Vacuum Bubble Nucleation and Black Hole Pair Creation

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Abstract

We study the nucleation of a vacuum bubble in the presence of gravity and consider the pair production of blak holes in the background of bubble solutions. This article is prepared for the proceedings of The 18th Workshop on General Relativity and Gravitation in Japan (JGRG18) in Hiroshima, Japan, 17-21 Nov 2008.

1 Introduction

Recently, we have investigated the possible types of the nucleation of vacuum bubbles [1]. We classified true vacuum bubbles in de Sitter background and present some numerical solutions. The nucleation rate and the radius of true and false vacuum bubbles are analytically computed using the thin-wall approximation. We obtained static bubble wall solutions of junction equation with black hole pair. In this article, we will summarize our results.

It has been shown that the first-order vacuum phase transitions occur via the nucleation of true vacuum bubble at zero temperature both in the absence of gravity [2] and in the presence of gravity [3]. This result was extended by Parke [4] to the case of arbitrary vacuum energy densities. An extension of this theory to the case of non-zero temperatures has been found by Linde [5] in the absence of gravity, where one should look for the $O(3)$-symmetric solution due to periodicity in the time direction β with period T^{-1} unlike the $O(4)$-symmetric solution in the zero temperature. These processes as cosmological applications of false vacuum decay have been applied to various inflationary universe scenarios by many authors [6]. The Hawking-Moss transition describes the scalar field jumping simultaneously at the top of the potential barrier [7]. A new method to calculate the tunneling wave function that describes vacuum decay was studied by Gen and Sasaki [8]. The effect of the Gauss-Bonnet term on vacuum decay was also studied [9]. Marvel and Wesley studied thin-wall instanton with negative tension wall and its relation to Witten's bubble of nothing [10]. In Ref. [11], the authors discussed the possible four different instantons in Euclidean de Sitter space.

As for the false vacuum bubble formation, Lee and Weinberg [12] have shown that if the vacuum energies are greater than zero, gravitational effects make it possible for bubbles of a higher-energy false vacuum to nucleate and expand within the true vacuum bubble in the de Sitter space which has a topology of 4-sphere. The false vacuum bubble nucleation is described as the inverse process of the true vacuum bubble nucleation. However, their solution is larger than the true vacuum horizon [13]. The oscillating bounce solutions, another type of Euclidean solutions, have been studied in detail by Hackworth and Weinberg [14]. On the other hand Kim *et al.* [15] have shown that false vacuum region may nucleate within the true vacuum bubble as global monopole bubble in the high temperature limit.

Next, we will consider black hole pair creation. Caldwell *et al.* studied black hole pair creation in the presence of a domain wall [16] using the cut-and-paste procedure, where the background has vanishing cosmological constant and the probability was obtained. The repulsive property of the domain wall give rise to black hole pair creation. However, our solutions give rise to the background for the black hole pair creation with a bubble wall more naturally.

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2 The vacuum bubble

The bubble nucleation rate or the decay rate of background vacuum is semiclassically given by

$$\Gamma/V = Ae^{-B/\hbar}, \tag{1}$$

where B is the difference between Euclidean action corresponding to bubble solution and that of the background and the prefactor A is discussed in Ref. [17], that with some gravitational corrections in Ref. [18]. We are interested in finding the coefficient B.

Let us consider the action

$$S = \int_{\mathcal{M}} \sqrt{-g} d^4 x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right] + \oint_{\partial\mathcal{M}} \sqrt{-h} d^3 x \frac{K}{\kappa}, \tag{2}$$

where $\kappa \equiv 8\pi G$, $g \equiv det g_{\mu\nu}$, and the second term on the right-hand side is the boundary term. $U(\Phi)$ is the scalar field potential with two non-degenerate minima with lower minima at Φ_T and higher minima at Φ_F, R denotes the Ricci curvature of spacetime $\mathcal{M}$, and K is the trace of the extrinsic curvature of the boundary $\partial\mathcal{M}$.

We will take $O(4)$ symmetry for both Φ and the spacetime metric $g_{\mu\nu}$, expecting its dominant contribution [19].

The Euclidean field equations for Φ and ρ have the form

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = \frac{dU}{d\Phi}, \tag{3}$$

$$\rho'' = -\frac{\kappa}{3}\rho(\Phi'^2 + U), \tag{4}$$

respectively and the Hamiltonian constraint is given by

$$\rho'^2 - 1 - \frac{\kappa\rho^2}{3}\left(\frac{1}{2}\Phi'^2 - U\right) = 0. \tag{5}$$

We will consider only the case with initial de Sitter space. The boundary conditions for the bounce are

$$\frac{d\Phi}{d\eta}\Big|_{\eta=0} = 0, \quad \frac{d\Phi}{d\eta}\Big|_{\eta=\eta_{max}} = 0, \quad \rho_{\eta=0} = 0, \quad \text{and } \rho_{\eta=\eta_{max}} = 0, \tag{6}$$

where η_{max} is a finite value in Euclidean de Sitter space.

In this article, we consider only several cases including true, false, and degenerate vacua. Firstly, we consider the large true anti-de Sitter bubble with small false de Sitter background. The size of the bubble and the nucleation rate are evaluated to be

$$\bar{\rho}^2 = \frac{\bar{\rho}_o^2}{D} \quad \text{and} \quad B = \frac{2B_o(\frac{\lambda_2}{\lambda_1})^2[\{1 + (\frac{\lambda_1}{\lambda_2})^2(\frac{\bar{\rho}_o}{2\lambda_2})^2\} - (\frac{\lambda_1}{\lambda_2})^2 D^{1/2} + E]}{[(\frac{\bar{\rho}_o}{2\lambda_2})^4\{(\frac{\lambda_2}{\lambda_1})^4 - 1\}D^{1/2}]}, \tag{7}$$

where $D = \left[1 + 2(\frac{\bar{\rho}_o}{2\lambda_1})^2 + (\frac{\bar{\rho}_o}{2\lambda_2})^4\right]$, $\lambda_1^2 = [3/\kappa(U_F + U_T)]$, $\lambda_2^2 = [3/\kappa(U_F - U_T)]$, $U_T < 0$, and $E = 2\{(\frac{\lambda_2}{\lambda_1})^4 - 1\}(\frac{\bar{\rho}_o}{2\lambda_1})^2[1 - (\frac{\bar{\rho}_o}{2\lambda_2})^2]D^{-1}$.

Secondly, we consider the large false de Sitter bubble with the small true de Sitter background. The size of the bubble has the same as above case. The nucleation rate is given by

$$B = \frac{2B_o[\{1 + (\frac{\bar{\rho}_o}{2\lambda_1})^2\} + \{1 + 2(\frac{\bar{\rho}_o}{2\lambda_1})^2 + (\frac{\bar{\rho}_o}{2\lambda_2})^4\}^{1/2}]}{[(\frac{\bar{\rho}_o}{2\lambda_2})^4\{(\frac{\lambda_2}{\lambda_1})^4 - 1\}\{1 + 2(\frac{\bar{\rho}_o}{2\lambda_1})^2 + (\frac{\bar{\rho}_o}{2\lambda_2})^4\}^{1/2}]}. \tag{8}$$

Thirdly, we consider the transition between degenerate vacua in de Sitter space. The critical radius of the wall is given by

$$\bar{\rho} = \frac{2}{\kappa\sqrt{\frac{S_o^2}{4} + \frac{4}{3\kappa}U_o}}. \tag{9}$$

The nucleation rate is given by

$$B = \frac{12\pi^2 S_o}{\kappa^2 U_o \sqrt{\frac{S_o^2}{4} + \frac{4}{3\kappa}U_o}}. \tag{10}$$

3 Black hole pair creation

We consider several types of the configuration as the background space for black hole pair creation. In this work the general Euclidean junction condition becomes

$$\sqrt{1-\frac{2GM}{r}\pm\frac{\Lambda_-}{3}r^2-\dot{r}^2}+\sqrt{1-\frac{2GM}{r}-\frac{\Lambda_+}{3}r^2-\dot{r}^2}=4\pi G\sigma r, \tag{11}$$

where $S_o=\sigma$ and $\cdot$ denotes the differentiation with respect to the proper time measured by the observer moving along with the wall. The signs $(-)$ and $(+)$ as a subscript of Λ represent left and right spacetime, respectively. After squaring twice, we define the effective potential for Euclidean junction equation to be

$$V_{eff}=\frac{1}{2}-\frac{1}{2}\left[\left(2\pi G\sigma+\frac{(\Lambda_+-\Lambda_-)}{24\pi G\sigma}\right)^2+\frac{\Lambda_-}{3}\right]r^2-\frac{GM}{r}, \tag{12}$$

with total energy is 0. The static bubble wall solution satisfies the following conditions

$$V_{eff}(r_b)=0, \quad \text{and} \quad \frac{dV_{eff}}{dr}\Big|_{r_b}=0. \tag{13}$$

The solution can exist at $r_b=3GM$. The masses of created black holes are uniquely determined by the given cosmological constant and surface tension on the wall:

$$M=\frac{1}{3G\sqrt{3\left[\left(2\pi G\sigma+\frac{(\Lambda_+-\Lambda_-)}{24\pi G\sigma}\right)^2+\frac{\Lambda_-}{3}\right]}}. \tag{14}$$

In this process the location of the wall with black holes is given by

$$r_b=3GM \quad \text{or} \quad r_b=\frac{1}{\sqrt{3\left[\left(2\pi G\sigma+\frac{(\Lambda_+-\Lambda_-)}{24\pi G\sigma}\right)^2+\frac{\Lambda_-}{3}\right]}}. \tag{15}$$

This case is that the left of the wall corresponds to flat space and the right corresponds to de Sitter space. The action turns out to be

$$\begin{aligned}
S_E &= S_E^C-\frac{\sigma}{4}\left(\sqrt{1-\frac{2GM}{r}}\int_0^{\tau_-}d\tau_-+\sqrt{1-\frac{2GM}{r}-\frac{\Lambda_+}{3}r^2}\int_0^{\tau_+}d\tau_+\right)\int_0^{\pi}d\theta\int_0^{2\pi}d\phi r^2\sin\theta \\
&- S_E(bulk)-S_E^B \\
&= S_E^C-\pi\sigma r_b^2\left(\frac{8\pi GM}{\sqrt{3}}+\left(4\pi G\sigma r_b-\frac{1}{\sqrt{3}}\right)\frac{2\pi r_h\sqrt{1-(9\Lambda_+G^2M^2)^{1/3}}}{1-\Lambda_+r_h^2}\right) \\
&- \frac{2\pi U_+}{3}r_b^3\left(\frac{2\pi r_h\sqrt{1-(9\Lambda_+G^2M^2)^{1/3}}}{1-\Lambda_+r_h^2}\right)+\frac{2\pi^2U_+}{\sqrt{3\Lambda_+}}r_{wob}^3 \\
&+ \sigma\pi r_{wob}^2\left(\frac{\pi r}{2}+2\sqrt{\frac{3}{\Lambda_+}}(4\pi G\sigma r-1)\right),
\end{aligned} \tag{16}$$

where $U_+>0$ and we use the Bousso-Hawking normalization for Schwarzschild-de Sitter space.

4 Summary and Discussions

In this paper we classified the possible types of vacuum bubbles and calculated the radius and the nucleation rate. We present analytic computation using the thin-wall approximation. There are nine types of true vacuum bubbles, three false vacuum bubbles, and Hawking-Moss transition in which the thin-wall approximation is not considered. Our results, including the half sized and the large bubble for true vacuum bubbles, false vacuum bubbles, and degenerate case, extend Parke's results. We considered the pair creation of black holes in some of these vacuum bubble backgrounds. These solutions give rise to the background for the black hole pair creation more naturally.

Our solutions, even if it has simple structure, can be used to understand the mechanism how the complicated spacetime structure could be created in the early universe as well as tunneling phenomena occur in the string landscape and eternal inflation.

Acknowledgements

We would like to thank Misao Sasaki for his hospitality at the 18th Workshop on General Relativity and Gravitation in Japan (JGRG18) in Hiroshima. We would like to thank Kei-ichi Maeda for helpful discussions. This work was supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government(MEST) through the Center for Quantum Spacetime(CQUeST) of Sogang University with grant number R11 - 2005 - 021. W.L. was supported by the Korea Research Foundation Grant funded by the Korean Government(MOEHRD)(KRF-2007-355-C00014).

References

[1] B.-H. Lee and W. Lee, arXiv:0809.4907.

[2] S. Coleman, Phys. Rev. D **15**, 2929 (1977); *ibid.* D **16**, 1248(E) (1977).

[3] S. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).

[4] S. Parke, Phys. Lett. B **121**, 313 (1983).

[5] A. D. Linde, Phys. Lett. B **100**, 37 (1981); Nucl. Phys. B **216**, 421 (1983).

[6] A. H. Guth, Phys. Rev. D **23**, 347 (1981); K. Sato, Mon. Not. R. Astron. Soc. **195**, 467 (1981); J. R. Gott, Nature **295**, 304 (1982); J. R. Gott and T. S. Statler, Phys. Lett. B **136**, 157 (1984); M. Bucher, A. S. Goldhaber, and N. Turok, Phys. Rev. D **52**, 3314 (1995).

[7] S. W. Hawking and I. G. Moss, Phys. Lett. B **110**, 35 (1982).

[8] U. Gen and M. Sasaki, Phys. Rev. D **61**, 103508 (2000).

[9] R.-G. Cai, B. Hu, and S. Koh, arXiv:0806.2508.

[10] K. Marvel and D. Wesley, arXiv:0808.3186.

[11] K. Marvel and N. Turok, arXiv:0712.2719.

[12] K. Lee and E. J. Weinberg, Phys. Rev. D **36**, 1088 (1987).

[13] R. Basu, A. H. Guth, and A. Vilenkin, Phys. Rev. D **44**, 340 (1991); J. Garriga, Phys. Rev. D **49**, 6327 (1994).

[14] J. C. Hackworth and E. J. Weinberg, Phys. Rev. D **71**, 044014 (2005).

[15] Y. Kim, K. Maeda, and N. Sakai, Nucl. Phys. B **481**, 453 (1996); Y. Kim, S. J. Lee, K. Maeda, and N. Sakai, Phys. Lett. B **452**, 214 (1999).

[16] R. R. Caldwell, H. A. Chamblin, and G. W. Gibbons, Phys. Rev. D **53**, 7103 (1996).

[17] C. G. Callan and S. Coleman, Phys. Rev. D **16**, 1762 (1977); E. J. Weinberg, Phys. Rev. D **47**, 4614 (1993); G. V. Dunne and H. Min, Phys. Rev. D **72**, 125004 (2005).

[18] D. Metaxas, arXiv:0804.1045.

[19] S. Coleman, V. Glaser, and A. Martin, Commun. Math. Phys. **58**, 211 (1978).