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Comment

Comment on Eur. Phys. J. C 77, 412 (2017) and Eur. Phys. J. C 81, 213 (2021)

W. E. V. Barker[1,2,a], **M. P. Hobson**[1,b], **A. N. Lasenby**[1,2,c]

[1] Astrophysics Group, Cavendish Laboratory, JJ Thomson Avenue, Cambridge CB3 0HE, UK
[2] Kavli Institute for Cosmology, Madingley Road, Cambridge CB3 0HA, UK

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1 Introduction

Certain papers [1–9] assert that overlooked effects are non-linearly implied by general relativity (GR) and may account for e.g. flat/rising galactic rotation curves without dark matter. If the gravitoelectric flux lines collapse under their own 'weight', regions of appreciably rarefied and enhanced force will supposedly appear: we use the broad term '*gravitoelectric flux collapse*' (GEFC) to refer to this claimed effect, and to flag the associated literature (e.g. '[1]', etc.). This comment *refutes* the proposed GEFC programme, by responding to [4,7].[1]

2 The matter coupling

The foundation of the GEFC approach is an expansion of the Einstein–Hilbert Lagrangian in the field $\varphi_{\mu\nu} \equiv M_{\text{Pl}} h_{\mu\nu}$, proposed in [4] as

$$\mathcal{L}_T = [\partial\varphi\partial\varphi] + \sqrt{2} M_{\text{Pl}}{}^{-1} [\varphi\partial\varphi\partial\varphi] + 2 M_{\text{Pl}}{}^{-2} \left[\varphi^2\partial\varphi\partial\varphi\right] - \sqrt{2} M_{\text{Pl}}{}^{-1} \varphi_{\mu\nu} \bar{T}^{\mu\nu} - M_{\text{Pl}}{}^{-2} \varphi_{\mu\nu}\varphi_{\lambda\sigma} \bar{T}^{\mu\nu}\eta^{\sigma\lambda} + \cdots , \tag{1}$$

where the notation $[\cdot]$ suppresses indices and $\bar{T}^{\mu\nu}$ is the background stress-energy tensor. More or less equivalent series to (1) are proposed in [1,3,5,6,8]. The lowest order terms in (1) are of course the massless Fierz–Pauli theory, coupled to a matter current. At higher perturbative orders we are concerned by the matter coupling, since a formal expansion [12,13] near the background $\bar{g}_{\mu\nu} (= \eta_{\mu\nu})$ is

$$S_M \equiv - \sum_{n=0}^{\infty} \frac{1}{n!} \left[\int \mathrm{d}^4x \frac{\varphi^{\mu\nu}}{M_{\text{Pl}}} \frac{\delta}{\delta \bar{g}^{\mu\nu}} \right]^n \bar{S}_M = \bar{S}_M - \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int \mathrm{d}^4x \frac{\varphi^{\mu\nu}}{M_{\text{Pl}}} \frac{\delta}{\delta \bar{g}^{\mu\nu}} \right]^{n-1} \int \mathrm{d}^4x \frac{\sqrt{-\bar{g}}\varphi^{\rho\sigma}}{2M_{\text{Pl}}} \bar{T}_{\rho\sigma} . \tag{2}$$

Even assuming (as sometimes applies) that $\mathcal{L}_M$ contains no derivatives of $g_{\mu\nu}$, it would then seem from Eq. (2) that (1) would require the radically restrictive matter condition

$$\left[\frac{\partial}{\partial g^{\mu\nu}}\right]^n \left(\sqrt{-g} T_{\rho\sigma}\right) = {}^{(n)}X_{\dots\rho\sigma}{}^{\kappa\lambda} T_{\kappa\lambda} , \tag{3}$$

where ${}^{(n)}X_{\dots\rho\sigma}{}^{\kappa\lambda}$ depends only on $g_{\mu\nu}$.

3 The non-relativistic scalar

Next, it is argued in [1,3,4,6,8] that for static spacetimes, the gravitational field may be represented by the *single* mode

$$\varphi^{\mu\nu} = 2\left(2\bar{u}^\mu\bar{u}^\nu - \bar{g}^{\mu\nu}\right)\varphi, \tag{4}$$

for $\bar{u}^\mu\bar{u}_\mu \equiv 1$, which in the static, perturbative context is the Newtonian scalar potential $2M_{\text{Pl}}\nabla^2\varphi \approx \rho$. However it is made clear in [4] that φ also dominates in some non-perturbative static regime, so we are effectively being invited to promote (4) to an isotropic Cartesian line element

$$\mathrm{d}s^2 = (1 + 2\varphi/M_{\text{Pl}})\,\mathrm{d}t^2 - (1 - 2\varphi/M_{\text{Pl}})\,\mathrm{d}\boldsymbol{x}^2, \tag{5}$$

assuming it is signature-preserving. The ansatz (4) is apparently *substituted directly into* (1), and the static assumption $\dot{\varphi} \equiv \partial_t\varphi = 0$ imposed to obtain a scalar Euclidean lattice action up to the required perturbative order [1]. But if the relevant solutions are indeed non-perturbative, why truncate (1)

[1] We note a certain parallel with a previous attempt to explain rotation curves using purely GR effects, by Cooperstock and Tieu [10] – that model was cogently shown to be non-viable by Korzyński [11].

[a] e-mail: wb263@cam.ac.uk (corresponding author)
[b] e-mail: mph@mrao.cam.ac.uk
[c] e-mail: a.n.lasenby@mrao.cam.ac.uk

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at all? By substituting the line element (5) *directly into* the fully nonlinear $\mathcal{L}_G$ we find that the lattice calculations in [4] are really attempting to probe the non-relativistic theory

$$\mathcal{L}_G \equiv -\frac{3\,(1-2\varphi/M_{\rm Pl})\,\dot{\varphi}^2 + (1-6\varphi/M_{\rm Pl})\,|\nabla\varphi|^2}{(1-2\varphi/M_{\rm Pl})\,\sqrt{1-4\varphi^2/M_{\rm Pl}{}^2}}. \tag{6}$$

The theory (6) entails only one vacuum equation of motion $c_j q_j = 0$, where $[q_j] \equiv \left(\ddot{\varphi}, \dot{\varphi}^2/\varphi, \nabla^2\varphi, |\nabla\varphi|^2/\varphi\right)$ and c_j is a rational function in $\varphi/M_{\rm Pl}$ whose components are

$$c_1 \equiv 6\,(1-2\varphi/M_{\rm Pl})\left(1-4\varphi^2/M_{\rm Pl}{}^2\right), \tag{7a}$$

$$c_2 \equiv 12\,(\varphi/M_{\rm Pl})^2\,(1-2\varphi/M_{\rm Pl}), \tag{7b}$$

$$c_3 \equiv 2\,(1-6\varphi/M_{\rm Pl})\left(1-4\varphi^2/M_{\rm Pl}{}^2\right), \tag{7c}$$

$$c_4 \equiv 4\,(\varphi/M_{\rm Pl})\left(1+\varphi/M_{\rm Pl}+6\varphi^2/M_{\rm Pl}{}^2\right). \tag{7d}$$

However, once a gauge such as (5) is chosen the Einstein field equations (EFEs) can impose up to *six* such equations, in addition to *four* constraints on the initial data: these had better all be consistent with $c_j q_j = 0$, otherwise we will no longer be studying gravity in any regime whatever. Taking for example the line element (5) substituted into $G^\mu_\mu = 0$ and accompanying constraint $G^{tt} = 0$, we obtain the system $c_{aj} q_j = 0$, where c_{aj} is a 2×4 matrix which can be diagonalised over the first 2×2 block. Now if we assume staticity, so $\dot{\varphi} = \ddot{\varphi} = 0$, we need retain only the second 2×2 block c_{ab}, writing $c_{ab} q_b = 0$ where $[q_b] \equiv \left(\nabla^2\varphi, |\nabla\varphi|^2/\varphi\right)$. However this yields [12]

$$\det c_{ab} \propto \frac{(\varphi/M_{\rm Pl})\,(7+6\varphi/M_{\rm Pl})}{(1-2\varphi/M_{\rm Pl})\left(1-4\varphi^2/M_{\rm Pl}{}^2\right)^3}, \tag{8}$$

so the EFEs do not actually admit any nontrivial static solutions under the GEFC ansatz. Thus (5) is too restrictive for nonlinear gravity, and this is not surprising. The field φ essentially corresponds to the principal PPN potential, which *cannot be considered in isolation at any PN order* [14]. Note that $\det c_{ab} \to 0$ as $\varphi/M_{\rm Pl} \to 0$. In this limit we recover the only link between the GEFC scalar and gravity, namely the static vacuum condition $\nabla^2\varphi = 0$. The discrepancy occurs because the GEFC ansatz is substituted *before* variations (or lattice path integrals). These are in general non-commuting operations.

4 Intragalactic lensing

Having considered the foundational GEFC scalar graviton of [1,3–6,8] we are ready to examine the main claims of [7], viz that the lensing of light rays in galaxies shows how gravitational field lines are distorted so that the (cylindrical) radial gravitational force near the edge of the galaxy declines like $1/R$ rather than the Newtonian $1/R^2$. We will use a Miyamoto–Nagai (MN) profile for the density and potential [15]

$$-\varphi/M_{\rm Pl} = GM\left[R^2 + \left(a+\sqrt{b^2+z^2}\right)^2\right]^{-1/2}. \tag{9}$$

Here a and b are characteristic scales in the R and z directions. Note that we will be using a system of units where length is measured in kiloparsecs, appropriate to galactic scales, and that instead of a smooth function like Eq. 9, [7] uses a non-analytic (cuspy) density distribution which is the product of exponentials. In Cartesian coordinates, we parameterise the photon momentum p^μ (with energy $E = p$) as $p^0 = p$, $p^1 = p\cos\alpha\cos\beta$, $p^2 = -p\sin\alpha\cos\beta$, $p^3 = p\sin\beta$. Then treating the lensing exactly, but within the linearised gravitational fields, one can demonstrate [12] in the (R, z) plane

$$\begin{aligned} \frac{\mathrm{d}\beta}{\mathrm{d}\lambda} &= 2p\left(-\sin\beta\frac{\partial\varphi}{\partial R} + \cos\beta\frac{\partial\varphi}{\partial z}\right), \\ \frac{\mathrm{d}p}{\mathrm{d}\lambda} &= p^2\left(\cos\beta\frac{\partial\varphi}{\partial R} + \sin\beta\frac{\partial\varphi}{\partial z}\right), \\ \frac{\mathrm{d}R}{\mathrm{d}\lambda} &= p\cos\beta, \quad \frac{\mathrm{d}z}{\mathrm{d}\lambda} = p\sin\beta, \quad \frac{\mathrm{d}\vartheta}{\mathrm{d}\lambda} = \frac{p\cos(\beta+\vartheta)}{\sqrt{R^2+z^2}}, \end{aligned} \tag{10}$$

where ϑ is the polar angle, and the affine parameter is λ. For the galaxy parameters, one can choose values yielding a similar ellipticity and overall dimensions as used in [7]; these are $M = 3\times 10^{11} M_\odot$, $a = 1.5$ kpc and $b = 0.045$ kpc. When the photon is launched with a starting inclination (to the x-axis) of 18 arcsec, the final (numerically integrated [12]) inclination angle β (the 'flattening') is by just $0.008''$, i.e. completely imperceptible. In contrast, the effect proposed in [7] requires the deflection to *cancel* the starting inclination.

5 Parameters for significant deflection

To explore the parameter space of this very small effect, we insert the MN results (9) for the φ derivatives into the expression for $\mathrm{d}\beta/\mathrm{d}\lambda$ in Eq. (10). Integrating along an affine path length λ and assuming an initial angle of β_0 we find [12] the deflection

$$\Delta\beta(\lambda) \approx -\frac{2M\beta_0 a\left(\sqrt{a^2+2ab+b^2+\lambda^2}-a-b\right)}{(a+b)b\sqrt{a^2+2ab+b^2+\lambda^2}}. \tag{11a}$$

For $1/R$ vs $1/R^2$ behaviour, [7] needs $\Delta\beta \to -\beta_0$. Solving this equation and assuming large λ, i.e. that this is happening for the eventual asymptotic motion of the photon, we find [12] that we need $M \to (a+b)\,b/2a$. For a highly flattened galaxy, with $a \gg b$, we have $M \to b/2$. This is very revealing. For a b of 0.045kpc as above, this means the mass needs to be $\sim 4.7 \times 10^{14} M_\odot$, i.e. of the scale of a large cluster of galaxies!

6 Possible origin of errors

In [7], it is claimed that '*the dominant bending comes from the rings with mid-planes at $z = 0$*'. Consider the lensing caused by an annulus of matter stretching from R' to $R' + \Delta R$ in the R direction, and infinitesimally thin in the z direction, with a surface density $\Sigma(R)$. For the Newtonian potential we can use [16]

$$-\varphi_{\rm ring}(R, z) \approx 2G\,\Delta R\,\sqrt{R'}\,\Sigma(R')\,m\,K(m)/\sqrt{R}, \qquad (12)$$

where $m^2 \equiv 4RR'/\left((R + R')^2 + z^2\right)$, and K is a complete elliptic function of the first kind, yielding [12]

$$\Delta\beta \approx -\Delta R\;\Sigma(R')\left(4\pi - 4\,(\ln 2 - \ln\theta)\,\theta - 3\pi\theta^2/2\right), \qquad (13)$$

where $\theta \equiv z/R'$. There will be a relatively small dependence of the deflection angle on either the z at closest approach or the R' of the annulus location, and therefore also on the initial inclination angle β_0. In fact, the value of the deflection is roughly constant with the z of closest approach [12], and such behaviour is widely known about already for lensing by *cosmic strings* [17]. The lensing is $|\Delta\beta| = 4\pi\mu$ where μ is the mass per unit length of the string, in our case $\Delta\beta = -4\pi\Delta R\;\Sigma(R')$. We now turn to the answer that [7] gets for this case, (the only explicit answer given), viz

$$\delta\beta(R, z) = GME(R, z)/\pi, \qquad (14)$$

where $E(R, z)$ (erroneously designated as '*the complete elliptical integral of the first kind*') readily evaluates to

$$E(R, z) = \left[4K\left(2R/\sqrt{z^2 + 4R^2}\right)\right]\left[z^2 + 4R^2\right]^{-1/2}, \quad (15)$$

and we are effectively finding the average inverse distance from a point $(R, z, \varphi = 0)$ to the infinitesimally thin ring $(R, z = 0, \varphi)$ as φ varies over 0 to 2π [12]. Indeed, comparing with Eq. (12), in which we need to set $R' = R$, we see that $-E(R, z)GM/(2\pi)$, where M is the mass of the ring, will be the Newtonian potential at the point $(R, z, \psi = 0)$. The [7] answer for the angular deflection (14) then appears to be twice this Newtonian potential. It is not clear why [7] believes that this is the way in which to get the deflection, and in particular it disagrees with our result (13) by being (logarithmically) *singular* as $z \to 0$. By contrast, our answer tends to a constant for small z. The effect of this discrepancy is small compared to the inexplicably overestimated lensing amplitude we uncovered above, but it does confirm that we understand the methods used by [7].

7 Conclusions

The scalar gravity model which underpins [1,3–6,8,9] is essentially arbitrary, not related to GR, and *inconsistent* with the nonlinear, static, vacuum EFEs. Moreover, the lensing effects claimed in [7], which are used as a heuristic for GEFC-type phenomena, appear to have been overstated by three orders of magnitude.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: No accompanying data.]

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