II. Tuesday Afternoon: Nucleon-Nucleon Scattering Below 500 Mev, G. C. Wick presiding.

D. FELDMAN presented a review of nucleon-nucleon scattering in the 30 - 500 Mev energy region, including some remarks on polarization effects in nucleon-nucleus scattering. The following represents a brief summary of the material presented. Feldman will publish an expanded version of his talk as a review article. (All values of the energy in the discussion of nucleon-nucleon experiments will be given in Mev in the laboratory system; scattering angles listed are in the center of mass system.)

I. Differential and total cross sections.

A. Experimental information

1. P-P scattering. The general features of the p-p scattering results are:

a. Isotropy. The angular distribution is, broadly speaking, flat outside the small angle Coulomb interference region for energies less than 350 Mev. Accurate experiments at Harvard indicate a departure from isotropy: $\sigma(40^o)/\sigma(\frac{\pi}{2}) = 1.06 \pm .03$ at 95 Mev. There is also a hint of a similar effect at 170 Mev. For energies above 350 Mev, anisotropy gradually sets in, as indicated in the following table:

Energy	$\sigma(\frac{\pi}{6})/\sigma(\frac{\pi}{2})$	Laboratory
380	$1.09 \pm .01$	Liverpool
437	$1.15 \pm .02$	Carnegie Tech.
460	$1.14 \pm .06$	Moscow
660	$2.7 \pm .2$	Moscow

Table 1

b. Constancy of cross section with energy. The variation of $\sigma(\frac{\pi}{2})$ with energy is illustrated by Fig. 1. In the region $150 < E < 500$, $\sigma(\frac{\pi}{2})$ seems to have a constant value of ~ 3.7 mb/ster. The hitherto unpublished data are indicated on the graph by points 3 (Taylor and Wood, 134 Mev), 5 (Cassels, 147 Mev), 8 (Holt, 380 Mev), 10, 11, 12 (Meshcheryakov et al., Nikitin et al., 460 Mev). The old Harvard, Harwell, and Rochester values have been omitted from the plot. These results were based on an incorrect value of the $C^{12}(p, pn)C^{11}$ cross section, used to calibrate the intensity of the incident beam, and the nature of the corrections to be made is uncertain.

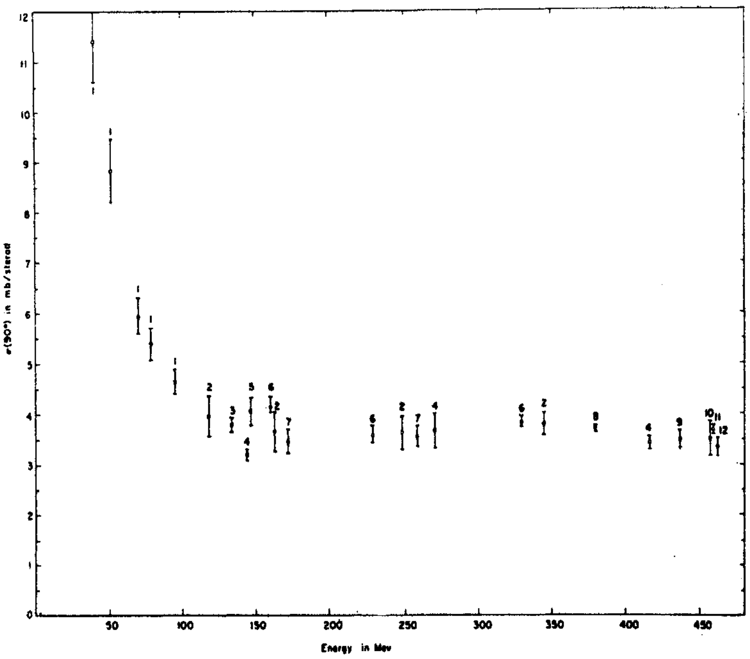

Fig. 1

c. Destructive Coulomb nuclear interference for small angle scattering. Measurements of small angle scattering are available in the energy region 142 - 330 Mev. If we measure the Coulomb interference in terms of the deviation of the cross section from the sum of pure Coulomb plus pure nuclear (isotropic) scattering, the indications are that the sign of the interference is negative.

2. N-P scattering

a. Symmetry. The general form of the angular distribution is that of a symmetric trough, with a minimum at around 90°. The asymmetry increases with increasing energy. The salient features of the energy variation of the angular distributions are summarized in the following table:

Energy Mev.	$\sigma(\pi)/\sigma(\frac{\pi}{2})$	$\sigma(\pi)/\sigma(0)$	θ_{min}
40	~1.6		
90	3.1	~1	~80°
140	4.4	1.1	85°
260	9.1	2.7	90°
400	9		100°

Table 2

Recent and unpublished results include the work of Griffith et al., University College, London, at 98 and 140 Mev, of Spital, Rochester, at 175 Mev, and of Randle and Skyrme, Harwell, at 140 Mev. The latter are final values and render previous cloud chamber results of the Harwell group obsolete.

b. 1/E energy dependence of σ_T. As illustrated by Fig. 2, such an energy dependence holds roughly below 200 Mev. Above this energy, σ_T flattens out.

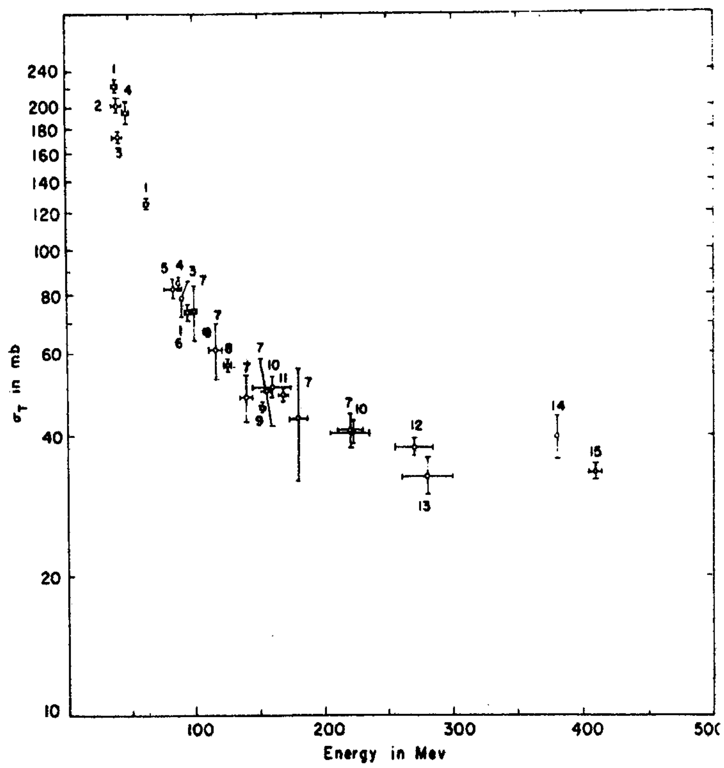

Fig. 2

B. Theoretical discussion of cross section data. A test of the equality of the n-n and p-p interactions has recently been described by Dzhelepov *et al.* They made a direct comparison of n-p and quasi-elastic n-d scattering at 300 Mev. Using an impulse approximation argument, they assert that for large scattering angles ($\theta \geq 50^{\circ}$), the n-d angular distribution can be expressed as the sum of the n-n and n-p angular distributions, plus a small interference term, which is estimated to be less than 15°/o of the n-p distribution. They deduce $\sigma_{nn}(\theta) = 3.5$ mb/ster for $\theta \geq 50^{\circ}$, in reasonable agreement with the p-p data.

A consequence of charge independence, shown by Feldman, is that $\sqrt{\sigma_{np}(\theta)}$, $\sqrt{\sigma_{np}(\pi-\theta)}$, and $\sqrt{\sigma_{pp}(\theta)}$ satisfy triangle inequalities (*i.e.* the sum of any two of them is equal to or greater than the third), where the σ-s refer to the nuclear part of the cross section alone. In particular, one obtains $\sigma_{np}(\frac{\pi}{2}) \geq \frac{1}{4}\sigma_{pp}(\frac{\pi}{2})$ and $\sqrt{\sigma_{pp}(0)} \geq \sqrt{\sigma_{np}(\pi)} - \sqrt{\sigma_{np}(0)}$.

Both of these inequalities are satisfied by experiments in the energy region under consideration. The latter inequality limits the forward to backward ratio for n-p scattering.

One may remark incidentally that it is the isotopic triplet scattering which is anomalously isotropic. The isotropic singlet scattering is strongly peaked in the forward and backward direction.

A number of potential models, some charge independent, some not, have been used in attempts to fit the experimental data. Their common feature is the need for non-central force terms in order to obtain agreement with experiment. The publication of these models some years ago therefore played a useful role in stimulating the initial experimental and theoretical work in the study of polarization at Rochester and at Berkeley.

II. Polarization.

A. Notation. A survey of various properties of polarization which can be deduced from considerations of symmetry and invariance (as derived by Wolfenstein and others in the published literature) was presented; also, the notation used in the discussion of experiments and theory was summarized. The polarization of a spin one half particle is defined as the expectation value of the Pauli spin: $\underset{\sim}{P} = \langle \underset{\sim}{\sigma} \rangle$. It is measured in a double scattering experiment in which the experimentally observed quantity is $\mathcal{E} = \frac{I_L - I_R}{I_L + I_R} = P_1 P_2$, where I_L is the intensity for scattering to the left and I_R the intensity for scattering to the right. P_i, the polarization in the i^{th} scattering, is a function of the energy and angle of that scattering.

Further information about nucleon-nucleon scattering may be obtained in triple scattering experiments. For these cases, the first scattering serves to polarize the beam, the second scattering changes the spin (both as to magnitude and direction), and the final scatterer serves as the analyzer. All three targets are unpolarized. If all scatters take place in the same plane, a measurement of appropriate left-right asymmetries in the scattering intensity yields the depolarization D as a function of polar angle. If the scatters take place in planes successively perpendicular to each other, the left-right asymmetry in the third scatter yields R, the rotation, as a function of polar angle. (See Proceedings of the Fifth Rochester Conference for more details on these experiments and the relation of measured asymmetries to D and R.)

More complicated experiments, such as quadruple scattering and spin correlation measurements, can yield still further results. In particular, Ypsilantis' report (see below) uses information obtained from the measurement of a function A, which requires that the first scattered beam be polarized in the direction of motion. The insertion of a magnetic field between the first and second scatters, perpendicular to both the proton's spin and its direction of motion, leads to a measurement of A.

B. Review of experiments.

1. Angular distributions. A set of typical experimental results for polarization is presented in the next two graphs. The first plot represents least square fits to the experimental data of polarization in p-p scattering at the energies indicated by a function of the form:

$$P\sigma = \sin\theta\cos\theta(a_1 + a_3\cos^2\theta + a_5\cos^4\theta).$$

This function represents the first terms in an expansion in odd powers of cosθ which follows from the general invariance considerations of Wolfenstein for the p-p system.

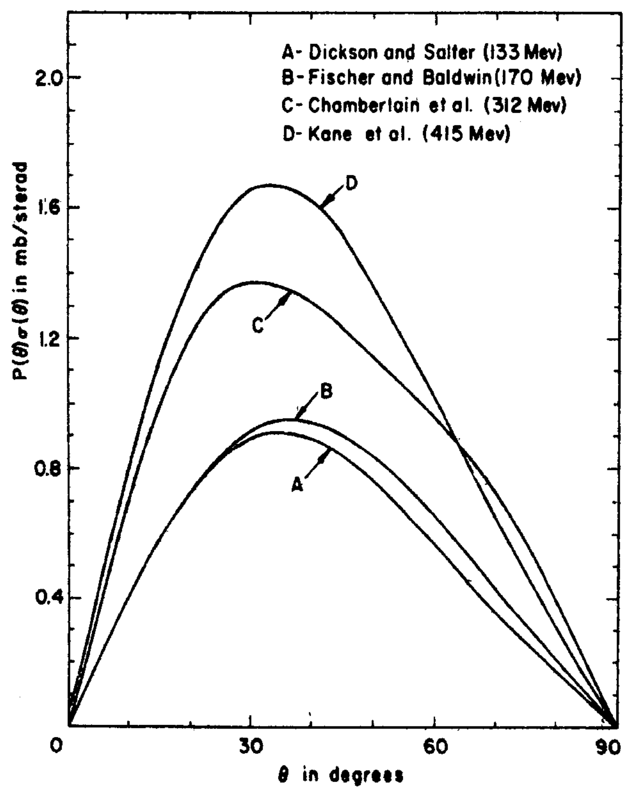

Fig. 3

Curves A and B have $a_5 = 0$. Measurements at Chicago and Rochester at various energies yield results which are consistent with the ones shown.

New measurements have been made at Harwell at 142 Mev which go down to small angles. (See Taylor's report.) Two conclusions can be drawn from the data as given. Obviously, the very existence of p-p polarization effects establishes the presence of non-central forces in the interaction. In fact, analysis of the data according to isotopic spin indicates that there are large polarizations, and therefore non-central forces, in both the isotopic triplet and singlet states. Secondly, the number of terms required in the least square fits indicate that F-wave interactions play a significant role at the energies for which the curves of Fig. 3 have been obtained. The maximum value of the polarization varies from 25°/o to 45°/o as the energy increases.

Fig. 4 indicates least square fits to typical experimental data for n-p polarization. The expression for $P\sigma$ now has the form: $P\sigma = \sin\theta \sum_{n=o}^{2L_{max}-1} a_n \cos^n\theta$. Curve A (see Taylor's report for details) has the first two, curve B the first four coefficients different from zero. An important consequence of these results is that both even

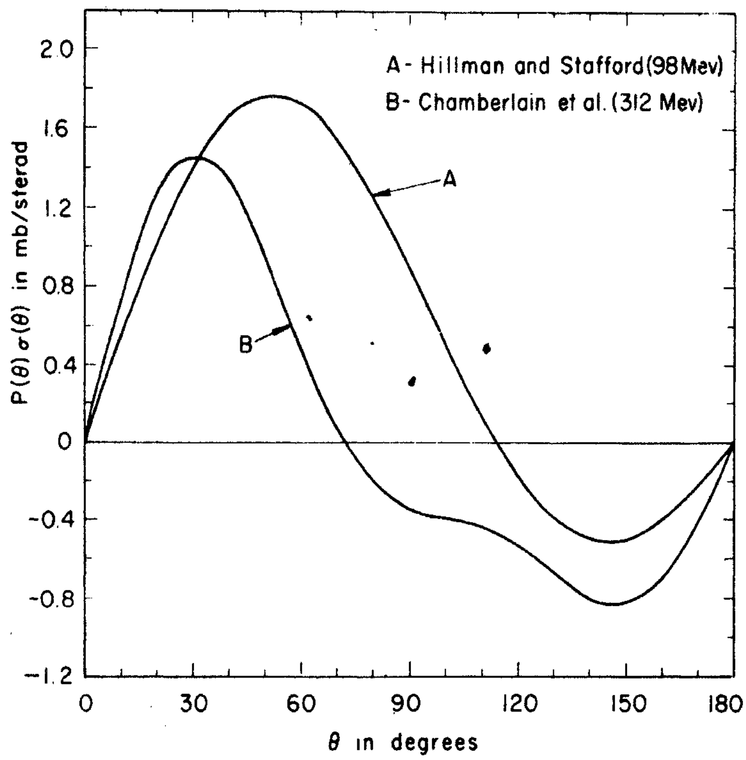

Fig. 4

and odd parity states enter in the n-p system, even at 98 Mev. The Serber force model of nuclear interactions is therefore ruled out. For illustrations of the kind of results obtained for the functions D and R from triple scattering experiments of protons by protons see Ypsilantis' report below.

2. Sign and magnitude polarization. The sign of the polarization of the proton beam (scattered elastically from C at Harwell with outgoing energy 135 Mev and off Be at Chicago with outgoing energy at 435 Mev) was measured by degrading the energy and scattering from the inverted P doublet of He. The polarization is determined to be positive in both experiments: the beam scattered to the left (facing in the direction in which the incident beam is going) has spin up. This sign agrees with the prediction of the shell model.

The sign and magnitude of the polarization of the 100 Mev neutron beam at Harwell (produced by a p-n exchange reaction in Be) was measured, taking advantage of the Coulomb interaction, by scattering it at small angles from uranium. The sign agrees with that of the proton polarization above.

The magnitude of the polarization of proton beams is determined by performing a double small-angle elastic scattering experiment in which the two scatters are similar so that $\mathcal{E} = P^2$. For the case that polarized neutrons are produced in a p-n exchange reaction, the usual procedure has been to assume that the reactions are quasielastic so that the latter formula is still applicable; where this method has been applied, it has led to consistent results.

3. Equivalence of the n-p and p-n polarization. The experiments at Berkeley and Carnegie Tech., which are at about the same energy,

indicate that one obtains similar angular distributions if one scatters polarized neutrons on unpolarized protons or polarized protons on unpolarized neutrons. This provides a direct piece of evidence for charge symmetry in the n-p system. In other words, nuclear forces of the type $(\underset{\sim}{\sigma}_1 - \underset{\sim}{\sigma}_2) \cdot \underset{\sim}{r} \times \underset{\sim}{p}$, which couple singlet and triplet states of a given angular momentum, are ruled out.

C. Theoretical discussion of polarization. Meson theory has contributed very little to the interpretation of high energy nucleon-nucleon scattering and polarization effects. As far as potential models are concerned, several calculations of p-p and n-p polarization distributions have been published. They are based on the models which had been utilized in the calculation of the scattering cross sections. Singular non-central potentials, which on more exact recalculation yield poor fits to the p-p cross sections, give the better fits to polarization data, while the hard core model of Jastrow, which results in reasonably correct cross sections, leads to a p-p polarization which is much too small. It appears that if one were to consider a hard core in singlet states and a singular tensor force in triplet states one could account for both the isotropy of the differential cross section and the large polarization in p-p scattering. Such a calculation has not actually been carried out.

A useful method of analyzing nucleon-nucleon scattering data, due to Wolfenstein and others, is to consider the most general form of the scattering amplitude, invariant under spatial rotations and reflections and time inversion. Such an amplitude for p-p scattering contains nine independent real functions of the cosine of the polar angle, possessing well-defined parities. At 90°, only five functions will appear. Though experimental data are not exhaustive enough to obtain the various parameters exactly, Wolfenstein has shown from the Berkeley 310 Mev p-p data that the contribution to the scattering at 90° from singlet states lie between 15°/o and 60°/o, that due to the spin-orbit-type term of the scattering amplitude lies between 35°/o and 70°/o, and that due to the tensor-like term between 2°/o and 20°/o.

As an introduction to the several reports on phase shift analysis which were to be given later, Feldman presented the results of an analysis he and Ohnuma made of p-p and n-p scattering and polarization at a nominal energy of 150 Mev. They assumed charge independence, ignored coupling between different orbital states, and restricted themselves to total angular momenta $J \leq 2$. There are therefore six isotopic triplet and four isotopic singlet phases. Coulomb interference criteria were also taken into account. The p-p and n-p scattering as well as p-p polarization was used to fix the phases; the predicted n-p polarization is then an incisive test of the remaining sets of phase shifts. The following three sets are in reasonable accord with experiment (phases in degrees):

Set	^1S	^1P	^1D	$^3S_1$	^3P_o	$^3P_1$	$^3P_2$	$^3D_1$	$^3D_2$	$^3F_2$
E	22.0	-8.0	-8.7	-45.0	-22.5	9.2	9.7	-5.0	-15.0	-5.4
F	24.8	15.0	-8.2	-40.0	-22.5	9.7	7.7	-10.0	-15.0	-6.6
L	-19.5	-20.0	7.1	8.0	-0.3	-20.5	7.7	27.0	25.0	-6.6

Feldman observed that isotropy of the p-p cross section requires the $^3P_2$, $^3F_2$ and the $^1S_1$, $^1D_2$ pairs of phase shifts to have opposite signs. However, the positive $^1S_1$, and the negative $^1D_2$ phase shifts obtained for some of the sets of p-p phases are opposite in sign from those gotten by the repulsive core model of Jastrow. Though relativistic corrections to the cross section are small, relativistic effects are significant in the fitting of the polarization at small angles, as Garren has pointed out.

Discussion among Brueckner, Weisskopf, Marshak, and Feldman brought out the fact that Feldman's phase shifts do not seem to correspond to any particularly simple potential model. Discussing potential models in general, Marshak stated that one cannot fit the data with a repulsive core static potential, in particular a Levy type potential with repulsive core, in all states. Perhaps a repulsive core in singlet but not triplet spin states might give correct results. Breit suggested that a study of the g^2 potential obtained from pseudoscalar meson theory with pseudo scalar coupling, though yielding no rigorous results, may indicate how to extend the types of potential models now in use. In particular, it may suggest ways of introducing velocity dependent potentials.

D. Polarization in the elastic scattering of protons by complex nuclei. Investigations of angular distributions have been made in the energy region 60 - 425 Mev and for a variety of elements. The experimental results may be summarized as follows:

1. Polarization effects are large above 130 Mev, but drop rapidly to zero below 100 Mev.

2. The sign of the polarization for p-nucleus elastic scattering is the same as that for p-p scattering.

3. The polarization varies smoothly as a function of angle for the light elements, but shows dips for the heavier elements. Similarly, Al is the lightest element for which a diffraction pattern is evident for the cross section.

4. The minima and maxima of the polarization curves lie close to the diffraction minima and maxima.

5. As A increases, the first polarization maximum is suppressed (Coulomb effect) and the pattern of the curves is compressed.

6. Neighboring elements lead to very similar polarizations, suggesting that the effect depends on the nuclear radius but not on the details of nuclear structure.

In contrast to point No. 3, however, it should be noticed that recently (a) Strauch and Titus at Harvard have separated out the elastic component in the scattering of unpolarized protons on C at 96 Mev and have gotten a diffraction pattern; (b) Chestnut, Hafner and Roberts at Rochester have observed three extrema in the angular distribution of the polarization for the elastic scattering of 210 Mev protons on C. The Rochester results for C and Ca, shown in Fig. 5, are quite similar, beyond the Coulomb region.

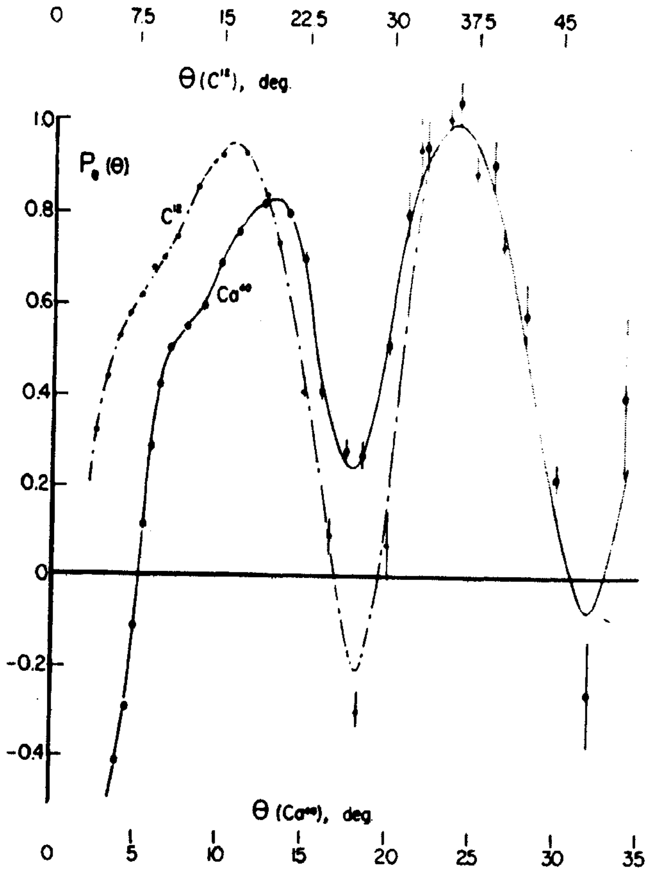

Fig. 5

Theoretical calculations of proton-nucleus polarization have been carried out by many authors, using an optical model potential, with a Thomas type spin-orbit term added, of the general form $U(r) = \left\{-(1+i\alpha) + \beta \frac{\Delta^2}{\hbar} \cdot L \frac{1}{2} \frac{d}{dr}\right\} V(r)$. The magnitude of the spin-orbit term is of the order of that required by nuclear shell theory. If an exact calculation is carried out, and the edges of the potential wells are rounded off, a reasonable agreement with experiment is obtained, but the detailed fit is not too good. It is not clear, at present, to what extent one can correlate the data of many elements in terms of the optical potential; in part, this may be due to the difficulty in separating the elastic from the inelastic scattering in the vicinity of the first diffraction minimum and beyond.

The initial experimental paper of the session was presented by TAYLOR of Harwell. First he reported on his work on the small angle scattering of unpolarized and polarized 142 Mev protons from liquid hydrogen. The angular region covered was between 5° and 40°. In order to avoid trouble with the high backgrounds, a counter telescope was used which did not see the windows of the target. The backgrounds fall from about 20°/o at the smallest angles to under 2°/o at 14°. The errors in this experiment are estimated to be between 4°/o and 5°/o. The values measured for the differential cross section indicate considerable destructive Coulomb-nuclear interference for small angles. The polarization at small angles was also measured, using a 46 ± 1°/o polarized incident proton beam, obtained by scattering at 6.3° from C. Dickson and Salter repeated their previous measurements (reported on at the last Rochester Conference) for angles greater than 40°. The energy and polarization of the incident beam were comparable to that of the beam used in the small angle measurement. The combined data can be fitted by the function:

$$P\sigma(\theta)/\sin 2\theta = (0.555 \pm .057) + (0.64 \pm .10) \cos^2 \theta.$$

Taylor next presented the experiment of Hillman, Stafford, and Whitehead on polarization in free n-p scattering at 98 Mev. The work will shortly be published. The incident beam had a polarization of 8.5 ± .6°/o (not 9.8°/o as indicated in the manuscript to be published). The measurements originally carried out for angles greater than 60° used carbon-polythene subtraction and the counters were rotated about the beam direction. The present work uses a liquid scintillator to detect the neutrons. The counter is kept fixed. Neutrons emerging from the cyclotron pass through a longitudinal magnetic field of ~4000 gauss for eleven feet, sufficient for the neutron magnetic moment to precess through 90°. Thus, using normal and reversed solenoid currents, it was possible to make measurements at the two customary azimuth angles of 0° and 180°. The least squares fit to the data obtained in this experiment, and in previous work at comparable energies but larger angles, is given by

$$P\sigma(\theta)/\sin \theta = (1.01 \pm .15) + (3.17 \pm .58) \cos \theta.$$

The estimated error is 6°/o on the absolute scale. Stafford and Whitehead plan to repeat the experiment at larger angles using the solenoid method, since they think that it will reduce previous uncertainties in the measurements.

Wolfenstein pointed out that the n-p polarization results as just reported further confirm Feldman's statement that there is evidence of polarization (and therefore of non-central forces) in isotopic singlet states. In particular the 98 Mev results just presented indicate definite effects of ^3S - ^3D interference terms.

HOLT of Liverpool next spoke briefly on a precise measurement of the differential cross section for p-p scattering at 383 Mev. His collaborators were Harting and Moore. A well-focussed, clean beam of protons was available for the experiment. The main uncertainty in the measurement of the cross section is in the determination of the flux of protons, measured by an ionization chamber calibrated in two ways. The first method of calibration involves the use of a Faraday cup; the second uses the comparison of direct counts in the scintillators of a reduced beam with current measurements in the ion chamber. The two methods of calibration differ by 2°/o. The scattering was measured by the polythene-carbon difference method. The value 3.70 ± .06 mb/ster was obtained for the differential cross section at 90°. Values of the cross section were obtained at other angles, down to 30°. In particular the value of the ratio of the cross sections at 30° and 90° is given by $\sigma(\frac{\pi}{6})/\sigma(\frac{\pi}{2}) = 1.09 \pm .01$. The calibrated ion chamber was also used by Parikh to measure the cross section for the $C^{12}(p,pn)C^{11}$ reaction at 383 Mev. The value obtained is $\sigma = 31.6 \pm 1.0$ mb.

Next, YPSILANTIS presented the result of phase shift calculations with the 310 Mev Berkeley data for p-p scattering and triple scattering measurements, carried out in collaboration with Stapp and Metropolis at Los Alamos. No assumptions were made about the phases, but the analysis was only carried up to, and including, H-waves. All mixing parameters which appear for the partial waves considered were included.

Most of the data that entered into the calculation were known before, or presented at, last year's conference. One additional measurement of the depolarization D has been made at a center of mass angle of 80°. The parameter A (see Feldman's talk), which requires that one have a longitudinal component of incident polarization, has since that time also been measured (by Simmons at Berkeley). With the exception of the introduction of a magnetic field, the general arrangements of triple scattering experiments were followed. A schematic outline of the geometry of the experiment is given by the adjoining figure. The magnetic field is applied to the beam which emerges from the cyclotron with a polarization perpendicular to its direction of motion. The field is perpendicular to both the direction of motion and the polarization, and rotates both. If the magnetic moment of the proton were one nuclear magneton, both the beam and the spin

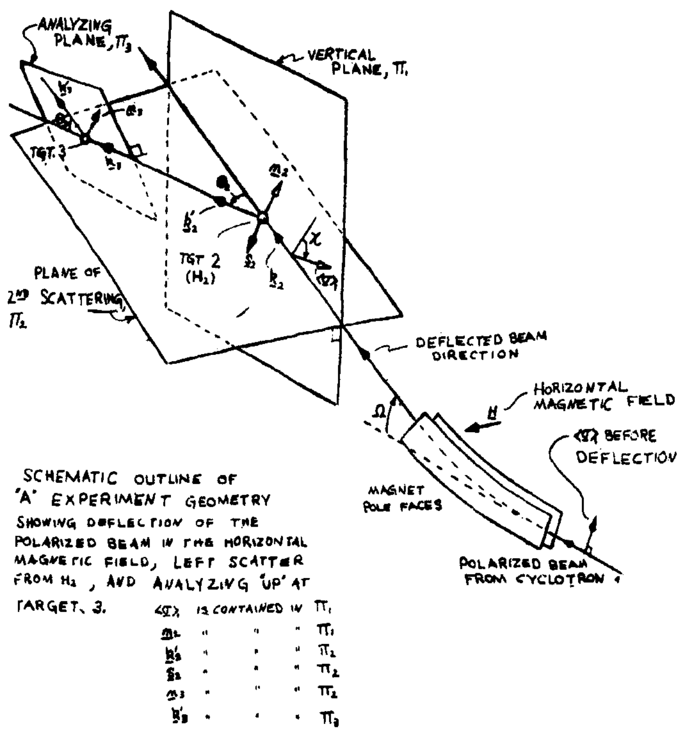

Fig. 6

would be rotated through the same angle and, on emerging from the field, would still be perpendicular to each other. However, the spin is rotated through a larger angle because of the anomalous magnetic moment of the proton, and on emerging from the field deviates by an angle χ from the normal to the beam direction, and therefore has a component along the beam. This allows one to measure the parameter A, which in Wolfenstein's formulation occurs with a coefficient, $\langle \underset{\sim}{\sigma} \rangle_1 \cdot k_2$ where $\langle \underset{\sim}{\sigma} \rangle_1$ is the polarization of the beam incident to the second scatter and k_2 is its momentum. The asymmetries in intensity in the third scatter were measured in the customary way, and at center of mass angles of 25.4°, 51.4°, and 76.3°.

All the data (36 measurements in all), including those of Fischer and Goldhaber for the Coulomb interference at 310 Mev, were processed by the Maniac. An attempt was made to minimize a quantity M, related to the accuracy of fit, starting with random phases. With 14 parameters and 36 measurements, M is expected to be 22. The four sets of solutions obtained are given in Table 3, together with an analysis of errors, when such an analysis was made.

310 Mev P-P Phase Shifts (in degrees)

State	"Nuclear" Phase Shift	Solution 1	Solution 2	Solution 3	Solution 4
^1S_o	δ_o	-10.9±4.9	-19.5±	-27.0±3.9	-10.1±4.9
$^1D_2$	δ_2	13.3±1.5	4.3±	4.8±1.2	12.8±1.4
$^1G_4$	δ_4	1.1±	1.3±	1.0±	1.0±
^3P_o	δ_{10}	-4.1±2.7	-36.0±	-25.4±3.8	-14.3±4.3
$^3P_1$	δ_{11}	-19.8±1.6	-11.7±	- 7.3±2.0	-26.7±2.6
$^3P_2$	$\bar{\delta}_{12}$	22.6±1.3	18.8±	23.1±1.5	-12.6±1.9
Mix(J=2)	$\bar{\varepsilon}_2$	- 1.8±2.0	9.3±	7.5±	- 0.8±4.0
$^3F_2$	$\bar{\delta}_{32}$	- 2.0±1.1	- 0.5±	- 1.4±1.8	- 1.3±2.0
$^3F_3$	δ_{33}	- 2.6±1.1	0.2±	1.5±0.7	2.1±1.5
$^3F_4$	$\bar{\delta}_{34}$	0.5±0.9	2.5±	2.6±1.5	3.2±1.0
Mix(J=4)	$\bar{\varepsilon}_4$	- 0.9±	1.5±	0.9±	1.1±
$^3H_4$	$\bar{\delta}_{54}$	- 1.1±	2.1±	- 0.7±	1.4±
$^3H_5$	δ_{55}	0.9±	- 1.4±	- 0.9±	0.1±
$^3H_6$	δ_{56}	- 0.6±	1.6±	- 0.8±	1.3±
Σ or M	23.8		21.6	24.5	17.9

Note: S_N - matrix for mixed case defined by

$$S_N = \begin{pmatrix} e^{i\bar{\delta}_{J-1,J}} & 0 \\ 0 & e^{i\bar{\delta}_{J+1,J}} \end{pmatrix} \begin{pmatrix} \cos 2\bar{\varepsilon}_J & i\sin 2\bar{\varepsilon}_J \\ i\sin 2\bar{\varepsilon}_J & \cos 2\bar{\varepsilon}_J \end{pmatrix} \begin{pmatrix} e^{i\bar{\delta}_{J-1,J}} & 0 \\ 0 & e^{i\bar{\delta}_{J+1,J}} \end{pmatrix}$$

Table 3

Coulomb interference effects are included to the extent to which Garren has calculated them. Figures 7 to 11 show the fit to the data.

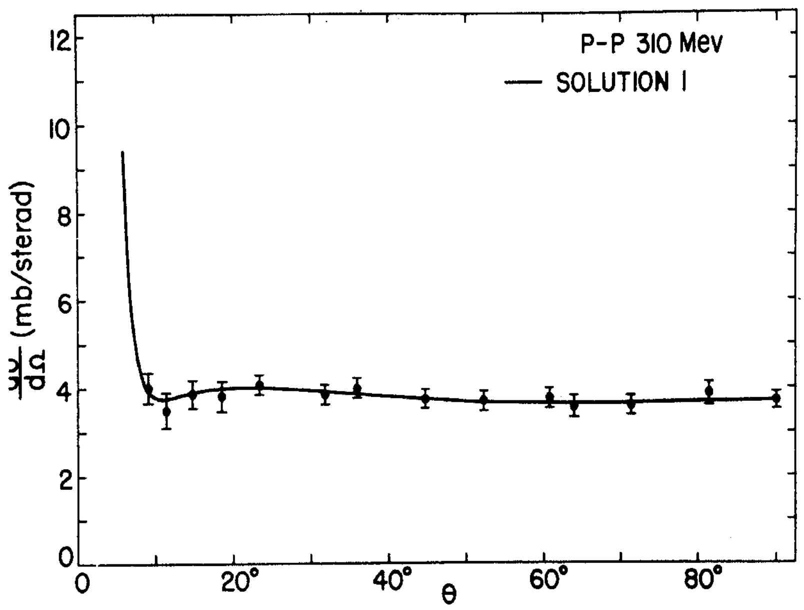

Fig. 7

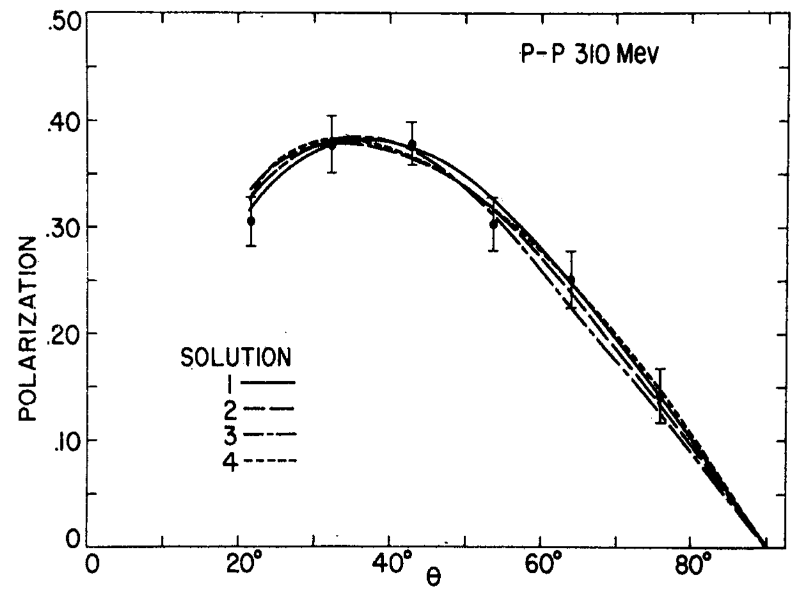

Fig. 8

(Editor's note: We thought it best to present one set of diagrams indicating the fit of phase shifts to experiment. It seemed natural to choose the set representing the most complete experiments. We apologize for the fact that limitations of space do not permit us to include corresponding figures from most of the other reports.) Solutions 2, 3, and 4 are essentially identical with solution 1 with respect to their fit of the cross section data. Two of the solutions do not fit well the small angle points of the rotation parameter R. There is some question about experimental results in this region and a check on them will be made. At this time, all four solutions represent essentially equivalent fits to the data.

Finally one might mention that the production cross section and the polarization of the deuteron in the reaction p+p⟶π+d, has been studied by various people at 310 Mev with both polarized and unpolarized proton beams incident on hydrogen. A theoretical analysis of these processes was carried out some time ago by Gell-Mann and Watson, and leads to loose inequalities among the phase shifts of the proton-proton states. The

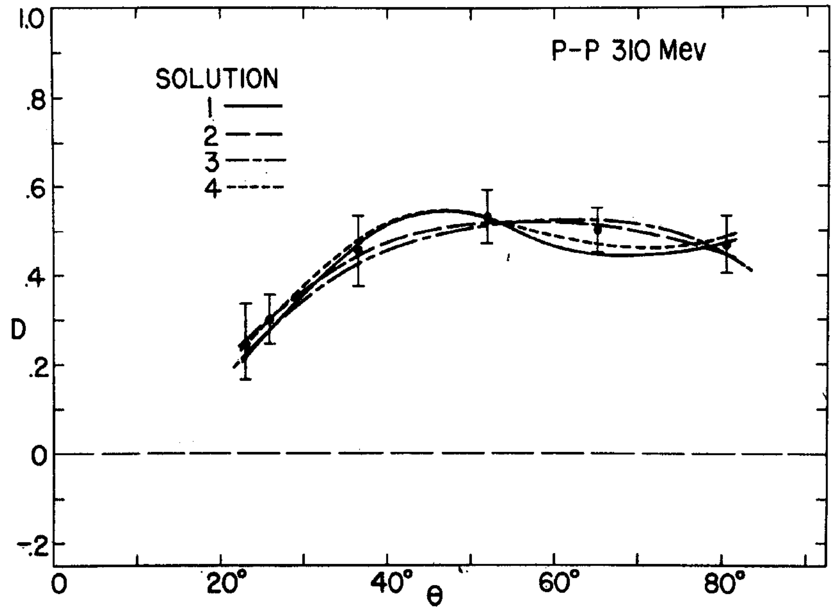

Fig. 9

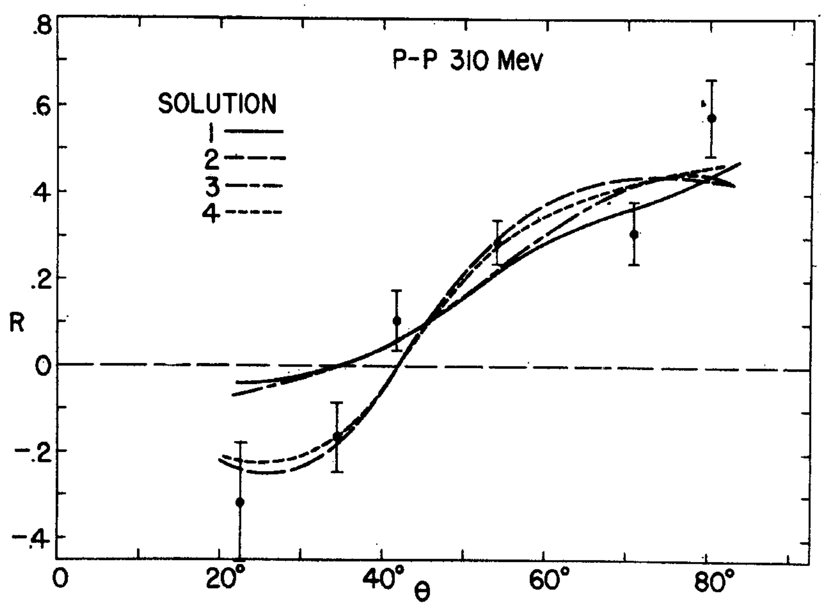

Fig. 10

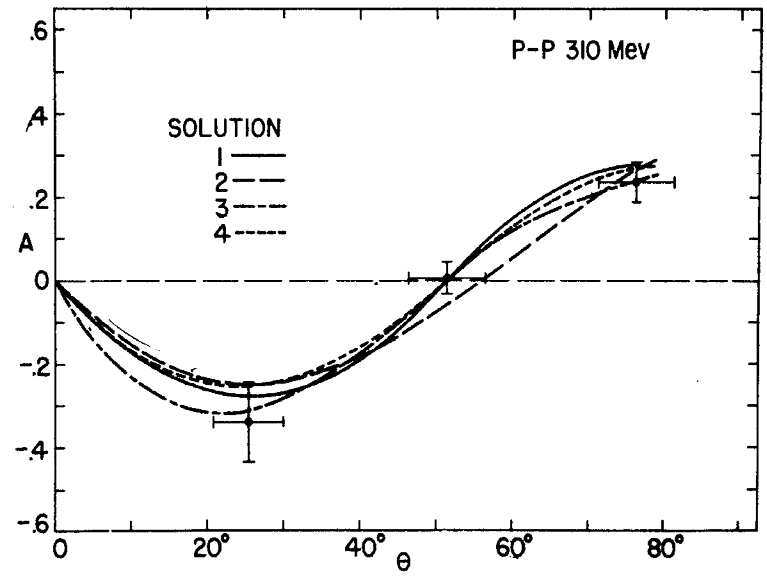

Fig. 11

phase shifts obtained are consistent with these inequalities.

Opening the discussion of Ypsilantis' paper, Wick commented that the work illustrates the difficulties in arriving at a definite picture from the measurements. Uncertainties remain even after this most detailed and careful investigation. Segré pointed out that the analysis was made at only one energy, 310 Mev, and for p-p scattering only. He suggested that phase shifts available, or to be calculated, at lower energies be used to help distinguish the correct solution. Conversely, the present solutions may help select the correct set of lower energy phase shifts.

Wick then asked if any specialist would like to "stick his neck out" and say if dispersion relations would help in distinguishing between the different solutions. Goldberger was elected spokesman. He felt that there were two major difficulties in applying the dispersion relations to this problem. In the first place, one is forced to study the unphysical region (see report of Goldberger's talk above) between zero projectile energy and the nucleon rest mass. This is actually a study of the exchange of any number of virtual mesons between the nucleons and the result is essentially the Fourier transform of the nucleon-nucleon potential. Finding it is a big problem. "It is unlikely that it will be worked out in the next week or two." The second difficulty is that the dispersion relations are, say for the forward scattering amplitude in n-p scatter, of the form

$$\text{Re } f(\omega)_{np} = \text{Mess}(\omega) + \int d\omega \left[\frac{\sigma_{np}(\omega')}{\omega' - \omega} + \frac{\sigma_{np^-}(\omega')}{\omega' + \omega}\right].$$

Mess (ω) is part of the Fourier transform of the potential. One should probably also include in the potential all of the anti-proton term and the inelastic part of the neutron proton term. It is the sum of these contributions that would appear to be most properly referred to as the potential.

Touschek asked if the relations could not be turned around and, perhaps at some later time when nucleon-nucleon scattering information as a function of energy were available, be used to study the nucleon-antinucleon cross section. Goldberger, Karplus, and Oppenheimer all felt that this was unlikely for various reasons: the energy denominator of the antinucleon term is too large (larger than twice the nucleon rest mass); the denominator at reasonable energies varies slowly with energy, making it indistinguishable from constant terms appearing in the relations; and the cross sections show no signs of diminishing with energy (the integrals appearing in the dispersion relation therefore very likely go up to the highest cosmic ray energies).

PHILLIPS of Harwell followed with a discussion of some preliminary results of phase shift calculations for the nucleon-nucleon interaction at 95 Mev. The coefficients of the expansions of the quantity $P\sigma$ in sines and cosines (see Feldman's talk for the form of these expressions) for n-p and p-p polarization indicate that there must be large contributions from partial waves with $L \geq 2$ at this energy. The p-p polarization data are available only at 130 Mev, but a rough guess at the order of magnitude of this polarization at 100 Mev leads to the conclusion that waves higher than P play a role in the interaction at this energy. Therefore it has been proposed to make a phase shift analysis, using only S, P, and D waves. The data available at this energy--the n-p differential cross section and polarization and the p-p scattering--plus a demand for destructive Coulomb interference and a guess of how the 130 Mev p-p polarization data extrapolate to 95 Mev, are sufficient to obtain at least a rough set of parameters. These could be refined as more experimental information becomes available. The set of phase shifts (in degrees) are given in Table 4. $\eta^{(1)}$ and $\eta^{(3)}$ are the eigenphases for the coupled ${}^3S_1$-${}^3D_1$ system and $\varepsilon^{(1)}$ is the admixture angle for this system. $\delta({}^3D_2)$ is negative or small and $\delta({}^3D_3)$ is small. A list of phases, obtained from the theory of Lomon and Feshbach (see below) is also given for comparison.

							Coupled ${}^3S$-${}^3D$ waves		
	${}^1S_0$	${}^1P_1$	${}^1D_2$	${}^3P_0$	${}^3P_1$	${}^3P_2$	$\eta^{(1)}$	$\eta^{(3)}$	$\varepsilon^{(1)}$
Approximate Fit	32	-27	0	-24	9	6	46	-12	26
Feshbach and Lomon	26	-14	1	-28	2	5.5	46	-10	25

Table 4

Wolfenstein questioned the validity of extrapolating the 95 Mev p-p polarization from the 130 Mev data. Phillips replied that the results are insensitive to the value of the p-p polarization.

BREIT next briefly summarized the work of his group. Hull, Ehrman, Hatcher, and Durant have analyzed the 300 Mev Berkeley p-p data on the differential cross section (not including Fischer and Goldhaber's results for small angles), and the polarization, with the partial aid of the Univac computing facility in New York. An attempt was made not to rely on the computer too heavily, which, if given a large number of parameters, might obtain the wrong solutions and still yield excellent agreement with experiment. The procedure has been to fit first the combined 300 Mev polarization data from several laboratories. No higher than F-waves are required for this. The notation K was used for the singlet and δ_L^J for the triplet phase shifts. The analysis did not use the gradient search procedure. Trial values of δ_2^F, δ_3^F, and δ_2^P were picked. The remaining phase shifts were then determined by the experimental data and the Coulomb interference criterion. The table below gives the solutions obtained. (Angles are in degrees):

	K_o	K_2	δ_0^P	δ_1^P	δ_2^P	δ_2^F	δ_3^F	δ_4^F
A	-17.5	-4	-19.8	-19.8	20	10	- 5	4.8
B	- 3.75	-6	-38.5	- 3.4	20	10	-10	2.8
C	0	0	-26.5	-19.7	19	10	-10	2.8
D	-30	-4	2.1	4.5	-15.5	0	- 5	16.3
E	- 2.5	-2	-33.6	- 4.1	21	10	- 5	4.8
F	35	0	16.6	9.6	2.2	-25	0	0.57

Table 5

Fits to the experimental data for p-p scattering are illustrated in Figures 12 and 13. (They are a sample of a somewhat larger group of slides shown by Breit.) An attempt was made to include higher angular momenta, leading to a slight improvement of the fit.

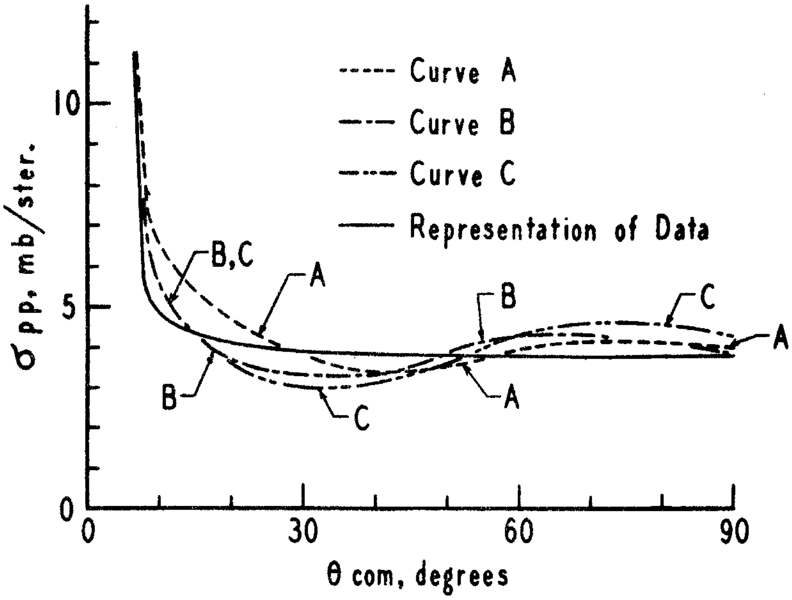

Fig. 12

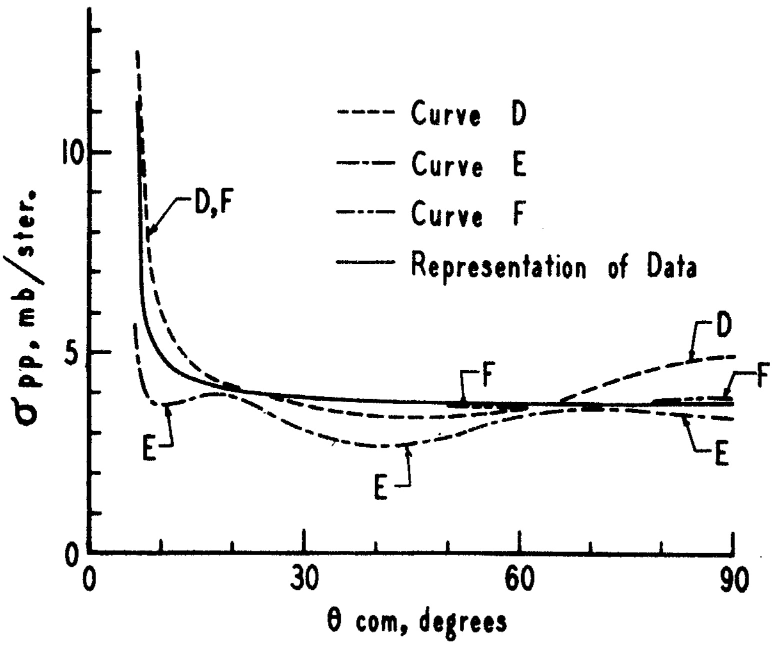

Fig. 13

The speaker next referred to a study by Saperstein of the detailed fit of the Lomon-Feshbach parameters (see below) at 130, 170, and 274 Mev. Even with some doctoring and addition of parameters, this model does not seem to be too successful. Breit mentioned the difficulty of making simultaneous fits to p-p and n-p data with phase shifts of approximately the magnitudes of those obtained by Feshbach and Lomon and the related argument due to Wertheim in connection with DePangher's 300 Mev n-p data. This argument identifies the difficulty with the wrong sign of the odd L-even L interference to fit the observed asymmetry. A boundary condition formulation, more closely along the lines of previous work by Breit and Bouricius represents the p-p data reasonably well, but the n-p data have not yet been tested.

Finally Breit referred to his calculations on meson production which indicate a non-negligible influence on polarization at the higher energies.

FESHBACH's talk concerned his work with Lomon. He termed it the "boundary condition approximation" in the phenomenological study of nucleon-nucleon interactions. The approximation, a generalization of the method of Breit and Bouricius and of effective range theory, involves the assumption of a critical radius r_0 such that the interaction between the nucleons vanishes for $r > r_0$, but is large, compared to the scattering energies of interest for $r < r_0$. The logarithmic derivative of the wave function satisfies the customary boundary condition,

$$r\frac{\partial \Psi}{\partial r}\Big|_{r=r_0} = F\Psi(r_0)$$

at r_o. r_o and F are real parameters which are, in general, functions of the angular momentum state. For singlet and J = L triplet states, and for ^3P_o, Ψ and F are numbers. For all other triplet states we have:

$$\Psi = \begin{vmatrix} \Psi_{J,J-1} \\ \Psi_{J,J+1} \end{vmatrix} ; \qquad F_J = \begin{vmatrix} f_{J,J-1} & f_J^{(t)} \\ f_J^{(t)} & f_{J,J+1} \end{vmatrix} .$$

The system of equations, given so far, is completely equivalent to a phase shift analysis, and near zero energy is just the effective range theory. The major assumption of the boundary condition approximation is that F is independent of the energy. This is a crude assumption. However, its very breakdown might give some clues as to the nature of a correct theory. One would suspect that the actual interaction contains a core--a region of very large interaction energies, surrounded by a weaker local potential which is predictable by low energy theorems. From this point of view, the energy independence of F will force at least some of the r_o-s to be energy dependent, and one would further expect that measurements involving distant collisions--such as $\sigma(0)$ and $\sigma(\pi)$--will be fitted poorly.

Charge independent fits were attempted for data available last summer in the energy range 0-274 Mev, including data on the deuteron. An order of magnitude fit was attempted to the p-p polarization, using only the first term of the $P\sigma$ expansion at all energies. Coulomb interference was not taken into account. In order to fit the isotropy of the p-p scattering, large ^1S_o and ^3P_o contributions are required. The former are fixed by the low energy data, and the latter estimated assuming energy independence. The sign of the ^3P_o phase is determined from the n-p distribution. The $^3P_2$ and $^3F_2$ states (the coupling between them was ignored) are determined from the polarization. The $^3S_1$ phase is determined by the n-p low energy data. Significantly, the low energy data essentially determine the total n-p cross section. The relevant parameters are given in Tables 6 and 7. The fits to experimental data are in general reasonable. Not too surprisingly, the fit to the n-p angular distribution for 100 Mev is not good in the vicinity of 0° and 180°.

Boundary parameters for the isotopic triplet states. States for which no entries were made were assumed to give negligible contributions to the scattering. The parameter $f_2^{(t)}$ was placed equal to zero. Energies E are expressed in Mev in the laboratory system. A and B refer to the two types of fit attempted.

State		$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	$^3F_2$	$^3F_3$
r_0 (10^{-13} cm)	A	$1.32e^{-0.03\sqrt{E}}$	1.32	1.32	0.88	0.88		
	B	$1.32e^{-0.02\sqrt{E}}$	1.32	1.32		1.1	1.32	1.32
f_J	A	−0.947	1.0	−16	0.45	0		
	B	−0.947	0.6	−16		0		1.5

Table 6

Boundary parameters for the isotopic singlet states.

State	$^3S_1+{}^3D_1$	$^1P_1$
r_0 (10^{-13} cm)	1.32	1.32
	$f_{01}=1.72=f_{12}$ $f_1^{(t)}=-3.80$	$f_1=2.5$

Table 7

HILL discussed next the work of Gammel, Thaler, and Christian at Los Alamos. The program falls into two parts. The first part is an unsuccessful effort to find a charge and velocity independent non-singular phenomenological potential giving a fit to nucleon-nucleon data in the range 0-310 Mev, and to the bound state of the deuteron. A potential, which contains no L.S part and in which no resonance occurs in the ^3P_o, $^3P_1$, $^3P_2$ + $^3F_2$ states does not seem to exist.

Marshak felt that the negative results obtained with these non-singular potentials were to be expected, since indications are that a singular tensor force is required to account for the magnitude of at least the polarization.

The second part of the program, still in progress, involves the testing of the nucleon-nucleon potential of Gartenhaus, computed from the pseudoscalar meson theory of Chew. Angular distributions for the scattering by this potential have been computed at various energies. Phase shifts were also computed up to $^3H_6$, and the results compared with the work of Feshbach and Lomon and with Stapp's phase shift analysis of the 310 Mev p-p data. The most striking feature of the Gartenhaus potential is that it yields a positive ^3P_o phase shift, corresponding to an attractive force in this state, while both the work of Feshbach and Lomon, and of Stapp and Ypsilantis yield negative ^3P_o (repulsive) phases. This agreement could be resolved provided one makes the assumption that the Gartenhaus potential leads to a bound ^3P_o state at zero energy. This would give a phase shift π at this energy, which could decrease and would therefore look like a repulsive phase shift.

In response to Gell-Mann's question, Hill stated that a search for this postulated zero energy ^3P_o resonance was now under way. Klein asked whether the predictions of the Gartenhaus potential agreed with experiment. Hill asserted that for n-p scattering, the agreement with observed σ_T and $\sigma(\theta)$ is good at 14.1 and 19.66 Mev, but that at 40 and 90 Mev the predicted angular distributions are peaked progressively too much in the forward direction.

Hill closed by inviting researchers to take advantage of the calculational machinery now in operation for the testing of any potential model desired.

ADDENDUM. The following is a brief abstract of a paper on the analysis of nucleon-nucleon scattering, submitted to the conference by RAPHAEL (introduced by Noyes). An expanded version of this material will be published shortly.

The first part of the work involves the development of what Raphael calls "the extended effective range description." It is based on the work reported on by Feshbach. If one allows the F defined by Feshbach to vary weakly with energy, one obtains the equation

$$\int_{r=r_o} d\omega\, r\, \Psi^{+} \frac{\partial(F/r_o)}{\partial E} r\Psi = \int_{r \geq r_o} (dr)\, \Psi^{+} \Psi^{-} \int_{\text{all space}} (dr)\, \Psi^{+} \Psi$$

where Ψ is the solution of the wave function with interaction and Ψ satisfies the free particle equation. If we demand that the left side of the above equation vanish at E = 0 in each scattering state (i.e., we demand the local energy independence of F/r_o), the right side will determine r_o.

This formulation is just a rephrasing of the shape independent approximation. Its usefulness is emphasized by the following example: there is evidence from nucleon-nucleon scattering analysis that the ${}^1S_0$ phase shift goes through zero at around 100 Mev. The k cot δ expansion therefore converges only below this energy. However the expression k cot (δ + k r_0), which follows from the present analysis, is roughly energy independent for a phase shift of this character in the range, say 0-300 Mev.

The second part of Raphael's contribution deals with the determination of some of the restrictions placed on the nuclear force by the scattering data over a limited energy region. The author feels that analyses based on the Gel'fand-Levitan equation involve unrealistic extrapolations of existing data to infinite energies. To illustrate his approach, Raphael restricts himself to S states. Assuming that the S phase shifts are known, one can write an integral equation for the wave function which involves, among other functions, the phase shift and integrations over the unknown potentials, which are assumed to be of short range. If one uses a Gaussian approximation procedure employing Laguerre polynomials to evaluate the integrals, one finds that for the purpose of the integral equation the potential may be approximated by

$$V(r) \cong \sum_{p=1}^{n} \gamma_{np} \; \delta(r - x_{np} R),$$

where x_{np} is the p^{th} zero of the Laguerre polynomial $L_n(x)$ and R is the range of the force. The constants γ_{np} can be determined by comparison with the experimentally observed S phases in the energy region under consideration. The greater the accuracy desired in a given energy range, or the larger the energy range one wishes to fit with a given accuracy, the larger the number n. The wave functions determined in this manner from the experimental data may be used in other problems in which two body forces are important. The approximation method has been checked with a number of monotonic potentials and has been found to be accurate.

Noyes has undertaken a detailed computational program, involving higher angular momenta and $n = 3$. This approximation is estimated to be valid in the energy range 0-300 Mev and therefore the 310 Mev p-p data can be used to help determine the parameters.