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Extending JT/SYK duality via so(2,2) Poisson Sigma Model

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A bulk plus boundary extension of Jackiw-Teitelboim Gravity (JT) coupled with non-abelian gauge fields is realized via a Poisson Sigma Model derived as a dimensional reduction of the AdS_3 Chern-Simons theory with WZW boundary terms. We show that the boundary action of the model can be written in terms of coadjoint orbits of an appropriate Virasoro-Kac-Moody group. The associated extended Schwarzian action matches the effective low energy action of recent SYK like tensor models.

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PoS(CORFU2024)307

1. Introduction

Two-dimensional dilaton gravity has proved to be a powerful tool in describing black hole physics at the classical and quantum level as an effective description of the near-horizon dynamics obtained from dimensional reduction [1–3]. Among these models, Jackiw-Teitelboim gravity (JT) [4–6] has attracted much attention recently, because of its holographic duality with the one-dimensional Sachdev–Ye–Kitaev model (SYK), which provides a concrete and solvable realization of the (near) AdS_2/CFT_1 correspondence [7–10]. In the JT/SYK case [11], the duality lies in a shared one-dimensional Schwarzian dynamical sector, which describes the low-energy/strong-coupling regime of the SYK model [12], as well as the boundary dynamics induced by the Gibbons-Hawking-York (GHY) term in the JT theory (see [13] for a recent review).

Besides the AdS/CFT duality, the beauty and power of such a framework lies in the generality of the symmetry-breaking scheme in low dimensional gravitational theories with a boundary (see also [18–20]). The presence of a boundary breaks the asymptotic group of symmetry $\mathcal{G}$ into some reduced symmetry group $\mathcal{H}$ which depends on the choice of boundary conditions. The boundary dynamics is governed by fields belonging to the quotient space $\mathcal{G}/\mathcal{H}$ which can be identified with an appropriate coadjoint orbit of $\mathcal{G}$. In the case of JT gravity, one has $\mathcal{G} = \mathrm{Diff}(S^1)$ and $\mathcal{H} = \mathrm{SL}(2,\mathbb{R})$. The boundary action can then be written as the geometric action on S^1 associated with the coadjoint action of a diffeomorphism F over an element of the Virasoro dual algebra and, with a suitable choice of this element, the Schwarzian action can be exactly derived in purely geometric terms.

The generality of the coadjoint orbits method provides a powerful tool for investigating extensions of the boundary JT/SYK dynamics starting from generalizations of the symmetry group of the dual models. Indeed, ever since the JT/SYK correspondence has been pointed out, much work has aimed at expanding the range of potential holographic relationships in 2d gravity [21–25]. Numerous variants and generalizations of the SYK model have been proposed in recent years [26–32], raising the natural question of whether an extended duality with generalized JT models would exist. In this sense, modifications of JT gravity have been introduced following diverse paths [33–36].

In particular, SYK tensor extensions have been shown to exhibit a broken $\mathfrak{diff}(S^1) \ltimes \hat{\mathfrak{g}}$ symmetry in the strong coupling limit, with $\mathfrak{g}$ the Lie algebra of some internal symmetry by which the tensor extension is realized [29] and the hat indicating, throughout the paper, the associated loop algebra. While preserving solvability in the large N limit, the effective dynamics of tensor generalizations of SYK models turns out to be a generalization of Schwarzian dynamics, which does not require disordered averaging [28]. Further interest in tensor SYK models is motivated by the idea of generalizing to higher dimensions the correspondence between matrix models and two-dimensional geometries [37, 38].

Since the emergent symmetry of tensor extensions of SYK models is generally characterized by the semidirect product of the group of reparametrisations of S^1 with an affine Kac-Moody algebra, a natural question in this context is whether a bulk gravitational theory exists whose boundary action is written in terms of coadjoint orbits of such semidirect product.

In our contribution, we propose a bulk plus boundary theory which reproduces a broken $\mathfrak{diff}(S^1) \ltimes \hat{\mathfrak{g}}$ symmetry at the boundary and can be regarded as a gauge extension of the JT gravity model, in the sense indicated in [36]. The way the extension is realized differs from what has been proposed so far in the literature, in many respects. In [36], for instance, the authors consider an $\mathfrak{sl}(2,\mathbb{R})$-BF theory for the gravitational sector together with an additional $\mathfrak{g}$-BF theory for the Yang-Mills part, so that the symmetry of the model is given by the direct sum $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{g}$. However, the proposed bulk duals, to date, do not have a clear gravitational interpretation and the role of the additional degrees of freedom extending the JY/SYK correspondence remains unclear.

We propose a class of extensions of Jackiw-Teitelboim gravity (JT) [4–6] where the presence of additional degrees of freedom is fully ascribed to the request that the 2d theory be obtained from a dimensional reduction of a Chers-Simons theory describing a maximally symmetric space in three dimensions, which includes a BTZ black hole solution (see e.g.[39, 40, 45]). This choice drastically reduces the space of possible duality relations, but at the same time allows the additional degrees of freedom to be clearly interpreted as Kaluza-Klein modes emerging from the above dimensional reduction. Notice that, although starting from three-dimensional theories limits the options on the possible symmetry groups for the corresponding 2d theories, the choice of the group does not unambiguously determine a gravitational dual for a given extension of SYK, as much depends on the boundary conditions. Consequently, when a symmetry group is fixed with a given criterion, i.e. compatibility with a pure gravity theory in 3d, as many duality relations can be realized as the allowed choices of boundary conditions.

Our construction can be summarized by the following steps. Building on the previous work which relates JT gravity, BF-theory and linear Poisson Sigma models (see e.g. [17] and references therein), we first derive the Poisson Sigma model with linear Poisson bracket of $\mathfrak{so}(2,2)$ type from the dimensional reduction of the $\mathfrak{so}(2,2)$ Chern-Simons theory with WZW boundary terms. This is a purely gravitational theory with a boundary dynamics, as well as JT gravity is in 2d. The two dimensional theory obtained in this way is an extension of JT gravity which also includes additional non-abelian gauge fields both in the bulk and at the boundary.

The choice of $\mathfrak{so}(2,2)$ allows us to interpret the model as a dimensional reduction of a purely gravitational 3d theory of a kinematical space-time, in the sense of [46], given that $\mathrm{SO}(2,2)$ is the isometry group for AdS_3.

The WZW boundary term for the 3d theory reduces to a boundary term in the form of a Casimir function for the $\mathfrak{so}(2,2)$ algebra in the 2d theory. Together with an appropriate choice of boundary conditions, this leads to a boundary dynamics governed by the $\mathfrak{diff}(S^1) \ltimes \hat{\mathfrak{g}}$ symmetry breaking. The boundary Casimir action is then identified with the action associated with coadjoint orbits of the Virasoro-Kac-Moody semidirect product. We explicitly compute the Kac-Moody terms and investigate their contribution as corrections to the nearly extremal entropy in JT. In particular, we discuss how the obtained results do not depend on the specific parameterization choices for edge fields, such as the highest weight gauge widely used in the literature [36].

The work is organised as follows. In Section 2, we recall the formulation of JT gravity as a 2d SL(2,$\mathbb{R}$)-BF theory and its derivation from a linear SL(2,$\mathbb{R}$)- Poisson Sigma Model (PSM). As we shall see, the latter formulation allows for a more natural introduction of boundary terms, differing from the BF action for a boundary contribution. Moreover, as we shall comment in the Conclusions, lends itself to generalizations. In the PSM framework, we review the construction of dynamical boundary actions in terms of a Casimir function and their relation with coadjoint orbits of infinite-dimensional groups. Finally we describe the dimensional reduction from 3d CS to our 2d model. Sections 3 and 4 contain the original results of the paper. In Sec. 3 we discuss the extension of the JT model via a $\mathfrak{so}(2,2)$-PSM and the need for the Virasoro-Kac-Moody semidirect product.

PoS(CORFU2024)307

Here, we also provide the explicit computation of the coadjoint orbits, including the Kac-Moody contributions. In section 4 we compute the leading order entropy in the Euclidean theory and compare it with the SYK results. We close in Section 5 with a summary of our results. We also further motivate the choice of working with a PSM with the indication of some future directions of research.

2. 2d JT Gravity as PSM/BF Theory

Before discussing the details of the $\mathfrak{so}(2,2)$-Poisson Sigma Model (PSM), it is worth to briefly review the topological gauge theory description of JT gravity in the first order formulation, the way it reduces to a BF model [47, 48] or equivalently to linear PSM [49] (see also [13] for a review). Upon reviewing the formalism, we will consider a two-dimensional manifold Σ with a boundary $\partial\Sigma$ and focus on the form of the boundary terms in the description of BF/PSM.

2.1 JT gravity in the BF/PSM formalism

The Jackiw Teitelboim two dimensional gravity theory is defined by the action [50]

$$S_{JT}[g,\Phi] = \frac{1}{16\pi G_N}\int_\Sigma \mathrm{d}^2x\sqrt{-g}\,\Phi(\mathcal{R}+2) + \int_{\partial\Sigma}\mathrm{d}u\sqrt{-h}(K-1)\Phi_b \tag{1}$$

where Φ is the dilaton and $\mathcal{R}$ the scalar curvature of the bulk, while Φ_b, K and $\mathrm{d}u\sqrt{-h}$ respectively refer to the restriction of the dilaton field on the boundary, the extrinsic curvature of $\partial\Sigma$ and the boundary volume form. The second term in (1), the Gibbons-Hawking-York (GHY) boundary term [51], is included to make the metric variational problem well defined. The bulk action in (1) can be explicitly derived from the near-horizon and near-extremality limit of the Reissner-Nordstrom solution in four dimensions, via dimensional reduction [11, 52]. The model is topological in the bulk and the dilaton field just acts as a Lagrange multiplier fixing the value of the curvature. Although there are no propagating degrees of freedom in the bulk, under suitable choices of boundary conditions, the boundary action is dynamical.

Let us first focus on the bulk. The bulk JT action can be formulated, in a first-order formalism, as a 2d topological $\mathrm{SL}(2,\mathbb{R})$ BF theory

$$S_{BF}[B,A] = \int_\Sigma \mathrm{Tr}(BF) \tag{2}$$

where F is the curvature of the Lie algebra valued connection 1-form $A = A^a_\mu\, dx^\mu J_a$, $B = B^a\, J_a$ a Lie algebra valued scalar field and J_a the $\mathfrak{sl}(2,\mathbb{R})$ Lie algebra generators. A detailed derivation can be found in [13, 14]

Equivalently, one can write the 2d BF theory as a linear Poisson sigma model (PSM) in terms of real fields (X, A), with $X : \Sigma \to M$ the usual embedding map and $A \in \Omega^1(\Sigma, X^*(T^*M))$ a one-form on Σ taking values in the pull-back of the cotangent bundle over M. The action of the general PSM is given by ($i, j = 1, \dots, \mathrm{dim}M$)

$$S(X,A) = \int_\Sigma A_i\wedge dX^i + \frac{1}{2}\Pi^{ij}(X)A_i\wedge A_j, \tag{3}$$

where $dX \in \Omega^1(\Sigma, X^*(TM))$ and the contraction of covariant and contravariant indices is induced by the natural pairing between T^*M and TM, yielding a two-form on Σ. To make contact with the BF formulation of JT gravity described above, one has to consider a linear Poisson tensor of Lie algebra type

$$\Pi^{ij}(X) = f_k^{ij} X^k \tag{4}$$

with f_k^{ij} the structure constants of the $\mathfrak{sl}(2,\mathbb{R})$ Lie algebra. In particular, integrating by parts the linear PSM action with Poisson tensor given by (4) we obtain the BF action (2) plus a boundary term:

$$S_{PSM} = S_{BF} - \int_{\partial\Sigma} X^i A_i . \tag{5}$$

The variation of the action with respect to X and A yields

$$\delta S_{PSM} = \int_\Sigma (E.L.)\, \delta X^i + (E.L.)\, \delta A_i - \int_{\partial\Sigma} \delta X^i A_i . \tag{6}$$

with Euler Lagrange equations (E.L)

$$D_A A = 0, \qquad dX + [X, A] =: \delta_X A = 0, \tag{7}$$

where D_A denotes the covariant derivative with respect to the gauge connection A. The first equation implies that A is pure gauge

$$A = g^{-1} dg, \tag{8}$$

with $g : \Sigma \to \mathrm{SL}(2,\mathbb{R})$, while the second equation states that the on-shell X field is a stabilizer of A. As suggested in (7), the dynamics of the dilaton corresponds to an infinitesimal gauge transformation that preserves the form of A along X on-shell. In particular, one can check that the boundary term in (5) is such that the gauge invariance restricts to the gauge transformations that satisfy $\delta_g A|_{\partial\Sigma} = 0$, which is exactly the equation of motion for the X field when restricted at the boundary. The reparametrization symmetry $\mathrm{Diff}(S^1)$ is broken and this breaking is responsible for the rise of dynamical boundary degrees of freedom (see [45] and references therein).

Now the presence of the boundary terms in (6) requires fixing the boundary values of the fields to have a well-defined variational principle. However, this choice would prevent any boundary dynamics. Alternatively, adding an extra boundary term allows for more general boundary conditions and preserves the variational principle. The PSM framework suggests a natural choice in this sense, consisting in the Casimir function X^2.[1] Indeed one can write

$$S_{(\Sigma+\partial\Sigma)} = S_{PSM} + \frac{1}{2}\int_{\partial\Sigma} X^i X_i \, \mathrm{d}u \ , \tag{9}$$

with $\mathrm{d}u$ the integration measure over $\partial\Sigma$, with the condition that

$$X_i|_{\partial\Sigma}\, \mathrm{d}u = A_i|_{\partial\Sigma} . \tag{10}$$

The presence of the extra boundary term in (9) allows for more general boundary conditions and preserves the variational principle. The condition in (10), together with the fact that A is pure gauge

[1] Any smooth function of X^2 is a Casimir function for the Poisson algebra $\mathcal{F}(\Sigma)$ with Poisson bracket (4), it being $\{X^2, -\} = 0$.

on-shell, and the required continuity of the fields at the boundary, leads to a boundary action which describes the dynamics of a particle on a group manifold, in this case on $\mathrm{SL}(2,\mathbb{R})$. Indeed, by continuing Eq. (8) to the boundary, the on-shell boundary action reads[2]

$$S|_{\partial\Sigma} = \int_{\partial\Sigma} \mathrm{Tr}(g^{-1}g')^2 \frac{1}{u'} \mathrm{d}\tau, \tag{11}$$

that is a particle on a group action where now g is an element of the loop group $\mathcal{L}\mathrm{SL}(2,\mathbb{R})$.

At this stage, it is not obvious that the boundary quadratic term derived above encodes the same boundary dynamics induced by the GHY term in JT gravity. To show that the boundary dynamics of the particle on the group manifold is related with the Schwarzian action. In the following section we review the elements of such a correspondence. Thereby, in Section 3, we propose a generalization of this result for the case of a $\mathfrak{so}(2,2)$ PSM.

2.1.1 Schwarzian Action from Coadjoint Orbits

As we already discussed in the previous section, the presence of a boundary action (11) breaks the reparametrisation invariance associated with $\mathrm{Diff}(S^1)$ to global SL(2,$\mathbb{R}$) on the boundary, while the gauge symmetry is automatically mod-out since the equations of motion (7) require the A connection 1-form to be pure gauge. Therefore, the particle on a group action shows a global $\mathrm{SL}(2,\mathbb{R})$ symmetry, modulo gauge transformation which are trivial on the boundary [54]. Accordingly, the dynamical boundary action is reduced to the coset space $\mathrm{Diff}(S^1)/\mathrm{SL}(2,\mathbb{R})$ defined by the aforementioned symmetry breaking pattern.

Such a reduction can be performed by means of the Kirillov coadjoint orbit method [55]. In brief, coadjoint orbits of the Virasoro group over the dual Virasoro algebra naturally realize homogeneous spaces $\mathrm{Diff}(\mathrm{S}^1)/\mathrm{Stab}(b)$, where $\mathrm{Stab}(b)$ indicates the little group associated with the dual element b over which the coadjoint action is computed. In the present case, we want $\mathrm{Stab}(b)$ to be the global residual $\mathrm{SL}(2,\mathbb{R})$. In particular, being $b = (f,t)$ an element in the dual of the Virasoro algebra and ϕ a finite diffeomorphism in the Virasoro group, we can compute the coadjoint action as follows [56, 57]

$$\tilde{b} = Ad^*_{\phi^{-1}}(f(\tau),t) = \left(\phi'^2 f(\tau) - \frac{t}{12}\{\phi(\tau),\tau\}_{\mathcal{S}}, t\right) \in \mathfrak{diff}^*(S^1) \tag{12}$$

For $f(\tau) = -\frac{tn^2}{24}$ one gets that $\mathrm{Stab}(b) = \mathrm{SL}^{(n)}(2,\mathbb{R})$ and the homogeneous space defined by the coadjoint orbit is $\mathrm{Diff}(\mathrm{S}^1)/\mathrm{SL}^{(n)}(2,\mathbb{R})$, where $\mathrm{SL}^{(n)}(2,\mathbb{R})$ denotes the n-fold cover of $\mathrm{SL}(2,\mathbb{R})$ (see e.g. [54]).

Now, a natural action functional on the coset space is given by the pairing of $\tilde{b}$ with an element $\xi \in \mathfrak{diff}(S^1)$, which gives [58]

$$\langle \tilde{b},\xi\rangle = \int_{S^1} \tilde{b}(\tau)\xi(\tau)\mathrm{d}\tau\,. \tag{13}$$

The latter can be identified with the reduction of the on-shell boundary action

$$S[g]|_{\partial\Sigma} = \int_{\partial\Sigma} \mathrm{Tr}(g^{-1}g')^2/u'\mathrm{d}\tau \tag{14}$$

[2]One might wonder why not to include the Casimir function directly in the bulk action. The reason, as explained in [49], is that the addition of a Casimir term in the bulk would break the topological nature of the model, resulting in a theory which is equivalent to a Yang-Mills in the bulk, which is not what we want.

on the coset space, provided $Xu'\mathrm{d}\tau = g^{-1}g'\mathrm{d}\tau$, where $g \in \mathcal{LG}$, by identifying $\tilde{b}$ with the Casimir X^2 and setting $\xi(\tau) = 1/u'$, for $\mathrm{d}u = u'\mathrm{d}\tau$ on $\partial\Sigma \sim S^1$. The relation between the Schwarzian action and the boundary action in (9) can then be understood purely from symmetry considerations.[3] The Schwarzian action is nothing but the action associated to the coadjoint orbits of the Virasoro group with global $\mathrm{SL}(2,\mathbb{R})$ symmetry and it can be identified with the particle-on-a-group-manifold action reduced on the coset space. A more direct way of deriving the equivalence between the action of a particle on a group manifold (14) and the natural action on the coadjoint orbits of $\mathrm{Diff}(S^1)$ can be found in [54].

In the following, we will seek an extension of the boundary Schwarzian dynamics of JT gravity by extending the derivation to a $\mathrm{SO}(2,2)$-PSM, which naturally encodes JT gravity plus extra gauge fields. The choice of $\mathrm{SO}(2,2)$ is motivated by the possibility of providing a natural gravitational origin to the extra gauge fields emerging in the $\mathrm{SO}(2,2)$-PSM, once the gravitational $\mathrm{SL}(2,\mathbb{R})$ sector has been singled out, in terms of a dimensional reduction from a 3d Chern-Simons (CS) theory with boundary [59]. We briefly recall the dimensional reduction from 3d CS theory to 2d BF/PSM model associated with JT gravity in the following. Thereby, we move to the proposed $\mathrm{SO}(2,2)$ model in Section 3.

2.2 Dimensional Reduction from 3d CS+WZW

Let us consider a CS theory in three dimensions, with $\mathfrak{so}(2,2)$-valued connection Ω. This yields a purely gravitational model describing the AdS_3 geometry. We shall see that the dimensional reduction of such a theory is equivalent to an $\mathfrak{so}(2,2)$ PSM in two dimensions. Moreover, we will show that the 2d boundary terms, necessary to derive the 1d Schwarzian dynamics, can be recovered by adding to the CS theory a WZW boundary term which dimensionally reduces to the action of a particle on a group manifold.

To this, let Σ_3 be a manifold with the structure $\Sigma_3 = \Sigma \times I$, where Σ is a two-dimensional manifold with boundary and I a suitably chosen one dimensional sub-manifold. We write

$$S_{CS}[\Omega] = \frac{1}{2}\int_{\Sigma_3} \langle \Omega \wedge d\Omega + \tfrac{1}{3}[\Omega,\Omega] \wedge \Omega\rangle. \tag{17}$$

It is convenient for the purpose of this section to write $\mathfrak{so}(2,2)$ as the direct sum $\mathfrak{so}(2,2) = \mathfrak{sl}(2,\mathbb{R})_L \oplus \mathfrak{sl}(2,\mathbb{R})_R$. Then $\Omega = \omega^L_i L_i + \omega^R_i R_i$, with L_i and R_i spanning $\mathfrak{sl}(2,\mathbb{R})$. The Killing form is given by

$$\langle L_i, L_j\rangle = \langle R_i, R_j\rangle = \eta_{ij}, \quad \langle L_i, R_j\rangle = 0 \tag{18}$$

[3]By setting $n = 1$, from (12) we get

$$Ad^*_{\phi^{-1}}(-\frac{t}{24}, t) = \left(-\frac{t}{12}\left(\frac{1}{2}\phi'^2 + \{\phi(\tau),\tau\}_S\right), t\right). \tag{15}$$

With the change of variables $F(\tau) \equiv \tan\left(\frac{1}{2}\phi(\tau)\right)$, the coadjoint action can be written in terms of a single Schwarzian derivative, that is

$$Ad^*_{\phi^{-1}}(-\frac{t}{24}, t) = \left(-\frac{t}{12}\{F(\tau),\tau\}_S, t\right). \tag{16}$$

Therefore, we recognise in (13) the Schwarzian action, while disregarding the contribution of the central extension terms in the pairing.

with $\eta = \text{diag}(1,-1,1)$. Therefore, the Chern-Simons theory splits into two copies of a single $\mathfrak{sl}(2,\mathbb{R})$-CS theory with connection $\omega = \omega^i \tau_i$. It is then sufficient to prove that each $\mathfrak{sl}(2,\mathbb{R})$-CS sector reduces to a 2d $\mathfrak{sl}(2,\mathbb{R})$-BF theory. The explicit form of each $\mathfrak{sl}(2,\mathbb{R})$ sector of (17) is given by

$$S_{CS}[\omega] = \frac{1}{2}\int_{\Sigma_3} d^3x\, \epsilon^{\lambda\mu\nu}\Big(\omega^h_\lambda \partial_\mu \omega^k_\nu + \frac{1}{3}\omega^h_\lambda\, f^k_{ij}\, \omega^i_\mu \omega^j_\nu\Big)\eta_{hk} \tag{19}$$

where f^k_{ij} are structure constants of $\mathfrak{sl}(2,\mathbb{R})$. Let (ϕ,τ,ρ) be local coordinates over Σ_3, with $\phi \in I$. The dimensional reduction scheme consists in identifying $\omega^i_\phi \to \Phi^i$, $\omega^i_{\tau,\rho} \to A^i_{\tau,\rho}$ and discarding any derivative with respect to ϕ, $\partial_\phi \to 0$. This results in the action

$$S_{CS}[\omega] = \frac{1}{2}\int_{\Sigma_3} d^3x\Big(\omega^h_\phi \epsilon^{\mu\nu}\partial_\mu \omega^h_\nu + \epsilon^{\mu\nu}\omega^h_\mu \partial_\nu \omega^k_\phi + \frac{1}{3}\epsilon^{\lambda\mu\nu}\omega^h_\lambda\, \epsilon^k_{ij}\, \omega^i_\mu \omega^j_\nu\Big)\,\eta_{hk}, \tag{20}$$

where the $\epsilon^{\mu\nu}$ only refers to the (ρ,τ) coordinates in Σ. The first two terms in the action are $\langle \Phi, dA\rangle$ and $\langle A\wedge d\Phi\rangle$, where the external derivative and the wedge product are now performed to be those over Σ, i.e. in the (τ,ρ) coordinates. The last term corresponds to $2\langle\Phi,[A,A]\rangle$. In order to recover a 2d BF theory, we have to recognize $F = dA + [A,A]$. From $d(\Phi A) = d\Phi\wedge A + \Phi dA$, we deduce that $\langle\Phi, dA\rangle + \langle A\wedge d\Phi\rangle = 2\langle\Phi, dA\rangle - \langle d(\Phi, A)\rangle$. One obtains then

$$S_{CS}[\omega] = \int_{\Sigma_3} d^3x\,\langle\Phi,F\rangle - \frac{1}{2}\int_{\partial\Sigma_3} d^2x\,\langle\Phi,A\rangle. \tag{21}$$

In order to make full contact with the 2d dilaton gravity on an AdS_2 disk Σ, we require that, after integrating out the redundant dimension,

$$\Sigma_3 \xrightarrow{\int_I d\phi} \Sigma, \qquad \partial\Sigma_3 \xrightarrow{\int_I d\phi} \partial\Sigma \tag{22}$$

Therefore, the dimensional reduction of the CS theory, not only reproduces a BF theory in the 2d bulk Σ, but also gives a boundary term. In order to also recover the dynamical boundary theory, a boundary term must be added to the 3d action, so that its dimensional reduction, together with the boundary term in (21), leads exactly to the $\mathfrak{so}(2,2)$-PSM with the boundary Casimir.

The full theory whose dimensional reduction leads to the two-dimensional $\mathfrak{so}(2,2)$-PSM model is a CS theory with the WZW boundary term as in [60]. We further refer to [61] for details. As explained in [60], the 3d theory has a clear interpretation in terms of BTZ black hole geometry, compatible with the near horizon physics description of JT gravity. The connection between dimensionally reduced AdS_3 theories and extremal black holes has been also explored in [40–44].

3. The $\mathfrak{so}(2,2)$ Poisson Sigma Model

As anticipated in the introduction, our goal is to define a generalized version of JT gravity from a topological gauge theory with a symmetry group containing $\text{SL}(2,\mathbb{R})$. The dimensional reduction of pure AdS_3-CS theory suggests the $\mathfrak{so}(2,2)$-Poisson sigma model over a two-dimensional manifold $\Sigma = \mathbb{R}\times S^1$ as natural candidate in this sense. In this section, we show how isolating the gravitational $\mathfrak{sl}(2,\mathbb{R})$ sector in the case of $\mathfrak{so}(2,2)$ naturally singles out the dynamics of the residual degrees of freedom, which result in additional non-abelian gauge fields that also become dynamical at the boundary. In particular, we show how the boundary dynamics is encoded in the Casimir functions of the full algebra, analogously to the case of the $\mathfrak{sl}(2,\mathbb{R})$-PSM, and it is naturally reduced on the $(\text{Diff}(S^1)\ltimes\mathcal{LG})/\text{SL}(2,\mathbb{R})$ coset space.

3.1 The bulk theory

We start by constructing the bulk theory. Let Ω be the $\mathfrak{so}(2,2)$-valued connection 1-form over Σ. The way we decompose the connection and the embedding maps in a given basis of the algebra and its dual is convenient for reasons that will be clear soon. The $\mathfrak{so}(2,2)$ algebra is isomorphic to two copies of $\mathfrak{sl}(2,\mathbb{R})$, so we have a natural chiral basis in $\mathfrak{so}(2,2) \simeq \mathfrak{sl}_L(2,\mathbb{R}) \oplus \mathfrak{sl}_R(2,\mathbb{R})$:

$$[L_i, L_j] = c_{ij}^k L_k, \qquad [R_i, R_j] = c_{ij}^k R_k \qquad [L_i, R_j] = 0 \tag{23}$$

and with $J_i = L_i + R_i$ we rotate the basis into a 'non-chiral' basis with a $\mathfrak{sl}_L(2,\mathbb{R})$ sector invariant under the action of the $\mathfrak{sl}_J(2,\mathbb{R})$ sub-algebra

$$[J_i, J_j] = c_{ij}^k J_k, \qquad [L_i, L_j] = c_{ij}^k L_k, \qquad [J_i, L_j] = c_{ij}^k L_k. \tag{24}$$

We will refer to the $\mathfrak{sl}(2,\mathbb{R})_J$ sub-algebra as the gravitational sector and to the $\mathfrak{sl}(2,\mathbb{R})_L$ as the "non-abelian gauge" sector. In the non-chiral basis, we then write the connection Ω as

$$\Omega = A^i J_i + B^i L_i. \tag{25}$$

We denote by $\mathfrak{Z}_i$ the embedding maps

$$\mathfrak{Z}_i : \Sigma \to \mathfrak{so}(2,2)^*$$

with the Poisson brackets

$$\{\mathfrak{Z}_i, \mathfrak{Z}_j\} = \Pi_{ij}(\mathfrak{Z}) = f_{ij}^k \mathfrak{Z}_k \tag{26}$$

where f_{ij}^k are now the $\mathfrak{so}(2,2)$ structure constants. The corresponding Poisson Sigma model takes the form

$$S_{P_\sigma} = \int_\Sigma d\Omega_i \wedge \mathfrak{Z}^i + \frac{1}{2}\Pi^{ij}(\mathfrak{Z})\,\Omega_i \wedge \Omega_j. \tag{27}$$

Where we used the invariant bi-linear form

$$\langle J_i, J_j \rangle = \langle L_i, L_j \rangle = k_{ij}, \qquad \langle J_i, L_j \rangle = \frac{1}{2} k_{ij}, \tag{28}$$

with k_{ij} the $\mathfrak{sl}(2,\mathbb{R})$ Killing form. A decomposition similar to that in (25) can be also be performed for the embedding maps. From here on, we denote the dual basis with lowered indices, thus we write

$$\mathfrak{Z} = \mathfrak{X}_i J^i + \mathfrak{Y}_i L^i. \tag{29}$$

It then follows that

$$\begin{aligned} S_{P_\sigma} = \int_\Sigma & dA_i \wedge \mathfrak{X}^i + dB_i \wedge \mathfrak{Y}^i + \frac{1}{2} dA_i \wedge \mathfrak{Y}^i + \\ & + \frac{1}{2} dB_i \wedge \mathfrak{X}^i + \frac{1}{2} c_k{}^{ij}\, \mathfrak{X}^k\, A_i \wedge A_j + \frac{1}{2} c_k{}^{ij}\, \mathfrak{Y}^k\, B_i \wedge B_j + \\ & + c_k{}^{ij}\, \mathfrak{Y}^k\, A_i \wedge B_j + \frac{1}{2} c_k{}^{ij}\, \mathfrak{X}^k\, A_i \wedge B_j + \\ & + \frac{1}{4} c_k{}^{ij}\, \mathfrak{X}^k\, B_i \wedge B_j + \frac{1}{4} c_k{}^{ij}\, \mathfrak{Y}^k\, A_i \wedge A_j . \end{aligned} \tag{30}$$

where we can recognize the $\mathfrak{sl}(2,\mathbb{R})_J$ Poisson-Sigma model appearing as the gravitational sub-sector of the theory. The equations of motion for the fields A_i and B_i are

$$\mathfrak{D}_A A = 0, \qquad \mathfrak{D}_\Omega B = c_i^{hk} B_h A_k L^i, \tag{31}$$

while the equations for the embedding maps read

$$\delta_{\mathfrak{X}} A = 0, \qquad \delta_{\mathfrak{Y}} \Omega = -c_i^{hk} \mathfrak{X}_h B_k L^i \tag{32}$$

This means that the on-shell A field is pure gauge with respect to the $\mathrm{SL}(2,\mathbb{R})_J$ sub-group. On the other hand, the on-shell $\mathfrak{X}$ field is stabilizer for A. Notice that the covariant derivative for the B field is computed with respect to the entire connection Ω, while the same is not true for the A field, whose equation of motion is equivalent to the case of the $\mathrm{SL}(2,\mathbb{R})$ BF/PSM. This shows that we are allowed to keep interpreting the A-sector of the model as equivalent to ordinary JT gravity. Differently, the covariant derivative acting on the B fields has both contributions from A and B. Therefore B behaves like a gauge field coupled to gravity.

We now introduce a boundary action term and characterize the boundary dynamics in S^1. The equation of motion for the gauge connection, that we derive from (27), is

$$\mathfrak{D}_\Omega \Omega = 0 \tag{33}$$

and, without adding any counterterm, the variation on the boundary is just

$$\delta S_{P_\sigma}|_{S^1} = -\int_{S^1} \Omega^i \delta \mathfrak{Z} \tag{34}$$

As it is the case for the $\mathfrak{sl}(2,\mathbb{R})$-PSM, if we insert a boundary Casimir counter-term $\mathfrak{Z}^2$ and set the boundary condition

$$\Omega|_{S^1} = \mathfrak{Z}|_{S^1} \mathrm{d}u \tag{35}$$

we get a particle on a group action (cfr (11)) :

$$\Omega|_{S^1} = g^{-1} dg \implies S_{P_\sigma}|_{S^1} = \int_{S^1} \frac{1}{2} \mathrm{Tr}\{(g^{-1} g')^2\} \frac{1}{u'} \mathrm{d}\tau, \tag{36}$$

where now g is an element in the SO(2,2) gauge group.

We know that the bulk theory has two $\mathfrak{sl}(2,\mathbb{R})_{J,L}$ sectors in interaction and we would like to make that manifest also at the boundary. We expect the boundary action to comprise a Schwarzian derivative term corresponding to the gravitational $\mathfrak{sl}(2,\mathbb{R})_J$-PSM sector ($\mathfrak{X}^2$) and a particle-on-a-group term for the non-abelian sector ($\mathfrak{Y}^2$), plus interactions. As we will see, this is indeed the case for a suitable choice of boundary conditions.

3.2 Asymptotic Symmetries

Let's now focus on the fate of the boundary symmetries and the form of the reduced boundary action. The equation of motion (33) fixes the connection Ω to be pure gauge

$$\Omega = g^{-1} dg, \qquad g : \Sigma \to \mathrm{SO}(2,2) \tag{37}$$

PoS(CORFU2024)307

This condition, together with (35), reproduces the same symmetry breaking mechanism of the $\mathrm{SL}(2,\mathbb{R})$-PSM, except for the presence of a global $\mathrm{SO}(2,2)$ symmetry in the action (36). The allowed gauge transformation are therefore the $\mathrm{SO}(2,2)$ gauge transformations which are constant at the boundary. Now we show that imposing further boundary conditions can dramatically change the symmetry breaking mechanism and the boundary dynamics can be understood in terms of coadjoint orbits of $\mathrm{Diff}(S^1) \ltimes \mathcal{L}\mathcal{G}$, for some $\mathcal{G}$ depending on the additional choices made.

Suppose that the embedding fields are chosen at the boundary such that,

$$\mathfrak{X}_{\mathrm{i}}|_{S_1} = -\mathfrak{Y}_{\mathrm{i}}|_{S_1} \tag{38}$$

Under these boundary conditions, and (35), the $\mathfrak{Z}$ field at the boundary is no longer $\mathfrak{so}(2,2)$-valued but $\mathfrak{sl}_R(2,\mathbb{R})$-valued.[4] As a consequence, we are reducing the unfixed fields at the boundary. In fact,

$$\mathfrak{Z} = (\mathfrak{X}_{\mathrm{i}} + \mathfrak{Y}_{\mathrm{i}})J^i + \mathfrak{Y}_{\mathrm{i}}(L^i - J^i) \xrightarrow{\mathfrak{X}_{\mathrm{i}} = -\mathfrak{Y}_{\mathrm{i}}|_{S^1}} \mathfrak{Z} = -\mathfrak{Y}_i R^i \tag{39}$$

This additional condition, together with Ω being pure gauge corresponds to a partial gauge fixing in the R-sector. Indeed, also Ω takes the form

$$\Omega|_{S^1} = h_R^{-1} dh_R, \qquad h_R \in \mathrm{SL}_R(2,\mathbb{R})\,. \tag{40}$$

Moreover, any $\mathrm{SL}_L(2,\mathbb{R})$ gauge transformation would not preserve the boundary conditions since it would force the boundary fields to get out of the R-sector. Therefore, the $\mathrm{SL}_L(2,\mathbb{R})$ gauge group must now be regarded as an actual broken gauge symmetry, while the global $\mathrm{SL}_L(2,\mathbb{R})$ symmetry is just trivial since global transformations in this sector are in the Little Group of the boundary connection. The only non-trivial global symmetry is then the $\mathrm{SL}_R(2,\mathbb{R})$ symmetry. With this in mind, we can conclude that the reduction of the on-shell boundary action involves the coadjoint orbit of $\mathrm{Diff}(S^1) \ltimes \mathcal{L}\mathcal{G}$, where the given boundary conditions identifies $\mathcal{L}\mathcal{G}$ with the $\mathrm{SL}_L(2,\mathbb{R})$ loop group. We can then understand the reduced on-shell particle on a group with the action associated to $\mathrm{Diff}(S^1) \ltimes \mathcal{L}\mathrm{SL}_L(2,\mathbb{R})$ and $\mathrm{SL}_R(2,\mathbb{R})$ global symmetry.

Since symmetries of the boundary theory are fixed by the additional boundary conditions it is important to ask how many nonequivalent choices of boundary conditions exist. Suppose then that the boundary embedding fields are chosen to be valued on a given sub-algebra $\mathfrak{h} \subseteq \mathfrak{so}(2,2)$

$$\mathfrak{Z}|_{S^1} = \mathfrak{H} \in \mathfrak{h} \tag{41}$$

Let then $\mathfrak{k}$ be the complement of $\mathfrak{h}$ in $\mathfrak{so}(2,2)$. In order to preserve the boundary condition (41) the following conditions must hold

$$[\mathfrak{h},\mathfrak{h}] = 0 \quad or \quad [\mathfrak{h},\mathfrak{h}] = \mathfrak{h}, \tag{42}$$

$$[\mathfrak{k},\mathfrak{h}] = 0 \quad or \quad [\mathfrak{k},\mathfrak{h}] = \mathfrak{h}. \tag{43}$$

By virtue of these conditions, we can list the allowed scenarios in the case of $\mathfrak{so}(2,2)$. The case $[\mathfrak{h},\mathfrak{h}] = 0$ is ruled out since $\mathfrak{so}(2,2)$ is semi-simple and it has no invariant abelian sub-algebras.

[4]Notice that the choice in (38), together with the $\Omega|_{S^1} = \mathfrak{Z}|_{S^1}\mathrm{d}u$ condition, is similar to the gauge fields paramentrisation in the Yang-Mills extension of JT gravity proposed in [36].

PoS(CORFU2024)307

The case $[\mathfrak{h},\mathfrak{h}] = \mathfrak{h}$ can be realized with $\mathfrak{h} = \mathfrak{sl}(2,\mathbb{R})_{L,R,J}$ or trivially with $\mathfrak{h} = \mathfrak{so}(2,2)$ which correspond to the absence of any further boundary condition apart from $\mathfrak{Z}|_{S^1} d\tau = \Omega|_{S^1}$. Any choice associated with $\mathfrak{h} = \mathfrak{sl}(2,\mathbb{R})_{R,L,J}$ or $\mathfrak{h} = \mathfrak{so}(2,2)$ leads to a well defined boundary condition, where $\mathfrak{Z}|_{S^1}$ takes values into a (sub)-algebra which can be either left invariant by its complement or commute with it.

3.3 Virasoro-Kac-Moody Coadjoint Orbits

At the end of the last section we showed that additional boundary conditions are allowed at the boundary and imply a partial breaking of the gauge symmetry. This means that if we want to make sense of the boundary action, we have to compute the coadjoint orbit of the Virasoro-Kac-Moody semidirect product as it is the case for SYK-like tensor models [29]. Generally speaking, the computation of coadjoint orbits for semidirect products of infinite dimensional groups is not at all an easy task, but the interesting feature of $\mathrm{Diff}(S^1) \ltimes \mathcal{LG}$ is that the Virasoro algebra and the Kac-Moody algebra are related by the Sugawara construction [62]. In particular, the Virasoro algebra acts naturally in a derivative way over the Kac-Moody sector. The algebra $\mathfrak{diff}(S^1) \ltimes \hat{g}$ is in fact given by [63]

$$\begin{aligned} &[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n}\,; \\ &[K_{i,m}, K_{j,m}] = c_{ij}^k K_{k,m+n} + m\langle K_i, K_j\rangle \delta_{m,-n}\,; \\ &[L_m, K_{i,n}] = -nK_{i,m+n}. \end{aligned} \tag{44}$$

In our case, we have $\hat{g} = \widehat{\mathfrak{sl}(2,\mathbb{R})}$. In order to compute the coadjoint orbit for the semidirect product $\mathrm{Diff}(S^1) \ltimes \mathcal{LG}$ we follow [64], where both the Virasoro and the Kac-Moody algebra elements are realized through functions. Let $(u(\tau), k(\tau), \alpha, \beta)$ be a generic element in the set $\mathfrak{diff}(S^1) \ltimes \widehat{\mathfrak{sl}(2,\mathbb{R})}$, with $u(\tau) \in \mathfrak{diff}(S^1)$, $k(\tau) \in \widehat{\mathfrak{sl}(2,\mathbb{R})}$ and α, β the respective central elements. Then, a basis independent way to write the commutation relation is simply given by

$$[(u,k,\alpha,\beta),(v,h,\gamma,\delta)] = ([u,v]_{Vir}, [k,h]_{KM} - uh' - vk', \Omega_{Vir}(u''',v), \Omega_{KM}(k,h')) \tag{45}$$

where the Ω's stand for the respective cocycles [65, 66]. The group multiplication in $\mathrm{Diff}(S^1) \ltimes \mathcal{LG}$ is defines as

$$(\phi, g)(\psi, h) = (\phi \circ \psi, g.h \circ \phi^{-1})\,, \tag{46}$$

where ϕ, ψ are finite diffeomorphisms and g, h are elements of the loop group. In order to compute the coadjoint action of $\mathrm{Diff}(S^1) \ltimes \mathcal{LG}$, we must as well introduce the pairing between the algebra and its dual space, which is given by the sum of pairings:

$$\langle (b,\rho,\alpha,\beta),(v,\lambda,\gamma,\delta)\rangle = \int b(\tau)v(\tau)d\tau + \int \langle \lambda(\tau)\rho(\tau)\rangle_{KM} d\tau + \alpha\gamma + \beta\delta. \tag{47}$$

Therefore, we have [64],

$$Ad^*_{(\phi,g)}(b,\rho,\alpha,\beta) = ((b\circ\phi)\phi'^2 + \alpha\{\phi,\tau\}_S + \langle g^{-1}dg, \rho\rangle + \frac{1}{2}\beta||g^{-1}dg||^2, \phi'(g^{-1}\rho g)\circ\phi + \beta g^{-1}dg, \alpha, \beta). \tag{48}$$

By pairing the coadjoint orbit with a Lie algebra element we can construct a natural action, as we already saw in 2.1.1 for the JT gravity case, and we can finally write the reduced expression for the on-shell action (36) :

$$S|_{S^1} = \int_{S^1} \{(b \circ \phi)\phi'^2 + \alpha\{\phi, t\}_S + \langle g^{-1}dg, \rho\rangle + \frac{1}{2}\beta||g^{-1}dg||^2\}d\tau \tag{49}$$

which matches with the results of [21]. The element $b(\tau)$ must be chosen in such a way that the global symmetry be $SL(2, \mathbb{R})$, which is realised through fractional transformations [67]. The geometric action (49), without loss of generality, can be thought of as the pairing of $Ad^*_{(\phi,g)}(b, \rho, \alpha, \beta)$ with a pure Virasoro element and, given the structure of the algebra in (45), this can be always rotated in a new element with non vanishing Kac-Moody component. Notice that this would not be true if we paired the coadjoint action with an element of the Kac-Moody subalgebra, because the latter is an invariant sub-algebra. Therefore, this last option could be regarded as a nonequivalent choice that kills the Schwarzian degree of freedom.

4. Black Hole Entropy

Once the boundary dynamics for the $\mathfrak{so}(2,2)$ PSM has been established with a given boundary condition, it is an interesting check to compute the leading order entropy. Indeed, it is well recognized that in JT gravity the latter can be interpreted as the black hole entropy since the theory admits a gravitational interpretation with the boundary playing the role of a near horizon surface. We shall see that we obtain a consistent result for our model.

Let $\mathcal{H}[\Omega]$ be the holonomy associated with the gauge connection Ω

$$\mathcal{H}[\Omega] = \mathcal{P} \exp\Big[- \oint \Omega\Big], \tag{50}$$

the equations of motion fix Ω to be pure gauge, i.e. $\Omega = g^{-1}dg$. This implies that, if the integration contour is the S^1 boundary and if we demand for smooth Euclidean solution, then

$$\mathcal{H} = g(\beta)g^{-1}(0) = \mathbb{1}, \tag{51}$$

with the inverse temperature β being the length of the boundary.

Suppose that no further boundary conditions are imposed apart from $\mathfrak{Z}|_{S^1}dt = \Omega|_{S^1}$, so that the boundary dynamics is that of a particle on the entire $SO(2,2)$ group. The global SO(2,2) symmetry for the boundary theory implies the presence of the conserved charges

$$J_i = \langle g^{-1}dg, \tau_i\rangle \tag{52}$$

where τ_i are $\mathfrak{so}(2,2)$ generators. This means that, up to constants, the on-shell connection satisfies $\Omega_i|_{o.s.} = J_i$, therefore

$$\mathcal{H}[\Omega] = \exp\Big[- \beta J\Big] = \mathbb{1}. \tag{53}$$

Let now Λ be the diagonalized $-\beta J$, i.e. $-\beta J = P\Lambda P^{-1}$. This implies

$$\mathcal{H}[\Omega] = \exp[P\Lambda P^{-1}] = P\exp[-\beta\Lambda]P^{-1} = \mathbb{1}. \tag{54}$$

Requiring $\exp[\Lambda] = \mathbb{1}$ implies that the eigenvalues must satisfy $\lambda_k = -2\pi i n_k/\beta$, with integers n_k. In the case of an $\mathfrak{so}(2,2)$-valued boundary connection, the 4 eigenvalues come in two pairs of eigenvalues with opposite sign:

$$\mathrm{Tr}(J^2) = -\frac{8\pi(n^2+m^2)}{\beta^2}. \tag{55}$$

In order to evaluate the entropy we need to compute the leading order free energy F which can be identified with the temperature times the on-shell boundary Euclidean action I

$$F = \beta^{-1} I|_{S^1, on-shell}. \tag{56}$$

The latter can be computed easily. Let C be the on-shell boundary Casimir function, then $C = \langle J, J\rangle$ and

$$I|_{S^1, on-shell} = \beta C = \beta\,\mathrm{Tr}(J^2) = -\frac{8\pi(n^2+m^2)}{\beta}. \tag{57}$$

The free energy then comes with the correct sign thus recovering a positive entropy which is linear in the temperature $T = \beta^{-1}$, that is

$$S = -\frac{dF}{dT} = 16\pi k\,(n^2+m^2)\,T \tag{58}$$

This result is consistent with the linear scaling of the entropy in SYK models [21, 68] and it shows that the extra $\mathrm{SL}(2,\mathbb{R})$ degrees of freedom contribute with a supplementary linear term $\propto \beta^{-1}m^2$.

5. Conclusions

We constructed a JT gravity bulk-plus-boundary generalization in terms of a $\mathfrak{so}(2,2)$-Poisson sigma model with a boundary Casimir action. It is a fact that 3d Chern-Simons theories with WZW term at the boundary, once dimensionally reduced, give 2d BF theories with the particle on a group action at the 1d boundary [61]. The dimensional reduction of SO(2,2) Chern-Simons-WZW theory, which is a 3d theory describing an AdS_3 geometry, leads to the proposed $\mathfrak{so}(2,2)$-PSM together with the dynamical particle on a group action.[5] The model provides a gravitational dual for SYK-like tensor models with internal symmetries [29, 31], whose low energy dynamics is characterized by a $\mathfrak{diff}(S^1) \ltimes \hat{\mathfrak{g}}$ symmetry. In our case, the $\mathfrak{diff}(S^1) \ltimes \hat{\mathfrak{g}}$ symmetry, with $\mathfrak{g} = \mathfrak{sl}(2,\mathbb{R})$, arises after a specific choice of boundary conditions. The additional Kac-Moody sector encodes the dynamics of extra edge modes, whose number is equal to $\dim(\mathfrak{g})$. In the near horizon interpretation, the standard computation of the near-extremal black hole entropy reproduces the JT result with an additional entropy contribution due to the presence of extra gauge fields. The additional contribution is equivalent to the bare JT case because of the particular structure of $\mathfrak{so}(2,2)$. The specific case of $\mathfrak{so}(2,2)$, seen from the CS perspective, allows to interpret the Kac-Moody modes, living at the S^1 boundary and expected to arise in the low-energy regime of SYK tensor models, as Kaluza-Klein modes associated with the dimensional reduction. The proposed $\mathfrak{so}(2,2)$-PSM provides a class of possible JT-Yang-Mills generalizations of the JT/SYK correspondence with a purely gravitational

[5] In three spacetime dimensions, $\mathfrak{so}(2,2)$ is in fact a kinematic algebra in the sense of [46], and a 3d Chern-Simons theory over this algebra provides a purely gravitational theory of AdS_3 geometry.

interpretation from a 3d perspective. This interpretation selects a class of SYK-like duals which naturally relates to 3d gravity.

The choice of working with a PSM, which in the present paper is linear, therefore equivalent to a BF theory up to boundary therms, lends itself to generalizations which could have interesting gravitational implications. The first generalization is related with the possibility of considering non-linear PSM; this has already been noticed in the literature [69], it being related to nonlinear generalizations of JT gravity [3, 70].

The second generalization, entirely novel up to our knowledge, would be to consider Jacobi sigma models [71–74], which are models with a deformed Poisson bracket on the target space, known as Jacobi bracket. The latter is constructed with a quasi-Poisson structure, violating Jacobi identity in a controlled manner. The explicit relation for the quasi-Poisson tensor Π reads $[\Pi, \Pi] = 2E \wedge \Pi$, with E a vector field on the target space, the so-called Reeb vector field, s.t. $L_E \Pi = 0$. This structure may be obtained by a homogeneous Poisson bracket in one dimension higher (technically a fiber bundle over the target space, with fiber $R - \{0\}$ [74]). Interestingly, the extra dimension, or, better to say, the generator of $R - \{0\}$, could be related with a dilaton field which could play a role in generalized JT gravity models.

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