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${}_{\Lambda_c\Lambda_c}^{4}He$ and ${}_{\Lambda_c\Lambda_c}^{4}H$ hypernuclei

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Abstract. In our article we consider charmed hypernuclei states having the number of baryons $B=4$ and containing two Λ_c hyperons. To do this we use the technique of the dispersion relations. We obtain the relativistic equations which describe these states. The relativistic amplitudes for 12-quark states, including the constituent quarks of three flavors u, d, c are considered. We find the approximate solutions of these equations and take into account main singularities of the amplitudes. We calculate masses and binding energies of the hypernuclei states ${}_{\Lambda_c\Lambda_c}^{4}He$, ${}_{\Lambda_c\Lambda_c}^{4}H$.

1. Introduction

Hadron spectroscopy is very important for studying the dynamics of strong interactions. The heavy hadrons containing charm or bottom quarks are particularly interesting.

In 1977, Jaffe studied three types of baryon-baryon bound states. These are deuteron-like states, ΔΔ-like states, and ΩΩ-like states. There are theoretical and experimental investigations of these states.

At present, the low-energy region cannot be consistently studied within the framework of QCD, which explains well the processes of hard interaction of quarks and gluons. Therefore, it becomes necessary to use various phenomenological models in elementary particle physics and nuclear physics. The most widely used are potential nonrelativistic models, QCD sum rules, and QCD lattice models [1, 2].

Various states of hypernuclei are of particular interest to researchers. There are many research papers on this topic [3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

In [13, 14, 15, 16] authors describe the method developed for working with relativistic multi-particle states. The physics of the such system can be described using a pair interaction between the particles. The authors use the *S*-matrix approach. Within the *S*-matrix approach, the following principles are used: analyticity, two-particle unitarity, relativistic invariance, and crossing symmetry. Taking into account the pair interaction of quarks and antiquarks, a relativistic generalization of the Faddeev-Yakubovsky equations was obtained. The approximate solution of these equations is based on the method of separating the main singularities of the scattering amplitudes. The amplitudes of pair interactions were previously obtained in the bootstrap quark model by the iteration method.

In the papers [14, 15] we calculated the masses of charm baryonia with one and two c quarks, and the masses of bottom baryonia with one b quark. Unitarity and analyticity are used in order to derive system of integral equations. The multiquark equations are derived using the framework of coupled channel formalism.

Heavy dibaryons and baryoniums can be produced at the LHC hadron collider. Experimental research to find heavy dibaryons is the main focus of the GSI, J-PARC, PANDA and Belle collaborations.

In previous work [16], the ^3He nucleus was considered as a quark-gluon interaction in the relativistic quark model. Relativistic 9-quark equations are constructed within the framework of the dispersion approach taking into account u, d - quarks. An approximate solution of these equations, using the separation of the main amplitude singularities, allows one to determine the poles of the 9-quark amplitudes and the mass of the ^3He nucleus.

In the present paper we consider charmed hypernuclei ${}_{\Lambda_c\Lambda_c}^{4}He$, ${}_{\Lambda_c\Lambda_c}^{4}H$, with the atomic (baryon) number *A=B=4*. We derive relativistic twelve-quark equations.

The poles of the multiquark amplitudes determine the masses of charmed hypernuclei with the *A=4*.

2. Twelve-Quark Amplitudes of the Charmed Hypernuclei

We obtain generalization of the relativistic Faddeev equations for 12-quarks hypernuclei in the dispersion relations technique.

We derive the relativistic 12-quark equations taking into consideration all possible subamplitudes. Then we represent a twelve-particle amplitude as the sum of the following subamplitudes:

$$A = \sum_i A_1^{(i)} + \sum_{i,j} A_2^{(ij)} + \sum_{i,j,k} A_3^{(ijk)} + \sum_{i,j,k,l} A_4^{(ijkl)} \tag{1}$$

Here i, j, k, l are quantum numbers of corresponding diquarks. The subamplitudes A_1^i determine the diquark and 10 quarks. The subamplitudes A_2^{ij} define the two diquarks and 8 quarks. The subamplitudes A_3^{ijk} correspond to the three diquarks and 6 quarks. The subamplitudes A_4^{ijkl} are the four-baryon state. We neglect some subamplitudes which represent the smaller contributions.

Table 1. The vertex functions and coefficients of Chew-Mandelstam functions.

n	J^P	$G_n^2(s_{kl})$	α_n	β_n	δ_n
1	0^+	$\frac{4g}{3} - \frac{8gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2}\frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
2	1^+	$\frac{2g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6}$

The total amplitude can be shown graphically as a sum of diagrams. In the Fig. 1 the graphical equation for the reduced amplitude $\alpha_4^{1^{uu}1^{uu}0^{uc}0^{uc}}$ for the state ${}_{\Lambda_c\Lambda_c}^{4}He$ $pp\Lambda_c\Lambda_c$ with the isospin projection $I_3 = 1$ and the spin-parity $J^P = 0^+$ is represented.

For interaction of two quarks we have amplitude:

$$\alpha_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})} \tag{2}$$

Here, s_{ik} is the two-particle subenergy squared, $G_n(s_{ik})$ are the quark-quark vertex functions, and $B_n(s_{ik})$ are the Chew-Mandelstam functions:

$$B_n(s_{ik}) = \int_{(m_i+m_k)^2}^{\Lambda_n} \frac{ds'_{ik}}{\pi} \frac{\rho_n(s'_{ik}) G_n^2(s'_{ik})}{s'_{ik} - s_{ik}} \tag{3}$$

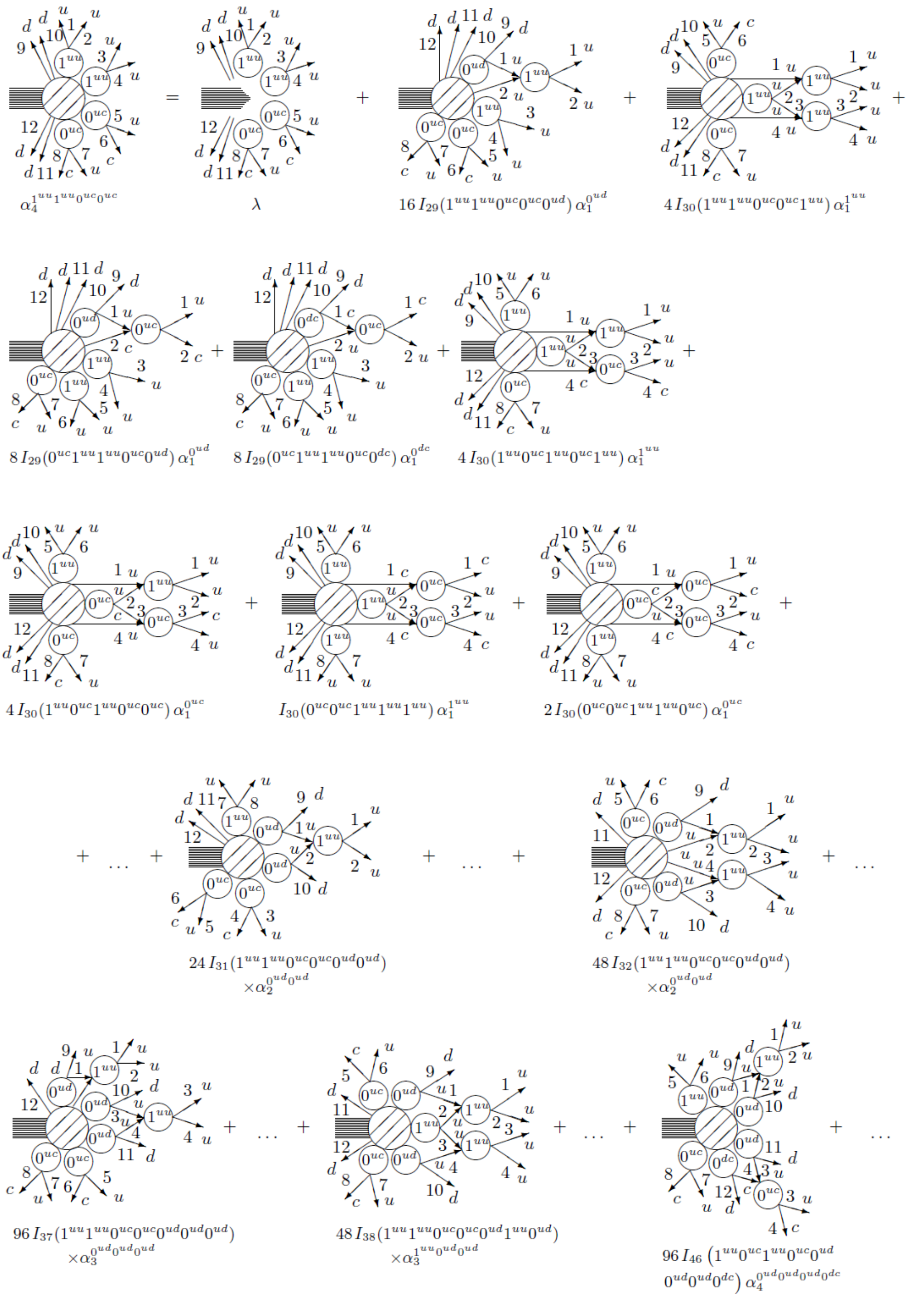

Figure 1. The graphical equation for the reduced amplitude $\alpha_4^{1^{uu}1^{uu}0^{uc}0^{uc}}$ for the state ${}_{\Lambda_c\Lambda_c}^{4}He$ $pp\Lambda_c\Lambda_c$ with the isospin projection $I_3 = 1$ and the spin-parity $J^P = 0^+$ (Table 2).

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$B_n(s_{ik})$ and $\rho_n(s_{ik})$ are the Chew-Mandelstam functions with cutoff Λ and the following phase spaces:

$$\rho_n(s_{ik}) = \left(\alpha_n \frac{s_{ik}}{(m_i+m_k)^2} + \beta_n + \delta_n \frac{(m_i-m_k)^2}{s_{ik}}\right) \sqrt{\left(1 - \frac{(m_i+m_k)^2}{s_{ik}}\right)\left(1 - \frac{(m_i-m_k)^2}{s_{ik}}\right)} \quad (4)$$

The coefficients α_n, β_n and δ_n can be found in the Table 1. We use *n=1* for the qq-pairs with $J^P = 0^+$, and *n=2* for the qq pairs with $J^P = 1^+$.

We consider only main singularities of our amplitudes, located near the region $s_{ik} \approx 4m^2$. We search for an approximate solution of equations, taking into account a definite number of leading singularities and neglecting the weaker ones.

In the case of the twelve-quark problem we take into account 3-th, 4-th, 5-th, 6-th particles singularities in the integrals and neglect weaker ones. The largest contribution to the twelve-quark amplitudes coming from the triangle singularities is given. We extract singularities in the coupled equations and obtain the reduced amplitudes α_i.

Let us consider the state ${}_{\Lambda_c\Lambda_c}^{4}He\ pp\Lambda_c\Lambda_c$ with the isospin projection $I_3 = 1$ and the spin-parity $J^P = 0^+$. This state is described by 47 equations for 47 amplitudes:

$$\alpha_1^{1^{uu}}, \alpha_1^{0^{ud}}, \alpha_1^{0^{uc}}, \alpha_1^{0^{dc}} \quad (5)$$

$$\alpha_2^{1^{uu}1^{uu}}, \alpha_2^{1^{uu}0^{ud}}, \alpha_2^{1^{uu}0^{uc}}, \alpha_2^{1^{uu}0^{dc}},$$
$$\alpha_2^{0^{ud}0^{ud}}, \alpha_2^{0^{ud}0^{uc}}, \alpha_2^{0^{ud}0^{dc}}, \alpha_2^{0^{uc}0^{uc}}, \alpha_2^{0^{uc}0^{dc}}, \alpha_2^{0^{dc}0^{dc}} \quad (6)$$

$$\alpha_3^{1^{uu}1^{uu}0^{ud}}, \alpha_3^{1^{uu}1^{uu}0^{uc}}, \alpha_3^{1^{uu}1^{uu}0^{dc}}, \alpha_3^{1^{uu}0^{ud}0^{ud}}, \alpha_3^{1^{uu}0^{ud}0^{uc}},$$
$$\alpha_3^{1^{uu}0^{ud}0^{dc}}, \alpha_3^{1^{uu}0^{uc}0^{uc}}, \alpha_3^{1^{uu}0^{uc}0^{dc}}, \alpha_3^{1^{uu}0^{dc}0^{dc}}, \alpha_3^{0^{ud}0^{ud}0^{ud}},$$
$$\alpha_3^{0^{ud}0^{ud}0^{uc}}, \alpha_3^{0^{ud}0^{ud}0^{dc}}, \alpha_3^{0^{ud}0^{uc}0^{uc}}, \alpha_3^{0^{ud}0^{uc}0^{dc}}, \alpha_3^{0^{ud}0^{dc}0^{dc}} \quad (7)$$

$$\alpha_4^{1^{uu}1^{uu}0^{ud}0^{ud}}, \alpha_4^{1^{uu}1^{uu}0^{ud}0^{uc}}, \alpha_4^{1^{uu}1^{uu}0^{ud}0^{dc}},$$
$$\alpha_4^{1^{uu}1^{uu}0^{uc}0^{uc}}, \alpha_4^{1^{uu}1^{uu}0^{uc}0^{dc}}, \alpha_4^{1^{uu}1^{uu}0^{dc}0^{dc}},$$
$$\alpha_4^{1^{uu}0^{ud}0^{ud}0^{ud}}, \alpha_4^{1^{uu}0^{ud}0^{ud}0^{uc}}, \alpha_4^{1^{uu}0^{ud}0^{ud}0^{dc}},$$
$$\alpha_4^{1^{uu}0^{ud}0^{uc}0^{uc}}, \alpha_4^{1^{uu}0^{ud}0^{uc}0^{dc}}, \alpha_4^{1^{uu}0^{ud}0^{dc}0^{dc}},$$
$$\alpha_4^{0^{ud}0^{ud}0^{ud}0^{ud}}, \alpha_4^{0^{ud}0^{ud}0^{ud}0^{uc}}, \alpha_4^{0^{ud}0^{ud}0^{ud}0^{dc}},$$
$$\alpha_4^{0^{ud}0^{ud}0^{uc}0^{uc}}, \alpha_4^{0^{ud}0^{ud}0^{uc}0^{dc}}, \alpha_4^{0^{ud}0^{ud}0^{dc}0^{dc}} \quad (8)$$

Here, for example, 1^{uu} means diquark with spin 1 consisting of two *u*-quarks. 0^{uc} means diquark with spin 0 consisting of one *u*-quark and one *c* quark. The α_1 are determined by the diquarks, and the α_2 includes the two diquarks and eight quarks. The α_3 defines the three diquarks and six quarks. The α_4 allows us to consider the ${}_{\Lambda_c\Lambda_c}^{4}He\ pp\Lambda_c\Lambda_c$ state.

The similar amplitudes are obtained for the ${}_{\Lambda_c\Lambda_c}^{4}H\ pn\Lambda_c\Lambda_c$ state.

The coefficients of the coupled equations can be calculated by counting the number of quark permutations.

There are 57 different diagrams in the case of the twelve-particle ${}_{\Lambda_c\Lambda_c}^{4}He$ state, but we only used the contributions of the 25 functions. The other functions I_i are small. These functions are integral functions, the explicit form of which is given in our previous works.

As an example we write down the equation for amplitude $\alpha_4^{1^{uu}1^{uu}0^{uc}0^{uc}}$ for the ${}_{\Lambda_c\Lambda_c}^{4}He$ state:

$$\alpha_4^{1^{uu}1^{uu}0^{uc}0^{uc}} = \lambda + 16I_{29}(1^{uu}1^{uu}0^{uc}0^{uc}0^{ud})\alpha_1^{0^{ud}} + 8I_{29}(0^{uc}1^{uu}1^{uu}0^{uc}0^{ud})\alpha_1^{0^{ud}}$$
$$+8I_{29}(0^{uc}1^{uu}1^{uu}0^{uc}0^{dc})\alpha_1^{0^{dc}} + 4I_{30}(1^{uu}1^{uu}0^{uc}0^{uc}1^{uu})\alpha_1^{1^{uu}} + 4I_{30}(1^{uu}0^{uc}0^{uc}0^{uc}1^{uu})\alpha_1^{1^{uu}}$$
$$+4I_{30}(1^{uu}0^{uc}1^{uu}0^{uc}0^{uc})\alpha_1^{0^{uc}} + I_{30}(0^{uc}0^{uc}1^{uu}1^{uu}1^{uu})\alpha_1^{1^{uu}} + 2I_{30}(0^{uc}0^{uc}1^{uu}1^{uu}0^{uc})\alpha_1^{0^{uc}}$$
$$+24I_{31}(1^{uu}1^{uu}0^{uc}0^{uc}0^{ud}0^{ud})\alpha_2^{0^{ud}0^{ud}} + 24I_{31}(0^{uc}1^{uu}1^{uu}0^{uc}0^{ud}0^{dc})\alpha_2^{0^{ud}0^{dc}}$$
$$+48I_{32}(1^{uu}1^{uu}0^{uc}0^{uc}0^{ud}0^{ud})\alpha_2^{0^{ud}0^{ud}} + 96I_{32}(1^{uu}0^{uc}1^{uu}0^{uc}0^{ud}0^{ud})\alpha_2^{0^{ud}0^{ud}}$$
$$+96I_{32}(1^{uu}0^{uc}1^{uu}0^{uc}0^{ud}0^{dc})\alpha_2^{0^{ud}0^{dc}} + 12I_{32}(0^{uc}0^{uc}1^{uu}1^{uu}0^{ud}0^{ud})\alpha_2^{0^{ud}0^{ud}}$$
$$+12I_{32}(0^{uc}0^{uc}1^{uu}1^{uu}0^{dc}0^{dc})\alpha_2^{0^{dc}0^{dc}} + 12I_{32}(0^{uc}0^{uc}1^{uu}1^{uu}0^{uc}0^{dc})\alpha_2^{0^{ud}0^{dc}}$$
$$+96I_{37}(1^{uu}1^{uu}0^{uc}0^{uc}0^{ud}0^{ud}0^{ud})\alpha_3^{0^{ud}0^{ud}0^{ud}} + 192I_{37}(1^{uu}0^{uc}1^{uu}0^{uc}0^{ud}0^{ud}0^{dc})\alpha_3^{0^{ud}0^{ud}0^{dc}}$$
$$+96I_{37}(0^{uc}1^{uu}1^{uu}0^{uc}0^{ud}0^{ud}0^{ud})\alpha_3^{0^{ud}0^{ud}0^{ud}} + 96I_{37}(0^{uc}1^{uu}1^{uu}0^{uc}0^{dc}0^{ud}0^{ud})\alpha_3^{0^{ud}0^{ud}0^{dc}}$$
$$+48I_{37}(0^{uc}0^{uc}1^{uu}1^{uu}0^{ud}0^{ud}0^{dc})\alpha_3^{0^{ud}0^{ud}0^{dc}} + 48I_{37}(0^{uc}0^{uc}1^{uu}1^{uu}0^{dc}0^{ud}0^{dc})\alpha_3^{0^{ud}0^{dc}0^{dc}}$$
$$+48I_{38}(1^{uu}1^{uu}0^{uc}0^{uc}0^{ud}1^{uu}0^{ud})\alpha_3^{1^{uu}0^{ud}0^{ud}} + 96I_{38}(1^{uu}0^{uc}1^{uu}0^{uc}0^{ud}1^{uu}0^{dc})\alpha_3^{1^{uu}0^{ud}0^{dc}}$$
$$+96I_{38}(1^{uu}0^{uc}1^{uu}0^{uc}0^{ud}0^{uc}0^{ud})\alpha_3^{0^{ud}0^{ud}0^{uc}} + 12I_{38}(0^{uc}0^{uc}1^{uu}1^{uu}0^{dc}1^{uu}0^{dc})\alpha_3^{1^{uu}0^{dc}0^{dc}}$$
$$+24I_{38}(0^{uc}0^{uc}1^{uu}1^{uu}0^{ud}0^{uc}0^{dc})\alpha_3^{0^{ud}0^{uc}0^{dc}} + 96I_{46}(1^{uu}0^{uc}1^{uu}0^{uc}0^{ud}0^{ud}0^{ud}0^{dc})\alpha_4^{0^{ud}0^{ud}0^{ud}0^{dc}}$$
$$+24I_{46}(0^{uc}0^{uc}1^{uu}1^{uu}0^{ud}0^{dc}0^{ud}0^{dc})\alpha_4^{0^{ud}0^{ud}0^{dc}0^{dc}} \quad (9)$$

3. Calculation results

In our paper we study hypernuclear states including light *u*, *d* quarks and two heavy *c* quarks. These are the hyperhelium state ${}_{\Lambda_c\Lambda_c}^{4}He$ and the hyperhydrogen state ${}_{\Lambda_c\Lambda_c}^{4}H$.

We consider the hyperhelium state ${}_{\Lambda_c\Lambda_c}^{4}He$ as the bound state of two protons and two Λ_c hyperons, and the hyperhydrogen state ${}_{\Lambda_c\Lambda_c}^{4}H$ as the bound state of proton, neutron, and two Λ_c hyperons.

We derive the relativistic integral equations for the amplitudes that describe these states and then obtain equations on reduced amplitudes. The system of relativistic equations for the hyperhelium state ${}_{\Lambda_c\Lambda_c}^{4}He$ consists of 47 equations, the system of relativistic equations for the hyperhydrogen state ${}_{\Lambda_c\Lambda_c}^{4}H$ consists of 63 equations.

The mass and binding energy of state ${}_{\Lambda_c\Lambda_c}^{4}He$ with the isospin projection $I_3 = 1$ and the spin-parity $J^P = 0^+$ are equal to *M=6448 MeV* and *6 MeV*, respectively.

The mass and binding energy of state ${}_{\Lambda_c\Lambda_c}^{4}H$ with the isospin projection $I_3 = 0$ and the spin-parities $J^P = 0^+, 1^+$ are equal to *M=6428 MeV* and *26 MeV*, respectively.

These states are presented in the Table 2.

Table 2. Masses and binding energies of hypernuclei. Parameters of model: $\Lambda = 6.162$, $g = 0.2122$, $m = 495\ MeV$, $m_c = 1655\ MeV$.

Hypernuclei	Quark content	Q	I_3	I	J^P	Masses, MeV	Binding energies, MeV
${}_{\Lambda_c\Lambda_c}^{4}He(pp\Lambda_c\Lambda_c)$	uud uud udc udc	4	1	1	0^+	6448	6
${}_{\Lambda_c\Lambda_c}^{4}H(pn\Lambda_c\Lambda_c)$	uud udd udc udc	3	0	0, 1	$0^+, 1^+$	6428	26

4. Conclusions

In our paper we obtain two bound states of hypernuclei ${}_{\Lambda_c\Lambda_c}^{4}He$ and ${}_{\Lambda_c\Lambda_c}^{4}H$. We calculate masses and binding energies of the states. Experimenters can use our predictions in experiments carried out at the collider to search for new particles. The experimental masses of charmed hypernuclei with the $A=4$ are absent.

It would also be interesting to consider other states of hypernuclei containing different numbers of heavy c-quarks, as well as states of hypernuclei containing heavy bottom quarks.

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