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Special Issue
Heavy-Ion Collisions and Multiparticle Production

Edited by
Prof. Dr. Qun Wang, Prof. Dr. Zuotang Liang and Prof. Dr. Enke Wang

MDPI

https://doi.org/10.3390/sym15030695

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T_{cs} and $T_{c\bar{s}}$ Family Production in Multi-Production Processes

Chuanhui Jiang [1], Yi Jin [2,3,*], Shi-Yuan Li [1,*], Yan-Rui Liu [1] and Zong-Guo Si [1]

1 School of Physics, Shandong University, Jinan 250100, China
2 School of Physics and Technology, University of Jinan, Jinan 250022, China
3 Guangxi Key Laboratory of Nuclear Physics and Nuclear Technology, Guangxi Normal University, Guilin 541004, China
* Correspondence: ss_jiny@ujn.edu.cn (Y.J.); lishy@sdu.edu.cn (S.-Y.L.)

Abstract: The production mechanism of multiquark exotic hadrons in high energy multiproduction processes lies in the structure of the relevant exotic hadrons as well as in some important aspects of high energy scattering, such as multi-parton interactions, underlying events, etc. At mass pole around 2900 MeV, a family of open charm tetraquarks, T_{cs}s and $T_{c\bar{s}}$s, are observed in B decay. They are also suitable for study in multiproduction processes to obtain more information on their structure. If these resonances are produced as compact four-quark states, one can predict the production properties based on the similarities in their production mechanism to those of Ξ_c, Σ_c, and Λ_c. Physics implies that the colour and baryon number fluctuations of the preconfinement system in high energy scattering can enhance both the baryon and four-quark state production rates via 'diquark fragmentation'. We calculate the production properties of the tetraquark family T_{cs}s and $T_{c\bar{s}}$s at LHC energy for the forthcoming LHC measurements.

Keywords: 2 tetraquark; multi-production; corresponding baryon

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Citation: Jiang, C.; Jin, Y.; Li, S.-Y.; Liu, Y.-R.; Si, Z.-G. T_{cs} and $T_{c\bar{s}}$ Family Production in Multi-Production Processes. *Symmetry* **2023**, *15*, 695. https://doi.org/10.3390/sym15030695

Academic Editor: Jorge Segovia

Received: 30 December 2022
Revised: 14 February 2023
Accepted: 6 March 2023
Published: 10 March 2023

1. Introduction

It has long been recognized that multiproduction at high energy is very important to understand various properties of QCD and its confinement mechanism. Its global property is related to the hadronic scattering total cross-section [1,2]. On the other hand, once the production of a specific hadron is considered, the structure of the hadron is also crucially relevant, besides its properties with respect to all the other hadrons. In the quark model of a hadron, mesons and baryons are quark-antiquark hadrons and three-quark hadrons, respectively. (Anti)quarks are bound by strong interactions and they are in colour singlet state. SU(3) colour symmetry also allows clusters of >3 (anti)quarks in colour singlet state. It is possible that these kinds of colour singlet clusters with modest mass are 'real particles', and they are called 'exotic hadrons' [3]. Among all the possibilities, four-quark states ($qq\bar{q}\bar{q}$, tetraquark) and five-quark states ($qqqq\bar{q}$, pentaquark), containing at least one heavy quark (c or b), have been observed in various experiments. Almost all the four- and five-quark states have been observed from the decay of heavier hadrons (e.g., bottom hadrons), rather than being promptly produced from multiproduction processes in high energy scattering. Theoretically, this fact can be understood by that their production rate is generally quite small in multiproduction processes because of the unitarity constraint [4,5], as well as the modest mass of the preconfinement clusters [6,7]. This conclusion is one of the most straightforward deductions from the quark model of a hadron. However, a few exceptions are observed from multiproduction at the LHC, and these exotics contain more heavy quarks and hence cannot be produced from heavier particle (e.g., B meson) decay [8–10]. Frankly, no rule in QCD says that only very heavy exotics can be produced from multiproduction, though one may complain that the "trigger" of the perturbative heavy quark production can modify the unitarity constraint of hadronization or that the preconfinement scale of heavy quark clusters can be different from pure light ones [6]. Therefore, it is a crucial goal to study exotic hadrons which are produced *both* from multiproduction and from B decay to understand more details of exotic hadron properties and structures. Recently, a group of tetraquark brothers and sisters of T_{cs} and $T_{c\bar{s}}$ were observed by LHCb from B decay (see Tables 1 and 2 and refs. therein). Their masses varied around 2900 MeV; hence, in this paper we call them the T(2900) family if they are not specified otherwise. It should also be possible to produce this T(2900) family in multiproduction process at the LHC. For the purpose of observing them in multiproduction, it is very important to predict their production rate as well as their kinematic distributions, since the initial state of multiproduction is different from, or rather more complex than, that of the B decay. In particular, without a simple and definite initial state, analysis methods such as partial wave analysis cannot be applied.

To investigate and answer such questions, the key problem is how to describe the production of these tetraquarks, i.e., how these four ingredient quarks are produced and how they become a hadron. This is a very complex problem and the mechanism could be model-dependent, i.e., relevant to the concrete hadronization models [11–13] adopted, which have yet not been well tuned for exotic hadrons. On the other hand, one may rely on other hadrons which have been well measured and hence can be tuned to be well described by a model, more concretely, an event generator. Once the production of such hadrons is fixed, one can deduce the corresponding kinematics for the tetraquark based on the relation of their production mechanism, which is not dependent on concrete hadronization models but only determined by the structure of the tetraquark. The idea is similar to our study on doubly heavy tetraquarks, T_{cc} [14]. In that case, the relation to Ξ_{cc} was mainly determined by the cc doubly heavy diquark. In the present case, we can refer to Λ_c, Σ_c and Ξ_c for the study of the kinematic spectra as well as production rate of the T(2900) family. The relations are mainly determined by the diquark which is constructed by a charm quark with a light quark. To explain this point, we again start from the colour structure of the tetraquark and demonstrate its relation with the above 'corresponding baryons'.

It is a common property of all kinds of multiquark hadrons that the bound (anti)quarks inside a multiquark hadron can be grouped into several clusters. The ways of grouping these (anti)quarks are not unique, as it is simply known from group theory that the reduction of a direct product of several representations is not unique. In some grouping ways, each cluster can be a colour singlet, indicating a picture of 'hadron molecule'. However, such a way of clustering the (anti)quarks is not necessary, since the only requirement is that the *whole* set of these clusters are a colour singlet. For example, the system $q_1\bar{q}_2q_3\bar{q}_4$ (the constituents of a four-quark state) can be decomposed/clustered in the following various ways:

$$(q_1q_3)_{\bar{3}} \otimes (\bar{q}_2\bar{q}_4)_3 \to 1 \tag{1}$$

$$(q_1\bar{q}_2)_{1\ or\ 8} \otimes (q_3\bar{q}_4)_{1\ or\ 8} \to 1 \tag{2}$$

$$\cdots$$

In the above examples, only the second case, when the two $q\bar{q}$ pairs are both colour singlets, could it potentially be considered as a hadron molecule. Group theory analysis here is applicable to the quark states as well as to the quark field operators [15]. If we go beyond such a static picture of the structure of multiquark hadrons, considering the situation more dynamically, the colour interactions in the system via exchanging gluons can change the colour state of each individual cluster, so each kind of grouping/reduction can change to the others and seems to have no special physical cause. Such an ambiguity has been considered in many hadronization and decay processes in the framework of 'colour recombination/rearrangement' [16–18]. The above fact makes the possibility of considering the multiquark hadron structure in a unique and uniform way difficult, while it leads to the possibility of introducing some phenomenological duality. It is easy to find that it is possible for multiquark hadrons to produce as 'hadron molecule formation', but the subsequent colour interactions in the system can eventually transit this 'molecule' into a 'real' (compact) multiquark hadron, at least by some probability, and vice versa. Hence, as is argued in [14], this ambiguity just shows that in cases where various kinds of model explain the data of the static properties well, such as masses and decay widths, and we cannot determine the structure from the static properties (a 'compact' multiquark state or a hadron molecule), we can still try to investigate the production mechanism of the T(2900) family relying on only their quark ingredients (necessarily requiring four quarks) and leaving out the details of the structure. If there are significant differences in the production properties among the T(2900) family members observed in the multiproduction processes, these signals are for sure a sound implication of their structure details. In this case, this family can be taken as an excellent example for the study of the production mechanism to gain insight into the exotic hadronic structure.

The T(2900) family has been long studied theoretically (for a theoretical review, see [19] and refs. therein). All T(2900) family members are listed in Tables 1 and 2 (with references). Some have been observed from B decay by the LHCb collaboration.

Table 1. Considered and observed tetraquark states for the P even case. The subscript J denotes the spin of the state: 0, 1, or 2.

Contents	States	Observed	Mass	Width
$cs\bar{u}\bar{u}$	T^{a-}_{csJ}			
$cs\bar{u}\bar{d}$	T^{a0}_{csJ}, T^{f0}_{csJ}	$T_{cs0}(2900)^0$ [20,21]	$2.866 \pm 0.007 \pm 0.002$ GeV	$57 \pm 12 \pm 4$ MeV
$cs\bar{d}\bar{d}$	T^{a+}_{csJ}			
$c\bar{s}u\bar{d}$	$T^{a++}_{c\bar{s}J}$	$T^{a}_{c\bar{s}0}(2900)^{++}$ [22,23]	$2.908 \pm 0.011 \pm 0.020$ GeV	$0.136 \pm 0.023 \pm 0.011$ GeV
$c\bar{s}u\bar{u}$, $c\bar{s}d\bar{d}$	$T^{a+}_{c\bar{s}J}$, $T^{f+}_{c\bar{s}J}$			
$c\bar{s}d\bar{u}$	$T^{a0}_{c\bar{s}J}$	$T^{a}_{c\bar{s}0}(2900)^0$ [22,23]	$2.908 \pm 0.011 \pm 0.020$ GeV	$0.136 \pm 0.023 \pm 0.011$ GeV

Table 2. Considered and observed tetraquark states for the P odd case. The subscript J denotes the spin of the state: 0, 1, 2, or 3.

Contents	States	Observed	Mass	Width
$cs\bar{u}\bar{u}$	$T^{\pi-}_{csJ}$			
$cs\bar{u}\bar{d}$	$T^{\pi0}_{csJ}$, $T^{\eta0}_{csJ}$	$T_{cs1}(2900)^0$ [20,21]	$2.904 \pm 0.005 \pm 0.001$ GeV	$110 \pm 11 \pm 4$ MeV
$cs\bar{d}\bar{d}$	$T^{\pi+}_{csJ}$			
$c\bar{s}u\bar{d}$	$T^{\pi++}_{c\bar{s}J}$			
$c\bar{s}u\bar{u}$, $c\bar{s}d\bar{d}$	$T^{\pi+}_{c\bar{s}J}$, $T^{\eta+}_{c\bar{s}J}$			
$c\bar{s}d\bar{u}$	$T^{\pi0}_{c\bar{s}J}$			

From the content of the quark components and considering their colour structure ambiguity, as discussed in the above paragraphs, we can conjecture the following correspondence between the T(2900) family and the charm hyperon for the study of the production in multiproduction processes:

(1) For the T_{cs} brothers and sisters, their production can be treated as the hadronization of the cs diquark and hence can be related to Ξ_c. The difference between a cs diquark transiting to a Ξ_c or a T_{cs} is that this diquark combines a light quark (u or d) or a light anti-diquark ($\bar{u}\bar{d}$, $\bar{u}\bar{u}$, or $\bar{d}\bar{d}$).

(2) According to a similar consideration, $T_{c\bar{s}}$ brothers and sisters can be considered as the fragmentation of a cq ($q = u$ or d, the same in the following) diquark. Once it combines with a $\bar{s}\bar{q}$ anti-diquark, it becomes one $T_{c\bar{s}}$, while it will become a Λ_c or Σ_c baryon by combining a light quark u or d.

We adopt the 'partially breaking $SU_f(3)$ symmetry' in hadronization, with the production ratio of the quark flavours created from vacuum by strong interactions as $u : d : s = 1 : 1 : \lambda$. At the same time, the light diquark production rate has a suppression factor

ζ [12]. By fixing these ratios with the above 'correspondence', we can just fully employ the production properties of Λ_c, Σ_c, and Ξ_c to predict the production of the T(2900) family.

2. Kinematic Spectra of Corresponding Baryons and Production of T(2900) Family at the LHC

As is mentioned in last section, the $T_{cs}/T_{c\bar{s}}$ four-quark states can be taken as the combination of a cs/cq cluster (diquark) with a light anti-diquark $\bar{q}\bar{q}'/\bar{s}\bar{q}'$ produced from strong interactions. In high energy processes such as proton–proton collisions at the LHC, charm quarks are produced from hard scattering (including the semi-hard multiparton interactions) and then evolve to softer scales via radiations. Before the hadronization process, preconfinement [7] clusters form, which contain charm quarks as well as light quarks produced during the evolution radiation and other softer strong interactions. The preconfined system is the end of perturbative QCD (PQCD) evolution, with the formation of colour singlet clusters. These clusters have modest masses independent from the centre-of-mass energy of the scattering process. Each colour singlet cluster will independently hadronize into hadrons. There is much evidecne for preconfinement cluster formation in high energy processes, one of which is the local parton hadron duality (for details, see [6] and refs. therein). It has long been argued that the colour structure of the preconfined system is not unique [17,18]. It has also been recognized that different colour structures will lead to different non-trivial baryon number distributions of the colour singlet clusters, which is referred to as the baryon number fluctuation of the preconfined system [6]. One of the colour structures (Equation (1)) favours the production of both charm hyperons and tetraquarks. In concrete calculations, the proton–proton scattering occuring at very high energy as the LHC suffers complexity of initial states, multi-parton scattering, and underlying events. It is impossible to employ simple analytical formulations to describe such complex processes with the special colour connections. Thus, we have to employ event generators as a practical framework to perform the calculation. Though most event generators can help to describe the initial states, multi-parton scattering, and underlying events (we will not repeat the details of the models here), we still have to solve two linked difficulties: The first is that no event generator has been sufficiently well developed to describe exotic hadrons. This difficulty, specified in our concrete problem, can be solved by the idea introduced in the above section, i.e., to employ the 'corresponding' baryons as a calibration to describe the T(2900) family. However, this idea at once encounters another difficulty, i.e., the generators fail to describe the baryon production data well. The calculated results are much lower than the data [24]. Therefore, here we must first try to find ways to tune the generator according to the baryon production data. The key point is to find the physics which is missing for the production of baryons (hence also of tetraquarks).

For the feasibility of the discussion, let us first take Ξ_c production as an example. Here, we employ the PYTHIA generator with the Lund string 'fragmentation' model [12,25] as the basic framework to calculate the differential cross-section. Its results are much lower than the experimental data [24], and we must add the missing contributions. As demonstrated in [26], a higher twist contribution which we refer to as 'combination' has to be taken into account and added to the lowest twist 'fragmentation', with the latter corresponding to the string fragmentation in the PYTHIA. As described in [26], a cs diquark combined with a light quark produced from the underlying events can be described by the following inclusive differential cross-section,

$$2E\frac{d\sigma}{d^3K} = \sum_{ab}\int dx_1 dx_2 f_1^a(x_1) f_2^b(x_2) \frac{d\hat{\sigma}_{ab}}{d\mathcal{I}} \frac{1}{\xi^2} \frac{(2\pi)^2}{(2M)^2} P(\xi_l) \tilde{F}(\xi, \xi_l)|_{\xi+\xi_l=1}. \tag{3}$$

In the above equation, $d\mathcal{I}$ is the dimensionless invariant phase space for the 'two-body' partonic final state $cs + x$, where x is treated as one particle. At present, the relevant matrix elements describing the combination contribution are yet unknown. Motivated by the results of [26], we can parameterize the combination contribution according to its exponential behaviour and add it to the PYTHIA result:

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}y\mathrm{d}p_T}\right)_{mod} = \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}y\mathrm{d}p_T}\right)_{pythia} \cdot a + \alpha e^{-\beta p_T}. \tag{4}$$

By fitting the best LHC experimental data, we obtain $a = 1.000$, $\alpha = 401.574$ μb GeV^{-1}c, and $\beta = 0.961$ (GeV/c)$^{-1}$, respectively. The result can be seen in Figure 1, and we obtain a good description of the data by adding the 'combination contribution'. Here, we would like to take the chance to clarify the terminology 'combination' in various circumstances:

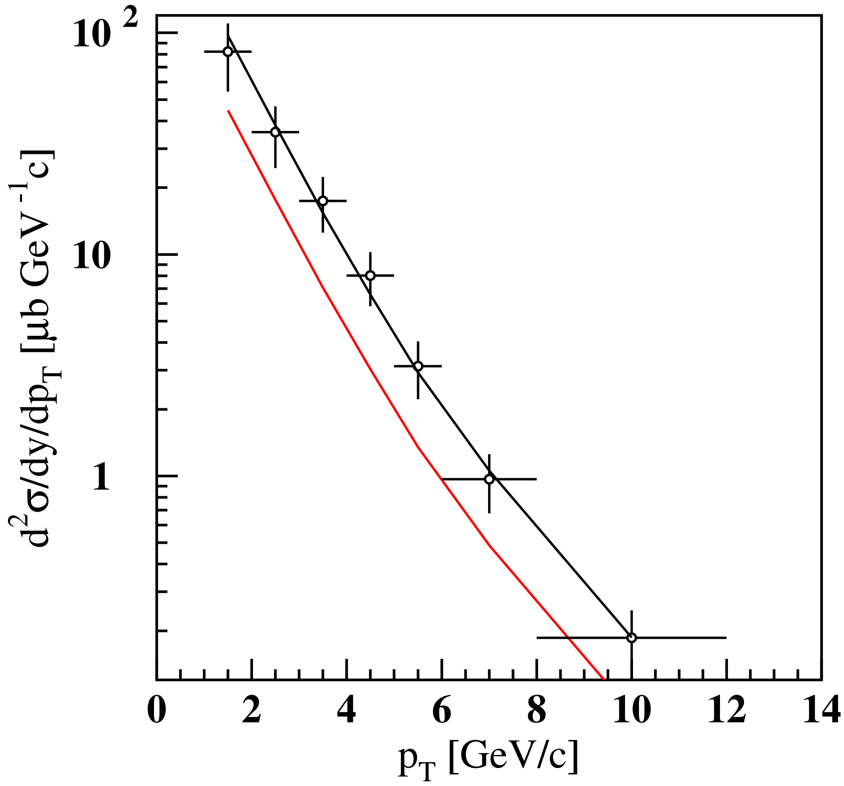

Figure 1. Cross-section of the prompt Ξ_c^0 as a function of p_T in pp collisions at $\sqrt{s}$ = 13 TeV. The black circles are the experimental data [24], while the black solid line is for Equation (4) with the fitted parameters. The red solid line is for the prediction of the distribution of the T_{cs}. The isospin and spin symmetry is adopted. We have averaged among all the isospin and spin states.

(1) In the case of PQCD factorization, this terminology refers to a higher twist contribution described by some matrix elements and can be added with the lowest twist 'fragmentation' contribution, as in [26]. At high energies, such as in the LHC, the underlying events highly contribute to the production of soft quarks; the combination contribution can hence be largely enhanced. The fitted results shown above are consistent with this consideration. Correspondingly, the result calculated by PYTHIA is only the lowest twist fragmentation contribution, which is described by the string fragmentation model, and one can find the fragmentation function is effectively that in [25]:

$$f(z) \propto z^{-1}(1-z)^d exp(-bm_\perp^2/z), \tag{5}$$

where d and b are free parameters. In our simulation program, we take $d = 0.3$ GeV^{-2} and $b = 0.58$ GeV^{-2}, as used in PYTHIA [25]. This part of the contribution is tuned with the e^+e^- data, etc.

(2) In the case of a hadronization model, combination refers to an intuitive picture to explain and describe the hadronization process. There are various quark combination models to describe hadronization (see [11] and references therein). However, introducing the production mechanism of exotic multiquark hadrons is a tough problem, in which the unitarity is a difficulty. One has to take a comprehensive account of all exotics. Here, to make it feasible to only study the T(2900) family inclusively, we choose the PYTHIA rather than the combination models. However, except in the fitting of the prompt charm hyperon by the above Formula (3), we refer to 'combination' always by the second meaning.

Similarly, we also calculate the differential cross-section of the prompt Λ_c^+ (there are not enough Σ_c data, so we neglect the discussion of it in the following), and obtain the corresponding equation as (4), while the parameters are $a = 0.214$, $\alpha = 2454.609$ µb GeV^{-1}c, and $\beta = 0.928$ (GeV/c)$^{-1}$, respectively. The results are shown in Figure 2.

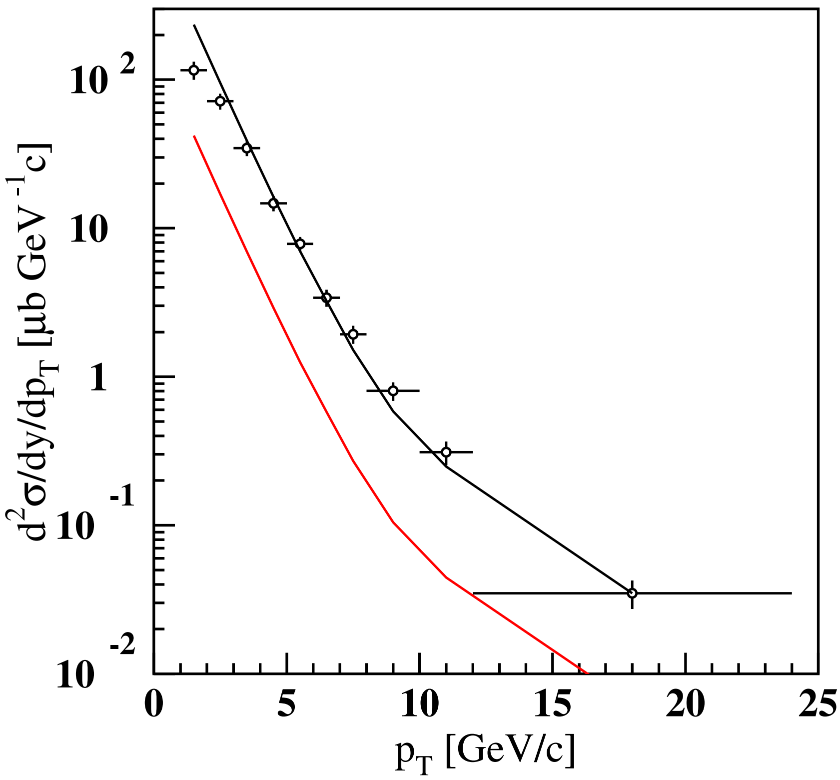

Figure 2. Cross-section of the prompt Λ_c^+ as a function of p_T in pp collisions at $\sqrt{s}$ = 13 TeV. The black circles are experimental data [27], while the black solid line is for Equation (4) with the fitted parameters. The red solid line is for the prediction of the distribution of the $T_{c\bar{s}}$. Isospin and spin symmetry are adopted. We have averaged among all these isospin and spin states.

With the charm hyperon differential cross-sections in hand, we can predict the tetraquark production. The similarity and difference between the charm hyperons, Ξ_c and Λ_c, and the tetraquark hadrons, T_{cs} and $T_{c\bar{s}}$, in their kinematic spectra lie in many aspects; here we only take the most simple model with the help of the Peterson formula [28]:

$$f(z) \propto \frac{1}{z(1 - 1/z - \epsilon_Q/(1-z))^2}, \tag{6}$$

where z is defined by $p_+^{\text{hadron}}/p_+^{\text{cluster}}$, with p_+ being the sum of the energy and the momentum projected in the moving direction of the cluster which hadronize to the correponding hadron, and the parameter ϵ_Q is expected to scale between flavours. In practice, different ϵ_Q values inversely proportional to the hadron square masses are used for tetraquarks T_{cs} and $T_{c\bar{s}}$ with respect to charm hyperons Ξ_c and Λ_c, respectively. The sampling of the tetraquark momenta hence is determined by the fitted charm hyperon momentum distribution and the above relative relation.

For the total production rate, we adopt the most general ansatz for production, the breaking of $SU_f(3)$ flavour symmetry, as mentioned in the Introduction. For the production of quark flavour, the ratio is $u : d : s = 1 : 1 : \lambda$, with $\lambda \sim 0.3$ for a vacuum. The quark to diquark production ratio is $1 : \zeta$, with ζ varying (increasing) with multiplicity [6]. For the multiproduction process of pp collision at high energies of up to 13 TeV at the LHC, the multiplicity is large, so we can take $\zeta \sim \lambda$ for simplicity. Based on this consideration, the T_{cs} has an extra suppression factor $\zeta \sim \lambda$ with respect to Ξ_c when sampling in the generator programme, while $T_{c\bar{s}}$ gets an extra suppression factor $\zeta \times \lambda \sim \lambda^2$. The results of T_{cs} and $T_{c\bar{s}}$ production at the LHC are displayed in the figures with red solid lines.

3. Conclusions and Discussion

In this paper, we study the T(2900) tetraquark family in multiproduction processes at the LHC. We first investigate the production mechanism of this kind of hadron, emphasizing its similarity with the charm hyperon production mechanism. By this similarity, we predict the production rate as well as the kinematics spectra. This is crucial information for the forthcoming relevant measurements at the LHC. Based on our prediction, this family can be measured at the LHC, since the corresponding charm hyperons have been well measured. The production rate of each one of the T(2900) family is only several times smaller than that of the relevant charm hyperon, and the kinematic spectra are almost the same. Furthermore, various decay channels have been carefully studied (see, e.g., [29,30]), and two body decays are dominant. A reconstruction of the tetraquark hadrons is practical.

In the above sections, to find the relation between charm hyperons and tetraquarks, our study is based on the concept of colour reconnection, as well as the intuitive physical picture of hadronization models. Prof. Qu-Bing Xie is one of the pioneers who initialized these ingenious concepts [11,12,17,18]. To further prove the validity of these physical pictures on the study of T(2900) family production, we propose that the light hadrons produced adjointly with the charm hyperon or T(2900) can provide additional clues via string effects (see a likely example in [14]). This measurement can be performed after the T(2900) tetraquark hadron(s) has (have) been observed.

In the analysis and calculations of this paper, we did not rely on the detailed structure of the T(2900) family, nor on that of the corresponding baryons, e.g., information on their spins, isospin/$SU_f(3)$ wave functions, or colour structures. One of the reasons is of course the lack of knowledge on the tetraquark structure. However, for our present purpose, it is more practical to ignore the details of the structure and give a 'baseline' prediction for the production of tetraquark hadrons via the best-fitted baryon production cross-sections. By comparing our prediction with the forthcoming data, one can take further steps to model the structure of the tetraquark more precisely. This is in fact a very general way to study the structure of hadrons with the help of the multiproduction processes, and we believe that a deeper understanding of the hadron structure and confinement mechanism lies in the comprehensive study not only of mesons and baryons but also of various exotic hadrons. This belief and ideas, as we know, were among the original motivations of Profs. Lian-Shou Liu, Qu-Bing Xie, and their collaborators when they began to study hadronzation and multiparticle dynamics (according to private conversations and their group seminar talks).

Author Contributions: Conceptualization, S.-Y.L.; Methodology, Y.J., Y.-R.L. and Z.-G.S.; Software, C.J. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Natural Science Foundation of China (NSFC) (grant nos. 12275157, 12235008, and 11775130). Y.J. is supported by the Open Project of Guangxi Key Laboratory of Nuclear Physics and Nuclear Technology under grant no. NLK2021-07.

Data Availability Statement: Not applicable.

Acknowledgments: We thank all the members of our particle theory group for discussions and collaborations.

Conflicts of Interest: The authors declare no conflict of interest.

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