PROTON-RADIOACTIVITY HALF-LIFE FORMULAS WITH ISOSPIN AND SHELL EFFECTS

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We have investigated the influence of isospin and shell effects on the proton decay half-lives of nuclei. In order to take into account these effects, new parameters related with isospin and shell effects have been added to the empirical formulas proposed in *Phys. Rev. C* **79**, 054330 (2009) and *Chin. Phys. C* **42**, 014104 (2018). The parameters of these new empirical formulas including isospin and shell effects have been fitted by 44 experimental available data comprising 29 ground states and 15 isomeric transitions of proton decay half-lives. The r.m.s. deviation between theory and experiment is decreased by inclusion of these modifications. The models have been applied to proton decaying nuclei whose experimental values are not yet known and then to actinide nuclei as well. Results consistent with the ones in the literature have been obtained. The role of the isospin and shell effects on the proton decay half-lives has been demonstrated.

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1. Introduction

For the proton-rich nuclei, β^+-decay is the most preferential decay mode. However, once the proton drip line is exceeded by the nucleus, nuclear force

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cannot stand against the electrostatic repulsion between the protons. Thus, another decay mode takes place and the existence of a nucleus beyond the proton drip line is restricted by proton decay or "proton radioactivity". Besides the importance of the proton radioactivity for the nuclear structure and unlike the neutron capture-decay processes, proton capture-decay reactions change the identity of the chemical elements. First experimental evidence of proton radioactivity was observed by Jackson *et al.* [1] from the isomeric state of ^{53}Co. Another experimental confirmation of the proton radioactivity was reported from the ground state of ^{151}Lu in 1981 [2]. In the following years, proton radioactivity from ground and isomeric states of the nuclei such as ^{147}Tm, ^{109}I, ^{185}Bi, ^{112}Cs, ^{141}Ho, ^{131}Eu and *etc.* has been proved [3–7]. Besides the experimental researches, several theoretical attempts have been performed to get the proton radioactivity properties such as half-life. Many theoretical models have been applied to study half-lives of proton decays of nuclei and different analytical formulas have been used to explain the experimental half-lives [8–10].

Buck *et al.* [11] extended the model of α and exotic decays in order to obtain the proton-emitted nuclei observable with the angular momentum term. An estimation of proton emission properties from the spherical nuclei has been studied by Basu *et al.* [12] within the Wentzel–Kramers–Brillouin (WKB) formalism, also the nucleus–nucleon interaction has been described microscopically. A general decay law formula has been proposed by Sahu *et al.* [13] to get the cluster, alpha and proton emission decay half-lives. Santhosh and Sukumaran [14–16] have used the Coulomb Proximity Potential Model (CPPM), its deformed version CPPMDN, and extended version of the Hatsukawa formula to achieve the half-lives of the various proton emitter nuclei. Different theoretical approaches such as wave function matching methods, fission-like methods and WKB-based methods and their applications can be found in Refs. [8, 17–21] (and references therein). Moreover, the Jeukenne, Lejeune and Mahaux (JLM) effective interaction [22], the generalized liquid drop model (GLDM) [23], the finite-range effective interaction of Yukawa form [24], the unified fission model [25, 26], the similarity renormalization group method [27] have been used in order to investigate the proton radioactivity of nuclei. More recently, the half-lives of one-proton emitters in the actinide region have been calculated by using the CPPM [28]. In that paper, authors have obtained that half-lives are in good agreement with the available experimental values.

In the present study, we have focused on two semi-empirical formulas to investigate the proton emission process. The first formula proposed by Dong *et al.* [29] for both the spherical proton emitters and deformed proton emitters is given by

$$\log\left(T_{1/2}(s)\right) = (aZ+b)Q^{-1/2} + c + c_0\frac{\ell(\ell+1)}{\sqrt{(A-1)(Z-1)A^{-2/3}}}\,, \quad (1)$$

the second one proposed by Zhang and Dong [21] is given as follows:

$$\log\left(T_{1/2}(s)\right) = a+bA^{1/6}Z^{1/2}+cZQ^{-1/2}+d\ell(\ell+1)A^{-1/6}Z^{-1/2}-\log_{10}S_p \quad (2)$$

to be valid in both equations A, Z are parent nuclei mass number and proton number, and Q — value of the reaction, respectively [21, 29]. With the approximation $\sqrt{(A-1)(Z-1)A^{-2/3}} \backsimeq A^{1/6}Z^{1/2}$ and the addition of the spectroscopic factor S_p, Eq. (2) has been designed to investigate the proton emission [21]. In literature, the effect of isospin that is an important effect in nuclear structure on the proton decay has been studied in different ways. In the macroscopic model such as the generalized liquid drop model, the isospin degree of freedom has been included [29]. Authors have considered the isospin in the microscopic theory such as Skyrme interactions [30] and the relativistic density functionals [31]. On the other hand, the analytical formula for proton decays has been derived successfully in microscopical way [32]. Therefore, in the framework of above semi-empirical proton decay formulas, we have systematically analyzed the proton emission by using six formulas. It is possible to divide the models presented in this article into three subgroups. The first subgroup contains the formulas that can be called as bare versions of Eqs. (1) and (2) and they consist of only the angular momentum (ℓ) term. In the second subgroup, to investigate the isospin effects on the proton radioactivity, we have added the $I = (N-Z)/A$ term to the both expressions. Finally, the last formulas are formed by including shell correction factor $E_{\rm shell}$ which is also important for the structure. It should be noted that the formulas given in this paper do not contain the spectroscopic factor. The available experimental data have been used for the fitting procedure to obtain the coefficients of the formulas. The present paper is organized as follows: the second section contains the proposed formulas for proton radioactivity calculations. Our compared numerical results with the experimental data and root mean square deviations can be found in Section 3, our conclusions are presented in the final section.

2. Models

The formulas in this section are divided into three subgroups, in the first one, we give the empirical formulas that are obtained from Eqs. (1) and (2), second one contains the formulas which have isospin effects, and the last one is formed by the shell-effect formulas. It should be noted that Eqs. (3), (5) and (7) are derived from Eq. (1), while Eqs. (4), (6) and (8) are derived from Eq. (2).

2.1. The empirical formulas for proton decay

The formula for proton decay is given by the following equations:

$$\log\left(T_{1/2}(s)\right) = (aZ+b)Q^{-1/2} + c\ell(\ell+1)A^{-1/6}Z^{-1/2} + d\,, \tag{3}$$

$$\log\left(T_{1/2}(s)\right) = a + bA^{1/6}Z^{1/2} + cZQ^{-1/2} + d\ell(\ell+1)A^{-1/6}Z^{-1/2}\,, \tag{4}$$

where A, Z are mass number and proton number of parent nuclei, Q — value for proton decay and ℓ — angular momentum; a, b, c, d are the parameters. These forms were proposed in Refs. [21, 29].

2.2. The formulas with isospin effect

As the isospin effect plays an important role in nuclear physics, its influence on the proton, alpha and cluster decays is investigated. For decay, the particle (alpha, cluster, proton) is assumed as surrounding the surface of the decaying mother nuclei. When nucleonic densities are different for protons and neutrons in nucleus, motion of particles on the surface may be affected by the asymmetry of the isospin. In this case, the interaction between the particle and the daughter would be isospin-dependent and then half-lives of decays are changed. The effects of isospin taking various approaches into account with different parameters have been examined in many different studies [33–36]. A positive value of isospin asymmetry for any nucleus means that the nucleus has the neutron number greater than the number of protons. In order to investigate the isospin effect on the proton decay, we have modified the formulas given above and have used the following equations:

$$\log\left(T_{1/2}(s)\right) = (aZ+b)Q^{-1/2} + c\ell(\ell+1)A^{-1/6}Z^{-1/2} + d + eI + fI^2\,, \tag{5}$$

$$\log\left(T_{1/2}(s)\right) = a + bA^{1/6}Z^{1/2} + cZQ^{-1/2} + d\ell(\ell+1)A^{-1/6}Z^{-1/2} + eI + fI^2\,, \tag{6}$$

where $I = (N-Z)/A$. We have added 2 terms to take into account the nuclear isospin effect on the proton decay. As together, I and I^2 terms give better results than other forms, this form has been used in the model. It should also be noted that these forms have been chosen similarly to their forms in the literature and phenomenologically to model the effect of isospin.

2.3. The formulas with shell effect

The influence of shell structure on the properties of heavy and superheavy nuclei have been investigated for a long time. In order to be able to apply Swiatecki's formula suggested for fission process to calculate the spontaneous fission half-lives of nuclei, many authors have considered a modified form of it. The shell effects considered in this way have had serious effects on the half-lives on the spontaneous fission half-lives of nuclei [37, 38]. From this point on, it might also be important to examine the effects of shell effects on proton decay of nuclei. In order to take into account the shell-effects on proton decays, we have modified Eqs. (3) and (4). This effect was considered by Santhosh *et al.* [37] and Bao *et al.* [38] for spontaneous fission decays. We have used the similar mechanism to investigate the shell effects on the proton decay as follows:

$$\log\left(T_{1/2}(s)\right) = (aZ+b)Q^{-1/2} + c\ell(\ell+1)A^{-1/6}Z^{-1/2} + d + eE_{\rm shell} + fE^2_{\rm shell}\,, \tag{7}$$

$$\log\left(T_{1/2}(s)\right) = a + bA^{1/6}Z^{1/2} + cZQ^{-1/2} + d\ell(\ell+1)A^{-1/6}Z^{-1/2} + eE_{\rm shell} + fE^2_{\rm shell}\,, \tag{8}$$

where $E_{\rm shell}$ is shell-correction factor and the values are taken from Ref. [39]. In this study, a fitting procedure has been done by using the module of curve fitting in Python 2.7 language, and the coefficients of the formulas are presented in Table I. It should be noted that we have tried a number of different forms phenomenologically, but we have found that the above equations are the forms that give the best r.m.s. values. One of the reasons for this may be that the terms $E_{\rm shell}$ plus $E^2_{\rm shell}$, I plus I^2 in the present forms obtained by fitting process, explain the effects of isospin and shell effects better than other forms.

TABLE I

The coefficients of the formulas.

Coefficients	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)
a	0.28724	−18.95294	0.31867	−20.20942	0.28831	−19.15679
b	7.22754	−0.49478	5.48957	−0.37396	7.16010	−0.49182
c	2.23746	0.38382	2.33102	0.39369	2.30189	0.38421
d	−28.97282	2.20594	−27.94171	2.33990	−29.0848	2.27732
e	—	—	−49.00030	−60.14092	0.20452	0.22926
f	—	—	352.70529	449.42227	−0.06873	−0.07556

3. Results and discussions

We have obtained the half-lives of proton decays of nuclei having the experimental measured values by using the formulas in the model section. The obtained and experimental values for the related nuclei are given in Table II. In this table, parent column shows the related nuclei decay via

TABLE II

$\log_{10} T_{1/2}$ (s) obtained by the formulas in the model section with experimental values of the proton decay of nuclei.

Parent	ℓ	$Q_{\rm exp}$	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)	Exp. [40, 41]
^{109}I	2	0.827	−3.44	−3.62	−3.54	−3.65	−3.51	−3.69	−4.029
^{112}Cs	2	0.823	−2.77	−2.93	−2.47	−2.50	−2.75	−2.90	−3.301
^{113}Cs	2	0.976	−4.84	−4.84	−4.84	−4.81	−4.79	−4.78	−4.777
^{117}La	2	0.814	−2.01	−2.17	−1.91	−1.99	−1.96	−2.10	−1.623
^{121}Pr	2	0.900	−2.70	−2.76	−2.52	−2.51	−2.66	−2.71	−2.000
^{130}Eu	2	1.039	−3.38	−3.33	−3.25	−3.20	−3.36	−3.31	−3.046
^{131}Eu	2	0.959	−2.36	−2.37	−2.41	−2.43	−2.33	−2.33	−1.670
^{135}Tb	3	1.200	−3.86	−3.76	−3.82	−3.75	−3.85	−3.76	−3.027
^{140}Ho	3	1.106	−2.36	−2.31	−2.37	−2.35	−2.39	−2.34	−2.222
^{141}Ho	3	1.190	−3.27	−3.20	−3.40	−3.39	−3.26	−3.19	−2.387
^{144}Tm	5	1.725	−4.85	−4.72	−4.77	−4.67	−4.79	−4.66	−5.569
^{145}Tm	5	1.753	−5.02	−4.89	−5.06	−4.98	−4.98	−4.86	−5.456
^{146}Tm	5	1.210	−0.86	−0.84	−0.92	−0.91	−0.87	−0.85	−0.930
^{147}Tm	5	1.073	0.66	0.64	0.57	0.56	0.65	0.63	0.577
^{150}Lu	5	1.283	−1.13	−1.10	−1.13	−1.11	−0.98	−0.93	−1.194
^{151}Lu	5	1.253	−0.84	−0.82	−0.90	−0.89	−0.71	−0.67	−0.896
^{155}Ta	5	1.468	−2.31	−2.28	−2.34	−2.33	−2.70	−2.70	−2.538
^{156}Ta	2	1.030	−0.51	−0.49	−0.60	−0.61	−0.77	−0.76	−0.609
^{157}Ta	0	0.947	0.00	0.02	−0.10	−0.11	−0.01	0.02	−0.523
^{159}Re	5	1.816	−4.29	−4.28	−4.31	−4.32	−4.60	−4.61	−4.678
^{160}Re	2	1.285	−2.93	−2.89	−3.01	−3.02	−2.98	−2.94	−3.060
^{161}Re	0	1.214	−2.86	−2.82	−2.97	−2.98	−2.82	−2.78	−3.357
^{164}Ir	5	1.844	−4.09	−4.12	−4.10	−4.14	−4.05	−4.07	−3.947
^{166}Ir	2	1.168	−1.17	−1.14	−1.18	−1.17	−1.10	−1.07	−0.818
^{167}Ir	0	1.096	−0.94	−0.91	−0.94	−0.91	−0.93	−0.90	−0.959
^{170}Au	2	1.488	−3.80	−3.81	−3.84	−3.87	−3.73	−3.74	−3.493
^{170}Au	0	1.464	−4.24	−4.24	−4.30	−4.33	−4.19	−4.19	−4.611
^{176}Tl	0	1.282	−2.04	−2.04	−1.96	−1.95	−2.10	−2.10	−2.284
^{177}Tl	0	1.180	−0.90	−0.88	−0.75	−0.70	−0.97	−0.97	−1.174
^{141m}Ho	0	1.255	−5.34	−5.24	−5.55	−5.53	−5.37	−5.28	−5.180
^{146m}Tm	5	1.140	−0.12	−0.11	−0.16	−0.16	−0.12	−0.12	−0.693
^{147m}Tm	2	1.133	−2.86	−2.82	−3.07	−3.09	−2.94	−2.92	−3.444
^{150m}Lu	2	1.306	−4.11	−4.04	−4.23	−4.22	−4.04	−3.96	−4.367
^{151m}Lu	2	1.332	−4.35	−4.28	−4.53	−4.53	−4.29	−4.21	−4.796
^{156m}Ta	5	1.127	0.97	0.97	0.98	0.98	0.79	0.78	0.930
^{159m}Re	5	1.831	−4.38	−4.37	−4.40	−4.41	−4.68	−4.69	−4.695
^{161m}Re	5	1.338	−0.78	−0.78	−0.77	−0.77	−0.64	−0.63	−0.650
^{165m}Ir	5	1.733	−3.41	−3.45	−3.43	−3.46	−3.28	−3.30	−3.469
^{166m}Ir	5	1.340	−0.36	−0.38	−0.30	−0.30	−0.22	−0.23	−0.076
^{167m}Ir	5	1.261	0.42	0.39	0.52	0.54	0.52	0.50	0.875
^{170m}Au	5	1.770	−3.27	−3.35	−3.25	−3.31	−3.13	−3.20	−2.980
^{171m}Au	5	1.719	−2.95	−3.03	−2.90	−2.94	−2.84	−2.92	−2.654
^{177m}Tl	5	1.984	−4.18	−4.33	−4.06	−4.12	−4.16	−4.32	−3.402
^{185m}Bi	0	1.624	−4.59	−4.71	−4.19	−4.14	−4.54	−4.66	−4.237

proton emission, ℓ shows angular momentum for the related decay, $Q_{\rm exp}$ presents an experimental Q-value that is taken form Ref. [10], obtained results have been indicated by using equation numbers. The last column shows the experimental $\log_{10} T_{1/2}$ (s) values for proton decay of the related parent nuclei [40, 41].

In order to compare the results, the r.m.s. deviations of the decimal logarithmic values are calculated by using the following equation:

$$\sigma = \left[\frac{1}{n-1}\sum_{k=1}^{n}\left[\log_{10}\left(T_p^{\rm cal}\right) - \log_{10}\left(T_p^{\rm exp}\right)\right]^2\right]^{1/2} , \tag{9}$$

where n denotes the number of the related nuclei. The obtained r.m.s. values for each expression as well as the other models are given in Table III. As one can see in this table, the r.m.s. values of present formulas are comparably good with the r.m.s. values of other models. According to r.m.s. values, equations (3), (5) and (7) derived from Eq. (1) can be said to give better results than equations (4), (6) and (8) derived from Eq. (2). It is noted that Eq. (5) including isospin effect has $\sigma = 0.4039$ that means that isospin effect reduces the r.m.s. value by 8.2% (0.4403 to 0.4039). Thus, it should be said that isospin effect on proton decay half-life is noticeable and it should be taken into account in the proton decay calculations. Equations (5) and (6) including the isospin terms in these forms could be used to calculate proton decays of nuclei. The ratio of the experimental obtained half-life values has also been given in figure 1. For the theoretical values, we have used the results obtained by Eqs. (5), (7) but it should be noted that

TABLE III

R.m.s. values for present models and empirical formula, UDLP, CPPM, Gamow-like models.

Eq. (3)	0.4207
Eq. (4)	0.4403
Eq. (5)	0.4039
Eq. (6)	0.4085
Eq. (7)	0.4026
Eq. (8)	0.4202
Empirical [43]	0.397
UDLP [43]	0.427
CPPM [43]	0.472
Gamow-like [43]	0.501

$\log_{10} T^{\rm exp}/\log_{10} T^{\rm theo}$ values for the element ^{157}Ta are 5.23 and 52.30 (not shown in the graphs), respectively. Thus, in the case of ^{157}Ta proton decay calculations, the desired theoretical value has not been obtained with not only shell-correction and isospin-dependent formulas but also other ones.

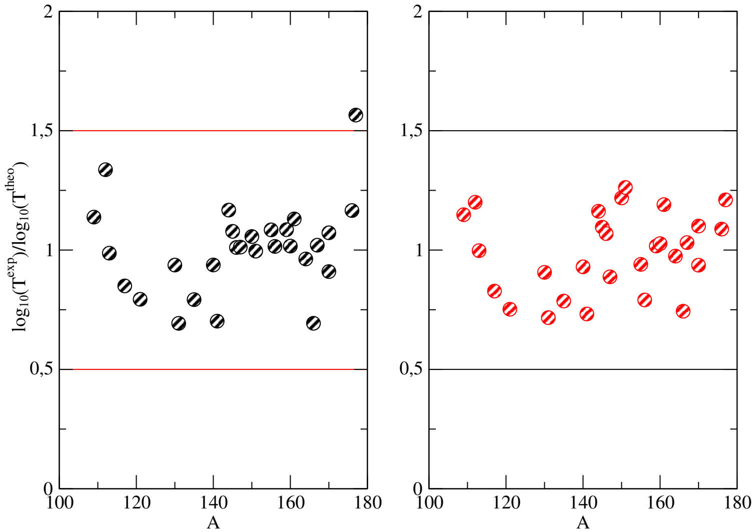

Fig. 1. Ratio between experimental and theoretical data for ground-state transitions. Theoretical data on the left-hand side have been obtained by Eq. (5) and theoretical data on the right-hand side have been obtained by Eq. (7).

After finding out that the best r.m.s. values for calculations of the proton decay half-lives are Eqs. (5) and (7), we have used these formulas to calculate the half-lives of possible proton decays of nuclei that do not yet have experimental values. Obtained results are given in Table IV. In this table, nucleus column shows the related nuclei decay, $\ell_{\rm min}$ shows minimum angular momentum for the related decay, Q_p presents Q-value that is taken form Ref. [43]. In the next columns, the results of other models have been listed. The lower experimental limit for some nuclei has been presented in the last column. Experimental values and the results of the other models are taken from Ref. [43]. As seen in Table IV, the present calculations give some new predictions for the related nuclei and they are consistent with other models and present experimental data. In particular, the calculation for $^{117}{\rm La}^m$ $-1.919, -1.983$ gives much better value than others and it is very close to experimental value. It is also underlined that the calculation for ^{108}I gives $0.805, 0.641$ which is better than the other results and also it is consistent with experimental value.

TABLE IV

$\log_{10} T_{1/2}$ (s) obtained by present study with other models and experimental values of the proton decay of nuclei that do not yet have experimental values.

Nucleus	ℓ_{min}	Q_p [MeV]	Eq. (5)	Eq. (7)	Ref. [43]	UDLP	Exp. [40, 41]
^{103}Sb	2	1.469	−9.541	−11.42	−9.902	−9.515	
^{104}Sb	2	0.519	2.329	1.683	1.890	1.278	> 0.827
^{105}Sb	2	0.331	9.638	9.679	9.216	7.980	> 3.049
^{108}I	2	0.610	0.805	0.641	0.433	−0.024	> 0.556
^{111}Cs	2	1.820	−10.43	−11.15	−10.751	−10.445	
^{116}La	2	1.091	−5.197	−5.517	−5.373	−5.456	
^{117m}La	4	0.951	−1.919	−1.983	−2.191	−2.155	≈ −1.989
^{127}Pm	2	0.922	−2.564	−2.399	−2.209	−2.514	
^{129}Pm	3	0.152	35.97	36.11	36.533	33.398	
^{137}Tb	5	0.843	2.835	3.032	2.977	2.717	
^{165}Ir	0	1.556	−5.569	−5.407	−5.593	−5.455	
^{169}Ir	5	0.780	7.876	7.549	8.088	7.404	
^{171m}Ir	5	0.402	21.41	20.69	21.952	20.396	
^{169}Au	0	1.947	−7.661	−7.508	−7.476	−7.569	
^{172}Au	2	0.877	3.818	3.583	3.991	3.586	
^{172m}Au	2	0.627	9.800	9.422	9.692	9.448	> 0.146
^{185}Bi	5	1.540	−0.301	−0.704	−0.721	−1.019	> −0.260
^{211}Pa	5	0.751	14.79	12.57	13.268	12.545	

Finally, the isospin and shell-dependent formulas developed here have been used to obtain the half-lives of actinide nuclei that have been recently studied with the CPPM in Ref. [28]. In that paper, authors have used the CPPM without including the isospin and shell terms directly to obtain proton decays of half-lives for actinide nuclei. The results obtained are as given in Table V. With the results obtained with the isospin-dependent (Eq. (5)) and shell-dependent (Eq. (7)) formulas in Table V, authors in Ref. [28] show the results obtained with the CPPM. The Q-values here are taken directly from Ref. [28]. In general, it is seen that the results of the formulas developed here are compatible with the results in Ref. [28]. Afterwards, the values in Table V were plotted by mass number in order to reveal which equation is better with the results obtained and to look at the general behavior of proton decay half-lives of actinide nuclei. As seen in Fig. 2, in the graphs obtained for 4 different actinide isotopes, even if the general behavior of graphs is the same, the isospin-dependent formula, Eq. (5), is closer to the results obtained with the CPPM model for actinide nuclei. This result may indicate that considering the isospin effects as in Eq. (5) may be important in explaining the proton decay of actinide nuclei. This situation supports the conclusion that the isospin effect is more dominant on the proton decay of the nucleus than the shell effect.

TABLE V

Proton decay $T_{1/2}$ (s) obtained by Eq. (5) and Eq. (7) of actinide nuclei with the results of CPPM model calculations in Ref. [28].

Parent	Q_{exp}	Eq. (5)	Eq. (7)	Ref. [28]	Parent	Q_{exp}	Eq. (5)	Eq. (7)	Ref. [28]
^{195}Ac	2.161	$3.123e-07$	$2.207e-07$	$1.892e-07$	^{219}Am	1.231	$9.725e+03$	$1.610e+02$	$3.264e+03$
^{196}Ac	1.591	$2.346e-03$	$1.145e-03$	$2.907e-03$	^{220}Am	0.971	$1.692e+08$	$1.342e+06$	$1.240e+08$
^{197}Ac	1.591	$2.822e-03$	$1.196e-03$	$2.923e-03$	^{221}Am	0.801	$1.214e+12$	$4.631e+09$	$1.700e+12$
^{198}Ac	1.321	$1.445e+00$	$4.169e-01$	2.294	^{222}Am	0.551	$3.183e+20$	$4.047e+17$	$3.189e+21$
^{199}Ac	1.331	$1.435e+00$	$3.273e-01$	1.791	^{223}Am	0.341	$5.947e+33$	$1.503e+30$	$7.923e+36$
^{200}Ac	1.441	$1.366e-01$	$2.539e-02$	$9.452e-02$	^{224}Am	1.181	$4.523e+05$	$5.935e+02$	$2.016e+04$
^{201}Ac	1.371	$9.550e-01$	$1.238e-01$	$6.087e-01$	^{215}Cm	0.221	$4.031e+47$	$1.051e+45$	$4.832e+55$
^{202}Ac	0.971	$3.701e+05$	$2.242e+04$	$6.797e+05$	^{218}Bk	2.241	$1.796e-05$	$2.606e-06$	$4.142e-06$
^{203}Ac	1.018	$8.389e+04$	$3.616e+03$	$8.762e+04$	^{219}Bk	2.231	$2.698e-05$	$3.259e-06$	$4.690e+06$
^{204}Ac	0.595	$2.675e+15$	$3.441e+13$	$2.889e+16$	^{220}Bk	1.841	$1.050e-02$	$8.709e-04$	$2.799e-03$
^{205}Ac	0.707	$9.595e+11$	$9.275e+09$	$2.616e+12$	^{221}Bk	1.871	$8.870e-03$	$5.494e-04$	$1.547e-03$
^{206}Ac	0.383	$4.099e+26$	$1.076e+24$	$8.225e+28$	^{222}Bk	1.611	$1.471e+00$	$5.494e-02$	$3.283e-01$
^{207}Ac	0.277	$2.718e+36$	$2.558e+33$	$9.965e+39$	^{223}Bk	1.461	$5.833e+01$	$1.342e+00$	$1.416e+01$
^{198}Th	0.291	$7.267e+33$	$2.636e+32$	$1.883e+34$	^{224}Bk	1.171	$2.885e+05$	$3.208e+03$	$1.300e+05$
^{199}Th	0.201	$6.471e+46$	$8.070e+44$	$3.350e+54$	^{225}Bk	0.841	$4.956e+11$	$1.855e+09$	$9.210e+11$
^{200}Pa	2.111	$1.748e-06$	$1.039e-06$	$1.073e-06$	^{226}Bk	0.501	$4.151e+23$	$3.561e+20$	$2.298e+25$
^{201}Pa	2.091	$2.682e-06$	$1.412e-06$	$1.381e-06$	^{227}Bk	0.771	$6.986e+13$	$1.052e+11$	$9.165e+13$
^{202}Pa	1.751	$5.343e-04$	$1.997e-04$	$3.988e-04$	^{221}Cf	0.281	$1.986e+40$	$7.298e+37$	$6.941e+46$
^{203}Pa	1.491	$1.020e-01$	$2.613e-02$	$1.099e-01$	^{224}Es	2.181	$1.593e-04$	$1.735e-05$	$2.976e-05$
^{204}Pa	1.221	$1.224e+02$	$1.923e+01$	$2.202e+02$	^{225}Es	2.181	$2.171e-04$	$1.748e-05$	$2.995e-05$
^{205}Pa	1.391	$1.749e+00$	$2.359e-01$	1.423	^{226}Es	1.771	$1.710e-01$	$7.417e-03$	$3.589e-02$
^{206}Pa	0.981	$9.075e+05$	$5.637e+04$	$2.375e+06$	^{227}Es	1.541	$2.461e+01$	$1.369e-01$	6.626
^{207}Pa	1.221	$3.138e+02$	$1.747e+01$	$2.244e+02$	^{228}Es	1.251	$6.788e+04$	$1.901e+02$	$3.454e+04$
^{208}Pa	0.801	$9.540e+09$	$1.892e+08$	$3.388e+10$	^{229}Es	0.821	$5.823e+12$	$2.547e+10$	$2.132e+13$
^{209}Pa	0.801	$1.412e+10$	$1.855e+08$	$3.412e+10$	^{230}Es	0.441	$6.945e+27$	$5.610e+24$	$4.157e+30$
^{212}Pa	0.42	$2.536e+25$	$3.925e+22$	$1.197e+27$	^{231}Es	0.421	$2.183e+29$	$1.012e+26$	$1.719e+32$
^{213}Pa	0.283	$1.693e+37$	$6.530e+33$	$8.563e-09$	^{229}Md	2.251	$1.974e-04$	$4.628e-06$	$2.883e-05$
^{203}U	0.381	$8.963e+26$	$4.233e+25$	$1.184e+31$	^{230}Md	1.921	$3.184e-02$	$4.242e-04$	$6.636e-03$
^{206}Np	1.911	$1.084e-04$	$4.291e-05$	$7.471e-05$	^{231}Md	1.871	$1.021e-01$	$3.184e-03$	$1.705e-02$
^{207}Np	1.881	$2.131e-04$	$6.757e-05$	$1.281e-04$	^{232}Md	1.441	$1.011e+03$	$1.474e+01$	$3.662e+02$
^{208}Np	1.751	$2.325e-03$	$5.217e-04$	$1.362e-03$	^{233}Md	1.381	$6.951e+03$	$6.354e+01$	$2.222e+03$
^{209}Np	1.691	$8.898e-03$	$1.467e-03$	$4.677e-03$	^{234}Md	1.001	$4.072e+09$	$1.619e+07$	$4.975e+09$
^{210}Np	1.301	$7.255e+01$	$6.280e+00$	$7.631e+01$	^{236}Md	0.911	$6.197e+11$	$1.052e+09$	$5.909e+11$
^{211}Np	1.561	$2.080e-01$	$1.451e-02$	$8.025e-02$	^{237}Md	0.561	$6.530e+22$	$2.721e+19$	$1.509e+24$
^{212}Np	1.151	$1.244e+04$	$4.130e+02$	$1.193e+04$	^{238}Md	0.591	$5.294e+21$	$1.642e+18$	$4.383e+22$
^{213}Np	1.181	$6.865e+03$	$1.809e+02$	$4.072e+03$	^{239}Md	0.251	$1.336e+48$	$2.806e+43$	$1.661e+54$
^{214}Np	0.771	$4.885e+11$	$5.462e+09$	$1.673e+12$	^{232}No	0.831	$9.133e+12$	$2.907e+10$	$1.936e+14$
^{215}Np	0.811	$7.424e+10$	$6.149e+08$	$1.286e+11$	^{233}No	0.331	$2.899e+37$	$3.442e+34$	$6.697e+42$
^{216}Np	0.471	$1.731e+23$	$3.611e+20$	$8.426e+24$	^{235}Lr	2.161	$2.786e-03$	$1.024e-04$	$3.273e-04$
^{217}Np	0.541	$1.029e+20$	$1.712e+17$	$9.050e+20$	^{236}Lr	1.781	$1.727e+00$	$3.627e-02$	$3.455e-01$
^{209}Pu	0.351	$3.331e+30$	$7.235e+28$	$1.432e+35$	^{237}Lr	1.731	$6.361e+00$	$9.442e-02$	1.017
^{212}Am	2.051	$5.542e-05$	$1.290e-05$	$2.267e-05$	^{238}Lr	1.361	$4.862e+04$	$3.900e+02$	$1.660e+04$
^{213}Am	1.951	$3.046e-04$	$5.113e-05$	$1.204e-04$	^{239}Lr	1.281	$7.370e+05$	$3.710e+03$	$2.455e+05$
^{214}Am	1.911	$7.408e-04$	$8.464e-05$	$2.451e-04$	^{240}Lr	0.811	$5.476e+14$	$8.902e+11$	$2.027e+15$
^{215}Am	1.931	$7.340e-04$	$6.027e-05$	$1.709e-04$	^{241}Lr	0.901	$5.542e+12$	$7.089e+09$	$6.610e+12$
^{216}Am	1.601	$3.427e-01$	$1.915e-02$	$1.176e-01$	^{242}Lr	0.611	$3.875e+21$	$1.575e+18$	$5.149e+22$
^{217}Am	1.531	$2.115e+00$	$8.833e-02$	$6.138e-01$	^{243}Lr	0.601	$1.566e+22$	$3.980e+18$	$1.512e+23$
^{218}Am	1.191	$2.257e+04$	$5.283e+02$	$1.318e+04$					

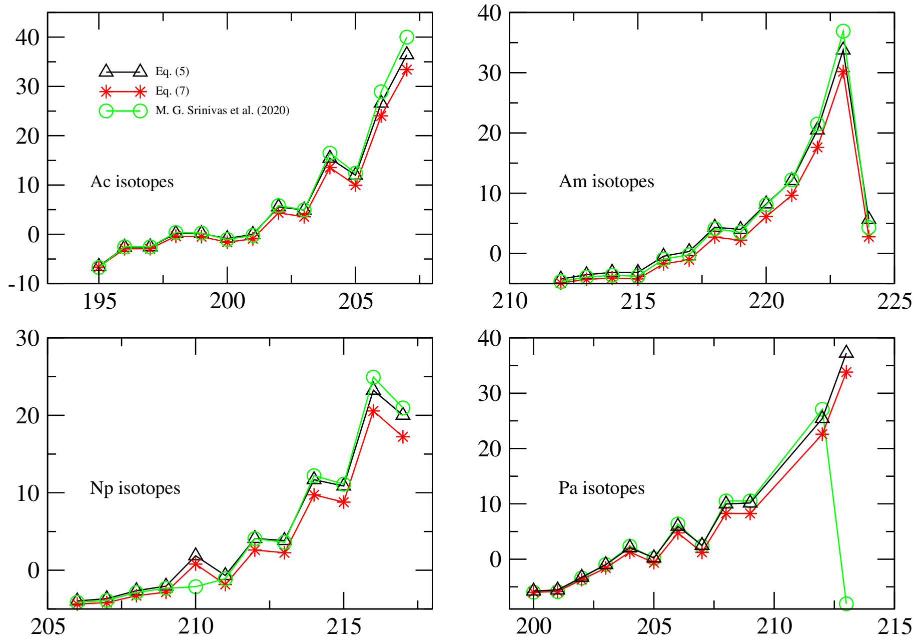

Fig. 2. Proton decay $\log_{10} T_{1/2}$ (s) obtained by Eq. (5) and Eq. (7) and the results of Ref. [28] of actinide nuclei *versus* mass number for Ac, Am, Np and Pa isotopes.

4. Summary

In this study, we have modified the formulas of proton decay half-lives by considering the isospin and shell effects. By using these formulas, we have calculated the proton decay half-lives of 44 proton emitters in the ground state or isomeric state. It has been shown that the isospin and shell effects reduce the r.m.s. of calculations, and isospin term is more effective than shell-effect term in particular cases. To the best of our knowledge, this has been the first study in the literature investigating the effect of isospin on the proton decay adding a term in the closed formula. Besides, it should be underlined that equations derived from Eq. (1) give better results than equations derived from Eq. (2). Moreover, we have used the formula proposed for the isospin to predict the proton decay half-lives of some proton emitters and actinide nuclei. For a few nuclei, much more consistent results with the experimental limits were obtained. The method presented here represents an alternative way to proton radioactivity calculations. However, there is still a need for a little discussion on deformation cases of the nucleus that can be taken into account for calculations. Since the nuclear deformation of the nucleus has also noticeable effects on proton radioactivity, it has been provided with a new formula by Dehghani and Alavi [10] very recently.

In their analysis, thirteen percent of reduction on the r.m.s. deviation has been reported. Besides, our calculations showed that the isospin term is also effective in the calculations. The evidence from this study intimates that a new equality which contains both isospin and deformation effects might be more functional for the proton radioactivity calculations. A considerable attention should be paid on advancing the present and previous methods. Finally, the results obtained in this paper would be important for future possible theoretical and experimental studies in this field.

REFERENCES

[1] K.P. Jackson *et al.*, «^{53}Co^{m}: A proton-unstable isomer», *Phys. Lett. B* **33**, 281 (1970).

[2] S. Hofmann *et al.*, in: P.G Hansen, O.B. Nielsen (Eds.) «Proceedings of the 4$^{\rm th}$ International Conference on Nuclei Far from Stability. Vol. 1», *Helsingør*, Denmark 1981, p. 190.

[3] T. Faestermann *et al.*, «Evidence for proton radioactivity of ^{113}Cs and ^{109}I», *Phys. Lett. B* **137**, 23 (1984).

[4] O. Klepper *et al.*, «Direct and beta-delayed proton decay of very neutron-deficient rare-earth isotopes produced in the reaction ^{58}Ni+^{92}Mo», *Z. Physik A* **305**, 125 (1982).

[5] R.D. Page *et al.*, «Decays of odd–odd $N-Z=2$ nuclei above ^{100}Sn: The observation of proton radioactivity from ^{112}Cs», *Phys. Rev. Lett.* **72**, 1798 (1994).

[6] C.N. Davids *et al.*, «Proton Decay of an Intruder State in ^{185}Bi», *Phys. Rev. Lett.* **76**, 592 (1996).

[7] C.N. Davids *et al.*, «Proton Radioactivity from Highly Deformed Nuclei», *Phys. Rev. Lett.* **80**, 1849 (1998).

[8] C. Qi, D.S. Delion, R.J. Liotta, R. Wyss, «Effects of formation properties in one-proton radioactivity», *Phys. Rev. C* **85**, 011303 (2012).

[9] E. Maglione, L.S. Ferreira, «New developments in the theory of proton radioactivity», *Eur. Phys. J. A* **15**, 89 (2002).

[10] V. Dehghani, S.A. Alavi, «Empirical formulas for proton decay half-lives: Role of nuclear deformation and Q-value», *Chin. Phys. C* **42**, 104101 (2018).

[11] B. Buck, A.C. Merchant, S.M. Perez, «Ground state proton emission from heavy nuclei», *Phys. Rev. C* **45**, 1688 (1992).

[12] D.N. Basu, P.R. Chowdhury, C. Samanta, «Folding model analysis of proton radioactivity of spherical proton emitters», *Phys. Rev. C* **72**, 051601 (2005).

[13] B. Sahu, R. Paira, B. Rath, «General decay law for emission of charged particles and exotic cluster radioactivity», *Nucl. Phys. A* **908**, 40 (2013).

[14] K.P. Santhosh, I. Sukumaran, «Description of proton radioactivity using the Coulomb and proximity potential model for deformed nuclei», *Phys. Rev. C* **96**, 034619 (2017).

[15] K.P. Santhosh, I. Sukumaran, «Predictions of proton emissions from the isotopes of nuclei with $Z = 100$–136 using the Coulomb and proximity potential model for deformed nuclei», *Eur. Phys. J. A* **54**, 102 (2018).

[16] Y. Hatsukawa, H. Nakahara, D.C. Hoffman, «Systematics of alpha decay half-lives», *Phys. Rev. C* **42**, 674 (1990).

[17] S.A. Alavi, V. Dehghani, M. Sayahi, «Calculation of proton radioactivity half-lives», *Nucl. Phys. A* **977**, 49 (2018).

[18] M. Balasubramaniam, N. Arunachalam, «Proton and α-radioactivity of spherical proton emitters», *Phys. Rev. C* **71**, 014603 (2005).

[19] E. Maglione, L.S. Ferreira, R.J. Liotta, «Nucleon Decay from Deformed Nuclei», *Phys. Rev. Lett.* **81**, 538 (1998).

[20] Y.Z. Wang *et al.*, «Competition between decay and proton radioactivity of neutron-deficient nuclei», *Phys. Rev. C* **95**, 014302 (2017).

[21] Z.X. Zhang, J.M. Dong, «A formula for half-life of proton radioactivity», *Chin. Phys. C* **42**, 014104 (2018).

[22] M. Bhattacharya, G. Gangopadhyay, «Microscopic calculation of half lives of spherical proton emitters», *Phys. Lett. B* **651**, 263 (2007).

[23] J.M. Dong, H.F. Zhang, G. Royer, «Proton radioactivity within a generalized liquid drop model», *Phys. Rev. C* **79**, 054330 (2009).

[24] T.R. Routray *et al.*, «Proton radioactivity with a Yukawa effective interaction», *Eur. Phys. J. A* **47**, 92 (2011).

[25] M. Balasubramaniam, N. Arunachalam, «Proton and α-radioactivity of spherical proton emitters», *Phys. Rev. C* **71**, 014603 (2005).

[26] J.M. Dong, H.F. Zhang, W. Zuo, J.Q. Li, «Unified fission model for proton emission», *Chin. Phys. C* **34**, 182 (2010).

[27] Q. Zhao, J.M. Dong, J.L. Song, W.H. Long, «Proton radioactivity described by covariant density functional theory with the similarity renormalization group method», *Phys. Rev. C* **90**, 054326 (2014).

[28] M.G. Srinivas *et al.*, «Proton decay of actinide nuclei», *Nucl. Phys. A* **995**, 121689 (2020).

[29] J.M. Dong, H.F. Zhang, G. Royer, «Proton radioactivity within a generalized liquid drop model», *Phys. Rev. C* **79**, 054330 (2009).

[30] T.R. Routray *et al.*, «Proton radioactivity half-lives with Skyrme interactions», *Eur. Phys. J. A* **48**, 77 (2012).

[31] L.S. Ferreira, E. Maglione, P. Ring, «Self-consistent description of proton radioactivity», *Phys. Lett. B* **701**, 508 (2011).

[32] D.S. Delion, R.J. Liotta, R. Wyss, «Systematics of Proton Emission», *Phys. Rev. Lett.* **96**, 072501 (2006).

[33] V.Y. Denisov, O.I. Davidovskaya, I.Y. Sedykh, «Improved parametrization of the unified model for decay and capture», *Phys. Rev. C* **92**, 014602 (2015).

[34] E. Shin *et al.*, «Nuclear isospin asymmetry in decay of heavy nuclei», *Phys. Rev. C* **94**, 024320 (2016).

[35] K.P. Santhosh, T.A. Jose, «Alpha and cluster decay using Modified Generalized Liquid Drop Model with iso-spin dependent pre-formation factor», *Nucl. Phys. A* **992**, 121626 (2019).

[36] D.T. Akrawy, H. Hassanabadi, S.S. Hosseini, K.P. Santhosh, «Nuclear isospin effect on α-decay half-lives», *Nucl. Phys. A* **975**, 19 (2018).

[37] K.P. Santhosh, C. Nithya, «Theoretical studies on the modes of decay of superheavy nuclei», *Phys. Rev. C* **94**, 054621 (2016).

[38] X.J. Bao *et al.*, «Competition between α-decay and spontaneous fission for superheavy nuclei», *J. Phys. G: Nucl. Part. Phys.* **42**, 085101 (2015).

[39] P. Möller *et al.*, «Nuclear ground-state masses and deformations: FRDM(2012)», *At. Data Nucl. Data Tables* **109–110**, 1 (2016).

[40] D.T. Joss *et al.*, «Probing the limit of nuclear existence: Proton emission from ^{159}Re», *Phys. Lett. B* **641**, 34 (2006).

[41] B. Blank, M.J.G. Borge, «Nuclear structure at the proton drip line: Advances with nuclear decay studies», *Prog. Part. Nucl. Phys.* **60**, 403 (2008).

[42] A.A. Sonzogni, «Nuclear Data Sheets for $A = 136$», *Nucl. Data Sheets* **95**, 837 (2002).

[43] J.L. Chen *et al.*, «New Geiger–Nuttall law for proton radioactivity», *Eur. Phys. J. A* **55**, 214 (2019).