. For example $R(Q^2) = \frac{\sigma_{e^+e^- \to hadrons}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$ can be written as

$$R(Q^2) = R_0(1 + \frac{\alpha_R(Q)}{\pi}) \quad (20)$$

a relation which defines the effective charge α_R corresponding to R. Suppose α_1 and α_2 are effective charges corresponding to observables 1 and 2. Then one can write[53]

$$\frac{\alpha_1(Q)}{\pi} = \frac{\alpha_2(Q^*)}{\pi} + r_2\left(\frac{\alpha_2(Q^{**})}{\pi}\right)^2 + r_3\left(\frac{\alpha_2(Q^{***})}{\pi}\right)^3 + \cdots \quad (21)$$

Q^*, Q^{**} etc. are determined by requiring that all running coupling effects occur in scales, not in the coefficients, r_i. One knows how to do this through order α^3, but a general procedure for defining scales at higher orders is lacking. The r_i then are coefficients in a confromally invariant QCD. There are some big surprises. For example,

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi}\right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi}\right)^3 + \cdots \quad (22)$$

where g_1 is the effective coupling corresponding to the Bjorken sum rule. Thus, the large coefficients appearing in (16), an $\overline{MS}$ calculation at scale Q, have disappeared. The factorials (19) due to infrared renormalons are now hidden in the scales. The n!'s that one in general would expect from graph counting ($instanton - \overline{instanton}$ pairs) are apparently absent from (22).

Gaining a deeper understanding of the QCD perturbation series and its relation to higher twist terms touches QCD in a very fundamental way. These are questions which cannot be asked in theories like QED or the Electroweak Theory. It should be very exciting to see how our insight develops here over the next few years.

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