

this time is ϕ_0 , whereas in the actual experimental apparatus, there is the time synchronization jitter between the microwave signal and the laser pulse. While the synchronous jitter between the phase of the microwave field and the laser pulse is σ_{RL} , the electron is actually emitted from the cathode face at a phase of $\Phi_0 + C\sigma_{RL}$, and the jitter between it and the reference electron emitted nominally at ϕ_0 is:

$$\begin{aligned}\Delta T_{f,RL} &= T_f(\Phi_0 + C\sigma_{RL}) - T_f(\Phi_0) \\ &= C\sigma_{RL} \left(\frac{dT_f}{d\Phi} \right)_{\Phi=\Phi_0} \\ &= F_{ij}(\Phi_0)\sigma_{RL}\end{aligned}\quad (4)$$

The influence factor $F_{ij}(\phi_0)$ introduced here describes the given time jitter between the microwave field and the laser at the different phase ϕ_0 , which ultimately leads to the arrival time jitter of the electrons, and it is the derivative of the time of flight $T_f(\phi_0)$. The position of $F_{ij} = 0$, that is, the T_f at this phase takes the extreme value, where the phase is 69.77° is shown in Figure 2.

LLRF CONTROLLER DESIGN

The schematic diagram of the LLRF system is shown in Figure 3. The RF system at UED consists of 1.4 cell RF gun and solenoid coil. Before its construction, it is crucial for us to build the system model to analyse.

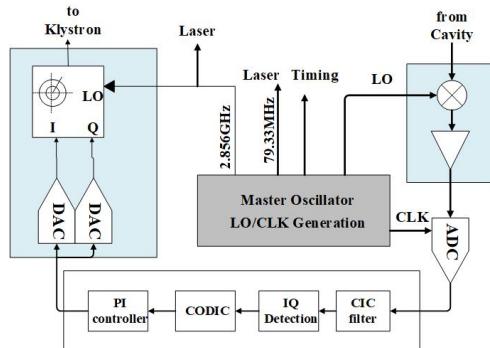


Figure 3: Overview of LLRF system.

Equivalent Circuit Model

In high-frequency electronic circuits, the input coupler can be equivalent to a transformer with a transformer ratio of $1:n$. The cavity can be equivalent to an RLC resonant circuit, and the parameters in the circuit can be represented by voltage and current shown in Figure 4. The circuit is simplified to be equivalent to the circuit shown in Figure 5.

Cavity Model Simulation

The circuit can be analysed by the equivalent circuit on the resonant cavity side shown in Figure 5. The differential equation of the cavity voltage obtained by Kirchhoff's law in the circuit theory is:

$$\ddot{V}_c(t) + \frac{1}{RL} \dot{V}_c(t) + \frac{1}{LC} V_c(t) = \frac{1}{C} \dot{I}(t) \quad (5)$$

Using microwave parameters instead of circuit parameters can be obtained:

$$\ddot{V}_c(t) + \frac{\omega_0}{Q_L} \dot{V}_c(t) + \omega_0^2 V_c(t) = \frac{\omega_0 R_L}{Q_L} \dot{I}(t) \quad (6)$$

In LLRF systems, high-power input and output signals are typically RF signals of a certain frequency. The voltage and current can be expressed as:

$$\begin{aligned}V_c(t) &= (V_r(t) + jV_i(t)) e^{j\omega t} \\ I(t) &= (I_r(t) + jI_i(t)) e^{j\omega t}\end{aligned}\quad (7)$$

where ω is the oscillating angular frequency of the RF signal. Bringing the differential equation to the equation, and opening the virtual part of the real part, the final simplification can be obtained [3]:

$$\begin{aligned}\dot{V}_r + \Delta\omega V_i + \omega_h V_r &= \omega_h R_L I_r \\ \dot{V}_i - \Delta\omega V_r + \omega_h V_i &= \omega_h R_L I_i\end{aligned}\quad (8)$$

Then the cavity model built with MATLAB/Simulink is shown in Figure 6.

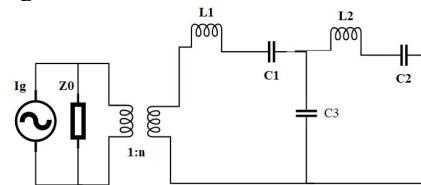


Figure 4: 1.4 cell RF gun equivalent circuit.

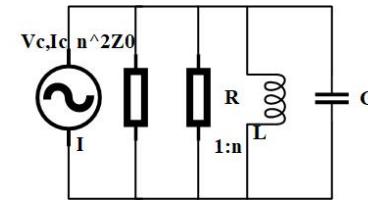


Figure 5: Equivalent circuit after simplification.

High-Voltage Modulator Fluctuation

Fluctuations in the high voltage modulator affect the amplitude and phase of the klystron output signal. Voltage fluctuations cause a change in the amplitude of the klystron output signal. The relationship between the amplitude and the klystron cathode voltage is as follows [4]:

$$\frac{dV_{out}}{V_{out}} = \frac{5}{4} \frac{dV}{V} \quad (9)$$

Assuming that the klystron output signal is $\cos(\omega t)$, the cathode voltage ripple of klystron is ΔV . The amplitude fluctuation and phase fluctuation of the RF output signal can be expressed as ΔA and $\Delta\theta$. The klystron gain is A , then its output signal can be expressed as:

$$V_{out} = (A + \Delta A) \cos(\omega t + \Delta\theta) \quad (10)$$

Considering relativity, it can be obtained:

$$\begin{aligned}d\theta &= -\frac{e\omega L}{mc^3 \beta^3 \gamma^3} dV \\ &= -\frac{e\omega L V}{mc^3 \beta^3 \gamma^3} \frac{dV}{V}\end{aligned}\quad (11)$$

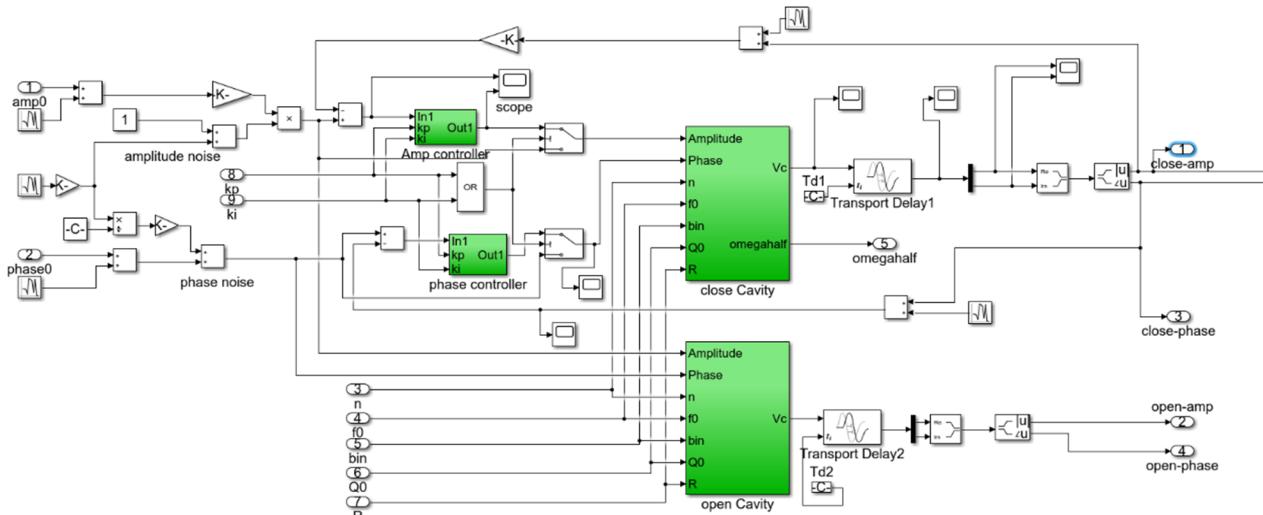


Figure 6: LLRF system simulated in MATLAB/Simulink, where work frequency $f_0 = 2856$ MHz, cavity quality factor $Q_0 = 12000$, cavity input coupling $\beta_0 = 0.9$.

PI Controller

The RF cavity is a first order low pass filter in base-band (assume that it is operated on-resonance). Then the cavity with half bandwidth $\Omega_{0.5}$ can be expressed as [5]:

$$H_{cavity}(s) = \frac{\Omega_{0.5}}{s + \Omega_{0.5}} \quad (12)$$

The suppression of the disturbance signal by the LLRF system is affected by the open-loop gain $L(s)$ (open-loop transfer function), which is manifested as follows: the larger the open-loop gain, the stronger the disturbance suppression capability. However, a large open-loop gain can cause system instability, which is a contradiction between loop anti-disturbance performance and stability margin. The open loop transfer function of the whole system is as follows:

$$\begin{aligned} L(s) &= G \cdot H_{cavity} \cdot E(s) \\ &= G \cdot \left(\frac{\Omega_{0.5}}{s + \Omega_{0.5}} \right) \cdot e^{-sT_d} \end{aligned} \quad (13)$$

where $E(s)$ is the delay module, T_d is the total delay, including the digital delay caused by the internal digital signal processing algorithm in the FPGA, the analogy delay caused by the cable and other devices. The general delay T_d is 1-1.5 μ s.

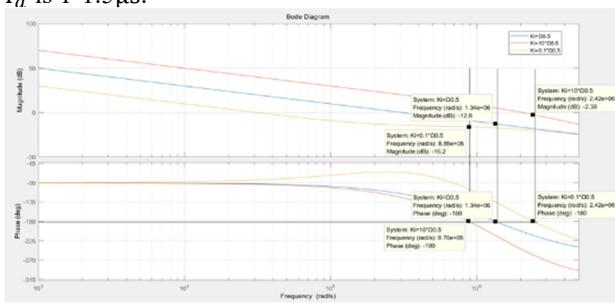


Figure 7: Bode plots of LLRF system.

In MATLAB simulation, the Pade function is needed to approximate the loop delay module. The bode plots is shown in Figure 7. Then we can predict the system stability of the LLRF system of the cavity by its Bode plots. The system performance was measured and evaluated via different proportional gain K_p and integral gain K_i . After gain-scanning, it can be obtained that $K_p = 0.1$, $K_i = 1000$.

Stabilities of LLRF

The typical LLRF system performance and parameters during experiment are listed above. The feedback gains were determined by the PI controller tuning, as stated above. Disturbing signals such as power supply ripples and microphonics can be suppressed well in the model shown in Figure 8.

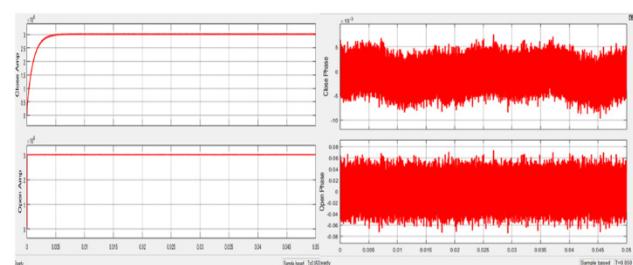


Figure 8: Operation results of LLRF system.

CONCLUSION

The basic design of the LLRF system for the HUST UED machine has been simulated. All of the simulated results show good performance to satisfy the fifty femtoseconds accuracy. The simulation of the LLRF system in UED can provide theoretical support for the construction of the whole UED system. We will improve the algorithm to satisfy the better precision.

REFERENCES

- [1] M. Titberidze *et al.*, "Present and Future Optical-to-Microwave Synchronization Systems at REGAE Facility for Electron Diffraction and Plasma Acceleration Experiments", in *Proc. 6th Int. Particle Accelerator Conf. (IPAC'15)*, Richmond, VA, USA, May 2015, pp. 833-836.
doi: 10.18429/JACoW-IPAC2015-MOPHA026
- [2] M. Hoffmann *et al.*, "Precision LLRF Controls for the S-Band Accelerator REGAE", in *Proc. 4th Int. Particle Accelerator Conf. (IPAC'13)*, Shanghai, China, May 2013, paper WEPME008, pp. 2938-2940.
- [3] X. Wang, X. Qiu, I. Ben-Zvi, "Experimental observation of high-brightness microbunching in a photocathode rf electron gun", *Phys. Rev. E*, vol. 54, no. 4, pp. R3121-R3124, Oct. 1996. doi: 10.1103/PhysRevE.54.R3121
- [4] C. Schmidt, "RF System Modeling and Controller Design for the European XFEL", Ph.D. thesis, Technischen Universitat Hamburg-Harburg, Hamburg, Germany, 2010.
- [5] F. Qiu *et al.*, "A Disturbance-Observer-based Controller for LLRF Systems", in *Proc. 6th Int. Particle Accelerator Conf. (IPAC'15)*, Richmond, VA, USA, May 2015, pp. 2895-2898.
doi: 10.18429/JACoW-IPAC2015-WEPMA054