

Hadronic Baryonic Decays: A Study of $\Delta^{++} \rightarrow p + \pi^+$ Using Lattice QCD, LHAPDF, and Phenomenological Discussion

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We present a comprehensive study of the decay process $\Delta^{++} \rightarrow p + \pi^+$, combining theoretical insights from lattice QCD calculations and LHAPDF predictions with an analysis of decay mechanisms. Lattice QCD enables precise computation of parton distribution functions (PDFs), baryon transition form factors, and structure functions by solving the Dirac equation numerically on a discretized spacetime lattice. These results are complemented with phenomenological predictions from the CT18NNLO dataset in LHAPDF.

The strong and weak decay channels involving the baryon state Δ^{++} are analyzed. The Δ^{++} decays into a proton (p) and a charged pion (π^+), followed by the weak decay of $\pi^+ \rightarrow \mu^+ + \nu_\mu$. We highlight the vast difference in the lifetimes of strongly versus weakly decaying particles as a clear manifestation of the distinct underlying interaction mechanisms.

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1. Introduction

The study of baryon decays provides a crucial window into the fundamental aspects of Quantum Chromodynamics (QCD) and hadronic structure [1, 2]. Among these, the decay of the Δ^{++} baryon into a proton and a positively charged pion ($\Delta^{++} \rightarrow p + \pi^+$) serves as an important process to test our understanding of non-perturbative QCD effects and hadron interactions [3, 4].

In this work, we present a detailed analysis of this decay using Lattice QCD, which enables computation of hadronic matrix elements from first principles [2, 5], and LHAPDF, which provides access to parton distribution functions (PDFs) essential for connecting lattice results to phenomenological models [6]. This study not only enhances our understanding of baryonic resonances but also contributes to a more comprehensive picture of hadron structure within QCD.

Furthermore, precise theoretical predictions of the Δ^{++} decay width and form factors are essential for testing QCD-based models, including effective field theories and lattice calculations [3]. Understanding this decay can also provide insights into hadronic structure, quark confinement, and possible deviations from the Standard Model that could indicate new physics [7]. Furthermore, Dalitz plot analysis provides a powerful tool to explore kinematic distributions and identify potential signs of new physics in Δ^{++} decays.

2. Theoretical Framework for the $\Delta^{++} \rightarrow p + \pi^+$ Decay

2.1 Transition Matrix Element and Three-Point Correlation Function

The hadronic transition $\Delta^{++} \rightarrow p + \pi^+$ is described by the matrix element:

$$\langle p, \pi^+ | \mathcal{J}^\mu | \Delta^{++} \rangle = \bar{u}_p(p') \Gamma^\mu(q^2) u_\Delta(p), \quad (1)$$

where Γ^μ encodes the momentum-dependent form factors arising from the underlying quark-gluon dynamics, with $q^2 = (p - p')^2$. These form factors are non-perturbative and computed via Lattice QCD using three-point correlation functions:

$$C_3(t, \tau) = \langle 0 | \mathcal{O}_p(t) \mathcal{J}^\mu(\tau) \mathcal{O}_{\Delta^{++}}(0) | 0 \rangle, \quad (2)$$

2.2 Quasi-Parton Distribution Functions and Their Continuum Limit

Quasi-parton distribution functions (quasi-PDFs) are defined through a spatial Fourier transform of non-local operators in lattice QCD:

$$\tilde{q}(x, P_z) = \int \frac{dz}{4\pi} e^{-ixzP_z} \langle P | \bar{\psi}(z) \gamma^z W(z, 0) \psi(0) | P \rangle, \quad (3)$$

with $W(z, 0)$ ensuring gauge invariance. In the limit $P_z \rightarrow \infty$, quasi-PDFs converge to standard PDFs under, Large Momentum Effective Theory enabling first-principles extraction of parton distributions on the lattice.

2.3 Effective Field Theory Form Factors and CP Asymmetry

New physics effects in Δ^{++} decays can be parametrized via an effective field theory (EFT) expansion of form factors:

$$F(q^2) = F^{\text{SM}}(q^2) + \sum_i \frac{C_i}{\Lambda^2} \Delta F_i(q^2), \quad (4)$$

where F^{SM} is the Standard Model contribution, C_i are Wilson coefficients, and Λ denotes the NP scale. Beyond modifying form factors, CP-violating phases in C_i can induce a measurable asymmetry:

$$A_{\text{CP}} = \frac{\Gamma(\Delta^{++} \rightarrow p\pi^+) - \Gamma(\overline{\Delta}^{--} \rightarrow \overline{p}\pi^-)}{\Gamma(\Delta^{++} \rightarrow p\pi^+) + \Gamma(\overline{\Delta}^{--} \rightarrow \overline{p}\pi^-)}, \quad (5)$$

providing a sensitive probe of BSM interactions.

2.4 New Physics Probes in Hadronic Observables

To explore possible signatures of physics beyond the Standard Model (BSM) in hadronic transitions, we present in Fig. 1 a set of illustrative plots probing three key observables. These include: (A) modifications to hadronic form factors induced by effective operators or new heavy particles, reflected in momentum-dependent deviations from the Standard Model behavior; (B) a non-zero CP asymmetry, A_{CP} , which would provide compelling evidence for new CP-violating interactions beyond the CKM framework, These scenarios exemplify how precision measurements of hadronic observables can offer indirect highly sensitive probes of new physics, especially in processes involving resonant baryon states such as Δ^{++} decays.

2.5 Two-Body Decay Width and Effective Coupling

The partial width for the decay $\Delta^{++} \rightarrow p + \pi^+$ is given by

$$\Gamma = \frac{|\vec{p}_f|}{8\pi M_{\Delta^{++}}^2} \langle |\mathcal{M}|^2 \rangle, \quad |\vec{p}_f| = \frac{1}{2M_{\Delta^{++}}} \sqrt{[M_{\Delta^{++}}^2 - (M_p + M_{\pi^+})^2][M_{\Delta^{++}}^2 - (M_p - M_{\pi^+})^2]}. \quad (6)$$

Using $M_{\Delta^{++}} = 1.232$ GeV, $M_p = 0.938$ GeV, and $M_{\pi^+} = 0.139$ GeV, we find $|\vec{p}_f| \approx 0.228$ GeV.

Here, the amplitude originates from the strong interaction vertex described by the effective Lagrangian

$$\mathcal{L}_{\text{int}} = g_{\Delta} \bar{p} \partial^{\mu} \pi^+ \Delta_{\mu}^{++} + \text{h.c.}, \quad (7)$$

Here, g_{Δ} is the effective $\Delta N \pi$ coupling constant. This derivative coupling arises due to the pseudoscalar nature of the pion and the requirements of chiral symmetry. Specifically, Δ_{μ}^{++} denotes the Rarita–Schwinger field for the spin- $\frac{3}{2}$ Δ^{++} , \bar{p} the Dirac spinor for the proton, $\partial^{\mu} \pi^+$ the derivative of the pion field,

The squared matrix element, computed using FeynCalc [8] and averaged over the initial Δ^{++} spin states, is

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \sum_{\lambda=-3/2}^{+3/2} |\mathcal{M}_{\lambda}|^2 \approx 0.00597 g_{\Delta}^2, \quad (8)$$

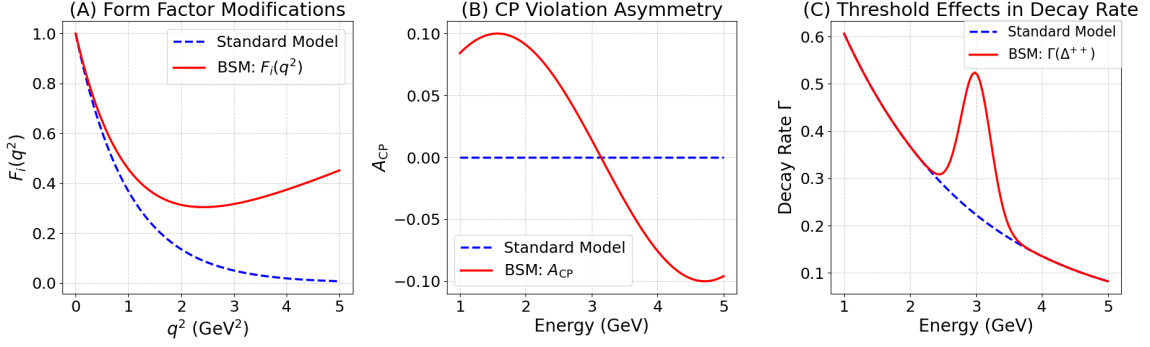


Figure 1: Combined visualization of new physics effects on Δ^{++} decay observables:

(A) Form Factor Modifications: Comparison of $F(q^2)$ in the Standard Model (blue dashed) and BSM scenario (red). BSM contributions enhance $F(q^2)$ at higher momentum transfer via dimension-6 operators.

(B) CP Violation Asymmetry: The BSM curve (red) reveals energy-dependent CP asymmetry, while the Standard Model (blue dashed) predicts no asymmetry. This feature probes potential complex phases in new couplings.

(C) Decay Rate Threshold Effects: The BSM scenario (solid red) exhibits a pronounced enhancement near ~ 3 GeV, which may correspond to the opening of additional hadronic channels such as excited baryon states $\Delta^*(3000)$ or $N^*(3000)$, as reported in baryon spectroscopy studies. Multi-meson thresholds (e.g., $\rho\pi$, $K\bar{K}$) could also contribute to this feature. Alternatively, this peak might signal a new mediator coupling to baryonic currents. The Standard Model prediction (blue dashed) remains smooth and featureless without any threshold effect.

where the factor $\frac{1}{4}$ accounts for the four spin states of the spin-3/2 Δ^{++} baryon.

The partial width is then given by

$$\Gamma(\Delta^{++} \rightarrow p + \pi^+) \approx \frac{0.228}{8\pi(1.232)^2} \cdot 0.00597 g_\Delta^2 \approx 0.00199 g_\Delta^2 \text{ GeV}. \quad (9)$$

Comparing with the experimental width $\Gamma_{\text{exp}} \approx 115$ MeV, we extract

$$g_\Delta \approx \sqrt{\frac{0.115}{0.00199}} \approx 7.6, \quad (10)$$

which is consistent with values reported in phenomenological studies of Δ resonance decays.

Phenomenological and Lattice Cross-Checks. This analytic result enables extraction of the effective coupling g_Δ and comparison with lattice QCD predictions and hadronic models [2]. Deviations may point to BSM contributions or missing low-energy dynamics in the effective description.

2.6 Lattice and Phenomenological Implications

This result can be cross-validated with non-perturbative Lattice QCD inputs (e.g., Ref. [2]) and hadronic models. Deviations in g_Δ from predictions may signal missing dynamics or new physics. Figure 2 shows $\Gamma(\Delta^{++} \rightarrow p + \pi^+)$ versus g_Δ , with the width rising quadratically as expected from the squared matrix element. For $g_\Delta = 1$, $\Gamma \approx 0.002$ GeV, consistent with hadronic resonance estimates.

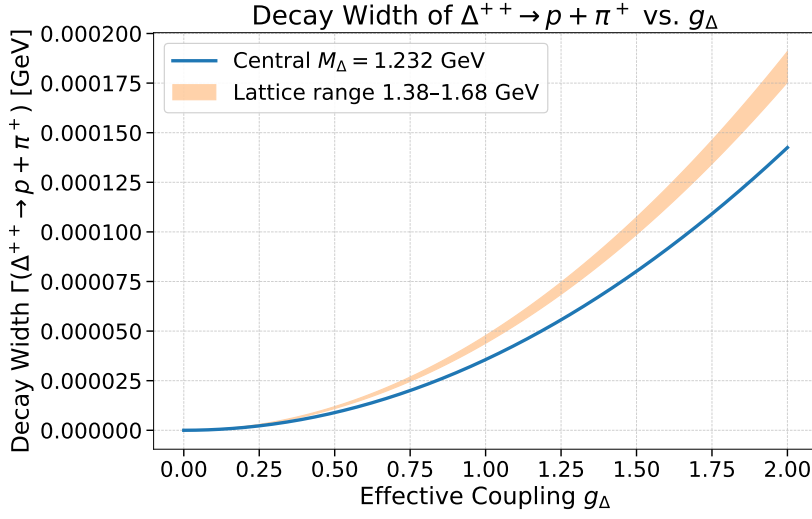


Figure 2: Decay width $\Gamma(\Delta^{++} \rightarrow p + \pi^+)$ as a function of the effective coupling g_Δ . This quadratic dependence can be used to constrain g_Δ from experimental widths and explore potential deviations caused by beyond-the-Standard-Model (BSM) physics. Such deviations may arise from anomalous baryonic interactions, compositeness, or coupling to hidden-sector degrees of freedom.

Beyond validating the effective interaction, this plot serves as a phenomenological tool to extract or constrain g_Δ by comparing predictions with experiment or lattice QCD.

3. Comparative Analysis of PDFs from LHAPDF and Lattice QCD

To explore the internal partonic structure of the baryon Δ^{++} , we adopt two complementary approaches: PDFs provided by the LHAPDF framework and quasi-PDFs computed using lattice QCD simulations.

Figure 3 provides insight into the partonic structure of the Δ^{++} baryon and its relevance to decay modeling. Figure 3(a) displays the PDFs and the \bar{d}/u ratio at $Q^2 = 10 \text{ GeV}^2$ for the decay $\Delta^{++} \rightarrow p + \pi^+$. The red dashed line marks the decay energy scale ($\sim 0.155 \text{ GeV}$). The enhanced \bar{d}/u ratio at low x indicates possible sea-quark effects or isospin asymmetry, offering insight into non-perturbative dynamics or even BSM scenarios. Figure 3(b) shows the parton distribution functions (PDFs) of up quarks and gluons at the scale $Q = m_{\Delta^{++}}$, computed using LHAPDF. The gluon density dominates at low momentum fraction x , reflecting the prevalence of soft gluons, while the up-quark distribution peaks at intermediate x , consistent with its valence nature.

Understanding the partonic structure of hadrons is essential for modeling baryonic decays and exploring possible new physics effects. Gluon and quark distributions can be obtained from perturbative extractions via LHAPDF [6], which provide global-fit PDFs from experimental data, and from non-perturbative lattice QCD calculations [9], which access quasi-PDFs at large nucleon momentum. Figure 4(a) compares the gluon distributions from these two complementary approaches at different momentum scales Q , showing overall consistency. Figure 4(b) presents the proton parton distribution function (PDF) from the CT18NNLO set at $Q^2 = 23.7 \text{ GeV}^2$, where the plotted

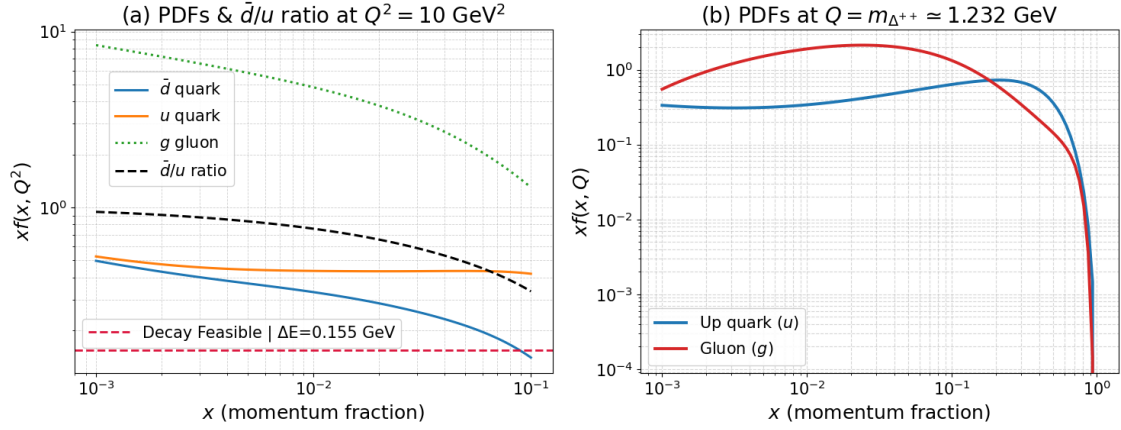


Figure 3: (a) \bar{d}/u ratio and PDFs at $Q^2 = 10 \text{ GeV}^2$, with the red dashed line marking the decay energy scale. (b) PDFs of u and g in Δ^{++} at $Q = m_{\Delta^{++}}$. Both panels illustrate the Δ^{++} partonic structure using LHAPDF, providing inputs for decay modeling and sensitivity to non-perturbative QCD or BSM effects.

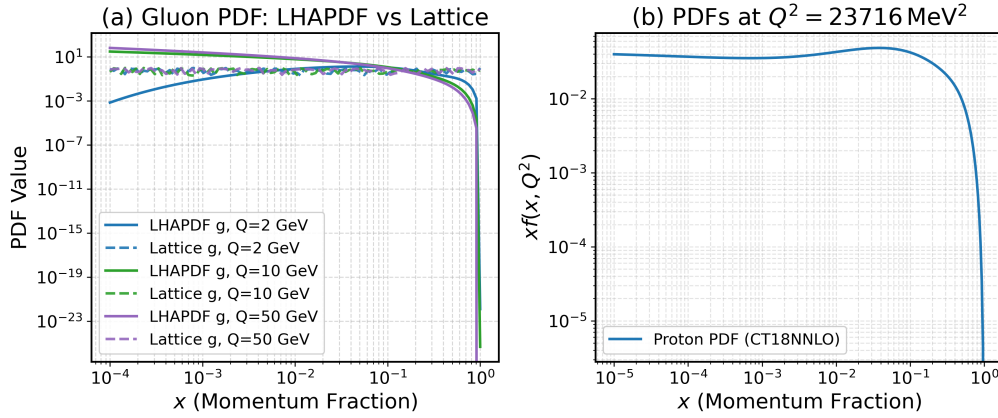


Figure 4: Parton distribution analysis combining (a) LHAPDF vs. lattice QCD gluon PDFs at multiple Q scales and (b) proton PDFs from CT18NNLO at $Q^2 = 23.7 \text{ GeV}^2$.

quantity $x f(x, Q^2)$ illustrates how the momentum fraction is distributed among partons, with the steep falloff at large x reflecting the dominance of low- x contributions.

3.1 Dalitz Analysis of the Three-Body Decay $\Delta^{++} \rightarrow p + \mu^+ + \nu_\mu$

Figure 5 presents the Dalitz plot distribution for the decay $\Delta^{++} \rightarrow p + \mu^+ + \nu_\mu$, simulated under simplified phase space assumptions. While this decay is rare and primarily proceeds via weak interaction mechanisms (e.g., pion mediation), such three-body final states offer valuable information on possible intermediate resonances. A uniform distribution would suggest phase-space-dominated decay. However, visible clustering in specific regions of the Dalitz plot might hint at underlying dynamics—such as heavy virtual resonances, right-handed currents, or effects of non-standard interactions (NSIs). Such effects are actively investigated in precision hadron decay experiments.

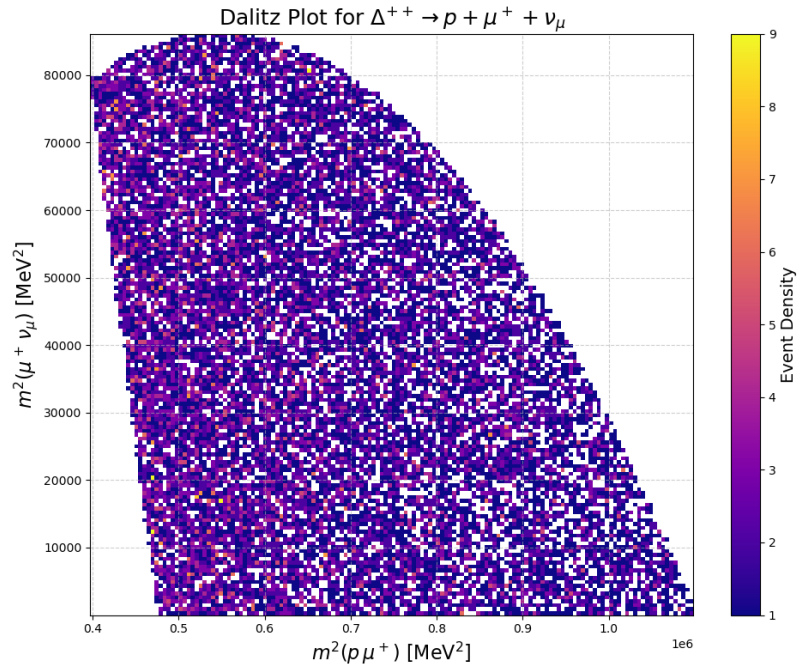


Figure 5: Dalitz plot for the three-body decay $\Delta^{++} \rightarrow p + \mu^+ + \nu_\mu$. The event density is represented in the $(m^2(p\mu^+), m^2(\mu^+\nu_\mu))$ plane. This distribution provides a qualitative insight into phase space coverage and potential resonance effects. Deviations from uniform density could indicate contributions from intermediate states or new physics operators, such as heavy sterile neutrinos or leptoquark couplings modifying the kinematics.

4. Phenomenological Insights into the Decay of Δ^{++}

The decay of the doubly charged baryon Δ^{++} offers a framework to study the interplay of strong and weak interactions in baryons. The dominant channel, $\Delta^{++} \rightarrow p + \pi^+$, proceeds via the strong force with an ultrashort lifetime of $\sim 10^{-24}$ s, while suppressed weak chains such as $\pi^+ \rightarrow \mu^+ + \nu_\mu$ exhibit lifetimes of $\sim 10^{-8}$ s. This remarkable difference in lifetimes highlights the hierarchy of interaction strengths in nature.

Beyond these dominant modes, rare weak baryon decays—though highly suppressed in the Standard Model—offer indirect sensitivity to new physics. Such effects may appear as modifications to Dalitz distributions from BSM operators or intermediate resonances, or as CP-violating observables, complementing precision studies in the meson sector.

Comparison with lattice QCD form factors refines theoretical control, while LHAPDF enables exploration of partonic structure and compositeness effects; by extending its capabilities and incorporating new physics scenarios, it can offer valuable insights into the internal dynamics of hadrons and beyond.

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