Fermi National Accelerator Laboratory

FERMILAB-Pub-87/30-A
February 1987

COSMIC STRING INDUCED PECULIAR VELOCITIES

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Abstract

We calculate analytically the probability distribution for peculiar velocities on scales from $10h^{-1}$ to $60h^{-1}$Mpc with cosmic string loops as the dominant source of primordial gravitational perturbations. We consider a range in the of parameters $\beta G\mu$ appropriate for both hot (HDM) and cold (CDM) dark matter scenarios. An $\Omega = 1$ CDM Universe is assumed with the loops randomly placed on a smooth background. It is shown how the effects can be estimated of loops breaking up and being born with a spectrum of sizes. It is found that to obtain large scale streaming velocities of at least 400 km/s it is necessary that either a large value for $\beta G\mu$ or the effect of loop fissioning and production details be considerable. Specifically, for optimal CDM string parameters $G\mu = 10^{-6}$, $\beta = 9$, $h = .5$, and scales of $60h^{-1}$ Mpc, the parent size spectrum must be 36 times larger than the evolved daughter spectrum to achieve peculiar velocities of at least 400 km/s with a probability of 63%. With this scenario the microwave background dipole will be less than 800 km/s with only a 10% probability. The string induced velocity spectrum is relatively flat out to scales of about $2t_{eq}/a_{eq}$ and then drops off rather quickly. The flatness is a signature of string models of galaxy formation. With HDM a larger value of $\beta G\mu$ is necessary for galaxy formation since accretion on small scales starts later. Hence, with HDM, the peculiar velocity spectrum will be larger on large scales and the flat region will extend to larger scales. If large scale peculiar velocities greater than 400 km/s are real then it is concluded that strings plus CDM have difficulties. The advantages of strings plus HDM in this regard will be explored in greater detail in a later paper.

1. Introduction

It has been suggested that topologically stable vortex lines, "cosmic strings," may have formed at a phase transition in the very early Universe[1]. In another area of cosmology is the fact that a satisfactory theory for the origin of galaxies has yet to be fully developed. Two basic, yet still very open issues are the origin and nature of the initial density fluctuations and the generation of large scale structure. Inspired by the possibility of a simple, origin-issue free solution to these problems, various people have been trying to generate a consistent scenario with cosmic strings[2-5]. To date a fatal flaw has yet to be found, although many issues are still open. In particuliar, do strings work better with hot (HDM) or cold (CDM) dark matter? This issue we will exlore vis-a-vis the question of large scale streaming velocities in this and a succeeding paper.

The basic string scenario we will follow is that after cosmic strings formed they straightened out and stretched. They also would occassionally hit each other and themselves. Albrect and Turok[6] have shown that it is necessary that such intersecting strings intercommute with a probability of at least a few tenths in order for the strings not to dominate the mass density of the Universe. Recent work suggests that such a probability is likely[7]. The picture we have then is a network of strings with dimensionless string tension $G\mu$ (we assume c=1 unless, other wise stated) with curvature on the order of the horizon in size and in dynamic equilibrium with loops being formed/reabsorbed from/onto each other and infinite strings. However, as the Universe expands so does the curvature of the strings. Loops near the horizon in size will frequently become subhorizon in size before being reabsorbed. This happens either through the simple expansion of the horizon or loop fissioning. Thus relative to the superhorizon strings the subhorizon loops shrink away, and it becomes increasingly less probable for them to reconnect. The division between these two stages in a loop's life we define to be when $t = R/b$, with R the size of the loop and b some constant of the order of $horizon/t$. These cast-off loops are hypothosized to be the seeds for galaxies and clusters[4]. Once subhorizon, the loops slowly gravitationally radiate away with a lifetime of about[9] $\gamma R/G\mu$, with $\gamma \approx 0.1$. While the details are still weak it seems this basic theory can easily account for the mass and number of galaxies as well as satisfy the requirement of a small microwave background anisotropy[10]. String theory also appears to be the only theory to date that naturally accounts[8] for the apparent scale free[14,15] nature of the cluster-cluster and galaxy-galaxy correlation functions.

A measurement that has recently gained some notoriety is the bulk motion of large regions ($\approx 50h^{-1}$Mpc in radius). While difficult to measure accurately, its lack of dependance on the assumption of light tracing matter makes it potentially a very useful discriminator of structure formation theories. Table 1 lists several measurements of our peculiar velocity relative to distant spheres and shells of galaxies around us. Presently there is still strong disagreement among the various groups but superior statistics are expected in the near future.

In anticipation of a clearer answer to this issue, Vittorio and Turner[11] have calculated the expected peculiar velocities on different scales in flat models of the Universe with various forms of dark matter assuming adiabatic gaussian primordial density fluctuations. In this paper we consider the scenario of a flat Universe with a network of heavy cosmic strings (produced at $kT \approx 10^{16}GeV$) as the primordial fluctuation spectrum (strings yield *non*-guassian primordial fluctuations[28]). For the bulk of this paper we will carry out calculations with a uniform background of CDM; however, we will also comment on the direction of differences for HDM scenarios. A later paper will look at the HDM scenario in full detail. It should be pointed out that on scales larger than the HDM Jeans radius the particles are effectively cold. Hence on scales larger than this the results discussed here still apply.

In the next section the effect of an isolated loop in a CDM background is discussed. The quantity we will be trying to get at is the number of times that, either isolated or conspiring in groups, loops would induce a peculiar velocity of at least v_p. Later this quantity will be used to determine the probability of inducing a given peculiar velocity.

In section 3 we discuss the effect of small loops which, when the individual contributions are added together randomly, add up to at least v_p.

In section 4 the loop density functions are defined, and in section 5 the effect of loops breaking up is discussed. The results are presented and discussed in section 6 and conclusions made in section 7.

2. Peculiar Velocities Induced by Isolated Loops

The perturbations in this scenario will be very small and isolated, hence one can use a simple spherical accretion model for the growth of perturbations around the loops. The picture we use here is that we are "falling" into every loop around us, but since our infall is still linear on large scales we can simply add up all of the contributions like steps in a random walk, $v_{pec}^2 = \sum v_i^2$, where v_i is the peculiar velocity induced by loop i.

We of course do not know the distribution of these loops, so what we actually calculate is the probability of inducing a peculiar velocity greater than some value v_p using an assumed loop density function.

The trajectory for a negative energy shell is given by

$$\dot{r}^2 = \frac{2GM}{r} - \frac{2GM}{r_i}$$

where r_i is the initial distance from the loop and M is the mass enclosed within radius r, and it is assumed that shells never cross. To first order in the perturbing mass this becomes

$$\dot{r} = \frac{2}{3}\frac{r}{t}\left(1 - \frac{1}{5}\Delta_i\left(\frac{t}{t_i}\right)^{2/3}\right)$$

with $\Delta_i = M_{loop}(\frac{4\pi}{3}\rho_b^{init} r_{init}^3)^{-1}$. Defining $\mu\beta R$ as the mass of the loop we have

$$v_p = \frac{3}{5}\beta G\mu R\frac{t_{now}}{rr_i}$$

where to first order $r_i = a_i r$. β relates the perimeter of the loop to its mean radius. The present value generally assumed[6] is $\beta = 9$, although this is not well established. In structure formation calculations $\beta G\mu$ always occur together so for simplicity we will usually treat them as one parameter in this paper.

Defining $D(R)$ as the present number density of R size loops we have then for the mean number of individual loops that induce a peculiar velocity of at least v_p:

$$N(v_p) = \int\int D(R)\theta(R - \frac{v_p a_i r^2}{\frac{3}{5}\beta G\mu t_{now}})4\pi r^2\, dr\, dR$$

For each case considered, the integral over R was evaluated then the r integral was performed numerically. This was found to be the best approach since while the integrals are trivial, the step functions breed new stepfunctions. In the end the final closed form solution is a rather unenlightening three pages long.

3. Induced Perturbation from Aggregates of Loops

The number of v to $v + dv$ loops is given by $-dN(v)$. Adding together these loops like steps in a random walk or random errors one finds

$$v^2_{induced} = \int_0^{v_p} -dN(v)v^2$$

The number of times v_p is induced is then

$$N(< v_p) = (\frac{v_{induced}}{v_p})^2$$

One might suspect a square root should be taken somewhere here. To see why this is not so suppose the above integral resulted in $v^2_{induced} = 4v_p^2$. Then we have 4 distinct collections of loops that induce v_p, not 2.

4. Distribution Functions Used

a) Loops Born Outside Our Horizon and Remain Outside Until $t > t_{eq}$

Two density functions are required since loops formed before t_{eq} do some of their expanding away during the radiation dominated era. Hence we have

$$D(R) = \frac{\omega}{R^4}\left(\frac{R}{bt_{eq}}\right)^{3/2} a^3_{eq} \qquad R < bt_{eq}$$

$$D(R) = \frac{\omega}{R^4}\left(\frac{R}{b't_{now}}\right)^2 \qquad R > bt_{eq}$$

In this model ω is a universal constant proportional to the number of loops formed per horizon volume, per horizon time. Albrect and Turok[6] found this form of the density function to work quite well for their simulations in the radiation dominated era. They only considered loops born before t_{eq} and thus could only fit the amplitude of the $R < bt_{eq}$ function: $\omega/b^{3/2} = \nu \approx .01$. It seems reasonable to introduce the extra parameters b and b' since the horizon scales differently after t_{eq} than it does before.

One must be careful with some of the limits of integration. The r limits are simply r_{min} to $3t_{now}$, where r_{min} is the size of the sphere being coherrently accelerated. R ranges from effectively zero to bt_{eq} for loops born before t_{eq}. For loops born afterwards things are more complicated since only loops less than $b't_i$ (t_i being the time when r_{min} was the horizon size) exist when a particular horizon is being crossed. Using $t_i = t_{now}a_i^{3/2} = a_i r/3$ one then gets the R limits to be bt_{eq} to $b't_{now}(\frac{r}{3t_{now}})^3$.

Finally we must also define a_i, the scale factor when we are first perturbed by the loop. Here a_i is set when the loop first enters the horizon: t_i. The horizon then we define as $b_H t_i$. Hence $a_i = b_H t_i / r$. Recall that r is always our present distance from the loop in question. Then $a_i = t_{now} a_i^{3/2} b_H / r$ or $a_i = (r/b_H/t_{now})^2$. b_H unfortunately varies from about 2 before t_{eq} to 3 well after. Since most of the space that will be integrated over will have $b_H \approx 3$ we assume it to be 3 for all $t > t_{eq}$.

b) Loops Born Before t_{eq} With Centers in Our Horizon Before t_{eq}

Here the density function is given by

$$D(R) = \frac{\omega}{R^4}\left(\frac{R}{bt_{eq}}\right)^{3/2} a_{eq}^3 = \frac{\omega a_{eq}^3}{(bt_{eq})^{3/2}} R^{-5/2}$$

and $a_i = a_{eq}$ since all perturbations start to grow at t_{eq} here.

The R limits are as above for loops born before t_{eq}. The r limits are r_{min} to $2.12t_{eq}$, the horizon at t_{eq}.

A complication here and in the last catagory are cases where the loop is "born" around us. To correct for this we first limit the above case to where $R < a_{eq} r$, i.e. only consider loops that we are outside of by t_{eq}. For loops with $R > a_{eq} r$ we set the time of accreation to be when $r_i = R$, i.e., just as we pass outside the loop. For this integral $a_i = R/r$.

c) Loops Born After t_{eq} With Centers Inside Our Horizon

The density function is as in the $R > bt_{eq}$ case of **a**):

$$D(R) = \omega(b' t_{now} R)^{-2}$$

However, in this catagory perterbations start to grow as soon as the loops are born, hence $a_i = (R/b'/t_{now})^{2/3}$, with the condition $R < a_i r$. For $R > a_i r$, then $a_i = R/r$, as in b).

The limits in this case are r_{min} to $3.t_{now}$ for r and bt_{eq} to $b' t_{now}$ for R.

Combining all of these cases gives the mean number of times a v_p perturbation is generated. The relation we will use is the inverse of this: for a given N we determine the velocity that is induced that many times. Choosing $N = 1$ and assuming the number of v_p perturbations is Poisson distributed, the probability of one or more v_p perturbations is $1 - e^{-1} = 63\%$. That is, the peculiar velocity on a given scale are at least v_p 63% of the time.

5. Effect of Loop Fissioning

There is substantial indication that once formed a loop breaks up into a number of daughters[6,13]. One can show that independent of how the loops are formed and or broken up, the spectrum remains unchanged up to a constant factor and up to loop sizes not too close to the largest parents[16]. The only assumption is that the mass of subhorizon loops generated per unit time per horizon volume goes as t^{-3}.

Albrect and Turok calculated the evolved spectrum in their simulations. However, for the calculation of peculiar velocities we do not care if the loops fission since we only need the gravitational potential outside the parent loop. As long as the loop does not shatter into pieces that fly apart so quickly that they envelop us, what a parent loop does after it forms is irrelevant for this calculation. This seems likely to be what generally happens, since for us to be "enveloped" would require us at one point to be close enough to the center of a parent loop such that it would take more than a few Hubble times (the time it would take for a loop to fission significantly) for us to get away. This seems especially unlikely in view of recent evidence that parent loops are born with sizes on the order of a fifth of the horizon size[13]. Thus to take into account loop fissioning all one has to do is "de-evolve" Albrect and Turok's coefficient by multipling it by some breakup factor, B_f.

In this paper the effect of parent loops being born with a spectrum different from a delta function is not considered, but as will be discussed in the next section, one can see that the effect can be well approximated with another modification of the naïve spectrum constant.

6. Results

There are four parameters that we are at present free to adjust: h (Hubble's constant divided by 100 km/s), $\beta G\mu$, f (the fraction of the horizon represented by b and b'), and B_f. Figures 2 through 5 show the effect of these parameters on the predicted peculiar velocities as a function of scale. The values chosen for the parameters are largely for display purposes. As will be discussed bellow, the effect of variation in the parameters can be estimated quite accurately. For future reference, the estimated values for $G\mu$[29] is 10^{-6} for CDM and 3×10^{-6} for HDM, assuming $\beta = 9$. These values enable galaxy mass objects to form by redshifts of $1 + z \approx 10$. The higher $G\mu$ for HDM is required because growth on galaxy scales doesn't occur until later in such models.

As expected $\beta G\mu$ has a linear effect, while B_f increased velocities by about $B_f^{\frac{1}{2}}$ (recall that increasing the number of sources of v_p by 4 increases the total induced peculiar velocity by only 2 when the contributions are added randomly). Unexpectedly, f has a quite uniform effect as well. One might expect reducing the maximum size of loops produced at any given time by a factor of 5 would have a similar effect on the velocities induced. However, in our spectrum we only demand that the radiation era spectrum coefficients satisfy $B_f^{-1}\omega/b^{3/2} = .01$, but the corresponding coefficient in the matter dominated era has a $1/b'^2$ dependance. Hence decreasing f by 5 *increases* the latter coefficient by $\sqrt{5}$. Further, on small scales, where the peculiar velocity is dominated by pre-t_{eq} loops the effect of reducing f is simply to cut off the high end of the loop spectrum, where the number density is very low. We thus get a less than expected effect because rather than reducing all of the loops by a factor f, we only loose a few big loops. The effect is not as big since beyond a certain size all loops get only one vote and those are very sparce.

The fact that these two effects of f on the peculiar velocities conspire to give a small and fairly uniform vertical shrinking of the curve is very fortunate for the following reason. One way to take into account a more realistic parent spectrum would be to divide it into narrow intervals, calculate the number of v_p or better perturbations induced by that interval of the spectrum and sum the results. However, over the entire range of scales each term in the sum will be related to the ones we have calculated in this paper times some factor. Hence the effect of using a delta function spectrum is again just an overall factor that can be determined and the effect negated once the real spectrum is known.

The Hubble constant is the only parameter whose variation produces dramatic changes in the peculiar velocity as a function of r. The plateau feature at small scales is a reflection of the fact that loops born before t_{eq} do not get to contribute until after t_{eq}. Further, the most significant part of a loop's perturbing effect occurs just after it is born or enters the horizon. Soon after that it is Hubble flowed away and bigger loops are formed. Consequently the dominant effect on scales smaller than $2t_{eq}/a_{eq}$ are loops formed outside that scale. There is an h^{-3} dependance to $2t_{eq}/a_{eq}$ thus larger plateaus occur for smaller h. The amplitude is smaller for smaller h since growth of perturbations starts latter. This plateau feature is a signature of string induced peculiar velocities and does not occur in gaussian density fluctuation scenarios. If our peculiar velocity relative to distant galaxies really does show little variation with scale out to some large distance this might be a strong argument in favor of strings.

7. Conclusions

It is perhaps worth pointing out here that there is no inconsistency between the large peculiar velocities one finds on small scales in a calculation such as this and the small relative peculiar velocities observed for nearby galaxies. For some reason some people have connected these two numbers when in fact they are almost completely unrelated. The former involves the integrated effect of all perturbing masses beyond some scale, while the latter involves a complicated summing of effects on scales less than or on the order of the separation of the galaxies. From the flatness of our peculiar velocity spectra one concludes that few significant perturbations occur near pairs of nearby galaxies. One would thus predict relatively small relative peculiar velocities.

The calculation made here is admittedly simple-minded; however, as has been shown, two of the most significant issues, the break-up of loops and the initial spectrum of parent loops, can easily be accounted for once their true nature is known. Other approximations, such as turning on the Newtonian potential of the perturbing loops as soon as they are created or cross the horizon, the transition through t_{eq}, and ignoring the skewing effect produced by perturbations accelerating one side of the sphere around us more than the other, are expected to change things by factors of order unity. The errors introduced by not knowing B_f or f make consideration of these other effects rather pointless. It must be remembered that the numbers quoted here are not intended to form the basis of some experiment. Rather they are simply intended to indicate the reasonableness, or lack of it, of the cosmic string theory of galaxy formation in the context of large scale steaming velocities, and perhaps give some sort of feel for where the velocities are big and where they are trivial. The quantity we are trying to estimate here, the likelihood of particular streaming velocities, may not be measured accuately for a long time, if ever. Single data points for those spheres that surround us are still plagued with errors of order the size of the data (except for that sphere that only includes us). With these limitations kept in mind we are left with the following conclusions:

1) With CDM and h much larger than .5, situations with 600 km/s streaming velocities on large and small scales become very rare: adjusting the parameters to get the large scales reasonable produces far too high velocities on smaller scales which presumably should be similar to that of the dipole anisotropy corrected for local motions.

2) To have streaming velocities on scales of $50h^{-1}$Mpc of at least 400 km/s with 63 % confidence, CDM, $h = .5$, and $f = .2$ requires $(\beta G\mu/9 \times 10^{-6})B_f^{\frac{1}{2}} \approx 6$. This would also produce velocities on small scales of about $600\sqrt{4}$ km/s with the same confidence, i.e., about 4 perturbations of 600 km/s or greater. Considered another way, the probability of having a peculiar velocity on small scales of less than 800 km/s would be 10%.

3) With hot dark matter the constraint on h would open up somewhat since non-relativistic matter domination would not come in until much later, hence the plateau would extend much further and lie lower. One would expect the constraint on $\beta G\mu B_f^{\frac{1}{2}}$ to still be necessary to give large scales large peculiar velocities. However, on small scales peculiar velocities of about 600 km/s should be possible with a more comfortable probability. Since a larger $\beta G\mu$ is required for galaxy formation in a HDM Universe[17] this would mean a smaller value for B_f. Better knowledge of the way loops form and break-up so that B_f could be calculated would add considerably to the constraints imposed by streaming velocities.

In summary, CDM and strings appear to have difficulties in producing large scale high peculiar velocities of the order reported by Burstein, Davies, Dressler, Faber, Lynden-Bell, Terlevich, and Wegner[26] with a high probability. However, the larger values of $\beta G\mu$ associated with HDM and strings could yield much more favorable results, assuming the correctness of large large scale streaming velocities. In addition the later accretion times would yield less extreme peculiar velocities on smaller scales and a flat spectrum out to higher distance scales, for a given value of h.

Thus HDM plus strings should be able to fit the Burstein et al. measurements as well as the microwave background with high probability. To our knowledge it is the only non-explosive $\Omega = 1$ theory to be able to do so. Thus we will explore this model in more detail in a later paper.

Acknowledgements

We wish to acknowledge useful discussions with N. Turok, A. Albrecht and D.P. Bennett. This research was supported by NSF and DOE grants at the University of Chicago, by the NASA/Fermilab Astrophysics Group and by A. van Dalen's fellowship from the Natural Sciences and Engineering Research Council of Canada.

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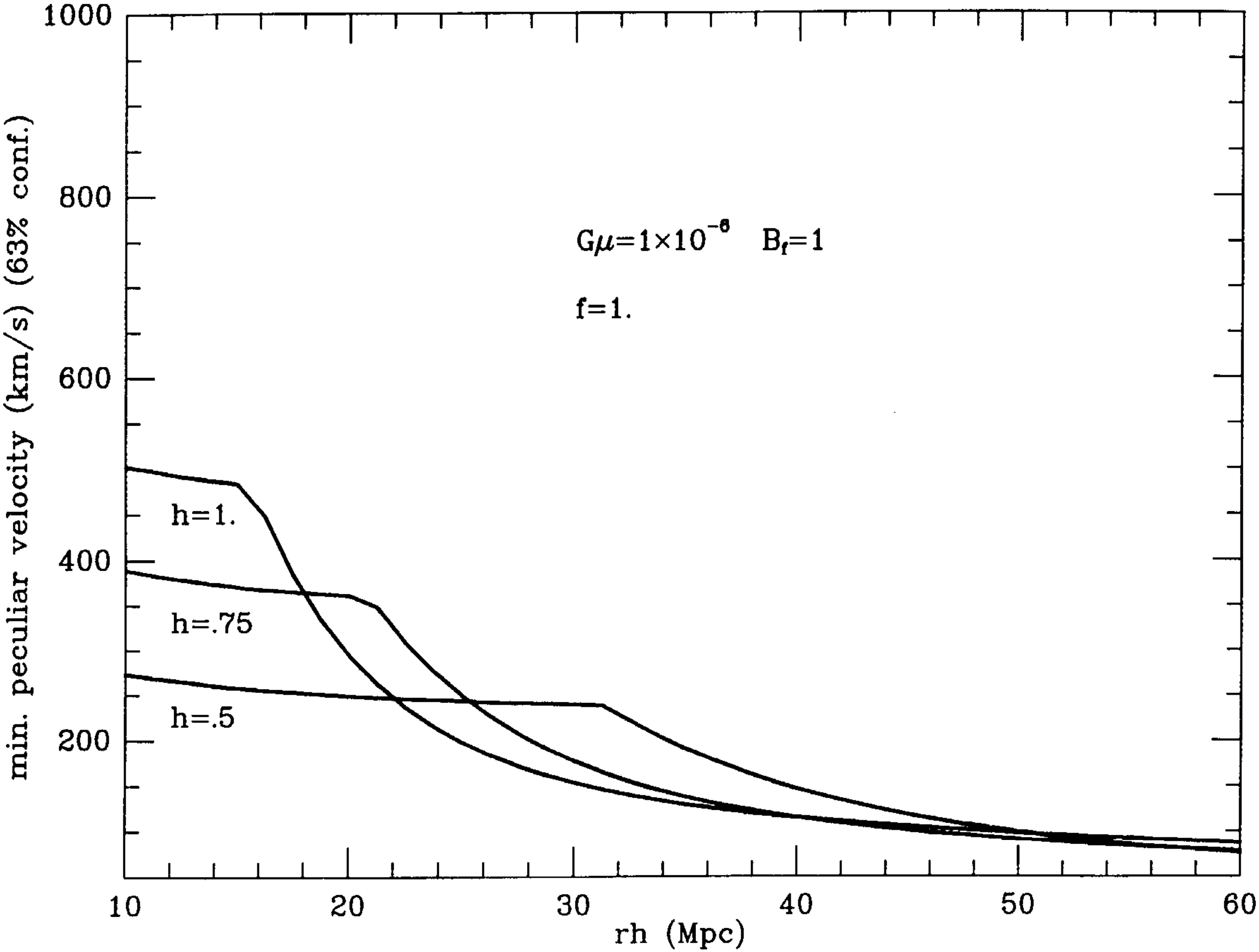
Table 1

Paper	Distance (Mpc)	Direction l°/b°	Velocity (km/s)	Agree with CBM?
Dipole CBM[18,19]	–	265/35	600 ± 50	–
Rubin et al.[20,21]	$35 - 65h^{-1}$	163/-11	454 ± 127	no
Hart and Davies[22]	$< 23h^{-1} >$	264/45	436 ± 55	yes
Different an. of same data		216/27	397 ± 90	close
de Vancouleurs and Peters[20]	$.65 - 36h^{-1}$	approaches CBM	160 to 790	yes on scales $> 35h^{-1}$
Colins et al.[24]	$35 - 65h^{-1}$	$186 \pm 30/-3 \pm 29$	621 ± 300	$\approx$ Rubin et al.
(various analysises with		$214 \pm 40/-33 \pm 24$	846 ± 334	"
parts of Rubin et al.		$184 \pm 35/-36 \pm 30$	680 ± 330	"
galaxies)		$190 \pm 20/-6 \pm 18$	662 ± 220	"
		$202 \pm 19/-11 \pm 17$	586 ± 200	"
Aaronson et al.(1986)[25]	$40 - 100h^{-1}$	$255 \pm 17/18 \pm 13$	780 ± 188	yes
Burstein et al.[26]	$< 60h^{-1}$	170/35	458	no
(same range of declination as Aaronson et al.(1986))	same	206/27	655	close
(re-analysis of Aaronson et al.(1982)[27])	$< 30h^{-1}$	228/42	282	no

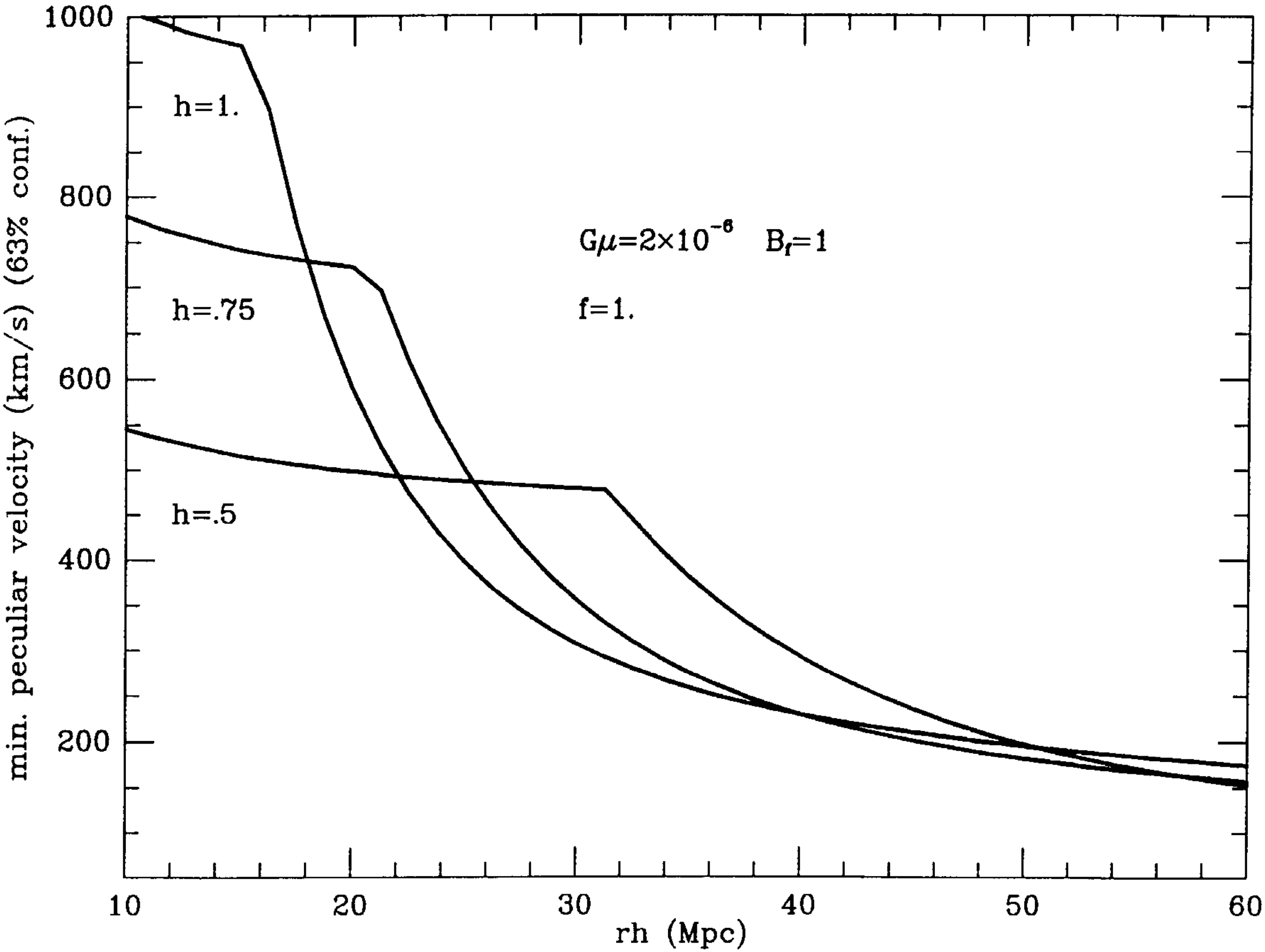
Figure Captions

1. Minimum expected peculiar velocities vs scale: $G\mu = 1 \times 10^{-6}$, $B_f = 1.$, and $f = 1.$
2. Minimum expected peculiar velocities vs scale: $G\mu = 2 \times 10^{-6}$, $B_f = 1.$, and $f = 1.$
3. Minimum expected peculiar velocities vs scale: $G\mu = 2 \times 10^{-6}$, $B_f = 1.$, and $f = .2$
4. Minimum expected peculiar velocities vs scale: $G\mu = 2 \times 10^{-6}$, $B_f = 3.$, and $f = .2$



min. peculiar velocity (km/s) (63% conf.)
1000
800
600
400
200
10
20
30
40
50
60
rh (Mpc)
Gμ=1×10^-6 B_f=1
f=1.
h=1.
h=.75
h=.5

Figure 1.



1000
800
600
400
200
min. peculiar velocity (km/s) (63% conf.)
h=1.
h=.75
h=.5
$G\mu=2\times10^{-6}$ $B_f=1$
f=1.
10
20
30
40
50
60
rh (Mpc)

Figure 2.

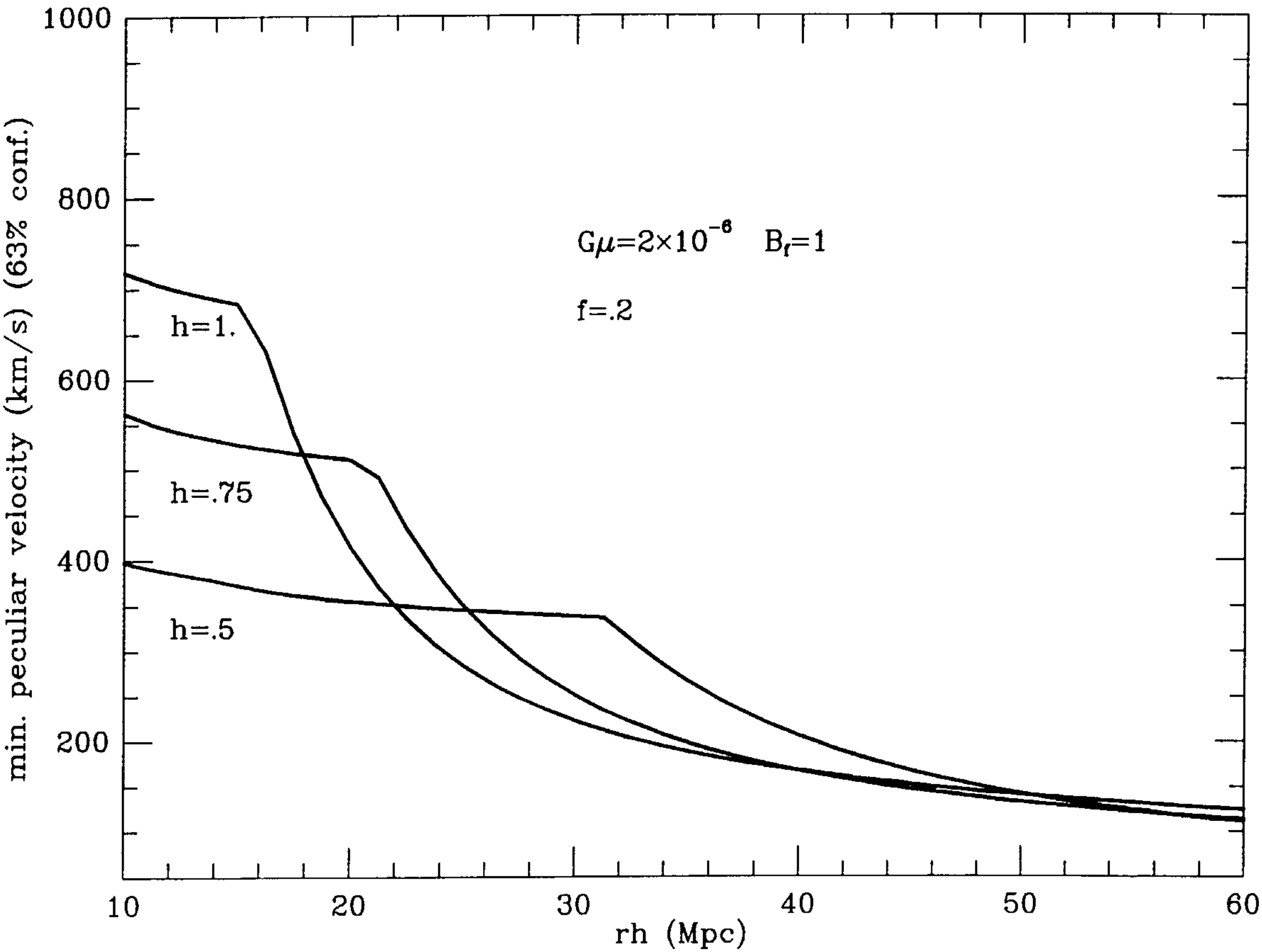
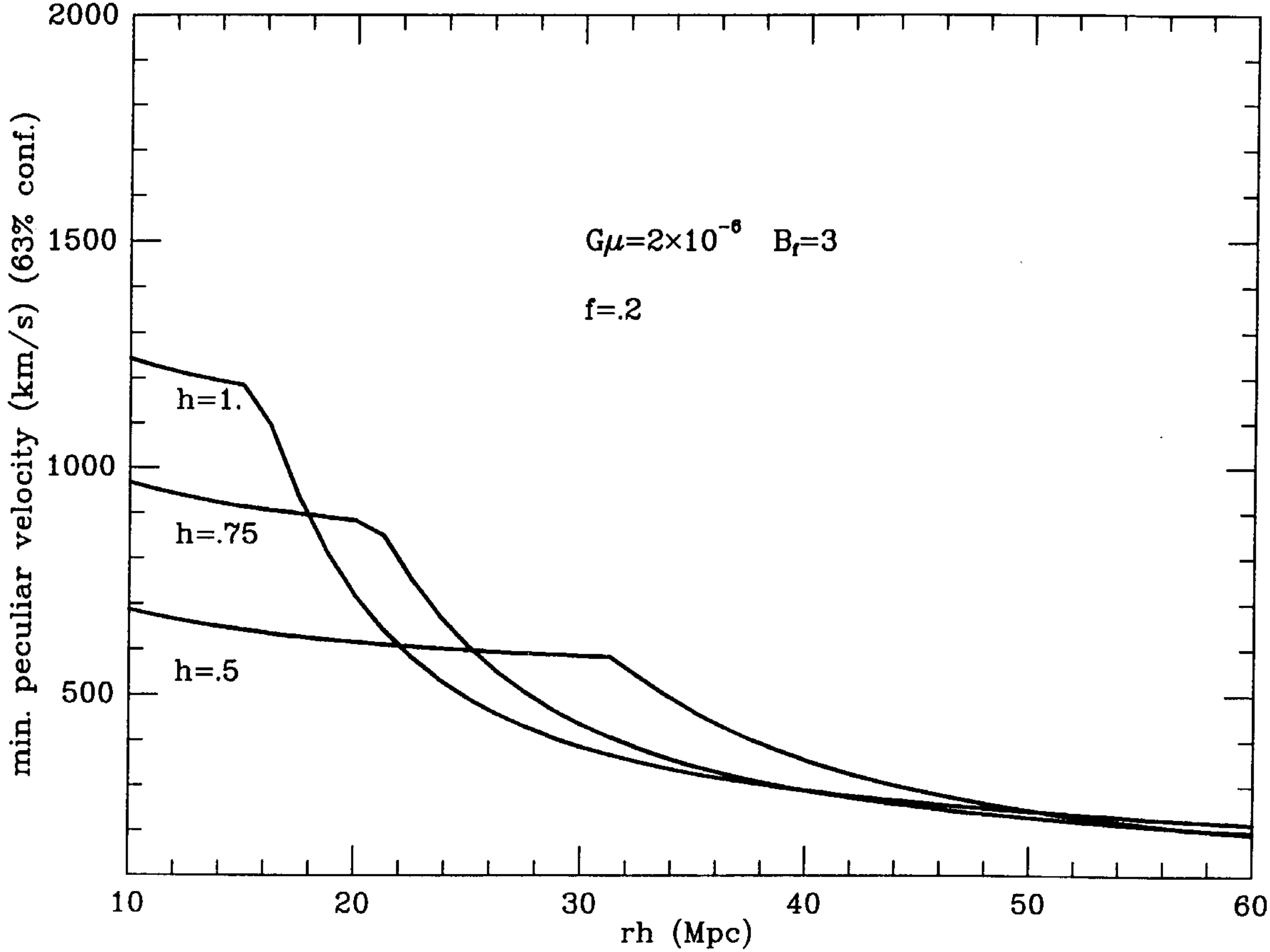

min. peculiar velocity (km/s) (63% conf.)
1000
800
600
400
200
10
20
30
40
50
60
rh (Mpc)
$G\mu=2\times10^{-6}$ $B_f=1$
f=.2
h=1.
h=.75
h=.5

Figure 3.



2000
1500
1000
500
min. peculiar velocity (km/s) (63% conf.)
$G\mu=2\times10^{-6}$ $B_f=3$
f=.2
h=1.
h=.75
h=.5
10
20
30
40
50
60
rh (Mpc)

Figure 4.