SELECTED TOPICS IN HIGH ENERGY PHYSICS: FLAVON, NEUTRINO AND EXTRA-DIMENSIONAL MODELS

by

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This manuscript is dedicated to:

Jan Doršner

1969–1992

Nek' ti je lahka teška zemlja Bosanska.

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ABSTRACT

There is already significant evidence, both experimental and theoretical, that the Standard Model of elementary particle physics is just another effective physical theory. Thus, it is crucial (a) to anticipate the experiments in search for signatures of the physics beyond the Standard Model, and (b) whether some theoretically preferred structure can reproduce the low-energy signature of the Standard Model. This work pursues these two directions by investigating various extensions of the Standard Model. One of them is a simple flavon model that accommodates the observed hierarchy of the charged fermion masses and mixings. We show that flavor changing and CP violating signatures of this model are equally near the present experimental limits. We find that, for a significant range of parameters, μ-e conversion can be the most sensitive place to look for such signatures.

We then propose two variants of an $SO(10)$ model in five-dimensional framework. The first variant demonstrates that one can embed a four-dimensional flipped $SU(5)$ model into a five-dimensional $SO(10)$ model. This allows one to maintain the advantages of flipped $SU(5)$ while avoiding its well-known drawbacks. The second variant shows that exact unification of the gauge couplings is possible even in the higher dimensional setting. This unification yields low-energy values of the gauge couplings that are in a perfect agreement with experimental values. We show that the corrections to the usual four-dimensional running, due to the Kaluza-Klein towers of states, can be unambiguously and systematically evaluated.

We also consider the various main types of models of neutrino masses and mixings from the point of view of how naturally they give the large mixing angle

MSW solution to the solar neutrino problem. Special attention is given to one particular "lopsided" $SU(5)$ model, which is then analyzed in a completely statistical manner. We suggest that this sort of statistical analysis should be applicable to other models of neutrino mixing.

Chapter 1

INTRODUCTION

Our current understanding of Nature entails the existence of four fundamental interactions: electromagnetic, weak, strong, and gravitational interaction. The need for their full theoretical description has been a driving force behind the development of the modern physical theories.

The branch of physics that addresses the fundamental building blocks of Nature and their interactions is elementary particle physics. Its goal is to accurately depict the physical phenomena involving the fundamental forces to an arbitrarily high energy scale. Due to the various difficulties, this goal is commonly replaced with a more modest one of obtaining an effective theoretical description that agrees with experiments up to a specific energy scale but depicts the physical reality less accurately.

The effective theory approach is not specific for the elementary particle physics only and its examples abound in other areas of physics. If one is interested in macroscopic properties of the solid state system that consists of a large number of atoms placed at the lattice sites with the lattice spacing a, one can switch from a discreet to a continuous description of the system as long as the macroscopic properties pertain to the length scale that is much greater than the intrinsic scale a. One does not expect an agreement between the theory and the experiment for the length scale smaller than a as the theoretical description at that scale needs to be modified. In the formal language of elementary particle physics the scale a is referred to as the ultraviolet cutoff scale.

It is possible the cutoff scale of some effective theory is not known *a priori* due to the lack of our understanding of the underlying structure of the physical system. In such a case there is a possibility the effective theory itself contains the hint of where the cutoff might be. Simply put, the cutoff scale will be the scale where the theory does not cut it anymore. To show how this works in practice we turn to the self-energy of the electric field of a point charge within the classical electrostatic theory described by Coulomb's Law. Our initial expectation, just on the grounds of naturalness, is to find the self-energy of the electric field of the electron to be of the order of its rest mass. However, the self-energy, being proportional to the integral over the whole space of the square of the electric field, will diverge at the lower boundary of integration if we treat the electron as truly point-like and integrate from zero to infinity over the radial distance r. To cure the divergence we can parameterize our inability to describe the physics at very short distances by introducing a cutoff $r_{\rm min}$ to be the fictitious radius of the electron, integrate, and finally compare the electrostatic energy with the rest energy of electron $m_e c^2$. We thus obtain $r_{\rm min} \sim 4\pi\epsilon_0 e^2/(m_e c^2) \approx 0.28 \times 10^{-14}\,$m, which corresponds to the energy scale[1] $\Lambda \approx 0.71 \times 10^8\,$eV, as the plausible limit on the applicability of the classical electrostatics.

Introduction of the cutoff in the previous example does not imply that Coulomb's Law is completely wrong; it simply emphasizes the fact that Coulomb's Law is valid only within a certain energy range in which it properly accounts for all the relevant degrees of freedom. The theory must be modified at length scales smaller than $r_{\rm min}$. In the electrostatics case, the modification should already take place at the scale $\Lambda = 2m_e c^2 \approx 1.0 \times 10^6\,$eV to allow the possibility of the electron-positron

[1] In the natural system of units, where $c = \hbar = 1$, we can treat the energy and the length on an equal footing. One is the inverse of the other, i.e. small lengths correspond to large energies and vice versa. The conversion factor is $\hbar c \approx 0.19 \times 10^{-6}\,$eVm.

creation and annihilation. The theory has to be taken from the classical to the quantum level to accommodate new degrees of freedom. It is generally expected that the cutoff scale represents the scale where new degrees of freedom enter the physical picture.

The prime theory of elementary particle physics is the Standard Model (SM). It is, by far, the most successful theoretical structure that we have. In the case of the anomalous magnetic moment of the electron the agreement between the experiment and the *prediction* of the SM has been verified at the level of one part in 10^8 (for a review see [1]). The modern view is that the SM itself represents an effective theory valid up to certain energy scale. Where that scale is could be revealed through (a) the careful analysis of the theoretical structure of the SM following the same line of reasoning as in the case of the self-energy of the electron, and (b) the comparison against the experimental signatures.

Accepting that the SM is just an effective theory, we are bound to modify it and go beyond in order to make the theory applicable in a wider energy range, and to make it more self-consistent, and more predictive. It is, in the end, this predictability that can help us falsify or confirm the correctness of the theoretical ideas we build into the various models using the experimental results.

The SM has been around for more than three decades. It has proved itself against the electroweak precision measurements (for a review see Ref. [2]) and has resisted any significant modification. To fully appreciate the effectiveness of the SM we briefly describe its structure in what follows.

1.1 The Standard Model

The Standard Model is a gauge theory that comprises the Glashow, Weinberg, and Salam theory of electroweak interactions [3, 4, 5] and the theory of strong interactions [6, 7, 8, 9, 10]. The form of the interactions is governed by the direct product $\mathcal{H} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ of gauge symmetries.[2] In addition, the model is Lorentz invariant and renormalizable. The strength of each interaction of the particular group is parameterized by the corresponding gauge coupling constant. In the case of $\mathcal{H}$, this amounts to specifying three gauge constants g_3, g_2, and g_1.

The carriers of the interactions are spin-1 particles called gauge bosons. They are always associated with the adjoint representation of the appropriate gauge group. The dimension of the adjoint representation for the special unitary group $SU(N)$ is $N^2 - 1$. Thus, there are eight carriers of the strong interactions—gluon fields[3] G^i—associated with an $\mathbf{8}$ of $SU(3)_c$, three W^i bosons associated with a $\mathbf{3}$ of $SU(2)_L$, and one B boson of $U(1)_Y$. Here we use the group theoretical language and specify the representation by its dimension. Going one step further we say that, under $\mathcal{H}$, the gluons transform as $(\mathbf{8}, \mathbf{1}, 0)$, W's transform as $(\mathbf{1}, \mathbf{3}, 0)$, and B transforms as $(\mathbf{1}, \mathbf{1}, 0)$. Note that for the Abelian gauge group $U(1)$ it suffices to specify only one number, i.e. the gauge quantum number the field carries under the $U(1)$. At this stage all the gauge bosons are massless.

The fermionic content of the theory is made out of spin-1/2 particles. In addition to their transformation properties under $\mathcal{H}$ we can also distinguish them by their spin orientation. If the spin of the fermion and its direction of motion are parallel (anti-parallel) the fermion is right-handed (left-handed). We can always project out the right-handed and the left-handed part of a Dirac spinor ψ using the projection operators $P_R = (1 + \gamma_5)/2$ and $P_L = (1 - \gamma_5)/2$ respectively. Namely, the right-handed (left-handed) four-component spinor is $\psi_{R(L)} = P_{R(L)}\psi$. Using the subscripts L and R to specify the handedness, the fermions of the Standard Model,

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[2] It took almost a decade for $SU(3)_c$ to be promoted from a global [6, 7] to a local symmetry [8, 9, 10]. For an extensive list of references on the development of the Standard Model see Ref. [11]

[3] The gauge bosons carry the group index (for gluons $i = 1, \ldots, 8$; for W's $i = 1, 2, 3$; for B $i = 1$) as well as the Lorentz index $\mu = 0, 1, 2, 3$. We will usually suppress these indices for brevity.

usually referred to as the matter fields, and their transformation properties under $\mathcal{H}$ are

$$L_{Li} \equiv (\mathbf{1}, \mathbf{2}, -1)_{Li} \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_{Li}, \tag{1.1a}$$

$$e_{Ri} \equiv (\mathbf{1}, \mathbf{1}, -2)_{Ri}, \tag{1.1b}$$

$$Q_{Li} \equiv (\mathbf{3}, \mathbf{2}, 1/3)_{Li} \equiv \begin{pmatrix} u \\ d \end{pmatrix}_{Li}, \tag{1.1c}$$

$$u_{Ri} \equiv (\mathbf{3}, \mathbf{1}, 4/3)_{Ri}, \tag{1.1d}$$

$$d_{Ri} \equiv (\mathbf{3}, \mathbf{1}, -2/3)_{Ri}, \tag{1.1e}$$

where $i = 1, 2, 3$ represents the generation index. The $SU(3)_c$ color index of the quarks ($\alpha = 1, 2, 3$) and the $SU(2)_L$ index of the doublets ($a = 1, 2$) are suppressed.

The first two rows in Eqs. (1.1) are the leptons and the last three rows are the quarks. The fermionic members of one generation make up a family. The SM contains three of them. The hypercharge—the $U(1)_Y$ quantum number—is normalized so that $Q = I_{3L} + Y/2$, where Q represents the electric charge operator and I_{3L} is the isospin, i.e. the eigenvalue of the third generator of the isospin group $SU(2)_L$. The matter fields are also massless at this point.

The gauge invariant kinetic energy terms for the fermions and the gauge bosons are

$$\mathcal{L}_{\text{fermion}} = \overline{L}_{Li} \mathrm{i} \not{D} L_{Li} + \overline{e}_{Ri} \mathrm{i} \not{D} e_{Ri} + \overline{Q}_{Li} \mathrm{i} \not{D} Q_{Li} + \overline{u}_{Ri} \mathrm{i} \not{D} u_{Ri} + \overline{d}_{Ri} \mathrm{i} \not{D} d_{Ri}, \tag{1.2}$$

and

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} - \frac{1}{4} W^i_{\mu\nu} W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{1.3}$$

respectively. The field strength tensors are

$$G^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_3 f_{ijk} G^j_\mu G^k_\nu, \tag{1.4a}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon_{ijk} W^j_\mu W^k_\nu, \tag{1.4b}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{1.4c}$$

where f_{ijk} and ϵ_{ijk} represent totally antisymmetric structure constants of $SU(3)$ and $SU(2)$ respectively. [The structure constants g_{ijk} of the $SU(N)$ group can be found for any nontrivial d-dimensional representation associated with the generators $X_i^{(d)}$ of that group through the commutation relations $[X_i^{(d)}, X_j^{(d)}] = \mathrm{i}g_{ijk}X_k^{(d)}$.] The gauge covariant derivative

$$D_\mu = \left(\partial_\mu + \mathrm{i}g_3 I_{ic} G^i_\mu + \mathrm{i}g_2 I_{iL} W^i_\mu + \mathrm{i}g_1 Y/2 B_\mu\right) \tag{1.5}$$

acts in the group space defined by the dimension of the representation of the appropriate matter field, where I_{ic} (I_{iL}) are $SU(3)_c$ ($SU(2)_L$) matrices. In the fundamental representation of $SU(3)_c$ ($SU(2)_L$), $I_{ic} = \lambda^i/2$ ($I_{iL} = \tau^i/2$), where λ^i (τ^i) represent the familiar Gell-Mann (Pauli) matrices. Recall that the dimension of the fundamental (defining) representation of $SU(N)$ is N. We normalize the generators of $SU(N)$ so that $\mathrm{Tr}[X_i^{(N)} X_j^{(N)}] = (1/2)\delta_{ij}$ for the fundamental representation.

We know from experiments that the matter fields have mass. How do the masses arise in the SM? We first show that the $SU(2)_L \otimes U(1)_Y$ gauge symmetries of the SM do not allow the usual bare masses for the matter fields and then introduce the mechanism that rectifies this shortcoming.

The bare mass m of the fermionic field ψ comes from the Lagrangian term

$$m\overline{\psi}\psi = m(\overline{\psi}P_L\psi + \overline{\psi}P_R\psi) = m\overline{\psi}_R\psi_L + m\overline{\psi}_L\psi_R, \tag{1.6}$$

which suggests that the fermions get their mass through the mating of the left- and the right-handed fields. The mass of the charged leptons thus comes from the mating of $\overline{L}_{Li}$ fields with e_{Ri} fields. The trouble is that their product in the group space is not $\mathcal{H}$ invariant due to the chiral nature of the SM, i.e. the left- and the right-handed fields transform differently under $\mathcal{H}$. More specifically, the product behaves as a singlet, i.e. gauge invariant, under $SU(3)_c$ but a doublet under $SU(2)_L$.

Moreover, the hypercharges of the fields add to -1[§] instead of zero. Any attempt to provide the mass for the up (down) quarks fails for the same reason. The product of the $\overline{Q}_{Li}$ with the u_{Ri} (d_{Ri}) yields the singlet of $SU(3)_c$, the doublet of $SU(2)_L$, and the hypercharges of the fields add to nonzero value 1 (-1). To summarize, the bare mass of the leptons and the quarks, though allowed by the color group, is explicitly *forbidden* by $SU(2)_L \otimes U(1)_Y$ gauge symmetry.

It is not only the matter fields that need to get mass. The gauge bosons of the weak interactions also have to be massive since the weak force is a short range one. The Higgs mechanism [12, 13, 14, 15, 16, 17] solves both of these problems. It bypasses the violation of the gauge invariance by replacing the bare mass in Eq. (1.6) with a complex spin-0[**] Higgs field H that transforms nontrivially under $\mathcal{H}$. The gauge properties of H are always chosen to yield a gauge singlet when contracted with the left-right field combination. In the SM case it obviously has to be a singlet under $SU(3)_c$, a doublet under $SU(2)_Y$, and its hypercharge must be 1 (-1) when coupled with the leptons and the down (up) quarks. This last requirement would call for two distinct Higgs fields if it was not for one special property of the fundamental representation of $SU(2)$. Namely, the $\mathbf{2}$ and the $\overline{\mathbf{2}}$ are related to each other via a similarity transformation. In our notation

$$H \equiv (\mathbf{1}, \mathbf{2}, 1) \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} H_1 - \mathrm{i}H_2 \\ H_3 - \mathrm{i}H_4 \end{pmatrix}, \tag{1.7}$$

where the superscript indicates the electric charge of the Higgs fields. The last form in Eq. (1.7) is written in terms of the Hermitian fields $H_i = H_i^\dagger$ observing that one complex scalar doublet has four degrees of freedom.

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§ Note that the hypercharge of $\overline{L}_{Li}$ is 1 (see Eq. (1.1a)). In general, if the field ψ is associated with the representation of dimension d, then its conjugate belongs to the conjugate representation $\overline{d} \equiv d^*$. In the case of the $U(1)$ symmetry this amounts to a change of sign of the $U(1)$ quantum number.

** This field carries no Lorentz indices and has the same dimensions as the mass.

The fermion masses arise from the gauge invariant Yukawa terms

$$\mathcal{L}_{\text{Yukawa}} = -\lambda^u_{ij}\epsilon^{ab}\overline{Q}^a_{Li}H^{\dagger b}u_{Rj} - \lambda^d_{ij}\overline{Q}^a_{Li}H^a d_{Rj} - \lambda^l_{ij}\overline{L}^a_{Li}H^a e_{Rj} + \text{H.c.}, \tag{1.8}$$

after H obtains a constant value. This happens only under the suitable circumstances which we investigate shortly. Here, $i, j = 1, 2, 3$ are the family indices, $a, b = 1, 2$ are $SU(2)_L$ indices, and ϵ^{ab} is a totally antisymmetric tensor. The matrices $\lambda^{u,d,l}$ are dimensionless and completely arbitrary. Their entries are referred to as the Yukawa couplings.

The gauge boson masses come from the Higgs field Lagrangian

$$\mathcal{L}_{\text{Higgs}} = (D^\mu H)^\dagger D_\mu H - V(H), \tag{1.9}$$

where the gauge covariant derivative $D_\mu H = \left(\partial_\mu + \mathrm{i}g_2 I_{iL} W^i_\mu + \mathrm{i}g_1 Y/2 B_\mu\right) H$ acts on H and $V(H)$ is the Higgs potential. The form of the potential, restricted by $SU(2)_L \otimes U(1)_Y$ invariance, is

$$V(H) = \lambda(H^\dagger H)^2 - \mu^2(H^\dagger H), \tag{1.10}$$

where the choice $\lambda, \mu^2 > 0$ makes H take on a nonzero vacuum expectation value (VEV). This simply means that the nonzero value of the field H minimizes the potential $V(H)$. If the VEV is chosen to be

$$\langle 0|H|0\rangle \equiv \langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{1.11}$$

where $v^2 = \mu^2/\lambda$ yields the minimum of $V(H)$, the generators I_{1L}, I_{2L}, and $I_{3L} - Y/2$ are spontaneously broken, i.e. the vacuum state is not invariant under the gauge transformations generated by these generators. However, the linear combination $I_{3L} + Y/2$ annihilates the vacuum since the vacuum carries no charge (see Eq. (1.7)) leaving residual $U(1)_{em}$ gauge symmetry behind. This residual gauge symmetry has made its *debut* in the Maxwell's equations of the classical electrodynamics. The gauge boson of the unbroken $U(1)_{em}$ is the familiar massless photon. The process of the spontaneous symmetry breaking can be schematically shown as $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_{em}$.

If $SU(2)_L \otimes U(1)_Y$ were a global symmetry we would expect to see three massless Nambu-Goldstone bosons [18, 19, 20, 21, 22] upon the symmetry breaking. This is not the case for the local (gauge) symmetry where these would-be massless fields get absorbed (eaten) by the gauge bosons associated with the generators of the broken symmetries that require these fields to become massive.[4] To see this process clearly we can use the freedom to rotate the Higgs field in the $SU(2)_L \otimes U(1)_Y$ space to absorb the three degrees of freedom visible in Eq. (1.7). This defines a so-called unitary gauge where the Higgs field takes the following form

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}. \tag{1.12}$$

The field h represents the fluctuations around the minimum and it is the actual physical Higgs scalar. At the same time the potential, in the unitary gauge, reads

$$V(h) = -\frac{\mu^4}{4\lambda} + \mu^2 h^2 + \lambda\sqrt{\frac{\mu^2}{\lambda}} h^3 + \frac{\lambda}{4} h^4, \tag{1.13}$$

yielding the Higgs scalar mass

$$m_h = \sqrt{2\mu^2} = \sqrt{2\lambda} v. \tag{1.14}$$

The physical gauge bosons and their masses follow from the first term of Eq. (1.9). The heavy bosons of weak interactions are

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp \mathrm{i} W_\mu^2), \tag{1.15a}$$

$$Z_\mu^0 = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3, \tag{1.15b}$$

[4] Massless vector field, such as a photon, has two transverse degrees of freedom while massive field has an extra (longitudinal) degree of freedom. Therefore, massless gauge boson has to gain one degree of freedom to become massive.

where θ_W is the electroweak mixing angle defined by $\tan\theta_W \equiv g_1/g_2$. Their masses are

$$m_W = \frac{g_2}{2} v, \tag{1.16}$$

and

$$m_Z = \frac{\sqrt{g_1^2 + g_2^2}}{2} v = \frac{m_W}{\cos\theta_W}. \tag{1.17}$$

Finally, the linear combination orthogonal to Z_μ^0 is the massless photon field

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3. \tag{1.18}$$

The fermion masses emerge upon the diagonalization of the couplings in Eq. (1.8) which prior to that reads

$$\mathcal{L}_{\text{Yukawa}} = -\frac{\lambda_{ij}^u}{\sqrt{2}} \overline{u}_{Li} u_{Rj}(v+h) - \frac{\lambda_{ij}^d}{\sqrt{2}} \overline{d}_{Li} d_{Rj}(v+h) - \frac{\lambda_{ij}^l}{\sqrt{2}} \overline{e}_{Li} e_{Rj}(v+h) + \text{H.c.}. \tag{1.19}$$

The Dirac mass matrices $U = \lambda^u v/\sqrt{2}$, $D = \lambda^d v/\sqrt{2}$, and $L = \lambda^l v/\sqrt{2}$ are in general completely arbitrary and must be diagonalized by bi-unitary transformations. For example, the redefinition of the matter fields in the flavor space $u_{Li} \to (U_u)_{ij} u_{Lj}$ and $u_{Ri} \to (V_u)_{ij} u_{Rj}$, where the unitary matrices U_u and V_u are chosen to diagonalize U ($U_u^\dagger U V_u = U^{\text{diag}}$), specifies the mass basis for the up quarks. The same procedure can be repeated for the down quarks ($U_d^\dagger D V_d = D^{\text{diag}}$) and the charged leptons ($U_l^\dagger L V_l = L^{\text{diag}}$) yielding the familiar up quark mass eigenstates (u, c, t), the down quarks mass eigenstates (d, s, b), and the charged leptons mass eigenstates (e, μ, τ). The only place where the change of the basis leaves a trace, generating the changing of the flavor, is in the Lagrangian term that describes the charged-current processes. Its form (see Eq. (1.2)), in terms of the gauge boson mass eigenstates, is

$$\mathcal{L}_{\text{cc}} = -g_2 \left(W_\mu^+ J_W^\mu + W_\mu^- J_W^{\mu\dagger} \right), \tag{1.20}$$

where the charged current reads

$$J_W^{\mu\dagger} = \frac{1}{\sqrt{2}} \left(\overline{\nu}_{Li} \gamma^\mu e_{Li} + \overline{u}_{Lj} \gamma^\mu d_{Lj} \right). \tag{1.21}$$

If we now redefine the matter fields as suggested above

$$\overline{u}_{Lj}\gamma^{\mu}d_{Lj} \rightarrow \overline{u}_{Li}\gamma^{\mu}U^{\dagger}_{uij}U_{djk}d_{Lk} \equiv \overline{u}_{L}\gamma^{\mu}V_{\text{CKM}}d_{L}, \tag{1.22}$$

we generate Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix $V_{\text{CKM}} \equiv U^{\dagger}_{u}U_{d}$ [23, 24]. The CKM matrix parameterizes the strength of the flavor changing as well as the amount of CP violation in the SM. The elements of the CKM matrix are labeled V_{ij}, where $i = u, c, t$ and $j = d, s, b$.

Since the neutrinos have no mass in the SM we are free to rotate them to match the change in the definition of e_{Li} fields in Eq. (1.21). This implies there is no analogue of the CKM matrix in the leptonic sector of the theory. Anticipating the extensions of the SM that accommodate experimentally observed neutrino masses we are led to introduce Pontecorvo-Maki-Nakagawa-Sakata mixing matrix $U_{\text{PMNS}} = U^{\dagger}_{l}U_{\nu}$ [25, 26, 27] where U_{ν} is the unitary matrix that diagonalizes the neutrino mass matrix M_{ν}. We will see later that the matrix M_{ν} is always symmetric which makes it qualitatively different from U, D, and L. The elements of the PMNS matrix are labeled U_{ij}, where $i = e, \mu, \tau$ and $j = 1, 2, 3$. The reason behind this notation is that the neutrino mass eigenvalues are labeled m_1, m_2, and m_3.

One of the successes of the SM is the explanation of the Fermi theory of beta decay on a more fundamental level. If we contract the two terms of Eq. (1.20) to obtain the tree level description of the four-fermion interaction, and integrate out the W boson, we obtain

$$\mathcal{L}_{\text{effective}} = \frac{g_2^2}{2m_W^2}(\overline{e}_{Li}\gamma^{\mu}\nu_{Li} + \overline{d}_{Lj}\gamma^{\mu}u_{Lj})(\overline{\nu}_{Li}\gamma_{\mu}e_{Li} + \overline{u}_{Lj}\gamma_{\mu}d_{Lj}). \tag{1.23}$$

Comparison with Fermi's four-fermion point interaction Lagrangian yields

$$\frac{G_F}{\sqrt{2}} \simeq \frac{g_2^2}{8m_W^2} = \frac{1}{2v^2}, \tag{1.24}$$

where G_F represents the Fermi constant. Since the value of G_F can be extracted from the experiments on the muon lifetime ($G_F = 1.16637(1) \times 10^{-5}\,\text{GeV}^{-2}$ [28])

we can determine the value of $SU(2)_L \otimes U(1)_Y$ breaking scale $v \simeq 246\,\mathrm{GeV}$. The Fermi theory is yet another example of an effective theory with a known cutoff. It is applicable in the regime of the length scales much larger than the length scales of the weak interactions (as determined by the electroweak breaking scale). Only when we resolve the gauge boson degrees of freedom we have to switch to the more fundamental description of Eq. (1.20).

An unexpected *prediction* of the SM is the conservation of B and L numbers. Namely, if we assign a baryon number B to all the quarks (1/3 (−1/3) for the quark (antiquark)) and a lepton number L to all the leptons (−1 (1) for the lepton (antilepton)) it turns out that all the processes with $\Delta B \neq 0$ and $\Delta L \neq 0$ are forbidden by the SM Lagrangian. This comes from an accidental symmetry of the SM. But it is this very accident that ensures the stability of the proton and thus the stability of our Universe.

We end this brief review of the SM by quoting some numerical values of the SM parameters.[5] The electroweak angle is determined from the measured mass of m_Z ($m_Z = 91.1876 \pm 0.0021\,\mathrm{GeV}$) and the fine structure constant $\alpha(m_Z) \equiv e(m_Z)^2/(4\pi)$ ($\alpha(m_Z)^{-1} = 128.92 \pm 0.03$). [The electric charge of the positron is identified with $e = g_1 g_2/\sqrt{g_1^2 + g_2^2} = g_2 \sin\theta_W$.] One obtains $\sin^2\theta_W = 0.23105 \mp 00008$ [29]. The strong coupling constant is $\alpha_3(m_Z) = 0.118 \pm 0.003$ and $m_W = 80.423 \pm 0.039\,\mathrm{GeV}$.

1.2 Open questions of the Standard Model

If the SM is truly an effective theory it has its own physical cutoff Λ. This cutoff, as we have seen in the Introduction, represents the energy scale where new degrees of freedom enter the physical picture. In our case the next obvious scale after the electroweak breaking scale v is given by the reduced Planck mass $M_{\mathrm{Pl}} =$

[5] The quoted values are from Ref. [28] unless specified otherwise.

$(8\pi G_{\text{Newton}})^{-1/2} = 2.4 \times 10^{18}$ GeV where the gravitational degrees of freedom start to compete with the SM ones. The disparity between the two scales is the very essence of the infamous hierarchy problem [30, 31, 32, 33].

To give a full flavor of the hierarchy problem let us consider the one-loop quantum corrections to the μ^2 parameter of the Higgs potential. These corrections stem from the fermion-fermion-Higgs couplings (see Eq. (1.8)) of the general form $\lambda_\psi \overline{\psi}\psi H$, where λ_ψ represents the Yukawa coupling of the Dirac fermion ψ. Assuming that the Higgs field H is just an ordinary complex scalar field the contribution of the fermion ψ reads (for a relevant Feynman diagram see Fig. 1.1 (a))

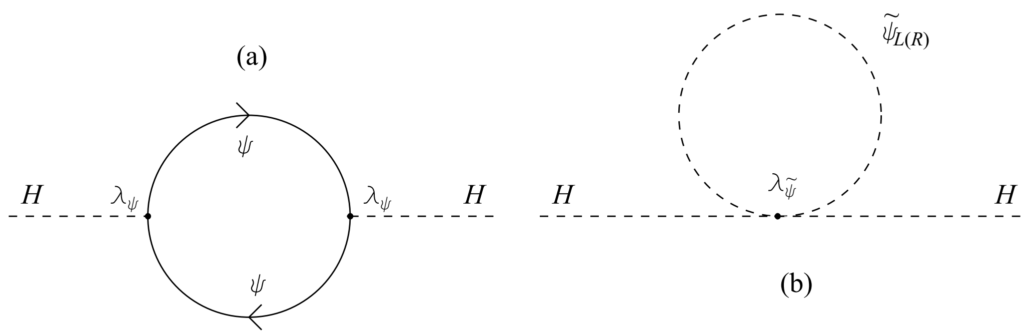


Figure 1.1: Feynman diagram of the one-loop correction to the Higgs potential parameter μ^2 via (a) the fermion loop; (b) the scalar loop(s).

$$\Delta\mu^2 = \frac{|\lambda_\psi|^2}{16\pi^2}\left[2\Lambda^2 - 6m_\psi^2 \ln\frac{\Lambda^2}{m_\psi^2} + \cdots\right], \tag{1.25}$$

where the ellipses represent finite terms that do not depend explicitly on Λ. The value that we prefer from the point of view of the electroweak symmetry breaking is $\mu^2 \sim (100\,\text{GeV})^2$ (assuming $\lambda \sim \mathcal{O}(1)$). But if the scale Λ was associated with the natural cutoff—the Planck scale—the quadratically divergent loop correction would drive μ^2 to the value that is 30 orders of magnitude away from the preferred value. Even without any correction we would expect the natural value of μ^2 to be of the order of the Planck mass since there is no symmetry that prevents the Higgs mass term in the Higgs potential. It is possible that some miraculous accidental cancellation takes place between the Higgs bare mass and the quantum corrections to explain μ^2 of the order of the weak scale but this is considered highly unnatural. The hierarchy problem becomes even more obvious if we note that all the quarks and the leptons and all the heavy gauge bosons get their masses through the Higgs mechanism. Any shift in the Higgs mass will manifest itself in the shift of all other masses. If the shift was infinite the theory would not be plausible anymore. The three main directions in elementary particle physics that address the stability of the Higgs mass under the quantum corrections and the disparity between the Planck and the weak scale are supersymmetry (see a review [34]), dynamical electroweak symmetry breaking (see a review [35]), and the theories with large extra spatial dimensions [36, 37, 38, 39]. We will describe the first of these ideas in some detail in Section 1.3.

Another shortcoming of the SM is the existence of a large number of free input parameters. There are three gauge groups with *three* distinct gauge couplings. There are *nine* seemingly unrelated masses of the quarks and charged leptons randomly spread over the energy range spanning six orders of magnitude. There are *four* parameters of the CKM mixing matrix (three angles and a phase) that come out of the diagonalization of the mass matrices. There are also *two* parameters of the Higgs potential that have to be fine-tuned to give the electroweak symmetry breaking. In addition to these, there is *one* more parameter of the SM: the θ parameter of QCD (for a review see [40]). Its existence is connected to the CP violating term of the SM Lagrangian and its smallness is what constitutes the strong CP problem. All in all, there are *nineteen* apparently unrelated parameters that one has to deal with. [Admittedly, the SM works well but only after we replace its parameters with the experimentally determined values.] What one would hope is to have an underlying principle that connects these seemingly unrelated parameters of the SM.

The shortcomings do not end here. There is also the question of the mysterious hypercharge assignment of the quarks and the leptons that gives both the neutrality of the ordinary matter ($Q_e = -Q_{\text{proton}}$) and the charge quantization. Moreover, the same hypercharge assignment is the backbone of the proof of the renormalizability of the SM that hinges on the fact that the triangular anomaly cancellation takes place. The condition for anomaly cancellation is that the sums of Y and Y^3 over the members of one family, taking the hypercharge of the right-handed field with an extra $(-)$ sign, vanish. Looking back at Eqs. (1.1) we have

$$\mathrm{tr}[Y] = 2(-1)^3 + (2)^3 + 6\left(\frac{1}{3}\right)^3 + 3\left(-\frac{4}{3}\right)^3 + 3\left(\frac{2}{3}\right)^3 = 0, \tag{1.26a}$$

$$\mathrm{tr}[Y^3] = 2(-1) + (2) + 6\left(\frac{1}{3}\right) + 3\left(-\frac{4}{3}\right) + 3\left(\frac{2}{3}\right) = 0, \tag{1.26b}$$

which represents a truly miraculous cancellation in view of the fact that it comes about through the conspiring of unrelated multiplets of the quarks and the leptons. The set of the theories that justifies the anomaly cancellation, relates the gauge couplings of otherwise unrelated gauge groups, and along the way arranges many other pieces of the puzzle goes under the generic name of Grand Unified Theories (GUTs). We will formally introduce the GUTs in Section 1.4.

All unresolved issues that we have mentioned so far are related to the purely theoretical considerations of the self-consistency of the SM. Their explanation is motivated in part by the quest for the naturalness of the theory, the aesthetics, and, in some instances, the experimental hints. There is, however, one pressing issue—the issue of the neutrino mass—that is raised by the experimental observations only. It stems from the fact that the SM fails to accommodate the neutrino mass contrary to an overwhelming body of experimental evidence (for a review see [41]). With the neutrino mass in place the palette of the SM would have to accept at least *seven* more parameters (three neutrino masses, and four parameters of the PMNS matrix) and the energy range would have to stretch additional six orders of magnitude to accept the neutrino mass of the order of $1/20\,\mathrm{eV}$ as suggested by the Super-Kamiokande experiment [42]. The smallness of the neutrino mass suggests that the entries of the matrix M_ν are much smaller than the entries of U, D, and L. This is another qualitative difference between M_ν and U, D, and L. Postponing any further discussion on neutrino mass until the Section 1.4 we now turn to the description of some of the most interesting extensions of the SM.

1.3 Supersymmetry

Supersymmetry (SUSY) is a symmetry that turns fermions into bosons, and vice versa. It is conceptually completely different from the familiar gauge symmetries. The generators responsible for the mutation of the bosons into the fermions are spin-1/2 fermionic operators. This makes SUSY a spacetime symmetry as oppose to the usual gauge symmetry that simply commutes with the Lorentz transformations and thus operates in its own gauge-space. Moreover, the fermionic operators anti-commute and so will the SUSY generators, in sharp contrast to the gauge symmetry generators that always satisfy commutation relations. This drastic departure from the usual gauge symmetries is worthwhile since SUSY ameliorates the hierarchy problem by stabilizing the Higgs mass against the radiative corrections as we soon demonstrate.

The most obvious change SUSY brings along is the doubling of the number of the degrees of freedom of the SM. For every boson (fermion) of the SM it introduces its SUSY partner—a fermion (boson). [We will use $\sim$ over the particle's SM symbol to explicitly mark its SUSY partner.] The doubling procedure is very special. It assigns the same gauge transformation properties and the gauge interactions to the SUSY particle its SM counterpart has by making them both share the same supermultiplet. The supermultiplets of the matter (gauge) fields and their partners are called the chiral (vector) supermultiplets. In addition, the Higgs field itself forms a chiral supermultiplet with its SUSY partner—the Higgsino. [The name of the spin-1/2 superpartner of the SM boson is obtained by appending "-ino" to the boson name.] The chiral nature of the SM requires two different supermultiplets for the left- and the right-handed parts of the Dirac fermion. It is thus more convenient to part with the usual Dirac four-component notation and treat the left- and the right-handed parts as being two different two-component Weyl spinors.[6] [The change in the notation will also make our discussion on the GUTs much more transparent.] In the Weyl notation the Dirac fermion ψ reads

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{1.27}$$

It is also more convenient to deal with the left-handed Weyl fields only, which is achieved by Hermitian conjugation of all the right-handed fields. [For example, the left-handed field form of ψ_R is $\psi_R^\dagger$.] In terms of the new notation the chiral supermultiplets are

$$L_i \equiv (\mathbf{1}, \mathbf{2}, -1)_i, \tag{1.28a}$$

$$\bar{e}_i \equiv (\mathbf{1}, \mathbf{1}, 2)_i, \tag{1.28b}$$

$$Q_i \equiv (\mathbf{3}, \mathbf{2}, 1/3)_i, \tag{1.28c}$$

$$\bar{u}_i \equiv (\overline{\mathbf{3}}, \mathbf{1}, -4/3)_i, \tag{1.28d}$$

$$\bar{d}_i \equiv (\overline{\mathbf{3}}, \mathbf{1}, 2/3)_i, \tag{1.28e}$$

where the bar over u, d, and e reminds us that we deal with the left-handed fields only. It is understood that each Weyl fermion in Eqs. (1.28) is accompanied with the complex scalar ($\bar{e}_i \equiv \left(e_{Ri}^\dagger, \tilde{e}_{Ri}^*\right)$). [The name of the spin-0 superpartner of the SM fermion is obtained by adding "s-" on the fermion name. For example, the superpartner of the electron is selectron.]

[6] For the alternative approach see Ref. [43].

The doubling will now generate a new contribution to $\Delta\mu^2$ at the one-loop level coming from the scalar superpartners $\tilde{\psi}_L^*$ and $\tilde{\psi}_R^*$ of the Dirac fermion ψ (see Fig. 1.1 (b) for the relevant Feynman diagram). The contribution reads

$$\Delta\mu^2 = \frac{\lambda_{\tilde{\psi}}}{16\pi^2}\left[-2\Lambda^2 + 2m_{\tilde{\psi}}^2 \ln\frac{\Lambda^2}{m_{\tilde{\psi}}^2} + \cdots\right], \tag{1.29}$$

where $m_{\tilde{\psi}}$ represents the scalar mass. SUSY not only ensures that $|\lambda_\psi|^2 = \lambda_{\tilde{\psi}}$ so that the quadratic divergences in Eqs. (1.25) and (1.29) cancel against each other but forces the cancellation to take place for all the fields and to all orders. Moreover, by imposing the same gauge transformations for the SM fields and their partners, unbroken SUSY *predicts* $m_\psi = m_{\tilde{\psi}}$.

The situation with the Higgs sector is a little different from the SM case. As we have already seen, the anomaly cancellation represents an external consistency condition imposed on the SM structure. In the case of SUSY it requires the existence of two Higgs supermultiplets $H_u \equiv (\mathbf{1}, \mathbf{2}, 1)$ and $H_d \equiv (\mathbf{1}, \mathbf{2}, -1)$ instead of one. If only one of them were present the contribution from the Higgs superpartner—Higgsino—would spoil the neat cancellation exhibited in Eq. (1.26). The situation is much better if there are two superpartners with the opposite hypercharges. It is only then that their contributions towards Y and Y^3 traces cancel. The subscripts u and d serve the bookkeeping purpose: H_u, with the VEV $\langle H_u \rangle = \binom{0}{v_u}$, gives the mass to the up quarks; H_d, with the VEV $\langle H_d \rangle = \binom{v_d}{0}$, gives the mass to the down quarks and the charged leptons. The ratio $\tan\beta = v_u/v_d$ plays an important role in the phenomenological considerations. The VEVs themselves are directly related to the electroweak breaking scale via $v = 2\sqrt{v_u^2 + v_d^2}$.

The gauge sector of the SM is promoted into the set of vector supermultiplets comprising the usual gauge bosons and their fermionic superpartners—gauginos. These vector supermultiplets and the chiral supermultiplets mentioned above specify the particle content of the Minimal Supersymmetric Standard Model (MSSM).

The most obvious problem with any supersymmetric theory, including the MSSM, is the mass degeneracy of the fields with the opposite statistics that inhabit the same supermultiplet. This degeneracy has to be lifted since we have not detected any bosonic particle with the mass corresponding to the mass of any of the matter fields of the SM. The lifting has to be done with two objectives in mind. First, we do not want to spoil the cancellation of the quadratic divergences that represented the strongest motivation for SUSY. Second, we need to keep the mass gap between the matter fields and their superpartners of the order of the electroweak breaking scale v. The first objective is accomplished if we break the SUSY via the "soft" breaking terms (for a detail classification see [44]). These terms have one thing in common: the couplings that multiply them have the mass dimension greater than or equal to one. The importance of the second objective is obvious. If violated, the second term in Eq. (1.25), proportional to the $(\mathrm{mass})^2$, would again drive the μ^2 parameter away from the preferred value. The second objective represents the so-called naturalness constraint on the superparticle masses. The justification for the presence of the soft terms in the Lagrangian and the exact scheme of the SUSY breaking represent the most active areas of the modern research on supersymmetric theories.

Besides the stabilization of the Higgs mass against the radiative corrections, SUSY makes two more improvements over the SM. One of them, which we find extremely significant, is on the unification of the gauge coupling constants. The running of the gauge coupling constants at the one-loop level, from one energy scale to the other, is given by

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_i(\mu')} + b_i \ln \frac{\mu'}{\mu}, \tag{1.30}$$

where μ and μ' are the energy scales in question, and, as before, $\alpha_i = g_i^2/(4\pi)$. Within the SM framework the beta coefficients b_i for the $SU(N)$ group read

$$b_i^{\mathrm{SM}} = -\frac{11}{3} C_2(SU(N)) + \frac{2}{3} T_f(d) + \frac{1}{3} T_b(d), \tag{1.31}$$

where C_2 is the quadratic Casimir ($g_{ilm}g_{jlm} = C_2\delta_{ij}$), and $T_{f(b)}(d)$ is the Dynkin index for the fermions (bosons) in the d-dimensional representation ($\mathrm{Tr}[X_i^{(d)}X_j^{(d)}] = T(d)\delta_{ij}$). On the other hand, the beta coefficients within SUSY simplify to

$$b_i^{\mathrm{SUSY}} = -3C_2(SU(N)) + T(d), \tag{1.32}$$

due to the fact that the chiral supermultiplets always have the same number of fermions and bosons ($T_f(d) = T_b(d)$) and the gauginos always live in the adjoint representation with the gauge bosons ($C_2(SU(N)) = T_f(N^2-1)$). The beta coefficient of the $U(1)_Y$ gauge group is

$$b_1 = \frac{3}{5}\sum_i \left(\frac{Y}{2}\right)_i^2, \tag{1.33}$$

where the index i runs over all the particles. It is now easy to run the gauge couplings in both the SM and the MSSM from the scale at which they are well known such as the M_Z scale to much higher scale where we suspect the more fundamental theory resides. It turns out that the gauge couplings almost meet but only in the case of MSSM (see Fig. 1.2). The scale where they meet lies below the Planck scale and it is usually referred to as the GUT scale ($M_{\mathrm{GUT}} \sim 2\times 10^{16}\,\mathrm{GeV}$) since it is only within the Grand Unified Theories that we expect solid interpretation of the gauge unification. The fact that the MSSM steers the couplings in the "right" direction is one more motivation to take SUSY seriously.

The other significant improvement of MSSM is on the justification of the electroweak symmetry breaking. As we have already seen, the presence of the spin-0 Higgs field in the SM is somewhat *ad hoc*. On the other hand, SUSY treats the Higgs field on an equal footing with the other matter fields placing it into just another chiral supermultiplet. This makes its appearance somewhat less special than it is in the SM. More importantly, the analysis of the renormalization group equations (RGEs) has shown that the running of the masses of the MSSM fields can drive the Higgs mass term towards the negative values. This makes the electroweak breaking an

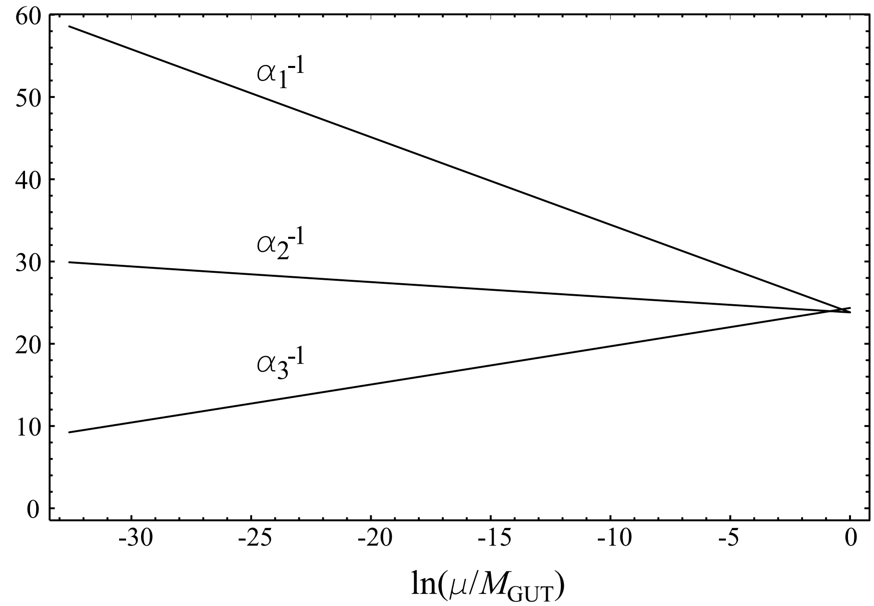


Figure 1.2: The two-loop running of the gauge couplings within the MSSM taking all superpartners to be degenerate at m_t. The relevant beta coefficients are summarized in Ref. [45].

ubiquitous process within the MSSM rather than an artificially introduced apparatus as it was in the SM.

For the sake of completeness of our exposition we proceed with the specification of the formalism of $\mathcal{N} = 1$ supersymmetry (for more details see Ref. [46]). The same formalism and notation will be used in the rest of this work. As the generic symbol for the chiral (vector) supermultiplet it is common to use Φ (V). Since the vector superfield belongs to the adjoint of the appropriate gauge group it obviously carries the group index a which we suppress. One then defines the chiral superfield $W_\alpha = -\frac{1}{4}\overline{DD}D_\alpha V$, where the differential operators D_α and $\overline{D}_{\dot{\alpha}}$ act in the superspace spanned by the variables $(x, \theta, \overline{\theta})$, and α and $\dot{\alpha}$ are the Weyl spinor indices in van der Waerden notation. In terms of these supermultiplets the most general supersymmetric renormalizable Lagrangian can then be written in the following form [46]:

$$\mathcal{L} = \frac{1}{16kg^2}\mathrm{Tr}\Big[W^\alpha W_\alpha\big|_{\theta\theta} + \mathrm{H.c.}\Big] + \Phi_i^\dagger \mathrm{e}^V \Phi_i\big|_{\theta\theta\overline{\theta\theta}} + \Big[\big(\underbrace{\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{6}g_{ijk}\Phi_i\Phi_j\Phi_k}_{W(\Phi)}\big)\big|_{\theta\theta} + \mathrm{H.c.}\Big]. \tag{1.34}$$

Here, $|_{\theta\theta} = \int \mathrm{d}^2\theta = -\frac{1}{4}\int \mathrm{d}^2\theta^\alpha\theta^\beta\epsilon_{\alpha\beta}$ is the shorthand for the integration over the Grassmann variables θ, and $V = V^a X_a$, where $\mathrm{Tr}[X_a X_a] = k\delta_{ab}$ in the adjoint. The holomorphic function $W(\Phi)$ is the superpotential. It contains all the information on the Yukawa couplings of the theory. Applying this formalism to the particle content of the MSSM we obtain the following form of the Lagrangian:

$$\begin{aligned}\mathcal{L} =& \frac{1}{16kg^2}\mathrm{Tr}\Big[W_i^\alpha W_\alpha^i\big|_{\theta\theta} + \mathrm{H.c.}\Big] \\ &+ \Big[L^\dagger \mathrm{e}^{V_L} L + \bar{l}^\dagger \mathrm{e}^{V_l} l + Q^\dagger \mathrm{e}^{V_Q} Q + \bar{u}^\dagger \mathrm{e}^{V_u}\bar{u} + \bar{d}^\dagger \mathrm{e}^{V_d}\bar{d} + H_u^\dagger \mathrm{e}^{V_{H_u}} H_u + H_d^\dagger \mathrm{e}^{V_{H_d}} H_d\Big]\Big|_{\theta\theta\overline{\theta\theta}} \\ &+ \Big[\big(\mu H_u H_d + \lambda_{ij}^u Q_i \bar{u}_j H_u + \lambda_{ij}^d Q_i \bar{d}_j H_d + \lambda_{ij}^l L_i \bar{l}_j H_d\big)\big|_{\theta\theta} + \mathrm{H.c.}\Big].\end{aligned} \tag{1.35}$$

In addition, there are the soft breaking terms which lift the SUSY degeneracy that we omit for simplicity.

As we have seen, SUSY has been tailored to make the theory stable under the radiative corrections ensuring its validity until we reach certain high energy scale where some more fundamental theory awaits. The new theory might show up at the GUT scale, as suggested by the MSSM running, introducing various relationships between the SM parameters valid only at that particular scale. In order to judge the credibility of the predictions and relationships of the new theory we need to propagate the values of the parameters that we observe at the weak scale up to the scale of the new theory. This we do in the case of the quark and the charged lepton masses, and the mixing angles, assuming the new theory emerges at the GUT scale. The results are summarized in Table 1.1. There are some quantitative

Table 1.1: The values of the quark and the charged lepton masses and the CKM angles V_{cb}, V_{us}, and V_{ub} at the m_t scale, compared to the experimental values extrapolated to the GUT scale ($M_{\rm GUT} = 2.37 \times 10^{16}$ GeV). Extrapolation is done taking all SUSY particles to be degenerate at m_t and assuming $\tan\beta = 3$. Masses are given in units of GeV.

	$m(m_t)$	$m(M_{\rm GUT})$
m_u	0.00127	0.000570
m_c	0.601	0.269
m_t	165	112
m_d	0.00288	0.000862
m_s	0.0501	0.0150
m_b	2.78	0.957
m_e	0.000502	0.000334
m_μ	0.104	0.0690
m_τ	1.75	1.16
$\|V_{us}\|$	0.22	0.22
$\|V_{cb}\|$	0.041	0.036
$\|V_{ub}\|$	0.0036	0.0032

relations that appear to hold at the GUT scale such as $m_b(M_{\rm GUT}) \simeq m_\tau(M_{\rm GUT})$, $m_s(M_{\rm GUT}) \simeq m_\mu(M_{\rm GUT})/3$, and $m_d(M_{\rm GUT}) \simeq 3m_e(M_{\rm GUT})$ that the new theory should try to account for. [This particular set of relations is referred to as the Georgi-Jarlskog [56] relations. The factors 1/3 and 3 are thus the Georgi-Jarlskog factors.] Bearing this in mind, we now turn our attention to the class of theories that are perfectly suited for explaining the quark and the lepton masses and mixings, as well as the gauge coupling unification.

1.4 Grand Unified Theories

The Grand Unified Theory aims at the unification of the quarks and the leptons under the group transformations of a certain unifying group $\mathcal{F}$ (for a review on group theory see [48]). The group $\mathcal{F}$ obviously contains the group $\mathcal{H}$ ($\mathcal{F} \supset \mathcal{H}$) and its gauging usually aids the understanding of the unification of the fundamental interactions already embedded in $\mathcal{H}$. The symmetry of the group $\mathcal{F}$ must be broken to yield the phenomenology in accord with the low-energy observations. The breaking $\mathcal{F} \to \mathcal{H}$ is usually assumed to happen at some very high energy scale. We then identify the breaking scale with the GUT scale where the couplings merge. We mostly review the supersymmetric form of GUTs (SUSY GUTs) unless explicitly stated otherwise.

It should be stressed from the onset that the group $\mathcal{F}$ does not have to be a single unifying group. It can also be a direct product of the groups. In that case the gauge coupling unification is not a prediction of the theory but rather an external aesthetic requirement. We will see examples of both cases in what follows.

1.4.1 Pati-Salam Model

The very first model of the quark-lepton unification is the model of Pati and Salam [49, 50, 51]. The model is based on the group $\mathcal{F} \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ with all the members of one family being placed in two multiplets: $F_{Li} \equiv (\mathbf{4}, \mathbf{2}, \mathbf{1})_{Li}$ and $F_{Ri} \equiv (\mathbf{4}, \mathbf{1}, \mathbf{2})_{Ri}$. More explicitly, we have

$$F_{L,Ri} = \begin{pmatrix} u^1 & u^2 & u^3 & \nu \\ d^1 & d^2 & d^3 & e \end{pmatrix}_{L,Ri}, \tag{1.36}$$

where we show the $SU(3)_c$ color indices only. [Note that F_{Li} and F_{Ri} are the conjugates of each other under the left-right discrete symmetry ($L \leftrightarrow R$).] The $SU(4)_c$ acts horizontally treating "lepton number as the fourth color" while both $SU(2)_L$ and $SU(2)_R$ act vertically on their respective multiplets. The charge operator in Pati-Salam model (PS) is given by a simple formula

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}, \tag{1.37}$$

which is to be compared with the SM one: $Q = I_{3L} + Y/2$. The hypercharge is clearly given by $Y/2 = I_{3R} + (B-L)/2$, where $B-L$ represents the generator of $SU(4)_c$. [In the fundamental representation of $SU(4)_c$ we have $B-L = \text{diag}(1/3, 1/3, 1/3, -1)$.] This time all the generators that enter the hypercharge definition originate from the non-Abelian groups. One of the properties of the non-Abelian groups is the quantization of the eigenvalues of their generators due to the nontrivial commutation relations that define the group algebra. Thus, Pati-Salam model justifies the peculiarity of the hypercharge assignment of the SM in a very natural way. This, in turn, ensures the neutrality of the ordinary matter and the anomaly cancellation. [Note that the right-handed neutrino makes no contribution to the anomaly.] The PS model introduces one right-handed neutrino for each family opening the possibility of generation of the experimentally observed neutrino mass. In addition to all that, the model promotes $B-L$ to a local symmetry. This turns out to be very important for the explanation of baryogenesis via leptogenesis [52]. The idea here is that the spontaneous violation of the local $B-L$ symmetry at some high temperature creates a lepton asymmetry. This then is converted to the observed baryon excess at lower temperatures by electroweak sphalerons [54, 55]. [It is the decay of the heavy right-handed neutrinos that can start the leptogenesis (for a review see [53]).]

All in all, the Pati-Salam model introduces a number of qualitatively new features with respect to the SM. These are:

- Unification of all the quarks and the leptons of one family within two multiplets.
- Introduction of the right-handed neutrinos.
- Justification of the hypercharge assignment and neutrality of the ordinary matter.
- Promotion of $B-L$ into a local symmetry.

Of course, one still needs to break the Pati-Salam model down to the SM. This is accomplished with the field $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$ that gets very large VEV in the right-handed neutrino direction. [To avoid SUSY breaking one actually needs the VEV for $(\mathbf{4},\mathbf{1},\mathbf{2})$, too.] This breaking however is not relevant for the quark and the charged lepton masses. They originate, together with the electroweak breaking, from the usual Higgs mechanism with “bi-doublet” of Higgs fields (H_u and H_d) transforming as $(\mathbf{1},\mathbf{2},\mathbf{2})_H$ under the PS group. To see the pattern of the masses and the mixings in the minimal PS model we switch again to the description in terms of the left-handed Weyl spinors: $(\mathbf{4},\mathbf{2},\mathbf{1})_L \to (\mathbf{4},\mathbf{2},\mathbf{1})$; $(\mathbf{4},\mathbf{1},\mathbf{2})_R \to (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$. The Yukawa term responsible for the mass, in group theoretical language, reads

$$\mathcal{L}_{\text{Yukawa}} = \lambda_{ij}(\mathbf{4},\mathbf{2},\mathbf{1})_i(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_j(\mathbf{1},\mathbf{2},\mathbf{2})_H \tag{1.38}$$

where, as before, λ represents arbitrary dimensionless matrix in the flavor space. After the Higgses in the bi-doublet get their VEVs we are left with the following *prediction* of the minimal supersymmetric Pati-Salam model: $U = N \propto D = L$, where N represents the Dirac mass matrix of neutrinos. This prediction is in gross disagreement with the experimental data. Moreover, it leads to the trivial CKM mixing matrix. [Note that the same bi-unitary transformations diagonalize U and D.] However, the most important thing is that we finally have the tool to relate mass matrices to one another. To create more realistic mass and mixing patterns one needs to extend the Higgs sector. For example, one can reproduce the Georgi-Jarlskog relations by introducing the Higgs in the adjoint of $SU(4)_c$ with the VEV pointing in the $B-L$ direction.

1.4.2 SU(5)

The smallest special unitary group that contains the SM group $\mathcal{H}$ is $SU(5)$. This fact prompted Georgi and Glashow [57] to use $SU(5)$ as a basis for the first true Grand Unified model.

The generators of $SU(5)$, in the fundamental representation, are 5×5 traceless Hermitian matrices acting on the five-dimensional vectors ψ^μ ($\mu = 1,\ldots,5$) we associate with the **5**. The embedding of $\mathcal{H}$ into $SU(5)$ is obvious if we perceive the 3×3 upper block matrices and the 2×2 lower block matrices of the $SU(5)$ generators as being the generators, in the fundamental representation, of $SU(3)_c$ and $SU(2)_L$ groups respectively. This means that the first three components of ψ^μ transform as the triplet of $SU(3)_c$ while the last two components transform as the doublet of $SU(2)_L$. To stress this fact it is customary to separate the index μ into two indices: $\alpha = 1,2,3$ of $SU(3)_c$ and $i = 4,5$ of $SU(2)_L$. The $U(1)_Y$ hypercharge generator must commute with all other generators and is chosen to be $Y \equiv \mathrm{diag}(-2/3,-2/3,-2/3,1,1)$. If we look back at Eqs. (1.28) we see that stacking the anti-triplet $\bar{d}_i$ and the doublet L_i into the five-dimensional vector yields appropriate representation and hypercharge assignments except for an extra field conjugation. To fix that we place $\bar{d}_i$ and L_i into the vector ψ_μ associated with the conjugate of the **5**—the $\overline{\mathbf{5}}$. In the group theoretical language, the decomposition $SU(5) \longrightarrow \mathcal{H}$ for the anti-fundamental reads

$$\overline{\mathbf{5}}_i \longrightarrow (\overline{\mathbf{3}},\mathbf{1},2/3)_i \oplus (\mathbf{1},\mathbf{2},-1)_i. \tag{1.39}$$

The next smallest representation of $SU(5)$ is the **10**. It is the antisymmetric product of the **5** with itself leading to the decomposition

$$\mathbf{10}_i \longrightarrow (\mathbf{3},\mathbf{2},1/3)_i \oplus (\overline{\mathbf{3}},\mathbf{1},-4/3)_i \oplus (\mathbf{1},\mathbf{1},2)_i, \tag{1.40}$$

which is just what we need to house the remaining ten particles of one family (Q_i, $\bar{u}_i$, and $\bar{e}_i$). Associating the **10** with a ten-dimensional antisymmetric tensor

$\psi^{\mu\nu} = \{\psi^{\alpha\beta}, \psi^{\alpha i}, \psi^{ij}\}$, we have

$$\psi_\mu = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \nu \\ e \end{pmatrix}, \text{ and } \psi^{\mu\nu} = \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & -u^1 & -d^1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u^2 & -d^2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -\bar{e} \\ d^1 & d^2 & d^3 & \bar{e} & 0 \end{pmatrix}. \tag{1.41}$$

The fact that the matter fields fit so neatly into two smallest representations of $SU(5)$ is nothing short of a miracle and serves as one of the main arguments for the GUTs.

The picture is not so perfect when it comes to the embedding of the Higgs doublets into $SU(5)$. It turns out that they abhor the unification. Assuming the MSSM content we need at least the $\mathbf{5}_H$ and the $\overline{\mathbf{5}}_H$ to accommodate H_u and H_d respectively. The problem is that they come with a triplet and an anti-triplet of "color" Higgses as can be seen from Eq. (1.39). Giving the large mass to these extra-fields to remove their signature from the low-energy phenomenology while keeping the doublets light is one of the most difficult tasks for model builders. This is the infamous "doublet-triplet splitting problem". We will address its implications and possible resolutions in great detail in Chapter 4.

The gauge fields, as always, reside in the adjoint representation—the $\mathbf{24}$. We can read off their transformation properties under $\mathcal{H}$ evaluating the product $\mathbf{5} \otimes \overline{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}$ in its decomposed form:

$$\mathbf{5} \otimes \overline{\mathbf{5}} \to \underbrace{(\mathbf{1}, \mathbf{1}, 0) \oplus (\mathbf{8}, \mathbf{1}, 0) \oplus (\mathbf{3}, \mathbf{2}, -5/3) \oplus (\overline{\mathbf{3}}, \mathbf{2}, 5/3) \oplus (\mathbf{1}, \mathbf{3}, 0)}_{24} \oplus \overbrace{(\mathbf{1}, \mathbf{1}, 0)}^{1}.$$

We now see that the gauge fields ($A^\mu{}_\nu$ ($A^\mu{}_\mu = 0$)), using the block matrix notation, transform as

$$A^{\mu}{}_{\nu} = \left(\begin{array}{c|c} \overbrace{A^{\alpha}{}_{\beta}}^{3\times 3} & \overbrace{A^{\alpha}{}_{i}}^{3\times 2} \\ \hline \underbrace{A^{i}{}_{\alpha}}_{2\times 3} & \underbrace{A^{i}{}_{j}}_{2\times 2} \end{array}\right) = \left(\begin{array}{c|c} (\mathbf{8},\mathbf{1},0) & (\mathbf{3},\mathbf{2},-5/3) \\ \hline (\overline{\mathbf{3}},\mathbf{2},5/3) & (\mathbf{1},\mathbf{3},0) \end{array}\right) + (\mathbf{1},\mathbf{1},0). \qquad (1.42)$$

Obviously, the fields $(\mathbf{8},\mathbf{1},0)$, $(\mathbf{1},\mathbf{3},0)$, and $(\mathbf{1},\mathbf{1},0)$ are the familiar gauge fields of the group $\mathcal{H}$. On the other hand the fields $(\mathbf{3},\mathbf{2},-5/3)$ and $(\overline{\mathbf{3}},\mathbf{2},5/3)$ represent completely novel interaction carriers. For example, the fields $(\mathbf{3},\mathbf{2},-5/3)$ can turn the anti-quark field in $(\overline{\mathbf{3}},\mathbf{1},2/3)$ into the lepton field in $(\mathbf{1},\mathbf{2},-1)$. This sort of process violates B and L numbers and represents the major prediction of any GUT—the *prediction* of the proton decay. The experimental limits on the lifetime of the proton are so severe that they represent the biggest stumbling block for any realistic GUT.

The breaking of $SU(5)$ down to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is facilitated by a Higgs field in an adjoint ($\phi^{\mu}{}_{\nu}$ ($\phi^{\mu}{}_{\mu} = 0$)). If the large VEV of the Higgs points in the direction of the hypercharge ($\langle \phi^{\mu}{}_{\nu} \rangle \equiv \mathrm{diag}(-2/3,-2/3,-2/3,1,1)$) the Higgs will not commute with the off-diagonal block matrices in Eq. (1.42) leaving the appropriate gauge bosons massive.

At this point, we observe that the Georgi-Glashow (GG) model has an extra desirable feature over the PS model. It introduces the notion of the gauge unification. Namely, since there is only one group there will be only one gauge coupling g. It is after the breaking that the familiar low-energy couplings g_3, g_2, and g_1 emerge. The ratio of these gauge couplings is now uniquely determined through the requirement that the appropriate generators in Eq. (1.5) all be normalized equally. In other words, we must have $g_3^2\,\mathrm{Tr}[I_{3c}^2] = g_2^2\,\mathrm{Tr}[I_{3L}^2] = g_1^2\,\mathrm{Tr}[(Y/2)^2] = g^2\,(1/2)$ for the fundamental representation of $SU(5)$. This condition, valid at the GUT scale, immediately gives $g_3^2 = g_2^2 = (3/5)g_1^2 = g^2$,[‡‡] *predicting* $\sin^2\theta_W = 3/8$.

‡‡ This explains the normalization of b_1 beta function in Eq. (1.33).

The unity of the quarks and the leptons always comes at a cost. We now have to use multiple stages of symmetry breaking as opposed to the SM case where there is only need for one. On the other hand, the enlarged symmetry relates the masses of the matter fields as we could have already witnessed in the PS model. In the case of $SU(5)$ model the minimal set of Yukawa terms is

$$\mathcal{L}_{\text{Yukawa}} = \lambda^u_{ij} \underbrace{\mathbf{10}_i \mathbf{10}_j \mathbf{5}_H}_{U} + \lambda^d_{ij} \underbrace{\mathbf{10}_i \overline{\mathbf{5}}_j \overline{\mathbf{5}}_H}_{D,L}, \tag{1.43}$$

which, after the Higgses get their VEVs, generates the following Dirac mass matrix relations: $U = U^{\mathrm{T}} = \lambda^u v_u$ and $D = L^{\mathrm{T}} = \lambda^d v_d$. More realistic mass and mixing patterns require larger Higgs sector. For example, we can reproduce the Georgi-Jarlskog relations that were mentioned a couple of times already with an extra $\overline{\mathbf{45}}_H$ of Higgs.

It is not difficult to extend the minimal $SU(5)$ model to accept the right-handed neutrinos $\bar{\nu}_i$. They will correspond to the gauge singlet fields—the $\mathbf{1}_i$. Their existence allows two qualitatively different types of the mass terms: (i) Majorana mass term, and (ii) Dirac mass term. The latter one is already familiar originating from $\lambda^\nu_{ij} \overline{\mathbf{5}}_i \mathbf{1}_j \mathbf{5}_H$, while the former one is allowed only for the particles that are their own antiparticles. If this indeed is the case for the electrically neutral neutrinos we can introduce the explicit Majorana mass term $(M_R)_{ij} \mathbf{1}_i \mathbf{1}_j$, where $(M_R)_{ij} = (M_R)_{ji}$ represents the Majorana mass matrix. Since no symmetry forbids M_R the magnitude of its elements is expected to be of the order of the cutoff scale of the theory. On the other hand, the Dirac mass matrix $N = \lambda^\nu v_u$ is of the order of the electroweak scale for $O(1)$ Yukawa couplings. The two terms, shown together as

$$(\nu \quad \bar{\nu}) \begin{pmatrix} 0 & N \\ N^{\mathrm{T}} & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix}, \tag{1.44}$$

can be brought through the redefinition of the fields into the following approximate block diagonal form

$$\begin{pmatrix} -NM_R^{-1}N^{\mathrm{T}} & 0 \\ 0 & M_R \end{pmatrix}. \tag{1.45}$$

Clearly, the mass eigenstates of the light neutrinos are obtained by diagonalizing the symmetric matrix $N_\nu = N_\nu^{\mathrm{T}} = -NM_R^{-1}N^{\mathrm{T}}$. The form of N_ν implies that it is the largeness of M_R that is responsible for the smallness of the observed neutrino masses. This mechanism is thus referred to as the "see-saw" mechanism [58, 59]. Taking $N \sim 200\,\mathrm{GeV}$ and $M_R \sim M_{\mathrm{GUT}} \approx 2 \times 10^{16}\,\mathrm{GeV}$ we get the light neutrino mass of the order of $1/10^3$ eV which is smaller but still very close to the value $1/20\,\mathrm{eV}$ deduced from the experiments. The see-saw naturally explains why M_ν has entries that are much smaller than the entries of U, D, and L.

One of the most aesthetically pleasing features of $SU(5)$ is the fact that the members of one family (including the right-handed neutrino) completely fill three smallest representations—the **1**, the $\overline{\mathbf{5}}$, and the **10**. Another one is the fact that $SU(5)$, being the simple group, naturally *predicts* the gauge coupling unification. One might hope that it is possible to go a step further and completely unify the members of the family while preserving the gauge coupling unification. We now turn our attention to the group that allows us to do just that.

1.4.3 SO(10)

The special orthogonal group $SO(10)$ is a rank-five[7] group. We thus expect both $SU(5)$ (a rank-four) and $SU(4) \otimes SU(2) \otimes SU(2)$ (a rank-five) to be possible candidates for subgroups of $SO(10)$. The exact decomposition turns out to be: (I) $SO(10) \to SU(5) \otimes U(1)$ and (II) $SO(10) \to SU(4) \otimes SU(2) \otimes SU(2)$. The

[7] The rank of any group $\mathcal{F}$ is equal to the total number of its commuting generators. The rank of any subgroup of $\mathcal{F}$, for obvious reasons, must be equal or smaller than the rank of $\mathcal{F}$ itself.

choice of the particular symmetry breaking chain has profound consequences for the mass relations and the low-energy phenomenology. The smallest representations of $SO(10)$ are the **10** (a fundamental), the **16** (a spinor), and the **45** (an adjoint). Before discussing any phenomenological signature of $SO(10)$ we show how each one of these look under the decompositions (I) and (II).

If the sixteen members of one family are to be united within the single multiplet of $SO(10)$ the family must reside in the **16**. Since we already know the representations that make the family in both the GG and the PS models we can immediately write the decomposition as

$$\mathbf{16} \xrightarrow{\text{(I)}} \mathbf{1}^5 \oplus \overline{\mathbf{5}}^{-3} \oplus \mathbf{10}^1, \tag{1.46a}$$

$$\mathbf{16} \xrightarrow{\text{(II)}} (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}), \tag{1.46b}$$

where the superscript represents the $U(1)$ charge. [$SO(10)$ ensures an automatic anomaly cancellation for every representation. This can help in determining the $U(1)$ charge since the sums over the charges and their cubes must vanish for every representation. All one has to decide on is how to fix the overall normalization.] The **45** decomposes as

$$\mathbf{45} \xrightarrow{\text{(I)}} \mathbf{24}^0 \oplus \overline{\mathbf{10}}^4 \oplus \mathbf{10}^{-4} \oplus \mathbf{1}^0, \tag{1.47a}$$

$$\mathbf{45} \xrightarrow{\text{(II)}} (\mathbf{15},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1},\mathbf{3}) \oplus (\mathbf{6},\mathbf{2},\mathbf{2}), \tag{1.47b}$$

where we use the fact that the adjoint is always a self-conjugate. Namely, in terms of decomposition (I) we know that the **45** must contain the **24** and the **1**. To keep it real we just account for the difference with the **10** and its conjugate—the $\overline{\mathbf{10}}$. As for the **10**, it decomposes as

$$\mathbf{10} \xrightarrow{\text{(I)}} \mathbf{5}^{-2} \oplus \overline{\mathbf{5}}^2, \tag{1.48a}$$

$$\mathbf{10} \xrightarrow{\text{(II)}} (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1}), \tag{1.48b}$$

which makes it a perfect candidate for the familiar Higgs fields of the MSSM.

We are now ready to construct the minimal $SO(10)$ model. Setting aside the questions pertaining to the actual breaking down to the SM group we observe that the families reside in the $\mathbf{16}_i$ ($i = 1, 2, 3$) and the Higgses live in the $\mathbf{10}_H$. The Yukawa term is simply $\lambda_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H$, where λ_{ij} represents a dimensionless symmetric matrix in the family space. It falls apart as

$$\lambda_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H \xrightarrow{\text{(I)}}$$
$$\lambda_{ij}\Big(\underbrace{\mathbf{10}_i^1\mathbf{10}_j^1\mathbf{5}_H^{-2}}_{U}+\underbrace{\mathbf{10}_i^1\overline{\mathbf{5}}_j^{-3}\overline{\mathbf{5}}_H^2}_{D}+\underbrace{\overline{\mathbf{5}}_i^{-3}\mathbf{10}_j^1\overline{\mathbf{5}}_H^2}_{L}+\underbrace{\overline{\mathbf{5}}_i^{-3}\mathbf{1}_j^5\mathbf{5}_H^{-2}+\mathbf{1}_i^5\overline{\mathbf{5}}_j^{-3}\mathbf{5}_H^{-2}}_{N}\Big),$$

which yields the following mass matrix relations: $U = N \propto D = L \propto \lambda$. Once again we are led to the trivial form of the CKM matrix.

The very fact that the right-handed neutrinos are unified with the quarks and the leptons prevents them from obtaining the simple $SU(5)$-like Majorana mass term that generates M_R. This is the problem that any realistic $SO(10)$ model has to address (for a review on $SO(10)$ models see [69]). There are two possible directions that one might take.

- First approach is to use renormalizable operators such as:

$$\text{(a)}\quad \lambda_{Rij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{126}}_H \xrightarrow{\text{(I)}} \lambda_{Rij}\mathbf{1}_i^5\mathbf{1}_j^5\langle\mathbf{1}_H^{-10}\rangle \tag{1.49}$$
$$\text{(b)}\quad \lambda_{Nij}\mathbf{16}_i\mathbf{1}_j\overline{\mathbf{16}}_H + M_{ij}\mathbf{1}_i\mathbf{1}_j \xrightarrow{\text{(I)}} \lambda_{Nij}\mathbf{1}_i^5\mathbf{1}_j^0\langle\overline{\mathbf{1}}_H^{-5}\rangle + \frac{1}{2}M_{ij}\mathbf{1}_i^0\mathbf{1}_j^0 \tag{1.50}$$

  where in the case (a) we introduce the $\overline{\mathbf{126}}_H$ of Higgs while in the case (b) we introduce, in addition to the $\overline{\mathbf{16}}_H$ of Higgs, some singlets of $SO(10)$. In both cases we expect the VEVs as well as the entries of M to be at or immediately below the cutoff scale of the theory. It is clear that the case (a) leads directly to the standard "see-saw" mechanism while the case (b) generates the "double see-saw" mechanism since we first have to rotate in the space spanned by the right-handed fields only. Namely, to obtain the light eigenstates, we have to diagonalize the matrix

$$\begin{pmatrix} \nu & \bar{\nu} & \mathbf{1}^0 \end{pmatrix} \begin{pmatrix} 0 & N & 0 \\ N^{\mathrm{T}} & 0 & M' \\ 0 & M'^{\mathrm{T}} & M \end{pmatrix} \begin{pmatrix} \nu \\ \bar{\nu} \\ \mathbf{1}^0 \end{pmatrix}, \qquad (1.51)$$

where we define $M' = \lambda_N \langle \overline{\mathbf{1}}_H^{-5} \rangle$. This can be achieved by two successive rotations. First, we can rotate in the "23 plane" to eliminate the 13 and 31 elements. Then, we can rotate in the "12 plane" to eliminate the 12 and 21 elements. The outcome is the light mass matrix of the form $M_\nu \simeq N(M'M^{-1}M'^{\mathrm{T}})^{-1}N^{\mathrm{T}}$.

- Second approach is based on the use of the higher dimension operators. The philosophy behind this approach is that $SO(10)$ represents just another effective theory. Introducing the cutoff, say M_{GUT}, then allows us to use the operator $\lambda_{Rij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H/M_{\mathrm{GUT}} \xrightarrow{\text{(I)}} \lambda_{Rij}\mathbf{1}_i^5\mathbf{1}_j^5\overline{\mathbf{1}}_H^{-5}\overline{\mathbf{1}}_H^{-5}/M_{\mathrm{GUT}}$ to generate M_R after the $\overline{\mathbf{16}}_H$ gets very large VEV in the right-handed neutrino direction ($\langle \overline{\mathbf{1}}_H^{-5} \rangle \sim M_{\mathrm{GUT}}$). This again brings us back to the standard "see-saw" mechanism.

1.4.4 Flipped SU(5)

One of the breaking chains of $SO(10)$ reproduces Georgi-Glashow $SU(5)$ model with the addition of an extra $U(1)$ symmetry. The electric charge Q is then the generator of the GG $SU(5)$ only, in accord with our previous discussion. There is however another avenue that one might take. One can embed the electric charge in such a manner as to have it come from the linear combination of the generators operating in both $SU(5)$ and $U(1)$. This is exactly what is done in a flipped $SU(5)$ [60, 61, 62] as we demonstrate below.

The flipped $SU(5)$ is best visualized through the following schematics:

$$SO(10) \to \overbrace{SU(5) \otimes U(1)_X}^{\text{flipped } SU(5)} \to$$
$$SU(3)_c \otimes SU(2)_L \otimes \underbrace{U(1)_Z \otimes U(1)_X}_{U(1)_Y} \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y.$$

Clearly, the hypercharge, and thus the electric charge, originates from the linear combination of the Z and the X generators:

$$\frac{Y}{2} = \alpha\frac{Z}{2} + \beta X, \tag{1.52}$$

where α and β are the coefficients to be determined. Note that we already know the values of Z ($Z \equiv \mathrm{diag}(-2/3, -2/3, -2/3, 1, 1)$ in the fundamental representation of $SU(5)$) while the values of X are fixed by the $SO(10)$ decomposition. With this in mind, it is now easy to find the values of α and β by imposing the condition that Eq. (1.52) reproduces the known hypercharges of the matter fields. We observe that the quarks in $(\mathbf{3}, \mathbf{2}, 1/3)_i$ must reside in the $\mathbf{10}_i^1$ since there is not enough room for them in the $\overline{\mathbf{5}}_i^{-3}$. This gives the first condition: $(1/6) = \alpha(1/6) + \beta(1)$. On the other hand, we expect the leptons from $(\mathbf{1}, \mathbf{2}, -1)_i$ to be in the $\overline{\mathbf{5}}_i^{-3}$ since the rest of the states will be occupied by the anti-triplet of quarks. This in turn provides the second condition: $(-1/2) = \alpha(-1/2) + \beta(-3)$. These two conditions provide two different embedding schemes for the matter fields, as shown by Barr in Ref. [62]:

1. $(\alpha, \beta) = (1, 0)$, corresponds to the GG $SU(5)$ embedding scheme.

2. $(\alpha, \beta) = (-\frac{1}{5}, \frac{1}{5})$, is the flipped $SU(5)$ embedding scheme.

We can now reconstruct the embedding of the rest of the matter fields of one family which in the $SU(5)$ language looks like

$$\psi_\mu = \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \\ e \\ \nu \end{pmatrix}, \quad \psi^{\mu\nu} = \begin{pmatrix} 0 & \bar{d}_3 & -\bar{d}_2 & -d^1 & -u^1 \\ -\bar{d}_3 & 0 & \bar{d}_1 & -d^2 & -u^2 \\ \bar{d}_2 & -\bar{d}_1 & 0 & -d^3 & -u^3 \\ d^1 & d^2 & d^3 & 0 & -\bar{\nu} \\ u^1 & u^2 & u^3 & \bar{\nu} & 0 \end{pmatrix}, \quad \text{and} \quad \psi = \bar{e}. \tag{1.53}$$

Flipped $SU(5)$ has certain appealing features that make it very attractive from the model building point of view. Since we plan to take advantage of these features in Chapters 4 and 5, where we describe them in great detail, we postpone any further discussion on flipped $SU(5)$.

We have seen that GUTs have introduced many qualitatively new notions into the realm of elementary particle physics. The ideas such as the gauge coupling unification, and the unification of the matter fields have become a reality. But one question still remains: Are these ideas enough to account for the mass and the mixing patterns?

1.5 Flavor Symmetry

The majority of the realistic GUT models in the literature utilizes the gauge symmetry in conjunction with some form of the flavor symmetry [63, 64, 65], i.e. the symmetry that acts in the flavor space, to create realistic patterns of the masses and the mixings. The idea of Abelian flavor symmetry in this context is especially simple.

It has been observed that all of the interfamily mass ratios and mixing angles can be written as powers of one or two small parameters. For example, the quark and the lepton mass ratios and the mixing angles (see Table 1.1.) at the GUT scale are

$$m_u : m_c : m_t \sim \epsilon^8 : \epsilon^4 : 1, \tag{1.54a}$$

$$m_d : m_s : m_b \sim \epsilon^5 : \epsilon^2 : 1, \tag{1.54b}$$

$$m_e : m_\mu : m_\tau \sim \epsilon^4 : \epsilon^2 : 1, \tag{1.54c}$$

$$|V_{us}| \sim \epsilon, \quad |V_{cb}| \sim \epsilon^2, \quad |V_{ub}| \sim \epsilon^4, \tag{1.55}$$

and so on, where $\epsilon \sim \lambda \sim 0.22$ is the Wolfenstein parameter [100]. This has suggested to many theorists the idea that there is a weakly broken Abelian symmetry which distinguishes fermions that are of the same type but of different families. Suppose, for instance, that there is a $U(1)_F$ flavor symmetry, under which the Standard Model Higgs has charge zero, the fermions $\bar{\psi}_i$ and ψ_j have charges $\bar{q}_i$ and q_j, and a "flavon" field S has charge -1. Then a Yukawa operator $\bar{\psi}_i \psi_j H$ is forbidden by the flavor symmetry, but the effective operator $\bar{\psi}_i \psi_j H (S/M_F)^{(\bar{q}_i + q_j)}$ is not. Such an effective operator might arise from integrating out vector-like fermionic fields whose mass is of order M_F, the "flavor scale". If one assumes that the breaking of $U(1)_F$ is weak, in the sense that $\langle S \rangle / M_F = \epsilon \ll 1$, then one has explained the fact that the effective mass parameter of the term $\bar{\psi}_i \psi_j$ is proportional to a power of the small quantity ϵ. This is the basic idea of Froggatt and Nielson [63], which has inspired a very large number of models in the literature.

The use of the flavor symmetry is closely tied to the expectation that the Yukawa couplings are $O(1)$ parameters. This indeed is the case for the top quark so we expect its Yukawa operator to be allowed by the flavor symmetry. All other Yukawa operators will usually be suppressed by the appropriate powers of the parameter ϵ. The flavor symmetry alone gives only order of magnitude estimates instead of the exact predictions for the matrix elements of the mass matrices. But then again, in conjunction with the GUTs it can be a powerful tool that relates the mass ratios and the mixing angles as we demonstrate next.

Concentrating our attention on the first two families of the quark sector we can posit the following form of the mass matrix [66, 67, 68]

$$D = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 1 \end{pmatrix} m. \tag{1.56}$$

The symmetric form might originate from the GUT with the hierarchical pattern ensured by an appropriate flavor symmetry. The task of bringing this matrix into the diagonal form is accomplished with the 2×2 orthogonal rotations ($O_d^{\rm T} D O_d = D^{\rm diag}$) with the rotation angle defined by $\tan 2\theta = 2\epsilon$. For small ϵ, $\theta \cong \epsilon$, the eigenvalues are $m_s \cong m$, and $m_d \cong -\epsilon^2 m$. [Note that $\det D = -\epsilon^2 m^2 = m_d m_s$.] This finally establishes the link between the mixing angle θ and the mass ratio, $\theta \cong \sqrt{|m_d/m_s|}$, we were hinting at. Applying the same idea to the mass matrix U and allowing for more realistic complex entries the Cabbibo angle turns out to be

$$\theta_c \cong \left| \sqrt{\frac{m_d}{m_s}} - {\rm e}^{{\rm i}\phi} \sqrt{\frac{m_u}{m_c}} \right|, \tag{1.57}$$

which is to be compared with the value of $|V_{us}|$. The agreement seems excellent for any value of the phase ϕ since at the GUT scale $\sqrt{m_d/m_s} \approx 0.24$, and $\sqrt{m_u/m_c} \approx 0.046$ (see Table 1.1.).

The synergy between the flavor symmetry and the GUTs can be nicely demonstrated in the GG $SU(5)$ model. As we have seen in Eq. (1.43), the U mass matrix comes from the term involving the product of two $\mathbf{10}$'s while both the D and the L matrices involve the product of only one $\mathbf{10}$ with the $\overline{\mathbf{5}}$. To use this to our advantage we can assign the same $U(1)_F$ charges to all the $\overline{\mathbf{5}}$'s while assigning successively greater charges to the $\mathbf{10}_3$, $\mathbf{10}_2$, and $\mathbf{10}_1$. This creates doubly suppressed hierarchy in the up sector compared to the down and the charged lepton sector as required by the empirical GUT relations in Eqs. (1.54).

All the mixing angles in the quark sector have turned out to be very small. This has suggested the idea of relating the small mixing angles of the CKM matrix to the small ratios of the entries of the U and D mass matrices of the quarks. As we have seen, this idea has been implemented by positing the symmetric "texture zero" form of these matrices. On the other hand, the mixing angles of the PMNS matrix in the lepton sector have not come out to be as small as was initially expected. Some of them are actually very close to their maximal value. More specifically, the measurements of neutrino oscillations (for a theoretical overview see [75]) have unambiguously shown that $|U_{\mu 3}| \cong 0.71$ [42], and $|U_{e2}| \sim 0.5$ [70, 71, 72]. Therefore the question is: How do we accommodate the maximal mixing? [The largeness of the mixing angles in the lepton sector has raised a number of theoretical puzzles. For a beautiful account of these puzzles and possible solutions see [74].]

It is very simple to accommodate the maximal mixing using the symmetric form of the mass matrix. For example, the maximal mixing in a two-state system is easily implemented with a following "pseudo-Dirac" matrix

$$\begin{pmatrix} \epsilon & 1 \\ 1 & \epsilon \end{pmatrix}, \tag{1.58}$$

where $\epsilon \ll 1$. We note that the mixing angle is very close to $\pi/4$, while the resulting masses are both of order one. It is clear from this simple example that the symmetric form implies the existence of the degenerate mass eigenvalues whenever there is a maximal mixing in the system. This approach, then, is not appropriate in the lepton sector where both charged leptons and neutrinos show certain mass hierarchy. This has suggested a somewhat novel approach based on the idea [73] to use the antisymmetric form of the mass matrices to explain the unexpected largeness of the mixing angles and the large hierarchy in the lepton sector. Obviously, the idea of the antisymmetry in the lepton sector can only be applied to the charged lepton mass matrix L since we know M_ν to be symmetric. We demonstrate the beauty of this idea in its natural setting of an $SU(5)$ GUT.

Recall that in $SU(5)$ there is a *prediction* for the relationship between D and

L mass matrices: $D = L^{\mathrm{T}}$. Since the physical mixing angles (the angles that enter CKM and PMNS mixing matrices) are the angles used to redefine the left-handed fields (see Eq. (1.22)), we can arrange things as follows. We can posit the form of the matrix L to require large rotations of the left-handed charge leptons via U_l. These large rotations in U_l are then related by $SU(5)$ to the large rotations in V_d, which are the rotations of the right-handed down quarks. But these right-handed mixing angles have nothing to do with the observed CKM angles. On the other hand, the small CKM angles originating from U_d are related by $SU(5)$ to small mixings of the right-handed leptons in V_l, which are irrelevant to neutrino oscillation phenomena since they do not enter PMNS mixing matrix. The models based on this idea are referred to as "lopsided" [77].

An example of how an $SU(5)$ lopsided model works is provided by the following mass matrices:

$$D = \begin{pmatrix} 0 & \epsilon \\ \sigma & 1 \end{pmatrix} m, \text{ and } L = \begin{pmatrix} 0 & \sigma \\ \epsilon & 1 \end{pmatrix} m. \tag{1.59}$$

Here, we consider only the last two families of the down quarks and the leptons, taking $\epsilon \ll 1$ and $\sigma \sim 1$. To bring L into the diagonal form we first perform a rotation from the left with an angle defined by $\tan\theta^l_L = \sigma$. Then, we rotate from the right with an angle defined by $\tan\theta^l_R = \epsilon/\sqrt{1+\sigma^2} \sim \epsilon$. This yields the mass eigenvalues ratio $m_\mu/m_\tau \approx \epsilon\sigma$ which is equal to the mass ratio m_s/m_b in the down quark sector. The angles θ^l_L and θ^l_R are equal to the angles θ^d_R and θ^d_L in the down quark sector repectively due to the $SU(5)$ symmetry. Being the left-handed mixing angle in the "23 plane", θ^l_L (θ^d_L) contributes towards the PMNS (CKM) matrix element $U_{\mu 3}$ (V_{cb}). Thus, we have $U_{\mu 3} \sim \sigma \sim \sqrt{\sigma/\epsilon}\sqrt{m_\mu/m_\tau}$ and $V_{cb} \sim \epsilon \sim \sqrt{\epsilon/\sigma}\sqrt{m_s/m_b}$. We can clearly see that the smallness of V_{cb} is directly related to the largeness of $U_{\mu 3}$.

We analyze one lopsided model in great detail in Chapter 3 and create another one in Chapter 4 to obtain realistic scheme of the masses and the mixings in extra-dimensional setting.

1.6 Outline

We have discussed the Standard Model structure in great length in this chapter. There is, however, one particular feature of the SM that has not been mentioned yet—the coupling of the Higgs scalar h to the fermions in the SM is flavor diagonal. We have taken it for granted since the mass matrices of the quarks and the leptons were proportional to the appropriate Yukawa coupling matrices ($\lambda^{u,d,l}$). This does not have to be the case always, especially if there are more Higgs-type scalars in the theory as we will see in Chapter 2. The appearance of these extra scalars is ubiquitous in the models with Abelian flavor symmetry. In these models the small mixing angles and mass ratios of quarks and leptons are typically given by powers of small parameters characterizing the spontaneous breaking of flavor symmetry by the Higgs-type "flavon" fields. The usual assumption that the spontaneous breaking takes place at some high energy scale makes all the effects of the flavon exchange virtually unobservable. But, if the scale of the breaking of flavor symmetry is near the weak scale, flavon exchange can lead to interesting flavor-violating and CP violating effects. These are studied in Chapter 2. We will put special emphasis on μ-e conversion since there are a number of experiments aiming at the improvement of existing limits. Some of them are already taking data (SINDRUM II Collaboration at PSI [79]), and some are planned to start in near future (MACO at BNL [80] and NUFACT at CERN [81]). In addition, we will investigate the effect of the flavon exchange on the processes such as d_e, and $\mu \to e + \gamma$.

Experiments on neutrino oscillations have unambiguously shown that the only viable solution of the solar neutrino problem is the large mixing angle MSW solution. On the other hand, the majority of the models on neutrino masses and mixings that has been published seems to have difficulty in explaining the values of the parameters required to produce that solution in a natural way. We investigate in Chapter 3 how well various neutrino models accommodate the values of the neutrino masses and mixings that are preferred by the recent global fit analyses. We also address this question in a statistical manner and propose to treat the entries of the mass matrices to be the random variables much in the spirit of the work of Hall, Murayama and Weiner [82] and subsequent analysis of Haba and Murayama [83]. We suggest this approach to be a very good indicator of how natural neutrino mass model is. Moreover, we claim this analysis to be applicable to a very large class of models of the neutrino masses and mixings.

The most appealing feature of the flipped $SU(5)$ is the way it allows one to solve the doublet-triplet splitting problem via the missing partner mechanism [84, 85]. The implementation of this mechanism however prohibits any further embedding of the flipped $SU(5)$ into $SO(10)$. This naturally means an automatic loss of the gauge coupling unification as the genuine *prediction* of the flipped $SU(5)$ model. So the question of whether it is possible to reconcile the gauge symmetry of flipped $SU(5)$ with the gauge unification becomes extremely important. We will show in Chapter 4 that embedding a four-dimensional flipped $SU(5)$ model in a five-dimensional $SO(10)$ model *à la* Kawamura [86], preserves the best features of both flipped $SU(5)$ and $SO(10)$. The missing partner mechanism, which naturally achieves both doublet-triplet splitting and suppression of $d = 5$ proton decay operators, will be realized as in flipped $SU(5)$, while the gauge couplings will be unified as in $SO(10)$. As promised before, we will also discuss in Chapter 4 the nature of the doublet-triplet problem.

If we believe in the gauge unification we can ask whether the gauge couplings truly unify at the GUT scale. The answer to this question was positive not so long ago since the uncertainty in their initial values extracted from the experiments were large enough to allow the three of them to meet. The situation has changed after the electroweak precision measurements and the improvements in measurements of α_3 since the error bars on the experimental values have become sufficiently small to prevent the exact unification. What we need is to identify the source that modifies the values of the gauge couplings sufficiently enough to lead to their perfect unification. At the same time we have to ensure that this source is not contradicting the existing experimental limits. This has been achieved in a number of ways. The most common one is to postulate the existence of the extra-matter fields within the usual four-dimensional framework. We are going to propose in Chapter 5 an extra-dimensional $SO(10)$ scheme that modifies the particle spectrum of the MSSM below $M_{\rm GUT}$ and generates the so-called Kaluza-Klein (KK) grand unification [87, 88]. The model will also be interesting from the phenomenological point of view. The cutoff of the model will be closer to $M_{\rm Pl}$ than was the case in four-dimensional GUTs. On the other hand, the mass of the heavy states will be below the usual $M_{\rm GUT}$ and yet there will be no problems in satisfying the constrains of proton decay experiments.

Chapter 2

FLAVOR EXCHANGING EFFECTS IN MODELS WITH ABELIAN FLAVOR SYMMETRY

2.1 Introduction

Flavor symmetry was first proposed to explain the structure of the quark and lepton mass spectrum and the CKM mixing of the quarks [63, 64]. More recently these ideas have been extended to account for the observed patterns of neutrino masses and mixings (see for instance [89, 78, 90, 91, 92]). In the context of SUSY, flavor symmetry has been invoked to solve the problem of flavor changing neutral currents, i.e. "the SUSY flavor problem" [93, 94, 95, 96, 97, 98, 99].

A wide assortment of flavor symmetries has been suggested. In particular, models based on both non-Abelian and Abelian symmetries have been constructed. One virtue of non-Abelian symmetries is that they can lead to degenerate masses, which have various theoretical uses. For example, one solution to the SUSY flavor problem is to posit a near degeneracy of the squark/slepton masses of the first two families. For another example, large neutrino mixing angles can be obtained by positing nearly degenerate neutrino masses. However, in this chapter we shall be interested in Abelian flavor symmetries.

Aside from having the potential to explain the hierarchies observed among fermion masses and mixing angles, the idea of a weakly broken Abelian flavor symmetry *à la* Froggatt and Nielson [63] can be used to construct models in which the dangerous flavor-changing effects in supersymmetric models are suppressed by "flavor alignment" [93]. The idea here is that in the preferred basis defined by the Abelian flavor charge assignments the off-diagonal elements of both the fermion mass matrices and the sfermion mass-squared matrices are suppressed by powers of the small parameters which characterize flavor breaking (i.e. parameters like ϵ). Thus the fermion and sfermion mass matrices are nearly "aligned" by flavor symmetry. The angles expressing their misalignment are suppressed by powers of the small parameters. If this suppression is strong enough it would solve the SUSY flavor problem.

In this chapter we examine some of the possible consequences for phenomenology of the exchange of the "flavon" fields themselves. A point that should be stressed from the outset is that there do not have to be such consequences at all. The reason is that the flavor scale M_F can be anything from the weak scale up to the Planck scale. All that matters is that the ratio $\langle S \rangle / M_F$ of the flavon expectation value (or values) to the flavor scale be somewhat smaller than 1. If the flavor scale is much larger than the weak scale, then the phenomenological effects of flavon exchange will be unobservable. In fact, many papers assume that the flavor scale is near the Planck scale, which is certainly a reasonable expectation. However, since we do not know *a priori* what the flavor scale is, it is interesting to investigate the phenomenology that would result from its being near the weak scale, and in particular to ask how low the flavor scale could actually be given present limits on flavor-changing and CP-violating processes. We would also like to know in which processes flavon-exchange effects would be likely first to show up.

There are many ways that new flavor physics just above the weak scale could affect low-energy phenomenology. For instance, if the Abelian flavor group is local, the exchange of the corresponding gauge bosons could cause flavor-changing neutral current processes. We will assume that the flavor group is either global, or breaks to a global symmetry at a sufficiently high scale that such gauge-boson-exchange effects can be ignored. We are only interested in this chapter in the exchange of the flavon fields themselves.

There are many models with Abelian flavor symmetry, and the number of parameters in such models can be large. What we shall do, therefore, is write down an effective low-energy theory that has a manageably small number of parameters and that has some of the typical features of models with Abelian flavor symmetry. Studying this toy model will give some idea of the likely magnitude of various effects. We will then look at some variations of the model to see how they would change the conclusions.

2.2 A simple effective theory of flavon physics

The model we shall study has a single flavon field S that is a singlet under the SM gauge group $\mathcal{H}$. The effective Yukawa couplings of the quarks and leptons to S and to the ordinary Standard Model Higgs field H, after integrating out all the fields whose mass is of order of the flavor scale M_F, is assumed to be

$$\mathcal{L}_{\text{Yukawa}} = -\hat{\lambda}^u_{ij}\epsilon^{ab}\overline{Q}^a_{Li}H^{\dagger b}u_{Rj} - \hat{\lambda}^d_{ij}\overline{Q}^a_{Li}H^a d_{Rj} - \hat{\lambda}^l_{ij}\overline{L}^a_{Li}H^a l_{Rj} + \text{H.c.}, \tag{2.1}$$

where $i, j = 1, 2, 3$ are family indices, and $a, b = 1, 2$ are $SU(2)_L$ indices. The $\hat{\lambda}$'s are given by the following expressions:

$$\hat{\lambda}^u = \begin{pmatrix} h^u_{11}\hat{\epsilon}^6 & h^u_{12}\hat{\epsilon}^4 & h^u_{13}\hat{\epsilon}^4 \\ h^u_{21}\hat{\epsilon}^4 & h^u_{22}\hat{\epsilon}^2 & h^u_{23}\hat{\epsilon}^2 \\ h^u_{31}\hat{\epsilon}^4 & h^u_{32}\hat{\epsilon}^2 & h^u_{33} \end{pmatrix}, \text{ and } \hat{\lambda}^d = \begin{pmatrix} h^d_{11}\hat{\epsilon}^6 & h^d_{12}\hat{\epsilon}^6 & h^d_{13}\hat{\epsilon}^6 \\ h^d_{21}\hat{\epsilon}^6 & h^d_{22}\hat{\epsilon}^4 & h^d_{23}\hat{\epsilon}^4 \\ h^d_{31}\hat{\epsilon}^6 & h^d_{32}\hat{\epsilon}^4 & h^d_{33}\hat{\epsilon}^2 \end{pmatrix}. \tag{2.2}$$

The corresponding matrix for the charged leptons, $\hat{\lambda}^l$, is assumed to have the same form as $\hat{\lambda}^d$ with $h^d_{ij} \to h^l_{ij}$. In these expressions the h_{ij} are all assumed to be of order unity, and the hierarchy among various masses and mixing angles therefore comes from the powers of $\hat{\epsilon}^2$, which is defined to be

$$\hat{\epsilon}^2 \equiv \frac{S}{M_F}. \tag{2.3}$$

The particular structure given in Eq. (2.2) is inspired by a model of Babu and Nandi [101], which has the same powers of $\hat{\epsilon}$, but where $\hat{\epsilon}^2 = (H^\dagger H)/M_F^2$ rather than S/M_F as here. Thus, their model is not a typical flavon model. However, the pattern of powers of ϵ is quite typical of many Abelian flavon models, and gives, as Babu and Nandi show (see below), an excellent fit to quark and lepton masses and CKM angles. If we call the vacuum expectation value of the flavon field $\langle S \rangle \equiv u$, then the small parameter that characterizes flavor changing is $\epsilon^2 \equiv u/M_F$.

The Higgs potential is assumed to have the form

$$\begin{aligned} V(H,S) =& \lambda(H^\dagger H)^2 - \mu^2(H^\dagger H) + \lambda_S(S^*S)^2 - \mu_S^2(S^*S) \\ &+ \lambda'(H^\dagger H S^* S) - \frac{1}{2}(\delta m^2 S^2 + \text{H.c.}). \end{aligned} \tag{2.4}$$

The last term has been put in to give a soft breaking of the global $U(1)_F$ under which $S \to e^{i\theta} S$, and thus to give mass to the pseudoscalar part of S. [This global $U(1)$ may ultimately come from a local flavor symmetry that is broken at a higher scale.] The parameter δm^2 is the only one in the Higgs potential that can have a phase. However, one can absorb this by a phase rotation of S. Having done so, the VEV of S is a real quantity. Minimizing this potential gives

$$S = u + \frac{1}{\sqrt{2}} s_1 + \frac{i}{\sqrt{2}} s_2, \tag{2.5a}$$

$$H = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} h \end{pmatrix}, \tag{2.5b}$$

where

$$v^2 = \left[2\lambda_S \mu^2 - \lambda'(\mu_S^2 + \delta m^2)\right] / \left(4\lambda\lambda_S - \lambda'^2\right), \tag{2.6}$$

$$u^2 = \left[2\lambda(\mu_S^2 + \delta m^2) - \lambda'\mu^2\right] / \left(4\lambda\lambda_S - \lambda'^2\right), \tag{2.7}$$

with $v \simeq 174\,\text{GeV}$, and $\langle s_1 \rangle = \langle s_2 \rangle = \langle h \rangle = 0$. We have changed the definition of v compared to the definition of Eq. (1.12) for later convenience. From Eqs. (2.3), and (2.5), we can write

$$\hat{\epsilon}^2 = \epsilon^2 \left[1 + \frac{s_1 + i s_2}{\sqrt{2} u}\right]. \tag{2.8}$$

Consequently, the couplings of s_1 and s_2 to the quarks and leptons are obtained by taking, in Eq. (2.1),

$$\hat{\lambda}_{ij}^{f} H = \lambda_{ij}^{f} \hat{\epsilon}^{2n_{ij}^{f}} \left(v + \frac{h}{\sqrt{2}} \right) \cong m_{ij}^{f} \left[1 + \frac{n_{ij}^{f}(s_1 + \mathrm{i} s_2)}{\sqrt{2}u} + \frac{h}{\sqrt{2}v} \right], \qquad (2.9)$$

where $f = u, d, l$, and where n_{ij}^{f} is the power of $\hat{\epsilon}^2$ that appears in $\hat{\lambda}_{ij}^{f}$. It turns out that for the interesting phenomenology one can ignore the terms higher than linear in the fields s_1 and s_2 in Eq. (2.9). Note that the coupling of h to the quarks and leptons will be made real and diagonal when the mass matrices m_{ij}^{f} are, but that the coupling of the flavon fields s_1 and s_2 will not be made real and diagonal because of the extra factor of n_{ij}^{f}. This is what will give the flavor-changing and CP-violating effects that we shall be interested in. We see also that s_2 couples in the same way to quarks and leptons as s_1 does but with a relative phase of i. This factor of i comes in squared in s_2 exchange and so does not lead to CP-violating effects to the order we are interested in.

Let us now look at how many parameters the model has. First, there are the large number of parameters that we have called h_{ij}^{f}. Because there are so many, there is no hope of making any sharp predictions. However, if we confine our ambition to making order of magnitude estimates of effects, then we can (for the most part) ignore the h_{ij}^{f}, since they are assumed all to be of order unity. This leaves the six parameters in the Higgs potential $(\lambda, \mu^2, \lambda_S, \mu_S^2, \lambda', \delta m^2)$, and the flavor scale M_F. These parameters can be traded for v, m_h, u, m_{s_1}, $\sin\phi$, m_{s_2}, and M_F. The VEV v is known precisely; the mass of the ordinary Higgs m_h is known approximately; and the parameter M_F is determined by the relation $\epsilon^2 \equiv u/M_F$. (The value of ϵ^2 is known approximately from the values of the quark and lepton mass ratios and the CKM angles.) Consequently, one is left with four free parameters: the masses of the scalar flavon m_{s_1} and the pseudoscalar flavon m_{s_2}, the VEV u of the flavon (which, as we have seen, controls the strength of the flavon couplings to matter), and the parameter $\sin\phi$ that describes the mixing between the ordinary Higgs scalar and the scalar flavon. This mixing is described by the mass matrix

$$\frac{1}{2}(h \quad s_1 \quad s_2)\begin{pmatrix} 4\lambda v^2 & 2\lambda' vu & 0 \\ 2\lambda' vu & 4\lambda_S u^2 & 0 \\ 0 & 0 & 2\delta m^2 \end{pmatrix}\begin{pmatrix} h \\ s_1 \\ s_2 \end{pmatrix}. \tag{2.10}$$

so that $\tan 2\phi = (\lambda' vu)/(\lambda_s u^2 - \lambda v^2)$. We will call the mass eigenstates $h' = \cos\phi\, h - \sin\phi\, s_1$, and $s' = \sin\phi\, h + \cos\phi\, s_1$, and their masses $m_{h'}$, and $m_{s'}$, respectively.

Turning to the diagonalization of the quark and lepton mass matrices, one finds the following masses and mixing angles

$$(m_u, m_c, m_t) \cong \left(|h^u_{11} - h^u_{12}h^u_{21}/h^u_{22}|\epsilon^6, |h^u_{22}|\epsilon^2, |h^u_{33}|\right) v, \tag{2.11a}$$

$$(m_d, m_s, m_b) \cong \left(|h^d_{11}|\epsilon^6, |h^d_{22}|\epsilon^4, |h^d_{33}|\epsilon^2\right) v, \tag{2.11b}$$

$$(m_e, m_\mu, m_\tau) \cong \left(|h^l_{11}|\epsilon^6, |h^l_{22}|\epsilon^4, |h^l_{33}|\epsilon^2\right) v, \tag{2.11c}$$

and

$$|V_{us}| \cong \left|\frac{h^d_{12}}{h^d_{22}} - \frac{h^u_{12}}{h^u_{22}}\right|\epsilon^2, \tag{2.12a}$$

$$|V_{cb}| \cong \left|\frac{h^d_{23}}{h^d_{33}} - \frac{h^u_{23}}{h^u_{33}}\right|\epsilon^2, \tag{2.12b}$$

$$|V_{ub}| \cong \left|\frac{h^d_{13}}{h^d_{33}} - \frac{h^u_{13}}{h^u_{33}} - \frac{h^u_{12}h^d_{23}}{h^u_{22}h^d_{33}} + \frac{h^u_{12}h^d_{23}}{h^u_{22}h^d_{33}}\right|\epsilon^4. \tag{2.12c}$$

Babu and Nandi [101] showed that this gives a reasonable fit to the data. They took $m_u(1\,\mathrm{GeV}) = 5.1\,\mathrm{MeV}$, $m_d(1\,\mathrm{GeV}) = 8.9\,\mathrm{MeV}$, $m_s(1\,\mathrm{GeV}) = 175\,\mathrm{MeV}$, $m_c(m_c) = 1.27\,\mathrm{GeV}$, $m_b(m_b) = 4.25\,\mathrm{GeV}$, $m_t^{\mathrm{phys}} = 175\,\mathrm{GeV}$, $m_\tau = 1.78\,\mathrm{GeV}$, $m_\mu = 105.6\,\mathrm{MeV}$, and $m_e = 511\,\mathrm{keV}$. Extrapolating, using the 3-loop QCD and one-loop QED beta functions, with $\alpha_s(M_Z) = 0.118$, they obtained the running masses in GeV evaluated at m_t: $m_t \simeq 166$, $m_c \simeq 0.6$, $m_u \simeq 0.0022$, $m_b \simeq 2.78$, $m_s \simeq 0.075$, $m_d \simeq 0.0038$, $m_\tau \simeq 1.75$, $m_\mu \simeq 0.104$, and $m_e \simeq 0.0005$. These are well fitted by $\epsilon^2 \simeq (1/6.5)^2 \cong 0.024$, if one takes $|h^u_{11} - h^u_{12}h^u_{21}/h^u_{22}| \simeq 0.95$, $|h^u_{22}| \simeq 0.14$,

$|h^u_{33}| \simeq 0.96$, $|h^d_{11}| \simeq 1.65$, $|h^d_{22}| \simeq 0.77$, $|h^d_{33}| \simeq 0.68$, $|h^l_{11}| \simeq 0.21$, $|h^l_{22}| \simeq 1.06$, and $|h^l_{33}| \simeq 0.42$. [The numerical values of the running masses at m_t of Babu and Nandi differ from our values, presented in Table 1.1., due to the difference in the initial conditions. We have used the central values of the quark masses at 2 GeV as given in Ref. [28] while Babu and Nandi used the initial values at 1 GeV as given in Ref. [102].]

Note that with the exception of h^u_{22} and h^l_{11} all these are of order unity. And as emphasized in [101] the smallness of h^u_{22} actually helps account for the values of $|V_{us}|$ and $|V_{ub}|$. From Eq. (2.12) one sees that with $h^u_{22} \simeq 1/7$, these mixings come out to be $|V_{us}| \sim 7\epsilon^2 \sim 0.2$, and $|V_{ub}| \sim 7\epsilon^4 \sim 3 \times 10^{-3}$.

As mentioned, in the basis where the mass matrices of the quarks and leptons are diagonal and real, the couplings of s_1 and s_2 remain with off-diagonal and complex elements, due to the extra factors of n^f_{ij} in Eq. (2.9). However, it is interesting that the flavor-diagonal couplings of s_1 are, in fact, real to leading order in the small parameter ϵ^2. That is, the imaginary part of these diagonal couplings is of order $\epsilon^2 \simeq 0.02$ times the real part. This is significant for the lepton and quark electric dipole moments, as we shall see. The reason that the diagonal couplings of s_1 are real to leading order can be seen by looking at a simple two-by-two example:

$$Y_h = \begin{pmatrix} h_{11}\epsilon^{2n_{11}} & h_{12}\epsilon^{2n_{12}} \\ h_{21}\epsilon^{2n_{21}} & h_{22}\epsilon^{2n_{22}} \end{pmatrix}, \quad Y_{s_1} = \begin{pmatrix} h_{11}n_{11}\epsilon^{2n_{11}} & h_{12}n_{12}\epsilon^{2n_{12}} \\ h_{21}n_{21}\epsilon^{2n_{21}} & h_{22}n_{22}\epsilon^{2n_{22}} \end{pmatrix} \frac{v}{u}. \tag{2.13}$$

In the basis where Y_h is diagonal and real, which we shall denote by primes,

$$(Y'_h)_{11} \cong \left[h_{11}\epsilon^{2n_{11}} - \frac{h_{12}h_{21}}{h_{22}} \epsilon^{2(n_{12}+n_{21}-n_{22})} \right] \mathrm{e}^{\mathrm{i}\alpha}, \tag{2.14}$$

$$(Y'_{s_1})_{11} \cong \left[h_{11}n_{11}\epsilon^{2n_{11}} - \frac{h_{12}h_{21}}{h_{22}} (n_{12}+n_{21}-n_{22}) \epsilon^{2(n_{12}+n_{21}-n_{22})} \right] \mathrm{e}^{\mathrm{i}\alpha} \frac{v}{u}. \tag{2.15}$$

The factor of $\mathrm{e}^{\mathrm{i}\alpha}$ is the phase rotation required to make $(Y'_h)_{11}$ real. [In the same basis, the matrix Y'_{s_1} is easily seen to be non-diagonal: $|(Y'_{s_1})_{12}| \cong |h_{12}(n_{12} - n_{22})\epsilon^{2n_{12}}|(v/u)$, and $|(Y'_{s_1})_{21}| \cong |h_{21}(n_{21} - n_{22})\epsilon^{2n_{21}}|(v/u)$.] In Eq. (2.14) one sees two terms in the expression for $(Y'_h)_{11}$. There are two cases to consider: either these two terms are of the same order in ϵ^2, or one is higher order in ϵ^2 than the other. If they are the same order, then $n_{12} + n_{21} - n_{22} = n_{11}$, which means that $(Y'_{s_1})_{11} = n_{11}(Y'_h)_{11}(v/u)$, a real quantity, to leading order in ϵ^2. If, on the other hand, one term in $(Y'_h)_{11}$ is of lower order in ϵ^2 than the other and dominates, then the corresponding term dominates in $(Y'_{s_1})_{11}$. Consequently, to leading order in ϵ^2, one has again that $(Y'_{s_1})_{11}$ is just an integer times $(Y'_h)_{11}(v/u)$ and therefore real.

This conclusion generalizes to more complicated situations. It is true for N-by-N matrices. It is also true if there are several Abelian flavon fields giving several ϵ parameters, as long as contributions to diagonal Yukawa couplings that are of different orders in the small parameters are not accidentally numerically comparable.

2.3 Flavor-changing and *CP*-violating processes

We are now ready to discuss various flavor-changing and CP-violating processes. The ones that shall be of chief interest are Δm_K^2 and ϵ_K in the neutral kaon system, the electric dipole moment of the electron d_e, the decay $\mu \to e + \gamma$, and μ-e conversion on nuclei $\mu + N \to e + N$. It is straightforward to calculate the contributions to these effects coming from flavon exchange in our toy model.

The relevant couplings for flavor-changing and CP-violating processes, in the physical basis of fermions and bosons, can be parameterized as

$$\mathcal{L} = -\frac{\sqrt{m_i m_j}}{v}\bar{\psi}_i \left(\Delta_{ij}^{a\,L} P_L + \Delta_{ij}^{a\,R} P_R\right) \psi_j H_a + g m_W \cos\varphi_a W^+ W^- H_a + \cdots\,, \quad (2.16)$$

where $a = h', s'$, and where indices i and j run over all quarks and charged leptons. We observe that due to the scalar nature of h' and s', to the leading order in ϵ^2, $\Delta_{ii}^{a\,L} \equiv \Delta_{ii}^{a\,R*} = \Delta_{ii}^{a\,R} \equiv \Delta_{ii}^{a}$ is real for all i's. [See the discussion after Eq. (2.14).] Acting on Yukawa coupling matrices with a set of bi-unitary transformations that brings fermion mass matrices into diagonal form, and simultaneously diagonalizing the Higgs sector one finds that

$$\Delta_{ee}^{h'\,L} = 4\chi^2\epsilon^2\frac{h_{12}^l h_{21}^l}{\sqrt{2}}\sin\phi\frac{v}{u}, \tag{2.17a}$$

$$\Delta_{ee}^{h'\,R} = 4\chi^2\epsilon^2\frac{h_{12}^{l*} h_{21}^{l*}}{\sqrt{2}}\sin\phi\frac{v}{u}, \tag{2.17b}$$

$$\Delta_{e\mu}^{h'\,L} = -\chi\epsilon\frac{h_{12}^l}{\sqrt{2}}\sin\phi\frac{v}{u}, \tag{2.17c}$$

$$\Delta_{e\mu}^{h'\,R} = -\chi\epsilon\frac{h_{21}^{l*}}{\sqrt{2}}\sin\phi\frac{v}{u}, \tag{2.17d}$$

and

$$\cos\varphi_{h'} = \cos\phi, \qquad \cos\varphi_{s'} = \sin\phi, \tag{2.18}$$

where we have omitted a term in Δ_{ee} which is real and leading order in ϵ^2, and introduced $\chi = (|h_{11}^l||h_{22}^l|)^{-1/2}$. The coefficients $\Delta_{ij}^{s'\,L,R}$ are obtained from $\Delta_{ij}^{h'\,L,R}$ by making the transformation $\cos\phi \to \sin\phi$, and $\sin\phi \to -\cos\phi$.

The electric dipole moment of the electron (d_e) comes from the familiar type of two-loop graph [103] shown in Fig. 2.1. In terms of the original fields s_1, s_2 and h

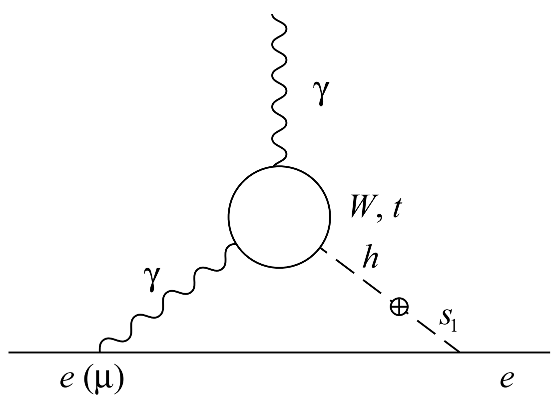


Figure 2.1: A two-loop Feynman diagram for electron electric dipole moment ($\mu \to e + \gamma$).

coming from S and H, rather than the mass eigenstates, one sees that the field that couples to the W or t loop must be h. This can be seen as follows. The s_i have no coupling to t at the leading order in ϵ^2, since the t mass comes from order $(S/M_F)^0$, i.e. $n^u_{33} = 0$. [See Eqs. (2.2) and (2.9).] The s_i also have no coupling to the $W^\pm$ since S does not participate in breaking $SU(2)_L \otimes U(1)_Y$. [If there were two Higgs doublets in the model, then the heavy loop could be a charged Higgs, in which case the field coupling to it in Fig. 2.1 could be an s_1.] However, the field coupling to the electron line must be either s_1 or s_2 in order to obtain a CP-violating phase, since the couplings of h are real and flavor diagonal in the physical basis of the leptons. However, the s_2, while it can give a CP-violating phase, does not mix with the h and therefore would not be able to attach to the W or t loop. The scalar line in the two-loop graph for d_e is thus s_1 where it attaches to the electron, and h where it attaches to the W or t loop. Consequently, the electron edm is proportional to the mixing $\sin\phi\cos\phi$. A significant point about the d_e diagram, which has already been alluded to, is that while the s_1 coupling to the electron has a CP-violating phase, that phase brings in an extra suppression of order ϵ^2. The electric dipole moment of a charged lepton is given by

$$d_i = \frac{eG_F\alpha}{8\sqrt{2}\pi^3} m_i \,\mathrm{Im}[A^L_{ii} - A^R_{ii}], \tag{2.19}$$

where the dominant, reduced amplitude [104], comes from W loop and reads

$$A^{L,R}_{ij} = -\sum_a \cos\varphi_a \Delta^{a\,L,R}_{ij} \left[3f\left(\frac{m_W^2}{m_a^2}\right) + \frac{23}{4}g\left(\frac{m_W^2}{m_a^2}\right) + \frac{3}{4}h\left(\frac{m_W^2}{m_a^2}\right)\right]. \tag{2.20}$$

We define $F(z) \equiv 3f(z) + \frac{23}{4}g(z) + \frac{3}{4}h(z)$, for short, where the functions f, g, and h are as defined in Eqs. (10), (11), and (15) of Ref. [104], respectively. The function $F(m_W^2/m^2)$ is plotted in the relevant region of scalar mass m in Fig. 2.2.

To determine the electron edm we need to determine the values of h^l_{12} and h^l_{21} coefficients that enter the expressions for $\Delta^{a\,L}_{ee}$ and $\Delta^{a\,R}_{ee}$. Since we cannot relate them to the charged lepton masses to the leading order in ϵ^2 we are prompted to estimate their values. In view of the fact that all but a couple of h_{ij} coefficients has

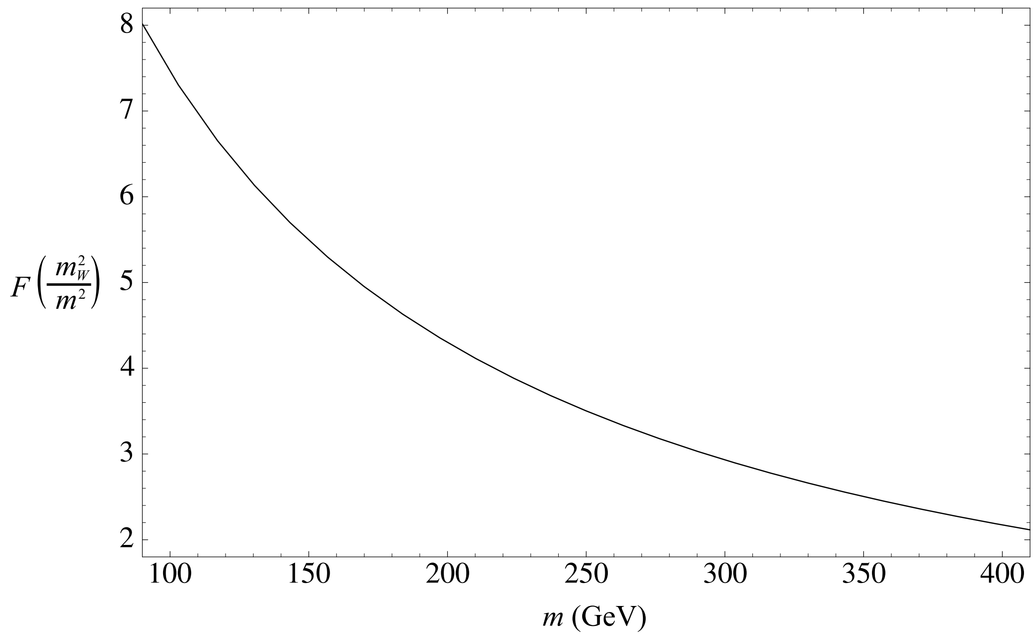


Figure 2.2: Plot of $F(m_W^2/m^2) \equiv 3f(m_W^2/m^2) + \frac{23}{4}g(m_W^2/m^2) + \frac{3}{4}h(m_W^2/m^2)$ as a function of scalar mass m.

come out to be of order unity we assume this to be the case for h_{12}^l and h_{21}^l, too. The same expectation applies to their phases. [We make the same assumption for h_{12}^d and h_{21}^d coefficients later on.] Therefore, setting $h_{12}^l = h_{21}^l = \mathrm{e}^{-\mathrm{i}\pi/4}$ in Eq. (2.17), the electron edm comes out to be

$$d_e = (1.5 \times 10^{-27}\,\mathrm{ecm}) \sin\phi \cos\phi \left(\frac{v}{u}\right) \left[F\left(\frac{m_W^2}{m_{h'}^2}\right) - F\left(\frac{m_W^2}{m_{s'}^2}\right)\right], \qquad (2.21)$$

Using the experimental value $d_e = 0.18 \times 10^{-26}\,\mathrm{ecm}$ [105] gives the following limit:

$$\sin\phi \cos\phi \left(\frac{v}{u}\right) \left[F\left(\frac{m_W^2}{m_{h'}^2}\right) - F\left(\frac{m_W^2}{m_{s'}^2}\right)\right] \leq 1.2. \qquad (2.22)$$

The diagram for $\mu \to e + \gamma$ is of the same two-loop type as the electron edm diagram, except that one of the external leptons is a μ rather than an e. As in the d_e case, the scalar which couples to the lepton line must be s_1 (here because it involves flavor-changing), while the scalar that couples to the $W^\pm$ or t loop must be

h. Thus the amplitude here is also proportional to $\sin\phi\cos\phi$. The branching ratio for the process $l_j \to l_i + \gamma$ is given by

$$B(l_j \to l_i + \gamma) = \frac{3}{4}\left(\frac{\alpha}{\pi}\right)^3 \frac{m_i}{m_j}\left(\frac{1}{2}\left|A_{ij}^L\right|^2 + \frac{1}{2}\left|A_{ij}^R\right|^2\right). \tag{2.23}$$

Again setting $h^l_{12} = h^l_{21} = \mathrm{e}^{-\mathrm{i}\pi/4}$, and imposing the experimental limit $B(\mu \to e + \gamma) \leq 1.2 \times 10^{-11}$ [106], one obtains

$$\sin\phi\cos\phi\left(\frac{v}{u}\right)\left[F\left(\frac{m_W^2}{m_{h'}^2}\right) - F\left(\frac{m_W^2}{m_{s'}^2}\right)\right] \leq 2.2. \tag{2.24}$$

The diagram relevant for μ-e conversion on nuclei is Fig. 2.3. The field

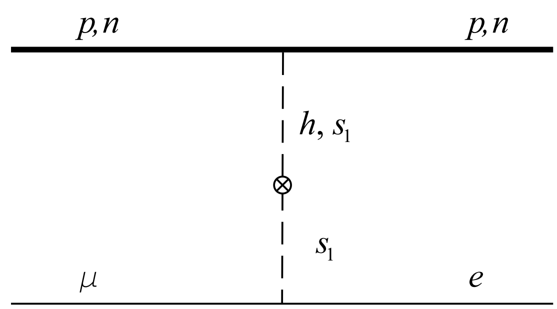


Figure 2.3: Tree level scalar exchange Feynman diagram for μ-e conversion on nuclei.

that couples to the lepton line must be s_1 or s_2, but the field that couples at the quark line may be h, s_1, or s_2. It is well known that the contributions of the pseudoscalar exchange to the coherent μ-e conversion on nuclei can be neglected [107] and will be ignored in our calculations. The contributions to the amplitude from diagrams where the scalar couples to the lepton as s_1 but to the quark as h go as $\sin\phi\cos\phi(1/m^2_{h'} - 1/m^2_{s'})$. Those in which the scalar couples to both the lepton line and the quark line as s_1 go as $\cos^2\phi(1/m^2_{s'}) + \sin^2\phi(1/m^2_{h'})$. We shall see these expressions emerge in the formulas that appear below.

The branching ratio of μ-e conversion $B(\mu^- + (A,Z) \to e^- + (A,Z))$, defined to be the ratio of decay widths $\Gamma(\mu^- + (A,Z) \to e^- + (A,Z))/\Gamma(\mu^- + (A,Z) \to$ capture), can be found using the procedure outlined in [108, 109]. We obtain

$$B = 2G_F^2 m_e m_\mu \frac{\alpha^3 m_\mu^5 Z_{\text{eff}}^4}{\pi^2 Z \Gamma_{\text{capt}}} A^2 F(q^2)^2 \left[\left| \sum_a \frac{\tilde{m}_N^a}{m_a^2} \Delta_{e\mu}^{a\,L} \right|^2 + \left| \sum_a \frac{\tilde{m}_N^a}{m_a^2} \Delta_{e\mu}^{a\,R} \right|^2 \right], \tag{2.25}$$

where $F(q^2)$ is the nucleon form factor, Z_{eff} is the effective atomic number, and where $\tilde{m}_N^a$ contains the heavy quark effects in effective scalar-nucleon-nucleon coupling [110] and is given by

$$\tilde{m}_N^a = \left\langle N \middle| \sum_{l=u,d,s} m_l \Delta_{ll}^a \bar{\psi}_l \psi_l + \sum_{h=t,b,c} m_h \Delta_{hh}^a \bar{\psi}_h \psi_h \middle| N \right\rangle. \tag{2.26}$$

We derive the most general, model independent, expression for $\tilde{m}_N^a$ using the approach of Shifman *et al.* [111], and subsequent improvements of inclusion of strange and heavy quark contributions discussed in [112, 113] as follows[1]

$$\tilde{m}_N^a = \left(\sum_h \Delta_{hh}^a \right) \frac{2}{27} \left[m_N - \sigma_{\pi N} \left(1 + \frac{y}{2} \frac{m_s}{\bar{m}} \right) \right] + \sigma_{\pi N} \left[\frac{\Delta_{uu}^a + \Delta_{dd}^a}{2} + \Delta_{ss}^a \frac{y}{2} \frac{m_s}{\bar{m}} \right], \tag{2.27}$$

where h runs over heavy quarks (t, b, c), $y = 2\langle N|\bar{s}s|N\rangle / \langle N|\bar{u}u + \bar{d}d|N\rangle$ is the strange content in the nucleon, $\sigma_{\pi N}$ is the pion-nucleon sigma term, m_N is the nucleon mass, and $\bar{m} = (m_u + m_d)/2$. In our model, the diagonalization procedure in quark sector, to the leading order in ϵ^2, leads to

$$\Delta_{ii}^{h'} = \left[\cos\phi - \kappa_i \frac{v}{u} \sin\phi \right] \Big/ \sqrt{2}, \tag{2.28}$$

where $(\kappa_u, \kappa_c, \kappa_t, \kappa_d, \kappa_s, \kappa_b) = (3, 1, 0, 3, 2, 1)$. Note that κ_i's are the powers of $\hat{\epsilon}^2$ of the appropriate diagonal elements that appear in $\hat{\lambda}_{ij}^f$ of Eq. (2.2).

[1] Our general expression for $\tilde{m}_N^a$ reproduces Eq. (3) of Ref. [110] but yields an additional term in Eq. (20) where authors analyze MSSM model. The additional piece is $\sigma_{\pi N}(\cot\beta + \tan\beta)/2$.

For μ-e conversion on ${}^{48}_{22}$Ti, we set $Z_{\text{eff}} = 17.6$, $F(q^2 = -m_\mu^2) = 0.54$, $\Gamma_{\text{capt}} = 2.59 \times 10^6\,\text{s}^{-1}$ [114], impose the experimental limit $B < 4.3 \times 10^{-12}$ [115], take $\bar{m} = 5\,\text{MeV}$, and use the set $(y, \sigma_{\pi N}) = (0.47, 60\,\text{MeV})$ [112], to obtain

$$\left(\frac{v}{u}\right)\left|\sin\phi\cos\phi\left(\frac{1}{m_{h'}^2} - \frac{1}{m_{s'}^2}\right)m - \left(\frac{v}{u}\right)\left(\frac{\sin^2\phi}{m_{h'}^2} + \frac{\cos^2\phi}{m_{s'}^2}\right)m'\right| \leq \frac{9\times 10^{-5}}{1\,\text{GeV}}, \tag{2.29}$$

where $m \simeq 350\,\text{MeV}$, and $m' \simeq 500\,\text{MeV}$.

The diagram relevant for the $\Delta S = 2$ processes is Fig. 2.4. Here, the field

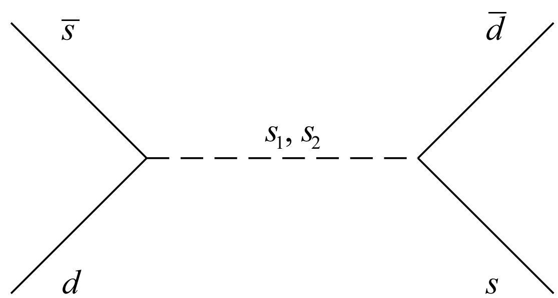


Figure 2.4: Tree level contribution to $K^0 - \bar{K}^0$ mixing.

that couples at both quark lines must be s_1 or s_2. Thus there are contributions that go as $\cos^2\phi(1/m_{s'}^2) + \sin^2\phi(1/m_{h'}^2)$ and as $1/m_{s_2}^2$. Noting that $\Delta_{ds}^{L,R}$ is obtained from $\Delta_{e\mu}^{L,R}$ by replacing h_{ij}^l with h_{ij}^d, and using the vacuum saturation approximation for the hadronic element [116], we find a new contribution coming from the scalar exchange to be

$$\begin{aligned}\epsilon_K^a \;\simeq\; & \frac{C_K}{m_a^2}\left\{\left(\frac{1}{6}\frac{M_K^2}{(m_d+m_s)^2} + \frac{1}{6}\right)\text{Im}\left[\left(\frac{h_{12}^{d*} + h_{21}^d}{\sqrt{2}}\right)^2\right]\right. \\ & \left. - \left(\frac{11}{6}\frac{M_K^2}{(m_d+m_s)^2} + \frac{1}{6}\right)\text{Im}\left[\left(\frac{h_{12}^{d*} - h_{21}^d}{\sqrt{2}}\right)^2\right]\right\}(1-\cos^2\varphi_a), \end{aligned} \tag{2.30}$$

while the exchange of pseudoscalar s_2, due to the extra factor of i, yields

$$\begin{aligned} \epsilon_K^{s_2} \;\simeq\; & \frac{C_K}{m_{s_2}^2}\left\{\left(\frac{1}{6}\frac{M_K^2}{(m_d+m_s)^2}+\frac{1}{6}\right)\mathrm{Im}\left[\left(\frac{h_{12}^{d*}-h_{21}^d}{\sqrt{2}}\right)^2\right]\right. \\ & \left. -\left(\frac{11}{6}\frac{M_K^2}{(m_d+m_s)^2}+\frac{1}{6}\right)\mathrm{Im}\left[\left(\frac{h_{12}^{d*}+h_{21}^d}{\sqrt{2}}\right)^2\right]\right\}, \end{aligned} \tag{2.31}$$

where we introduce

$$C_K = \frac{f_K^2 M_K B_K \epsilon^{12}}{8\sqrt{2}\,\Delta M_K}\left(\frac{v}{u}\right)^2. \tag{2.32}$$

Using $B_K = 0.75$, $\Delta M_K \simeq 3.49\times10^{-12}\,\mathrm{MeV}$, $f_K \simeq 160\,\mathrm{MeV}$, $M_K \simeq 497.67\,\mathrm{MeV}$, $m_s = 175\,\mathrm{MeV}$, $m_d = 8.9\,\mathrm{MeV}$, $h_{12}^d = h_{21}^d = \mathrm{e}^{-\mathrm{i}\pi/4}$, and requiring the terms involving $m_{h'}$, $m_{s'}$, and m_{s_2} separately to contribute to ϵ_K an amount less than the experimental value of that quantity ($|\epsilon_K| = 2.26\times10^{-3}$ [105]) give the limits

$$\left(\frac{v}{u}\right)^2\frac{\sin^2\phi}{m_{h'}^2},\ \left(\frac{v}{u}\right)^2\frac{\cos^2\phi}{m_{s'}^2} \le \frac{3.9\times10^{-6}}{1\,\mathrm{GeV}^2},\quad \text{and}\quad \left(\frac{v}{u}\right)^2\frac{1}{m_{s_2}^2} \le \frac{3.8\times10^{-5}}{1\,\mathrm{GeV}^2}. \tag{2.33}$$

If we take $m_{h'} \simeq 10^2\,\mathrm{GeV}$, as suggested by experiment, then Eq. (2.33) implies that $(v/u)\sin\phi \le 1/5$, which is not a very stringent bound. Substituting this into Eq. (2.21), one sees that the electron edm can easily be near the present published experimental limit. For instance, taking $(v/u)\sin\phi \simeq 0.1$, so that flavon exchange contributes of order 1/5 of the experimental value of ϵ_K, and taking $m_{s'} \simeq 300\,\mathrm{GeV}$, Eq. (2.21) gives $d_e \sim (0.6\times10^{-27}\,\mathrm{ecm})\cos\phi$.

Comparing Eqs. (2.22) and (2.24) (in which the unknown parameters, $\sin\phi$, $m_{s'}$, and u, enter in exactly the same way) reveals that the present limits on the decay $\mu\to e+\gamma$ and the electron edm are about equally sensitive to flavon exchange in this model. For example, if the CP-violating phases are large and all h_{ij} are close to one, as was assumed in deriving Eqs. (2.21) and (2.24), and d_e is just below the present limit, then the rate for $\mu\to e+\gamma$ is roughly a forth of the present limit.

One sees here the importance of the fact that the diagonal Yukawa couplings of the flavon field s_1 have phases suppressed by $\epsilon^2 \simeq 2\times10^{-2}$. Were it not so, then the present limit on the electron edm would imply that the rate for $\mu \to e + \gamma$ was at least four orders of magnitude below present limits (unless parameters were fine-tuned). $\mu + N \to e + N$ would also be suppressed.

Turning to μ-e conversion on nuclei, one sees from Eq. (2.29) that the present limit on this is also, for a wide range of parameters, about as sensitive to flavon exchange as are the present limits on d_e and $\mu \to e+\gamma$. For example, if $(v/u)\sin\phi \simeq 1/5$, then the first term on the left-hand side of Eq. (2.29) (i.e. the term proportional to $\sin\phi\cos\phi/m_{h_0}^2$) gives a contribution to the rate for μ-e conversion that is about an order of magnitude below the present limit. However, in some regions of parameter space, $\mu + N \to e + N$ can be the most sensitive to flavon exchange. Suppose, for example, that v/u is smaller, but not much smaller, than one, and that $\sin\phi \ll 1$. Then both d_e and $\mu \to e+\gamma$ are highly suppressed, whereas $\mu+N \to e+N$ need not be because of the term that goes as $\cos^2\phi/m_{s'}^2$ on the left-hand side of Eq. (2.29).

We have only considered the effects arising from the effective Yukawa terms in Eq. (2.1). However, there is another source of flavor violation from flavon exchange that can be very important. To get the effective low energy Yukawa terms in Eq. (2.1), fermions having mass of order M_F are integrated out. There are diagrams involving these heavy fermions that can be important. The most important such diagram is that shown in Fig. 2.5, which is a contribution to $K - \overline{K}$ mixing. The internal fermion has mass of order M_F. The external fermion is the s_0 quark, i.e. the s quark in the original basis in which the Yukawa matrices of Eq. (2.2) are written. When one goes to the physical basis of the light quarks, s_0 will contain a small admixture of the physical d quark: $s_0 = s + O(\epsilon^2)d$. Consequently, there will be from Fig. 2.5 a $\Delta S = 2$ piece that goes as ϵ^4. The Yukawa couplings in Fig. 2.5 may be assumed to be of order unity. [The only reason the effective Yukawa couplings of the known light quarks are small is that they are suppressed by powers of ϵ^2, since they arise from integrating out heavy fermions. However, in

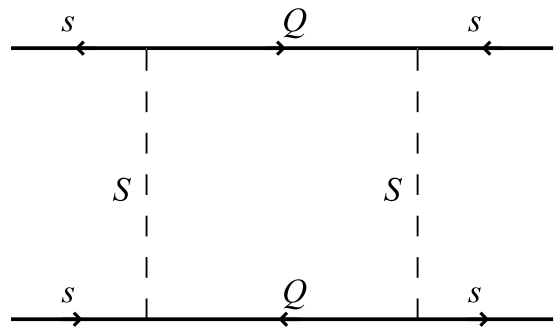


Figure 2.5: Box diagram contribution to $K - \overline{K}$ mixing. The internal fermion Q has mass of order M_F.

the underlying theory containing those heavy fermions there is no reason for the Yukawa couplings to be small.] The coefficient of the $\Delta S = 2$ operator arising from Fig. 2.5 should therefore typically be of order $(16\pi^2)^{-1}\epsilon^4(1/M_F^2) = (16\pi^2)^{-1}\epsilon^8 u^{-2}$. Using $\epsilon^2 \sim 2 \times 10^{-2}$ and $u \sim 300\,\mathrm{GeV}$, one has that the coefficient of the $\Delta S = 2$ term is of order $10^{-14}\,\mathrm{GeV}^{-2}$. With some of the phases or couplings being assumed somewhat smaller than one, the contribution from Fig. 2.5 can easily be within the limit set by ϵ_K.

2.4 Conclusions

We presented a simple flavon model that can accommodate the observed hierarchy of the charged fermion masses and mixings in terms of the powers of one small parameter. It has been shown that the flavor-diagonal couplings of the flavon field, under a general set of assumptions, are real to the leading order in that parameter. This implies that flavor changing and *CP* violating signatures, d_e, $\mu \to e + \gamma$, and μ-e conversion on nuclei, can be equally near the present experimental limits with all other low energy constraints satisfied. For a significant range of parameters μ-e conversion can be the most sensitive place to look for such signatures.

Chapter 3

OBTAINING THE LARGE ANGLE MSW SOLUTION TO THE SOLAR NEUTRINO PROBLEM IN MODELS

3.1 Introduction

The main solutions to the solar neutrino problem are the small mixing angle MSW[1] solution (SMA), the large mixing angle MSW solution (LMA), the LOW solution [119, 120, 121], and the vacuum oscillations solution (VAC) [122, 123]. The experimental situation has been very ambiguous until the recent results of the Sundbury Neutrino Observatory [124] (SNO) and the Kamioka Liquid scintillator AntiNeutrino Detector [72] (KamLAND) experiments. The results of the SNO experiment have firstly singled out the LMA and LOW solutions as the most likely oscillation solutions [125]. The more recent results of the KamLAND experiment have then eliminated even the LOW solution [126], leaving the LMA solution as the only viable one.

On the other hand, a survey of the hundreds of published models of neutrino masses and mixings shows that most of them yield the SMA or VAC solution, and even some that claim to obtain the LMA solution are only marginally consistent with the latest global analyses of the data. The purpose of this chapter is to look at the main types of models of neutrino masses and mixing angles that have been proposed

[1] MSW stands for Mikheev-Smirnov-Wolfenstein, in honor of the co-discoverers of the matter effect [117, 118] on solar neutrinos. This effect allows for the conversion of solar neutrinos into neutrinos of another flavor due to the interaction with the matter thus explaining their observed deficit.

in the literature from the point of view of their ability to yield the LMA solution in a natural way. There are two aspects of this question that can be distinguished. First, one can ask whether a certain scheme or model can *fit* the LMA solar solution with some choice of model parameters that is not too badly fine tuned. Second, one can ask whether the model *explains* the LMA values of the neutrino masses and mixing angles. In order to say that a theoretical model really explains them, something close to the LMA best-fit values should emerge automatically when the parameters of the theoretical model take their most "natural values". If a model parameter that is *a priori* of order one must be set to a value of ten or a tenth in order to fit the neutrino masses and mixings, one has accommodated them but not really explained them. What our survey will show is that of the great number of models that now exist in the literature, few can be said to provide an explanation (in this sense) of the LMA values of $\tan^2\theta_{\rm sol}$ and $\Delta m^2_{\rm sol}$.

What are the LMA values that are to be explained? A recent global analysis of Fogli, Lisi, Marrone, Montanino and Rotunno [131] have shown that the LMA solution splits into two sub-regions, LMA-I and LMA-II. The region LMA-I is preferred by the global fit with the best fit of $\Delta m^2_{\rm sol} \sim 7\times 10^{-5}\,{\rm eV}^2$ and $\sin^2\theta_{\rm sol} \sim 0.3$, where $\Delta m^2_{\rm sol} = m_2^2 - m_1^2$ and $\sin\theta_{\rm sol} \equiv |U_{e2}|$. The region LMA-II is characterized by the best fit with the twice the value of $\Delta m^2_{\rm sol}$ and almost the same value of $\sin^2\theta_{\rm sol}$. The two regions merge at 99.37% confidence level giving $\Delta m^2_{\rm sol} \sim 3\times 10^{-4}\,{\rm eV}^2$. The 95% confidence-level allowed region given in [131] extends in $\Delta m^2_{\rm sol}$ from about $5\times 10^{-5}\,{\rm eV}^2$ to $8\times 10^{-4}\,{\rm eV}^2$, and in $\tan^2\theta_{\rm sol}$ from about 0.3 to 0.7. Similar results have been presented by Gonzalez-Garcia and Pena-Garay [133]. Their 90% confidence-level region extends in $\tan^2\theta_{\rm sol}$ from about 0.3 to 0.8. A significant aspect of the fits is that they *exclude* exactly maximal mixing for the LMA solution.

The experimental signal reported by the LSND Collaboration [127], when combined with the results of the experiments on solar and atmospheric neutrinos, requires at least four light neutrino species. Their signal, however, has not been corroborated (nor completely excluded) by the results reported by the KARMEN Collaboration [128]. Therefore, we limit our survey to the models with the three light neutrinos. There is also a whole new class of neutrino models where neutrinos have extra-dimensional origin (for a brief review see [129]). These models are still not as predictive as the four-dimensional ones even though they have certain promising features (see [130]). For example, we have seen in Section 1.4.2 that the see-saw formula yields the neutrino mass of the order of $1/10^3\,\mathrm{eV}$ which is smaller by a factor of ten than the experimentally preferred value. In the extra-dimensional setting one can identify the scale of the right-handed Majorana neutrinos to be the compactification scale M_C of the extra-dimension instead of the usual assumption that $M_R \sim M_{\mathrm{GUT}}$. The compactification scale, as we demonstrate in Chapter 5, is usually of the order of $10^{14}\,\mathrm{GeV}$, giving the light neutrino mass in the right range. We do not discuss these models in this chapter any further.

As we shall see, the issue of how close to exact maximal mixing $\tan^2\theta_{\mathrm{sol}}$ is allowed to be is crucial for deciding whether several kinds of models can naturally give an acceptable LMA solution. In this chapter we shall say that a model gives an LMA solution that is in comfortable agreement with the data if it predicts $\tan^2\theta_{\mathrm{sol}} \leq 0.8$. It is convenient to express the mass-squared splitting $\Delta m^2_{\mathrm{sol}}$ as a fraction, which we shall call r, of the mass-squared splitting relevant to the atmospheric neutrino oscillations: $r \equiv \Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$. The value of $\Delta m^2_{\mathrm{atm}} (\equiv m_3^2 - m_2^2)^{\ddagger}$ has recently been decreased from $2.6 \times 10^{-3}\,\mathrm{eV}^2$ [132, 133] to $2 \times 10^{-3}\,\mathrm{eV}^2$ by the preliminary reanalysis of the existing data by the Super-Kamiokande Collaboration [134]. This, however, has made no impact on the values of the $\Delta m^2_{\mathrm{sol}}$, $\tan^2\theta_{\mathrm{sol}}$ [135], and the value of the atmospheric angle $\tan^2\theta_{\mathrm{atm}} = 1$, where $\sin\theta_{\mathrm{atm}} \equiv |U_{\mu 3}|$. We take the

‡ The sign of $\Delta m^2_{\mathrm{sol}}$ is known from solar matter effect. The sign of $\Delta m^2_{\mathrm{atm}}$, on the other hand, is still not determined giving the rise to two possible mass hierarchies in the neutrino sector.

best fit values before the reanalysis that give $r \simeq 2.8 \times 10^{-2}$ for the LMA-I region.

The rest of this chapter is organized as follows. In Section 3.2, we shall look at the basic non-see-saw approaches to neutrino mass. We shall see that models based on such approaches have been constructed which can fit the LMA solution, but for the most part not comfortably, either because r tends to come out too small or because the solar mixing angle tends to come out too close to maximality. In other words, most of the non-see-saw models that can fit the LMA solution do not really explain it in the sense that we have defined. In Section 3.3, we look at see-saw models. Here too, most of the published models give the SMA or VAC solutions. However, we show that there are some reasonably simple "textures" that can reproduce nicely the LMA values of the neutrino masses and mixings. However, it remains unclear whether these simple textures can arise in simple models.

In Section 3.4, we look at a well-known model that is particularly interesting for two reasons: (a) it is very simple in conception, and (b) it can explain at least the LMA value of the solar neutrino angle, although it does not explain the value of the neutrino mass splitting. It is an $SU(5)$ grand unified model with an Abelian family symmetry. We shall analyze this model in some detail both analytically and numerically. We shall show how the predictions of this model can be studied statistically in a completely analytic way by assuming that the unknown parameters of the model have Gaussian distributions. This method should be easily applicable to many other kinds of models.

Section 3.5 is a brief summary.

3.2 Non-see-saw models

3.2.1 Non-see-saw models where $\theta_{\rm atm}$ comes from M_ν.

In non-see-saw models the mass matrix M_ν of the three light neutrinos is typically generated by new low-energy physics. It therefore has no relation, or only a very indirect relation, to the Dirac mass matrices of the charged leptons, the down quarks, and the up quarks. This has the great advantage of making it easy to explain why the atmospheric neutrino mixing angle is very large ($|U_{\mu 3}| \equiv \sin\theta_{\rm atm} \cong 0.7$) while the corresponding quark mixing is so small ($|V_{cb}| \cong 0.04$). If the Dirac matrices are assumed to be hierarchical, then they would naturally give the small mixing angles seen in the quark sector. But if M_ν is unrelated to the Dirac mass matrices, it could easily have a very different form with large off-diagonal elements that gives large mixing angles. Non-see-saw models based on this idea are called Type I(1) in [136].

The tricky question for this type of model is to explain why $\Delta m^2_{\rm sol} \ll \Delta m^2_{\rm atm}$. If the large mixing $U_{\mu 3}$ comes from diagonalizing the 2-3 block of M_ν, one would expect that m_2 and m_3, the second and third eigenvalues of M_ν, would have similar magnitudes, in which case typically so would $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$. The challenge then is to reconcile the hierarchy seen in the eigenvalues of M_ν with the large atmospheric mixing angle. To do this requires a special form of M_ν. Two special forms have been found viable in constructing realistic models, one leads to a so-called "inverted hierarchy" $m_1 \cong m_2 \gg m_3$, and the other to the ordinary hierarchy $m_1 \ll m_2 \ll m_3$. We shall consider these in turn.

3.2.1.1 Inverted hierarchy models.

Inverted hierarchy models have the following special form for M_ν:

$$M_\nu = \begin{pmatrix} m_{11} & cM & sM \\ cM & m_{22} & m_{23} \\ sM & m_{23} & m_{33} \end{pmatrix}. \tag{3.1}$$

Here $c \equiv \cos\theta$ and $s \equiv \sin\theta$, where $\theta \sim 1$, and $m_{ij} \ll M$. One can diagonalize this matrix in stages, the first step being to rotate by angle θ in the "23 plane", bringing the matrix to the form

$$M'_\nu = \begin{pmatrix} m_{11} & M & 0 \\ M & m'_{22} & m'_{23} \\ 0 & m'_{23} & m'_{33} \end{pmatrix}. \tag{3.2}$$

One sees immediately that $m_1 \cong m_2 \cong M \gg m_3$. The mass-squared splitting relevant to atmospheric oscillations is $\Delta m^2_{\rm atm} \cong M^2$, whereas the splitting relevant to solar oscillations is $\Delta m^2_{\rm sol} \cong 2(m_{11}+m'_{22})M$, which is much smaller, as required by all the viable solar solutions. The atmospheric angle gets a contribution $\theta \sim 1$ from diagonalizing the 2-3 block of M_ν, so in the absence of some unlikely cancellation it will be large, as observed. But what of the solar angle? From the fact that the 1-2 block of Eq. (3.2) has a pseudo-Dirac form of Eq. (1.58) it is apparent that the solar mixing angle will be close to maximal. Consequently, inverted hierarchy models cannot give the SMA solar solution, but rather give "bimaximal" mixing.

The inverted hierarchy form of Eq. (3.1) can arise in several plausible ways. One example is the Zee type of model [137, 138, 139]. In the Zee model [140, 141] there is a singly charged singlet scalar field h^+, which is allowed by the Standard Model quantum numbers to couple (antisymmetrically) to both a pair of lepton doublets ($h^+ L_i L_j$) and a pair of Higgs doublets ($h^+ \Phi_a \Phi_b$), assuming that more than one Higgs doublet exists. If both types of coupling are present, a conserved lepton number cannot be consistently assigned to h^+, and consequently $\Delta L = 2$ Majorana masses for the left-handed neutrinos arise at one-loop level. The resulting one-loop mass matrix has the form

$$M_\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}. \tag{3.3}$$

For $c \ll a \sim b$, this has the desired inverted hierarchy form.

The inverted hierarchy form can also arise in models with an approximately conserved $L_e - L_\mu - L_\tau$ lepton number. If this quantum number is exactly conserved, then only the 12, 21, 13, and 31 elements of M_ν can be nonvanishing. If there are small violations of $L_e - L_\mu - L_\tau$ the form in Eq. (3.1) can result [142, 139, 143, 144, 145, 146, 147, 148].

The question of present interest to us is whether the inverted hierarchy can give an acceptable LMA solution. To answer this one must look more closely at the solar neutrino mixing angle. This is given by $\theta_{\rm sol} = \theta_{12}^\nu - \theta_{12}^l$, where the two angles on the right-hand side are the contributions that come from diagonalizing M_ν and L respectively. From Eq. (3.2) it is easily found that $\tan 2\theta_{12}^\nu \cong 2M/(m'_{22} - m_{11})$, so that $\theta_{12}^\nu \cong \pi/4 - (m'_{22} - m_{11})/4M$. We have already seen that $r \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} \cong 2(m'_{22} + m_{11})/M$. Requiring that this be of order 10^{-2} as needed for the LMA-I solution, and assuming that there are no accidental cancellations, one has that $\theta_{12}^\nu \cong \pi/4 + O(10^{-3})$. If θ_{12}^l vanished, this would give $\tan^2\theta_{\rm sol} = 1 + O(10^{-3})$, which is too close to maximal mixing to be in comfortable accord with the global fits. However, one expects that $\theta_{12}^l \sim \sqrt{m_e/m_\mu} \cong 0.07$. This contribution can have any complex phase relative to the contribution from M_ν, and can therefore increase or decrease $\tan^2\theta_{\rm sol}$ from unity. If one assumes that $\theta_{12}^l = 0.07$ and has a relative minus sign to θ_{12}^ν, then $\tan^2\theta_{\rm sol} \cong 0.75$, which is consistent with the global LMA fits. However, one can see that the tendency of inverted hierarchy models is to give solar mixing that is closer to maximality than to the best-fit LMA value of $\tan^2\theta_{\rm sol} \approx 0.4$. This is one reason why many of the published inverted hierarchy models claim a better fit to the VAC solution than to the LMA solution [143, 144, 145, 146, 147] A significant reduction of the experimental upper limits on $\tan^2\theta_{\rm sol}$ would make the inverted hierarchy idea much less plausible as an explanation of the LMA solution. For example, a value of $\tan^2\theta_{\rm sol} = 0.5$, would imply in the inverted hierarchy context that $\tan\theta_{12}^l \cong 0.17 \cong 2.5\sqrt{m_e/m_\mu}$, which would require a very special form of the 1-2 block of L.

3.2.1.2 Ordinary hierarchy models.

The other possibility for non-see-saw models that gives a large atmospheric neutrino mixing angle coming from M_ν and a hierarchy in the mass-squared splittings is

$$M_\nu \cong \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & s^2 M & scM \\ m_{13} & scM & c^2 M \end{pmatrix}. \tag{3.4}$$

Here, again, $c \equiv \cos\theta$ and $s \equiv \sin\theta$, where $\theta \sim 1$, and $m_{ij} \ll M$. As written, the 2-3 block of the matrix has vanishing determinant; however, it is assumed that there are small corrections to these elements, which we have not written.

As in the case of the inverted hierarchy models, one can diagonalize this in stages, starting with a rotation by angle θ in the 23 plane. The result of such a rotation is to bring the matrix to the form

$$M'_\nu = \begin{pmatrix} m_{11} & m'_{12} & m'_{13} \\ m'_{12} & m'_{22} & 0 \\ m'_{13} & 0 & M \end{pmatrix}. \tag{3.5}$$

Because of the small corrections to the 23 block that were just mentioned, the 22 element in Eq. (3.5) does not vanish, but is small compared to M. This matrix gives $\Delta m^2_{\rm sol} = O(m^2_{ij})$ and $\Delta m^2_{\rm atm} \cong M^2$. Thus the right hierarchy of splittings for any of the solutions can be achieved for the appropriate values of m_{ij}/M. In contrast to the inverted hierarchy form, this form can give either small or large $\theta_{\rm sol}$, and in the large-angle case there is no preference for values of $\theta_{\rm sol}$ that are very close to maximal.

The form in Eq. (3.4) is clearly special in the sense that the 2-3 block is approximately of rank one. This would be unnatural unless some symmetry or mechanism guaranteed it. One possibility is that this form arises from a non-Abelian flavor symmetry [149], however, this is difficult to achieve. Rather, almost all published models that achieve this form in a natural way use the idea of factorization. The idea of factorization is that the dominant contribution to the neutrino mass matrix has the form $(M_\nu)_{ij} = f_i f_j$, which is obviously of rank one. If $f_1 \ll f_2, f_3$, this dominant term reproduces the large elements in Eq. (3.4). The condition that f_1 is small compared to f_2 and f_3 is necessary to satisfy the experimental constraint that $U_{e3} \leq 0.15$. One drawback of most models based on factorization is that they do not explain why f_1 is small.

A factorized form can arise in various ways in non-see-saw models. A much studied example is supersymmetry with terms in the superpotential that violate both lepton number and R-parity. Cubic terms of this type are $\lambda_{ijk} L_i L_j \bar{e}_k$ and $\lambda'_{ijk} L_i Q_j \bar{d}_k$. The latter leads to one-loop $\Delta L = 2$ neutrino mass diagrams, in which a neutrino converts into a virtual quark-squark pair. Assuming that the LR squark masses are proportional to the corresponding quark masses, this diagram gives $(M_\nu)_{ij} \propto \lambda'^*_{ikl} \lambda'_{jlk} m_{d_k} m_{d_l}$. Consequently, the b-quark/b-squark loop dominates, and gives a contribution that is proportional to $\lambda'^*_{i33} \lambda'_{j33} m_b^2$, which obviously has a factorized form. This gives only the heaviest neutrino mass, m_3. The second largest neutrino mass comes from a similar diagram with both b and s quarks/squarks in the loop. Consequently, one has that $r \cong (m_2/m_3)^2 \sim (m_s/m_b)^2 \sim 3 \times 10^{-4}$. This is much smaller than the value of 2.8×10^{-2} preferred by experiment; however, there are several unknown parameters that come into this calculation, such as the couplings λ'_{ijk}, so that nothing prevents the right LMA value of r from being obtained [150]. However, the model does not really explain the magnitude of r.

We have only considered the effects of the cubic lepton-number-violating and R-parity-violating terms in the superpotential. There are also in general bilinear terms of the form $L_i H_u$. These have the effect of mixing leptons and Higgs fields, and so allow the sneutrino fields to acquire non-vanishing vacuum expectation values. That, in turn, through the sneutrino-neutrino-neutralino coupling gives a tree-level neutrino mass in which the neutralino plays the role of "right-handed neutrino". It is easily seen that this tree-level mass has a factorized form and gives mass only to one neutrino, i.e. m_3. The other neutrino masses, m_2 and m_1, arise from the one-loop diagrams previously discussed. In consequence, in such models where both cubic and bilinear R-parity-violating terms contribute to M_ν one expects that $r \cong (m_2/m_3)^2 \sim (loop/tree)^2 \ll 10^{-2}$. For this reason, most analyses of supersymmetric models in which the bilinear R-parity-violating terms contribute to M_ν conclude that there is much more parameter space for the VAC solution than for the LMA solution, i.e. the LMA solution requires special choices or tuning of parameters [151, 152, 153, 154, 155]. However, in [156] it is shown that under certain assumptions (specifically, that there are only bilinear R-parity-violating terms and that the SUSY-breaking terms are non-universal) the LMA solution can be achieved without fine tuning. Nevertheless, it seems, on the whole, that the SUSY models with R-parity breaking do not do well in explaining the LMA value of $\Delta m^2_{\rm sol}$.

Another possibility for obtaining an approximately factorized form for M_ν that has been much studied in the literature is called "single right-handed neutrino dominance" (SRHND) [157]. As the name suggests, the idea here is that instead of there being three right-handed neutrinos, one in each family, as there are in typical grand unified theories or Pati-Salam models, there is just one right-handed neutrino, $\bar{\nu}$, which can have mass terms $M_R \bar{\nu}\bar{\nu} + f_i(\nu_i \bar{\nu})\langle H \rangle$. Integrating out $\bar{\nu}$ gives a rank-1 factorized contribution to M_ν. If one assumes that $\bar{\nu}$ couples with almost equal strength to the μ and τ neutrinos, and (for some reason not generally explained) only weakly to the electron neutrino, the large terms in Eq. (3.4) are reproduced.

One way to explain the smallness of the coupling of $\bar{\nu}$ to the electron neutrino would be to impose a symmetry that distinguishes ν_e from the ν_μ and ν_τ (but does not distinguish the latter from each other). Such a symmetry would also tend to suppress mixing between the ν_e and the heavier neutrinos, and thus give the SMA solar solution, as in the model of [158].

Since the right-handed neutrino only gives mass to one neutrino, some other mechanism must be found to give mass to the other neutrinos. In [159] the lighter two neutrino masses arise from loop effects. In [160] they arise at tree level from integrating out other heavy states that have different flavor quantum numbers than $\bar{\nu}$. In [161] they arise from operators of the form $\nu_i \nu_j H_u H_u / M_{\rm Pl}$, which it is argued are generally there anyway, the idea being to avoid having to invent new beyond-the-standard-model physics to account for each kind of neutrino mass. In all these cases, m_1 and m_2 are much less than m_3, though the specific reason is different in each case: in [159] they are suppressed by loop factors, in [160] by small flavor-breaking parameters, and in [161] by $M_R/M_{\rm Pl}$. That SRHND models tend to give a strong hierarchy in neutrino masses is what one would naturally expect. Since the mechanisms that generate the largest neutrino mass and the other neutrino masses are different, there is no reason *a priori* that they should yield masses of similar scale. Rather, it would be a coincidence calling for an explanation if they did. Because most SRHND models give $m_2 \ll m_3$ they yield the VAC solution or SMA solution to the solar neutrino problem rather than the LMA solution [159, 160, 161].

To obtain the LMA solution, one wants m_3 and m_2 to be only about a factor of ten in ratio. This suggests that they arise from the same basic mechanism. One possibility is that all three neutrino masses arise from integrating out right-handed neutrinos, but that one of those right-handed neutrinos is somewhat lighter than the others and so dominates to some extent, but not by a large factor. However, this would really be just a special case of the ordinary see-saw mechanism, which we will discuss in the next section. In fact, the structure given in Eq. (3.13) is really based on this idea, which was proposed in [162].

3.2.2 Non-see-saw models where $\theta_{\rm atm}$ comes from L.

There is another class of non-see-saw models in which the large atmospheric neutrino angle comes predominantly from the diagonalization of the charged lepton mass matrix L. [This class of models is called Type II(1) in [136].] This has the advantage that it becomes easy to reconcile the largeness of $\theta_{\rm atm}$ with the smallness of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$, since the former comes from L while the latter comes from M_ν. On the other hand another issue arises for this class of models, namely explaining why the CKM angles are small. Since the form of L is such as to give a large mixing angle $\theta_{\rm atm}$, one would naturally expect that the Dirac mass matrices D and U of the quarks would be such as to give similarly large contributions to the CKM angles. The point is that in many kinds of models the Dirac mass matrices L, D, and U are closely related to each other.

One possibility is that there are indeed large contributions to the CKM angles coming from U and D, but that these nearly cancel. This possibility is realized in a much-studied class of models based on the idea of "flavor democracy" [165, 166, 167, 168, 169, 170, 171, 172]. In flavor democracy models it is assumed that all the Dirac mass matrices have approximately the "democratic" form

$$\left(\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right). \tag{3.6}$$

This form can be enforced by permutation symmetries among the three families. In the limit of exact flavor democracy, the matrices U and D are exactly of the same form, so that flavor mixing in the quark sector cancels out. On the other hand, it is assumed that the neutrino mass matrix M_ν has a very different form. In most papers it is assumed to be approximately proportional to the identity matrix, though in some papers it is only assumed to be nearly diagonal. As a result, for the leptonic mixing angles there is no cancellation such as makes the CKM angles small.

If L has exactly the democratic form, then

$$U_l^\dagger = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \equiv U_{FD}. \tag{3.7}$$

If the mass matrix of the neutrinos is exactly diagonal, then $U_{\rm PMNS} = U_l^\dagger = U_{FD}$. This would give $\sin^2 2\theta_{\rm atm} = 8/9$, which is consistent with the data, and $\tan^2\theta_{\rm sol} = 1$, i.e. exactly maximal mixing for solar neutrinos. However, the matrix L clearly cannot have exactly the democratic form, as that is rank one and would give $m_e = m_\mu = 0$. There must therefore be small corrections to L coming from the breaking of the permutation symmetries. These corrections not only generate masses for the electron and muon but also make the angle $\theta_{\rm sol}$ deviate from maximality. For the simplest and most widely assumed form of these corrections to L, one can calculate the corrections to $\theta_{\rm sol}$ in terms of $\sqrt{m_e/m_\mu}$. One finds, still assuming that M_ν is diagonal, that

$$\tan^2\theta_{\rm sol} = 1 - \frac{4}{\sqrt{3}}\sqrt{m_e/m_\mu} \cong 0.84,$$

or equivalently $\sin^2 2\theta_{\rm sol} = 0.993$. This is too close to unity to be in comfortable agreement with the LMA global fits. Almost all published models based on flavor democracy have $\tan^2\theta_{\rm sol} \cong 1$, or else obtain smaller values by fine-tuning. However, Tanimoto, Watari, and Yanagida have a version in which there are small corrections to M_ν that can reduce $\tan^2\theta_{\rm sol}$ to the region preferred by the LMA fits [173]. While this shows that it is possible within the flavor democracy framework to construct LMA models that can fit the data, it does not appear that flavor democracy does a good job of explaining the LMA value of $\tan^2\theta_{\rm sol}$. Flavor democracy is more naturally compatible with the VAC or LOW solutions.

We may summarize the situation by saying that most schemes that have been proposed based on non-see-saw mechanisms neither very comfortably fit nor really do much to explain the values of the neutrino parameters required for the LMA solution to the solar neutrino problem. The great majority of non-see-saw models in the literature more naturally give the SMA or VAC solutions. There are exceptions, which we have noted above. How close $\tan^2\theta_{\rm sol}$ is to 1 is a crucial issue.

3.3 See-saw models

The see-saw mechanism is usually associated with grand unification. In $SO(10)$ grand unified models, and in most other unified schemes except $SU(5)$, the existence of one right-handed neutrino for each family is required to make up complete multiplets of the unified group. Moreover, the see-saw formula $M_\nu = -N^T M_R^{-1} N$ gives neutrino masses in the range required by experiment if the scale of M_R is near the grand unified scale. Thus both the existence of neutrino masses and their magnitude are elegantly accounted for by the related ideas of grand unification and the see-saw mechanism. In this section, we shall therefore assume that we are dealing with a grand unified model.

3.3.1 See-saw models where $\theta_{\rm atm}$ comes from M_ν.

In models based on $SO(10)$, there is generally a close relationship among the four Dirac mass matrices N, U, D, and L. Indeed, in the minimal $SO(10)$ model of Section 1.4.3 (which is too simple to be realistic) $N = U \propto D = L$. The smallness of the CKM angles and the small interfamily mass ratios of the quarks can be explained by assuming that the matrices U and D are "hierarchical" in form. There are two simple kinds of hierarchical mass matrix that are frequently encountered in models

$$\begin{pmatrix} (\epsilon')^2 & \epsilon\epsilon' & \epsilon' \\ \epsilon\epsilon' & \epsilon^2 & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} \epsilon' & \epsilon' & \epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix}, \tag{3.8}$$

where $\epsilon' \ll \epsilon \ll 1$. The entries in these matrices as written are to be understood as giving only the order in the small parameters of the entries. The first form in Eq. (3.8) has what may be called a "geometric hierarchy", since an off-diagonal element is of the same order as the geometric mean of the corresponding diagonal elements. The second form in Eq. (3.8) has what may be called a "cascade hierarchy", since the matrix is made up of successive tiers, a 1-by-1, a 2-by-2, and a 3-by-3, of ever smaller magnitude. Both forms in Eq. (3.8), if applied to the quark masses, give $V_{cb} \sim \epsilon$, $V_{us} \sim \epsilon'/\epsilon$, and $V_{ub} \sim \epsilon' \sim V_{us}V_{cb}$.

While there is as a rule a close relation among the four Dirac mass matrices in $SO(10)$, the Majorana mass matrix M_R of the right-handed neutrinos can be quite different in form. For example, in minimal $SO(10)$ the Dirac mass matrices all come from the same term, $\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H$, whereas the matrix M_R comes from different terms, either $\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{126}}_H$ or $\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H$ as was discussed in Section 1.4.3. A reasonable hypothesis is that the CKM angles are small because of the hierarchical nature of the Dirac matrices, while the largeness of $\theta_{\rm atm}$ and possibly of $\theta_{\rm sol}$ has to do with the very different form of M_R. Models based on this idea were classified in [136] as Type I(2).

A potential difficulty with this idea is that if the Dirac mass matrix of the neutrinos N has a hierarchical form it tends, through the see-saw formula, to make M_ν also have a hierarchical form, indeed a more strongly hierarchical form. For example, suppose $N = {\rm diag}(\epsilon', \epsilon, 1)$, and we parameterize M_R^{-1} as $(M_R^{-1})_{ij} = a_{ij}$. Then

$$M_\nu = \begin{pmatrix} (\epsilon')^2 a_{11} & \epsilon\epsilon' a_{12} & \epsilon' a_{13} \\ \epsilon\epsilon' a_{12} & \epsilon^2 a_{22} & \epsilon a_{23} \\ \epsilon' a_{13} & \epsilon a_{23} & a_{33} \end{pmatrix}. \tag{3.9}$$

If all the a_{ij} are of the same order, then $\tan 2\theta_{23}^\nu \cong 2\epsilon a_{23}/(a_{33} - \epsilon^2 a_{22}) \sim \epsilon$. In order for $\theta_{\rm atm}$ to come primarily from diagonalizing M_ν, one must have $\theta_{23}^\nu \sim 1$. Clearly, this is only possible with some special form of M_R. One possibility is that

$a_{23}/a_{33} \sim \epsilon^{-1}$. If this is true, then there is a hierarchy among the elements of M_R that is related to the hierarchy among the elements of N. That is, the atmospheric neutrino mixing angle is of order unity because of a conspiracy between the Majorana and Dirac neutrino mass matrices. This would appear to be somewhat unnatural in a theory in which M_R and N have different origins, as is typically the case in unified models. On the other hand, this "Dirac-Majorana conspiracy" might not be unnatural in a model in which the same flavor symmetry, and the same small parameter characterizing the breaking of that symmetry, controlled the structure of both matrices. A good example of such "correlated hierarchies" is the model of [174].

A very important question is whether $\theta_{\rm atm}$ can naturally be of order unity even if the Dirac matrices are hierarchical and the parameters in M_R have no direct relationship to those of N. The answer is yes. In [175] and [162] interesting examples were found that satisfy these criteria. The specific forms given in those papers happen to lead to the SMA solar solution, but with some modifications they can also yield a satisfactory LMA solution, as we will now see.

Example 1: The following structure is closely related to that in [175]:

$$N = \begin{pmatrix} d\epsilon' & e\epsilon' & f\epsilon' \\ g\epsilon' & a\epsilon & b\epsilon \\ h\epsilon' & c\epsilon & 1 \end{pmatrix} m_N, \quad M_R = \begin{pmatrix} 0 & 0 & A \\ 0 & 1 & 0 \\ A & 0 & 0 \end{pmatrix} m_R. \tag{3.10}$$

Here a, ..., h are of order one, $\epsilon' \ll \epsilon \ll 1$, and $(\epsilon'/\epsilon)\epsilon^{-1} \ll A \ll \epsilon^{-1}$. Keeping only the significant terms, the resulting light neutrino mass matrix $M_\nu = -N^T M_R^{-1} N$ is

$$M_\nu \cong - \begin{pmatrix} O(\epsilon'^2) & O(\epsilon\epsilon') & d\epsilon'/A \\ O(\epsilon\epsilon') & a^2\epsilon^2 & ab\epsilon^2 + e\epsilon'/A \\ d\epsilon'/A & ab\epsilon^2 + e\epsilon'/A & b^2\epsilon^2 + 2f\epsilon'/A \end{pmatrix} \frac{m_N^2}{m_R}. \tag{3.11}$$

A rotation in the 23 plane by angle $\theta \cong \tan^{-1}(a/b) \sim 1$ diagonalizes the 2-3 block and brings the matrix to the form

$$M'_\nu \cong - \begin{pmatrix} O(\epsilon'^2) & -\frac{ad}{\sqrt{a^2+b^2}}\epsilon'/A & \frac{bd}{\sqrt{a^2+b^2}}\epsilon'/A \\ -\frac{ad}{\sqrt{a^2+b^2}}\epsilon'/A & \frac{2a(af-be)}{a^2+b^2}\epsilon'/A & 0 \\ \frac{bd}{\sqrt{a^2+b^2}}\epsilon'/A & 0 & (a^2+b^2)\epsilon^2 \end{pmatrix} \frac{m_N^2}{m_R}. \tag{3.12}$$

If one assumes that $\epsilon'/A \sim \epsilon^2/10$, then it is apparent that $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} \sim 10^{-2}$ as required for the LMA solution. It is to be observed that the 12 and 21 elements of this matrix are of the same order as the 22 element. This is just what is needed to get the right value of $\theta_{\rm sol}$ for the LMA solution, i.e. a value that is of order unity, but *not* very close to maximal. For example, if the 12 element is *exactly* equal to the 22 element, then $\tan^2\theta_{\rm sol} \cong 0.39$, which is in excellent agreement with the LMA-I best-fit value given in [131]. An examination of this matrix reveals that in obtaining the LMA solution a crucial role is played by the "cascade hierarchy" form of N. In particular, it is important that d be of the same order as e and f, which would not be the case if N had a "geometric hierarchy" form. It should also be noted that the largeness of the atmospheric angle is also traceable to the cascade hierarchy form of N, and specifically to the fact that b is of the same order as a.

Example 2: The following structure is closely related to that given in [162]

$$N = \begin{pmatrix} d\epsilon' & e\epsilon' & f\epsilon' \\ g\epsilon' & a\epsilon & b\epsilon \\ h\epsilon' & c\epsilon & 1 \end{pmatrix} m_N, \quad M_R = \begin{pmatrix} B & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R. \tag{3.13}$$

As in the last example, a, ..., h are of order one, and $\epsilon' \ll \epsilon \ll 1$. If one also assumes that $\epsilon^2/A \gg \epsilon'^2/B \gg 1$, then the light neutrino mass matrix $M_\nu = -N^T M_R^{-1} N$ takes the form (keeping only the important terms):

$$M_\nu \cong \begin{pmatrix} O(\epsilon'^2/A) & ga\epsilon\epsilon'/A + de\epsilon'^2/B & gb\epsilon\epsilon'/A + df\epsilon'^2/B \\ ga\epsilon\epsilon'/A + de\epsilon'^2/B & a^2\epsilon^2/A + e^2\epsilon'^2/B & ab\epsilon^2/A + ef\epsilon'^2/B \\ gb\epsilon\epsilon'/A + df\epsilon'^2/B & ab\epsilon^2/A + ef\epsilon'^2/B & b^2\epsilon^2/A + f^2\epsilon'^2/B \end{pmatrix} \frac{m_N^2}{m_R}.$$

A rotation in the 23 plane by angle $\theta \cong \tan^{-1}(a/b) \sim 1$ diagonalizes the 2-3 block and brings the matrix to the form

$$M'_\nu \cong \begin{pmatrix} O(\epsilon'^2/A) & \frac{d(be-af)}{\sqrt{a^2+b^2}}\epsilon'^2/B & O(\epsilon\epsilon'/A) + O(\epsilon'^2/B) \\ \frac{d(be-af)}{\sqrt{a^2+b^2}}\epsilon'^2/B & \frac{(be-af)^2}{a^2+b^2}\epsilon'^2/B & 0 \\ O(\epsilon\epsilon'/A) + O(\epsilon'^2/B) & 0 & (a^2+b^2)\epsilon^2/A \end{pmatrix} \frac{m_N^2}{m_R}.$$

The same remarks apply as in the previous example. The largeness of both the atmospheric neutrino angle and the solar neutrino angle can be traced to the "cascade hierarchy" form of N. Because the 12, 21, and 22 element are of the same order, the solar angle is (as required for the LMA solution) of order one, but *not* very close to maximal. The right ratio of mass splittings for the LMA solution can be obtained if $\epsilon'^2/B \sim 10^{-1}\epsilon^2/A$.

These two examples show that there are reasonable forms or "textures" for the mass matrices in the context of the see-saw mechanism that can quite naturally yield the LMA solution. However, actual detailed models based on these textures have not been constructed. It is also not clear how simple it is for the seemingly required "cascade hierarchy" form to arise in the framework of grand unified models. Finally, it should be noted that while some forms for N and M_R can be identified which would naturally give the LMA solution, most of the viable forms give the SMA or VAC solutions, and indeed the great majority of see-saw models published in the literature give the latter solutions rather than the LMA solution.

It has been recently suggested by Hall, Murayama and Weiner [82] that even the structureless mass matrices in the neutrino sector can lead to θ_{atm} of order unity. In their approach, dubbed "anarchy", the entries of the mass matrices are taken to be random numbers of constant probability in a given range. The ratio r is then explained through the fact that any hierarchy factor that might show up in the Dirac mass matrix of the neutrinos tends to be quadratically enhanced via see-saw mechanism. The presence of the M_R mass matrix further helps in smoothing out the distribution. The major prediction of the anarchy-type models [82, 83, 163, 164] is that all three angles in the lepton sector are close to their maximal values (barring any accidental cancellations). This, then, represents a serious problem due to the experimental limits on the value of $|U_{e3}|$. One might say that the anarchy models give "trilarge" mixing.

3.3.2 See-saw models where $\theta_{\rm atm}$ comes largely from L.

As we saw in Subsection 3.2.2, there are advantages to models in which the large atmospheric angle comes primarily from the charged lepton mass matrix L. In particular it becomes easy to reconcile the largeness of $\theta_{\rm atm}$ with the smallness of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$, since they come from different matrices: the former from L and the latter from M_ν. As we also saw, however, having a large angle arise from the diagonalization of L raises the question of why a large CKM angle does not arise from the diagonalization of the quark mass matrices D and U. The answer in the "flavor democracy" models, was that the CKM angles are small by a cancellation caused by an approximate symmetry. The possibility of a different and very elegant answer arises in the context of grand unified models, especially if an $SU(5)$ symmetry plays a role in the form of the fermion mass matrices.

We have seen in Section 1.5 that within $SU(5)$ one can realize the idea that the matrix L is such as to give large left-handed mixings and small right-handed mixings, so that $\theta_{\rm atm}$ can be large while V_{cb} is small. In other words, L must be highly asymmetric or lopsided [77]. [It should be noted that in $SU(5)$, L is related to $D^{\rm T}$, but not in general to U or N. Thus while the lopsidedness of L entails the lopsidedness of D, there is no reason to expect N and U to be lopsided, and in the examples we give below they are not.]

Many models have been proposed based on this idea of lopsided mass matrices [73, 76, 176, 77, 78, 177, 178, 179, 180, 181, 182, 90, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192]. These are classified as Type II(2) in [136]. The great majority of these models give the SMA solution or the VAC solution to the solar neutrino problem. However, it is possible to obtain the LMA solution as well [190, 191, 192]. In a lopsided model in which the LMA, LOW, or VAC solution arises, the large atmospheric angle can come primarily from the matrix L while the large solar angle can come from the matrix M_ν. This actually has the virtue of simplicity, since the form of M_ν is less constrained than in models where it must give rise both to a large $\theta_{\rm atm}$ and large $\theta_{\rm sol}$.

An example of how the LMA solution might arise in a see-saw model with lopsided L is provided by the following matrices:

$$N = \begin{pmatrix} d\epsilon' & e\epsilon' & f\epsilon' \\ g\epsilon' & a\epsilon & b\epsilon \\ h\epsilon' & c\epsilon & 1 \end{pmatrix} m_N, \quad M_R = \begin{pmatrix} 0 & A & 0 \\ A & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R. \qquad (3.14)$$

As before, it is assumed that a, ..., h are of order one, and that $\epsilon' \ll \epsilon \ll 1$. Suppose the value of A is such that $\epsilon^2,\ \epsilon' \ll \epsilon\epsilon'/A \ll 1$. Then, keeping only the most important terms, the light neutrino mass matrix has the form:

$$M_\nu \cong - \begin{pmatrix} O(\epsilon'^2/A) & ad\epsilon\epsilon'/A & bd\epsilon\epsilon'/A \\ ad\epsilon\epsilon'/A & 2ae\epsilon\epsilon'/A & (af+be)\epsilon\epsilon'/A + c\epsilon \\ bd\epsilon\epsilon'/A & (af+be)\epsilon\epsilon'/A + c\epsilon & 1 \end{pmatrix} \frac{m_N^2}{m_R}.$$

Here the 23 and 32 elements are small compared to the 33 element, leading to a small contribution to $\theta_{\rm atm}$; but that is alright, since $\theta_{\rm atm}$ is supposed to arise primarily from diagonalizing L in this class of models. As in the previous examples, one sees that the 12, 21 and 22 elements are of the same order, giving a large, but not nearly maximal, value of $\theta_{\rm sol}$ as required by the LMA solution. To get the right ratio of neutrino mass-squared splittings one needs $\epsilon\epsilon'/A \sim 10^{-1}$.

3.4 An $SU(5)$ pattern

A particularly interesting kind of pattern can arise very simply in the context of $SU(5)$ with Abelian flavor symmetry. Consider an $SU(5)$ model with a $U(1)$

flavor symmetry under which the quark and lepton multiplets have the following charge assignments: $\mathbf{10}_1(2)$, $\mathbf{10}_2(1)$, $\mathbf{10}_3(0)$, $\overline{\mathbf{5}}_1(1)$, $\overline{\mathbf{5}}_2(0)$, $\overline{\mathbf{5}}_3(0)$, Let the breaking of the $U(1)$ flavor symmetry be done by a field S having $U(1)$ charge -1, and an expectation value $\langle S \rangle / M_F = \epsilon \ll 1$. Then the mass matrices of the quarks and charged leptons will have the following forms:

$$D \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} m_D, \quad U \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} m_U,$$

$$L \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix} m_D. \tag{3.15}$$

Note that L and D have the lopsided form. It has been pointed out in several papers in the literature that these forms give a very good account of the mass ratios and mixing angles of the quarks and leptons [76, 193, 194].

One can see that the quantities $m_\mu/m_\tau \simeq 1/17$, $m_s/m_b \simeq 1/50$, and $V_{cb} \simeq 1/25$ all are of order ϵ. Thus, ϵ is roughly of order $1/20$. A consistent value of ϵ is obtained from the fact that $m_c/m_t \simeq 1/400$, $m_u/m_c \simeq 1/200$, $m_e/m_\mu \simeq 1/200$, and $V_{ub} \simeq 1/300$ are all of order ϵ^2. From the Cabibbo mixing and the ratio m_d/m_s, one would get the somewhat larger value $\epsilon \sim 1/5$.

The light neutrino mass matrix M_ν arises from the see-saw mechanism; so to know this matrix exactly it would be necessary to know M_R. However, to know merely the order in ϵ of the elements of M_ν it is not necessary to know the $U(1)$ family charges of the right-handed neutrinos at all, since the effective mass term in which M_ν appears involves only the left-handed lepton doublets, which are in the

$\overline{\mathbf{5}}_i$. Knowing the $U(1)$ charges of the $\overline{\mathbf{5}}_i$ tells us that

$$M_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} m_D^2/m_R. \tag{3.16}$$

From the forms of L and M_ν it is obvious that the mixing $U_{\mu 3}$ of the second and third family neutrinos will get $O(1)$ contributions from both these matrices, thus explaining the largeness of the atmospheric neutrino mixing angle. Let us imagine now diagonalizing the 2-3 block of M_ν to get

$$M'_\nu \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & m_{2(0)} & 0 \\ \epsilon & 0 & 1 \end{pmatrix} m_D^2/m_R. \tag{3.17}$$

The entry $m_{2(0)}$ would naturally be expected to be of order 1. However, for the ratio $r \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ to come out to be of order 10^{-2}, as required by the LMA solution, $m_{2(0)}$ should rather be of order 1/10. If we accept this rather mild fine-tuning, and assume that $m_{2(0)} \sim 1/10$, something interesting can be observed, namely that the 12 and 21 elements of M'_ν are of the same order as the 22 element, since $\epsilon \sim 1/20$. Recall that this is just what is needed for $\tan^2\theta_{\rm sol}$ to come out to be near the best-fit LMA value of about 0.3 or 0.4.

This model, then, naturally explains both the value of $\theta_{\rm atm}$ and the LMA value of $\theta_{\rm sol}$, provided that $r \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ is set to the LMA value.

Let us now imagine diagonalizing M'_ν. The rotation needed to eliminate the 13 and 31 elements will give a contribution to U_{e3} that is of order ϵ, quite consistent with the present experimental limit of 0.15. This leaves the diagonalization of the 1-2 block. In doing this one may neglect the 11 element, since it is of order ϵ^2. One then finds the simple relations (a) $\tan 2\theta_{\rm sol} \sim 2\epsilon/m_{2(0)}$, and (b) $m_2 = m_{2(0)}/(1-\tan^2\theta_{\rm sol})$. [Here we have ignored the $O(\epsilon)$ contribution to $\theta_{\rm sol}$ coming from diagonalizing L, since we are interested in large values of $\theta_{\rm sol}$.] From these relations one can infer roughly what region this model gives in the standard $\log(\tan^2\theta_{\rm sol})$-$\log(\Delta m^2_{\rm sol})$ plot. One sees from (a) that $\tan^2\theta_{\rm sol} \sim \epsilon^2 (m_{2(0)})^{-2}$, and from (b) that $\Delta m^2_{\rm sol} \sim (m_{2(0)})^2$. In other words, in the standard plot the region corresponding to this model lies roughly along a line with slope -1 going through the LMA allowed region. We shall see shortly, both by a much more careful analytic calculation and by a Monte Carlo numerical calculation, that this conclusion is correct. The form of (b) tells us something else that is interesting. As the value of the solar angle approaches maximality, i.e. $\tan^2\theta_{\rm sol} \longrightarrow 1$, the denominator in (b) approaches zero. Therefore, to maintain a finite value of $\Delta m^2_{\rm sol}$ the value of $m_{2(0)}$ must be tuned to be extremely small. Thus, one expects the region of greatest probability in this model to fade away as $\tan^2\theta_{\rm sol}$ approaches 1. This is confirmed by the analytic and Monte Carlo calculations, as we shall see.

We shall now study the predictions of this model in a statistical way, much in the spirit of [83]. Similar statistical analyses have been done in several recent papers [194, 164, 195], and our results are consistent insofar as they can be compared with theirs. However, our analysis differs in some respects. We do not treat ϵ as a free parameter, and seek to find its optimal value for the various solar solutions. Rather, we fix ϵ to the value that best reproduces the mass ratio m_μ/m_τ and then derive the full region of the $\tan^2\theta_{\rm sol}$ -$\Delta m^2_{\rm sol}$ plane which results. We also show that by treating the random variables as having a Gaussian distribution the statistical predictions of the model can be obtained analytically. We also carry out a numerical simulation using both a Gaussian and a non-Gaussian distribution similar to those used in previous analyses and show that it agrees remarkably well with the analytic results obtained using a Gaussian distribution.

To carry out the statistical analysis we parameterize the neutrino and charged lepton mass matrices as follows:

$$M_\nu = \begin{pmatrix} f\epsilon^2 & d\epsilon/\sqrt{2} & e\epsilon/\sqrt{2} \\ d\epsilon/\sqrt{2} & b & c/\sqrt{2} \\ e\epsilon/\sqrt{2} & c/\sqrt{2} & a \end{pmatrix} m_D^2/m_R, \tag{3.18}$$

and

$$L = \begin{pmatrix} O(\epsilon^3) & O(\epsilon^2) & O(\epsilon^2) \\ O(\epsilon^2) & D\epsilon & C\epsilon \\ O(\epsilon) & B & A \end{pmatrix} m_D. \tag{3.19}$$

We will take the unknown order-one parameters a, b, c, d, e, f, A, B, C, and D, to be complex random variables whose real and imaginary parts have Gaussian distributions with standard deviation σ. For example, if $a = |a|\mathrm{e}^{\mathrm{i}\theta_a}$, then $P(a)\,\mathrm{d}a = (2\pi\sigma^2)^{-1}\exp(-|a|^2/2\sigma^2)|a|\,\mathrm{d}|a|\,\mathrm{d}\theta_a$. It should be noted that we have put factors of $1/\sqrt{2}$ in the off-diagonal elements of M_ν. This is the appropriate normalization to use for a symmetric matrix.

What we want to calculate is the probability distribution $P(r,t)\,\mathrm{d}r\,\mathrm{d}t$, where $r \equiv \Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$, as before, and $t \equiv \tan^2\theta_{\mathrm{sol}}$, given that the order-one unknown parameters in the mass matrices have Gaussian distributions as described. We will first describe and give the results of an analytic calculation of $P(r,t)$, and then present the results of a Monte Carlo numerical calculation of $P(r,t)$.

A very important point in what follows is that if one does unitary changes of basis

$$\begin{pmatrix} \nu_{L2} \\ \nu_{L3} \end{pmatrix} \longrightarrow V \begin{pmatrix} \nu_{L2} \\ \nu_{L3} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \ell^-_{L2} \\ \ell^-_{L3} \end{pmatrix} \longrightarrow V \begin{pmatrix} \ell^-_{L2} \\ \ell^-_{L3} \end{pmatrix},$$

the resulting parameters a', ..., e', A', ..., D', have exactly the same Gaussian distributions as the parameters in the original basis. (This would not be true without the factors of $1/\sqrt{2}$ in M_ν.) This is one fact that makes the analytic calculation tractable using Gaussian distributions. Moreover this basis independence is more consistent with the group-theoretical approach advocated in [83].

The first thing to do is diagonalize L. For our purposes, we need only diagonalize the 2-3 block to find m_μ/m_τ and the contribution of L to $\theta_{\rm atm}$. This involves multiplying the 2-3 block of L from the right (which in our convention is a transformation on the *left*-handed leptons) by a unitary matrix

$$U_\ell^{[23]} \cong \left(\begin{array}{cc} A & B^* \\ -B & A^* \end{array} \right) (|A|^2 + |B|^2)^{-1/2}.$$

This eliminates the large element B in L. At the same time, the 33 element becomes $\sqrt{|A|^2 + |B|^2}$. The new 22 and 23 elements, which can be written $D'\epsilon$ and $C'\epsilon$ respectively, have the same Gaussian distribution as do A, B, C, and D. Consequently, one has that

$$(m_\mu)_{rms}/(m_\tau)_{rms} = \epsilon |D'|_{rms}/(\sqrt{|A|^2 + |B|^2})_{rms} = \epsilon/\sqrt{2}.$$

Thus, the most reasonable value to choose for the small parameter, from the point of view of lepton physics, is $\epsilon/\sqrt{2} = m_\mu/m_\tau$.

The first constraint that we shall impose is that the atmospheric neutrino mixing comes out to be very close to maximal, as found experimentally. This angle gets contributions from the diagonalizations of both L and M_ν. It would seem, then, that we must, in computing $P(r,t)$, take into account the random variables in both mass matrices. However, a great simplification occurs because of the use of Gaussian distributions and the resulting basis independence of the probability distributions. A little thought shows that one can compute $P(r,t)$ in a basis where the contribution to the atmospheric neutrino mixing coming from L has some fixed value, and the result will not depend on that value. Thus the parameters in L are irrelevant to $P(r,t)$. It is simplest in practice to choose the basis where the entire atmospheric neutrino mixing comes from L. A further simplification is achieved by neglecting the parameters e and f. The parameter e comes into calculating U_{e3} (which is predicted to be of order ϵ, and so consistent with the experimental bound $|U_{e3}| \leq 0.15$), but has a negligible effect on $\theta_{\rm sol}$, $\theta_{\rm atm}$, and the neutrino masses. The parameter f is multiplied by ϵ^2, and so is negligible also. Finally, one can choose a basis where the parameters a, b, and d are real. That means that the only parameters that come into the calculation of $P(r,t)$ are $|a|$, $|b|$, $|c|$, $|d|$, and $\theta_c \equiv \arg c$. From now on, we shall drop the absolute value signs and denote $|a|$, for example, simply by a.

One begins, then, with a matrix

$$M_\nu = \begin{pmatrix} 0 & d\epsilon/\sqrt{2} & 0 \\ d\epsilon/\sqrt{2} & b & ce^{i\theta_c}/\sqrt{2} \\ 0 & ce^{i\theta_c}/\sqrt{2} & a \end{pmatrix} m_D^2/m_R, \tag{3.20}$$

and a probability distribution

$$P(a,b,c,\theta_c,d) = \frac{abcd}{2\pi\sigma^8}\,\mathrm{e}^{-\frac{1}{2\sigma^2}(a^2+b^2+c^2+d^2)}. \tag{3.21}$$

The first step is to diagonalize the 1-2 block of the matrix given in Eq. (3.20), which gives $\tan 2\theta_{\rm sol} \equiv s = \sqrt{2}d\epsilon/b$. This allows the elimination of the random variable d in favor of the measurable parameter s or equivalently t. The 11 and 22 elements of the matrix then become $\frac{1}{2}(1-\sqrt{1+s^2})b$ and $\frac{1}{2}(1+\sqrt{1+s^2})b$ respectively. The latter quantity we shall denote as b'. The next step is to impose the atmospheric angle constraint. Since we are working in a basis where the contribution to this angle from M_ν is vanishingly small, the imposition of this constraint sets the parameter c to zero. More precisely, if one requires that the (complex) contribution to the atmospheric angle from M_ν have magnitude bounded by some arbitrary small cutoff $\Delta_{\rm atm} \ll 1$, the condition on c becomes $(c/\sqrt{2})/(a-b') \leq \Delta_{\rm atm}$. This means that the integration over $\mathrm{d}c\,\mathrm{d}\theta_c$ in the probability distribution can be done, yielding $\int c\,\mathrm{d}c\,\mathrm{d}\theta_c = \pi(\sqrt{2}(a-b')\Delta_{\rm atm})^2$. The only remaining variables are then a, b, and s. The random variable a can be eliminated in favor of the measurable ratio r of mass-squared splittings using the relation $r = (b^2\sqrt{1+s^2})/(a^2-b'^2)$. It is a very good approximation here to replace b' by b, since for the whole region of interest either s or r is very small as we shall see. After all these steps, one is left with a probability distribution $P(r, b, s)$. The final step is simply to integrate over the random variable b. Since this integral is a Gaussian it is easily done. The final result is

$$P(r,s)\,\mathrm{d}r\,\mathrm{d}s = N\,\frac{rs\,\mathrm{d}r\,\mathrm{d}s}{1+s^2}\,\frac{\left[\sqrt{1+\frac{r}{\sqrt{1+s^2}}}-\sqrt{\frac{r}{\sqrt{1+s^2}}}\right]^2}{\left[1+\frac{2r}{\sqrt{1+s^2}}+\frac{rs^2}{2\epsilon^2\sqrt{1+s^2}}\right]^4}, \qquad (3.22)$$

where N is just a normalization constant. Changing variable from $s \equiv \tan 2\theta_{\rm sol}$ to $t \equiv \tan^2\theta_{\rm sol}$ one finds that

$$P(r,t)\,\mathrm{d}r\,\mathrm{d}t = N\,\frac{2r\,\mathrm{d}r\,\mathrm{d}t}{1-t^2}\,\frac{\left[\sqrt{1+r\left(\frac{1-t}{1+t}\right)}-\sqrt{r\left(\frac{1-t}{1+t}\right)}\right]^2}{\left[1+2r\left(\frac{1-t}{1+t}\right)+\frac{2rt}{\epsilon^2(1-t^2)}\right]^4}. \qquad (3.23)$$

One can see the qualitative behavior of this function rather easily. The crucial term is the one containing ϵ^2 in the denominator. This term forces the product rt to be of order ϵ^2. This is consistent with what we argued above, namely that the region of greatest probability in this model has $rt \sim$ constant, i.e. a line of slope -1 in the $\log(\tan^2\theta_{\rm sol})$-$\log(\Delta m^2_{\rm sol})$ plane. Moreover, we see that as $t \longrightarrow 1$, the product rt is forced to be less than or of order $\epsilon^2(1-t^2) \longrightarrow 0$, so that the probability is suppressed for $t \cong 1$.

In Fig. 3.1, we give a contour plot of the probability function $P(r,t)$ just computed analytically, and compare it to the results of a Monte Carlo calculation. For the Monte Carlo calculation we used the forms in Eqs. (3.18) and (3.19), with $\epsilon/\sqrt{2} = m_\mu/m_\tau$, and assumed that the magnitudes of the complex random variables had a Gaussian distribution. The phases of the complex variables were also treated as random numbers and were varied from 0 to 2π. We then diagonalized randomly generated matrices to obtain the corresponding PMNS mixing matrices $U_{\rm PMNS}$ and neutrino masses and analyzed the results by imposing the conditions $\sin^2 2\theta_{\rm atm} \geq 0.9$

and $|U_{e3}| \leq 0.15$. These conditions reduced our initial set of 50,000 data points to 15,718 that were compatible with both the CHOOZ reactor data [196] and the atmospheric neutrino experiments. Only the points that passed the cuts are given in Fig. 3.1.

In Fig. 3.2, we generate the points assuming that the magnitudes of the complex variables, instead of having the Gaussian distribution, have constant probability in the interval 0.5 to 2.0 and zero probability outside that interval. The phases are again treated as the random numbers in the interval from 0 to 2π. This time 20,860 points, out of the initial set of 50,000 points, have passed all the cuts.

We expect Gaussian distribution to be in better agreement with the analytic solution than the constant probability distribution. But, one can see from the excellent agreement between the analytic and Monte Carlo results presented in Fig. 3.2 that the exact form used for the probability distributions of the random variables themselves makes little difference. This has also been found in other papers [83, 194, 164, 195]. One point that should be noted is that since Figs. 3.1 and 3.2 are the log-log plots, the correct thing to plot and what has been plotted, is $P(\log r, \log t) \sim P(r,t)rt$.

In Figs. 3.3(a) and 3.3(b), we have fixed the value of r at $r = 2.8 \times 10^{-2}$, which comes from using the best-fit values from experiment, and plotted the resulting $P(\log t)$ against a normalized, binned distribution that was extracted from the data set produced with (a) the Gaussian and (b) the constant probability distribution respectively. We have obtained the binned distributions by counting the number of data points in the strip $\log r = \log(2.8 \times 10^{-2}) \pm 0.1$, the width of one bin being 0.2. This strip is shown in Figs. 3.1 and 3.2. The normalization has been carried out with respect to the maximum of $P(\log t)$. Note the excellent agreement between the analytic and Monte Carlo solution in Fig. 3.3(a). On the other hand, the binned distribution in Fig. 3.3(b) lies slightly below the analytic curve. We expect this behavior since the data set created by the constant probability distribution limits the scope of the magnitudes of the matrix elements in Eqs. (3.18) and (3.19). In both cases the most probable value for $\tan^2\theta_{\rm sol}$ is about 0.1, with a very substantial part of the area under the curve being in the region [0.3, 0.8] preferred by both the LMA-I and LMA-II solution global fits. This region is shaded in Fig. 3.3 for convenience.

The one weakness of this model is that it does not explain the value of $r \equiv \Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ as required by the LMA-I solution fits. From Figs. 3.1 and 3.2 one sees that a value of 10^{-1} for this ratio is near the peak of the probability distribution $P(r,t)$. However, the same figure shows that a value of 2.8×10^{-2} is near the edge of the preferred region, and so requires a mild fine-tuning. On the other hand, this model accommodates the value of r as preferred by the LMA-II solution fits. [If the preliminary reanalysis of the existing data by the Super-Kamiokande Collaboration [134] proves to be correct the value of r will change from 2.8×10^{-2} (5.8×10^{-1}) to 5.8×10^{-2} (7.5×10^{-1}) for the LMA-I (LMA-II) solution. This would put the predicted value of r of the model in the middle of the preferred region.] However, once r is constrained to be the right value, the value of $\tan^2\theta_{\rm sol}$ needed for both the LMA-I and LMA-II solution emerges quite naturally, as can be seen from Figs. 3.3(a) and 3.3(b). The atmospheric mixing angle is, of course, also naturally explained.

3.5 Conclusions

One can see from the foregoing that it is considerably easier to build satisfactory models of the VAC, LOW, or SMA type than of the LMA type. That is reflected in the models that have actually been constructed in the literature. One problem is that in many models which predict large solar mixing angle, notably inverted hierarchy schemes and flavor democracy schemes, this angle tends to come out very close to maximal. They do not naturally explain why $\tan^2\theta_{\rm sol}$ should come out in the range 0.3 to 0.8 preferred by the data. Other non-see-saw schemes, such as SUSY with R-parity breaking and single-right-handed-neutrino-dominance (SRHND) models, tend to predict a value of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ significantly less than that preferred by the LMA solution. While the LMA value of this ratio can be fitted, it is not really explained.

The situation seems more promising for the see-saw approaches, although here also the great majority of published models give the small angle or vacuum solar solutions. We showed that certain fairly simple textures exist that would naturally reproduce the neutrino masses and mixings required by the LMA solution. Whether these textures can be implemented in simple models remains to be seen.

One of the few existing schemes that shows a natural preference for the LMA solution is the lopsided $SU(5)$ model studied in Section 3.4. The value of $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ requires a mild fine-tuning, but given that, both the atmospheric angle and the LMA value of the solar angle emerge quite naturally. We studied the predictions of this model in a statistical way, and found that by using Gaussian distributions the analysis could be carried out very simply and accurately by purely analytic means. We believe that the same methods should be applicable to many other models. The advantage of such statistical analyses is that they allow one to estimate in a somewhat objective and quantitative way how "fine tuned" models must be to reproduce the data.

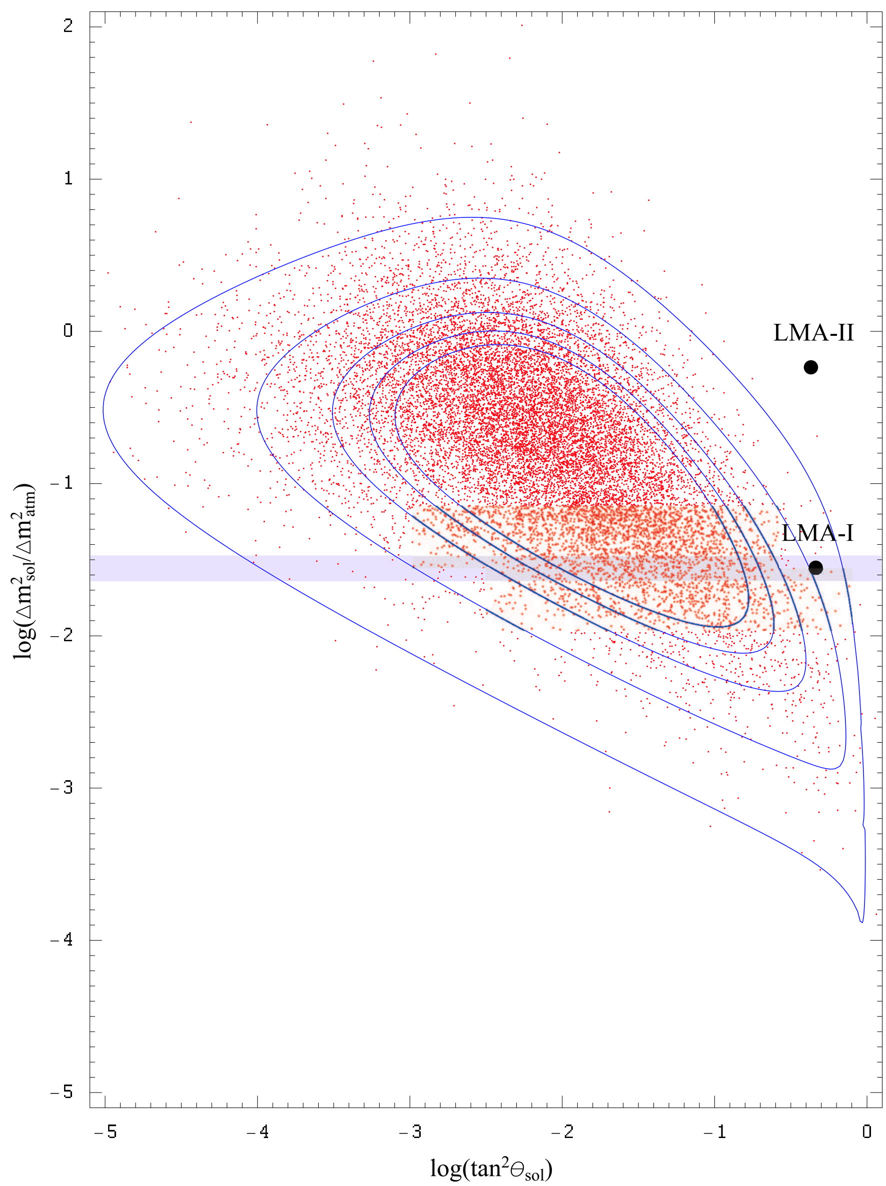


Figure 3.1: Contour plot of the normalized probability distribution P in $\log(\tan^2\theta_{\rm sol})$-$\log(\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})$ plane with the contour values 0.002, 0.02, 0.06, 0.1, and 0.14 superimposed on the numerically generated distribution of points. The points are generated using the Gaussian distribution for the magnitudes of the mass matrix entries. Large dots represent the best-fit values for LMA-I and LMA-II solutions as indicated.

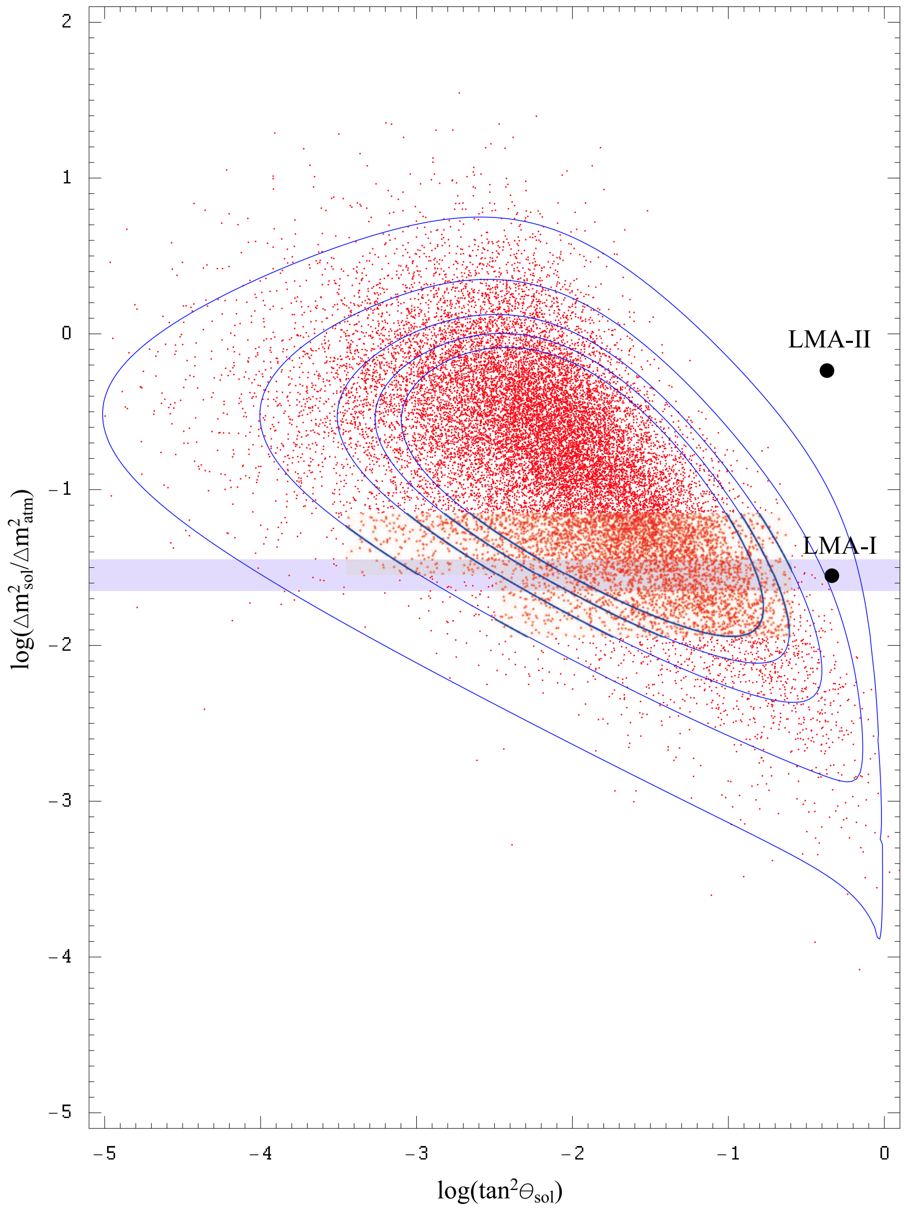


Figure 3.2: Contour plot of the normalized probability distribution P in $\log(\tan^2\theta_{\rm sol})$-$\log(\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm})$ plane with the contour values 0.002, 0.02, 0.06, 0.1, and 0.14 superimposed on the numerically generated distribution of points. The points are generated using the constant probability distribution. Large dots represent the best-fit values for LMA-I and LMA-II solutions as indicated.

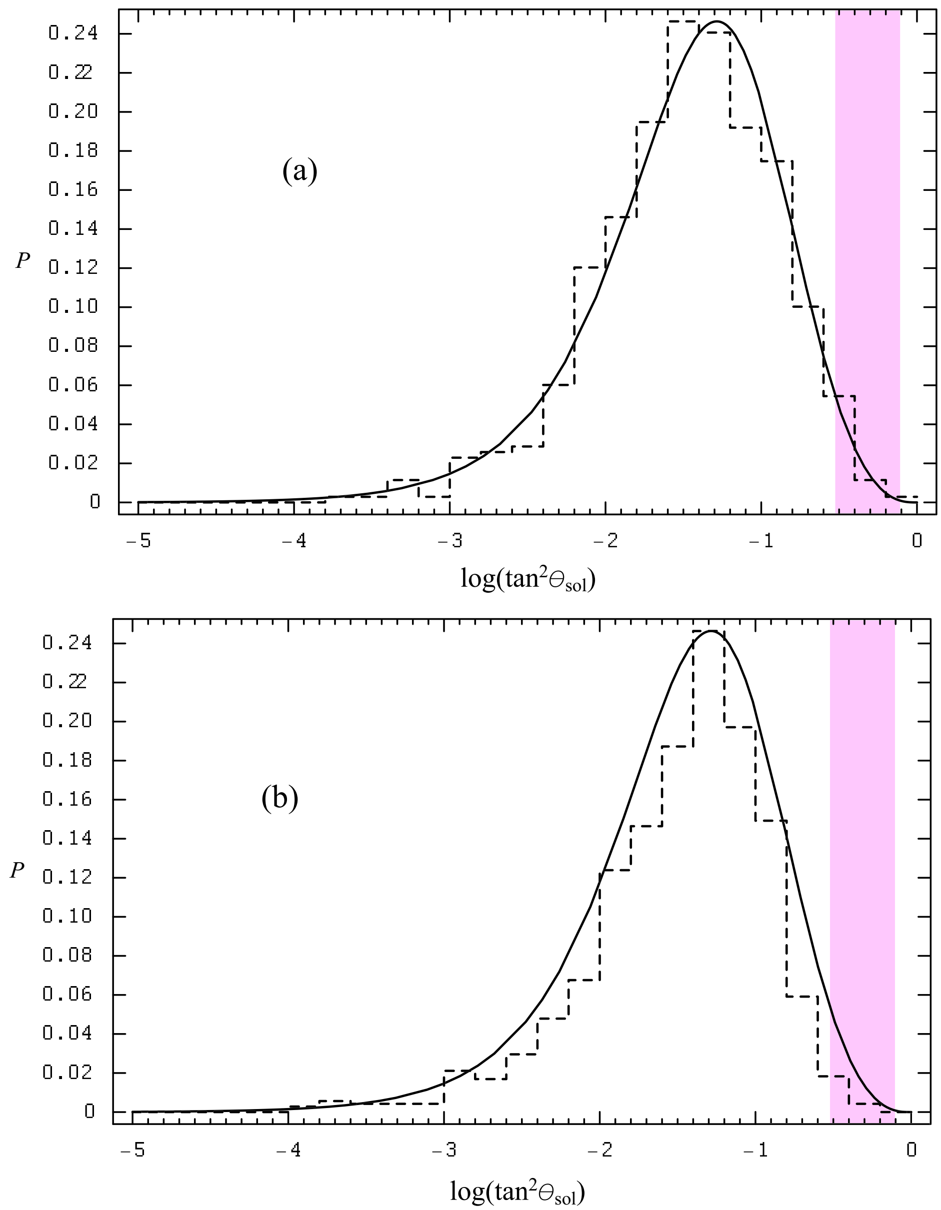


Figure 3.3: Normalized probability distribution P (solid line) as a function of $\log(\tan^2\theta_{\rm sol})$ for the best-fit LMA-I solution value $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm} = 2.8\times10^{-2}$ plotted against the normalized, binned distribution (dashed line) extracted from (a) the Gaussian and (b) the constant probability data sets.

Chapter 4

UNIFYING FLIPPED $SU(5)$ IN FIVE DIMENSIONS

4.1 Introduction

A beautiful feature of flipped $SU(5)$ [62, 84, 85] is that it provides a natural setting for the missing partner mechanism. This mechanism, when implemented in flipped $SU(5)$, not only solves the doublet-triplet splitting problem but also allows one to avoid entirely the Higgsino-mediated proton decay that is such a difficulty for SUSY GUTs. On the other hand, flipped $SU(5)$ gives up one of the most attractive features of grand unification, namely unification of gauge couplings, because it is based on the group $SU(5) \otimes U(1)$. Another drawback of flipped $SU(5)$ models is that the masses of down quarks and charged leptons come from different operators, so that one does not obtain the relation $m_b(M_{\rm GUT}) = m_\tau(M_{\rm GUT})$. The unification of gauge couplings and relations between down quark masses and charged lepton masses could be recovered by embedding the group $SU(5) \otimes U(1)$ in the simple group $SO(10)$. However, in that case, the missing partner mechanism no longer works, since the partner that was missing from the $SU(5)$ multiplets is present in the larger $SO(10)$ multiplets.

One is thus in somewhat of a quandary. The point of this chapter is that a way out of this quandary is provided by unification in five dimensions. We show that if the group $SO(10)$ in five dimensions is broken by orbifold compactification to the group $SU(5) \otimes U(1)$ in four dimensions it is possible to have at the same time the good features of both flipped $SU(5)$ and of $SO(10)$. The essential reason is that if $SO(10)$ is broken by the orbifold compactification then the fields of the effective four-dimensional theory need not be in complete $SO(10)$ multiplets. This means that at the four-dimensional level the famous missing partners can still be missing and the doublet-triplet splitting can be achieved without the dangerous Higgsino-mediated proton decay. On the other hand, because there is $SO(10)$ at the five-dimensional level, there is approximate unification of gauge couplings, and there is also the possibility of getting $SO(10)$-like Yukawa couplings for the quarks and leptons.

By now there are many models that use orbifold compactification of extra dimensions to break grand unified symmetries. The first such models [86, 197, 198, 199, 200, 87, 201] showed that with one extra dimension it is possible to construct $SU(5)$ models which have natural doublet-triplet splitting and no problem with the $d = 5$ proton decay operators that plague four-dimensional SUSY GUTs. The breaking of grand unified symmetries by orbifold compactification of a single extra dimension does not reduce the rank of the group [202]. Thus to break $SO(10)$ all the way to the Standard Model by orbifold compactification requires at least two extra dimensions. Interesting six-dimensional $SO(10)$ models have been constructed in several papers [203, 204, 205]. However, it is also possible that the breaking from the grand unified group to the Standard Model is achieved by a combination of orbifold compactification and the conventional four-dimensional Higgs mechanism. That allows the construction of realistic $SO(10)$ models with only a single extra dimension, as was first shown by Dermíšek and Mafi [206]. In their model the theory in the five-dimensional bulk has $\mathcal{N} = 1$ supersymmetry and gauge group $SO(10)$. Orbifolding breaks $SO(10)$ to the Pati-Salam [51] symmetry $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. The orbifold has two inequivalent fixed points O and O'. On O there is a full $SO(10)$ symmetry, but on O' only the Pati-Salam group. On the brane at O the conventional Higgs mechanism breaks $SO(10)$ down to $SU(5)$. Thus the unbroken symmetry in the low-energy theory in four dimensions is the intersection of $SU(5)$ and the Pati-Salam group, i.e. the Standard Model group. Another, more recent, example of realistic $SO(10)$ model in five dimensions is that of Albright and Barr [207]. Their model also harbors the Pati-Salam group on the hidden brane. For more examples of $SO(10)$ based models in five dimensions see [208, 209].

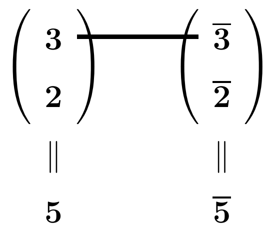
The model we shall present is similar in some ways to that of Dermíšek and Mafi but differs from it in several important respects. Whereas they use orbifold compactification to break to the Pati-Salam group and Higgs fields on the brane O to break to $SU(5)$, we shall use orbifold compactification to break to $SU(5) \otimes U(1)$ and Higgs fields in the bulk to break the rest of the way to the Standard Model. They use orbifold breaking to split the doublets from the triplets, whereas we use the four-dimensional flipped-$SU(5)$ missing partner mechanism.

4.2 Missing partners in four dimensions

Before we consider higher dimensional theories we shall briefly review the missing partner mechanism in four-dimensional theories, showing why it works in flipped $SU(5)$ but not in $SO(10)$.

4.2.1 Flipped SU(5)

First recall what happens in ordinary (i.e. Georgi-Glashow) $SU(5)$ [57]. In ordinary $SU(5)$ the two Higgs doublets of the MSSM, which we shall denote $\mathbf{2}$ and $\overline{\mathbf{2}}$, have color-triplet partners, which we shall denote $\mathbf{3}$ and $\overline{\mathbf{3}}$. [We use this short-hand notation for Standard Model representations: $\mathbf{2} \equiv (\mathbf{1}, \mathbf{2}, 1)$, $\overline{\mathbf{2}} \equiv (\mathbf{1}, \mathbf{2}, -1)$, $\mathbf{3} \equiv (\mathbf{3}, 1, -2/3)$, $\overline{\mathbf{3}} \equiv (\overline{\mathbf{3}}, \mathbf{1}, 2/3)$.] These are contained in fundamental and anti-fundamental multiplets of $SU(5)$: $\mathbf{5} = \mathbf{2} + \mathbf{3}$ and $\overline{\mathbf{5}} = \overline{\mathbf{2}} + \overline{\mathbf{3}}$ (see Eq. (1.39)). A combination of an $SU(5)$-singlet mass term and a Yukawa coupling to a Higgs in the adjoint representation, can (with suitable fine-tuning) give GUT-scale mass to the triplet partners while leaving the MSSM Higgs doublets light. This can be represented schematically as



where the solid horizontal line represents a large Dirac mass M_3 connecting the colored Higgsinos $\mathbf{3}$ and $\overline{\mathbf{3}}$. It is well-known that the exchange of these colored Higgsinos gives a dangerous $d = 5$ proton-decay operator, as shown in Fig. 4.1. From the figure one sees that this proton decay amplitude is proportional to the

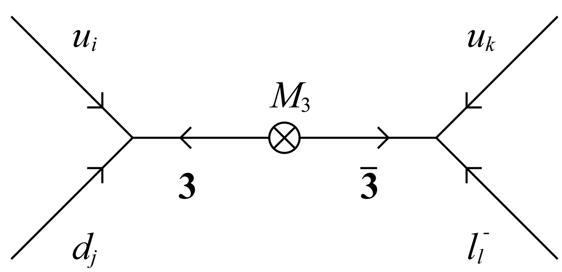
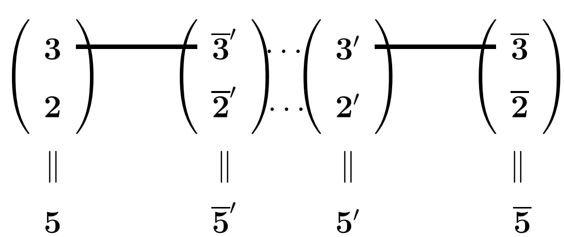


Figure 4.1: The kind of graph that gives rise to $d = 5$ proton decay operators.

mass connecting $\mathbf{3}$ to $\overline{\mathbf{3}}$. Suppressing this proton decay therefore requires severing this connection. This can be done by introducing an extra pair of Higgs multiplets $\mathbf{5}' + \overline{\mathbf{5}}'$, so that the triplets in the unprimed multiplets get mass not with each other but with the triplets in the primed multiplets as shown in the following diagram



If the MSSM Higgs doublets are the unprimed ones, then one sees that their colored partners are not connected to each other by a mass term, so that the $d = 5$ proton-decay amplitude vanishes. Unfortunately, however, there is an extra pair of doublets that remains light, namely the primed ones. The effect of these on the renormalization group equations would destroy gauge coupling unification. To give the needed superheavy mass to these doublets one could introduce a term $M\overline{\mathbf{5}}'\mathbf{5}'$; however, this would give mass terms connecting not only $\mathbf{2}'$ to $\mathbf{2}$ but also $\mathbf{3}'$ to $\overline{\mathbf{3}}'$ (indicated by dotted lines in the previous diagram) and thus indirectly (after the primed triplets were integrated out) reconnecting $\mathbf{3}$ to $\overline{\mathbf{3}}$ and bringing back the dangerous $d = 5$ proton decay amplitude.

Now let us turn to flipped $SU(5)$ and see how it avoids these problems very elegantly [85]. In flipped $SU(5)$ models one has Higgs fields in the following representations of $SU(5) \otimes U(1)$: $h = \mathbf{5}^{-2}$, $\overline{h} = \overline{\mathbf{5}}^{2}$, $H = \mathbf{10}^{1}$, and $\overline{H} = \overline{\mathbf{10}}^{-1}$. Under the Standard Model group these decompose as follows, $h = \overline{\mathbf{2}} + \mathbf{3}$, $\overline{h} = \mathbf{2} + \overline{\mathbf{3}}$, $H = \overline{\mathbf{3}} + (\mathbf{3}, \mathbf{2}, 1/3) + (\mathbf{1}, \mathbf{1}, 0)$, and $\overline{H} = \mathbf{3} + (\overline{\mathbf{3}}, \mathbf{2}, -1/3) + (\mathbf{1}, \mathbf{1}, 0)$. The Higgs superpotential contains the terms $h\,H\,H + \overline{h}\,\overline{H}\,\overline{H}$. When the Standard Model singlets $(\mathbf{1}, \mathbf{1}, 0)$ in the H and $\overline{H}$ acquire VEVs they break $SU(5) \otimes U(1)$ down to the Standard Model group and they also give mass to the triplet Higgs. Schematically,

$$
\begin{array}{cc|cc}
\begin{pmatrix} \mathbf{3} \\ \overline{\mathbf{2}} \end{pmatrix} \!\!-\!\!-\!\!- & \begin{pmatrix} \overline{\mathbf{3}} \\ \mathbf{other} \end{pmatrix} & \begin{pmatrix} \mathbf{3} \\ \overline{\mathbf{other}} \end{pmatrix} \!\!-\!\!-\!\!- & \begin{pmatrix} \overline{\mathbf{3}} \\ \mathbf{2} \end{pmatrix} \\
\| & \| & \| & \| \\
h & H & \overline{H} & \overline{h}
\end{array}
\tag{4.1}
$$

where, for simplicity, $(\mathbf{3}, \mathbf{2}, 1/3) + (\mathbf{1}, \mathbf{1}, 0) \equiv \mathbf{other}$. The triplets in h and $\overline{h}$ get mass with those in H and $\overline{H}$. However the doublets in h and $\overline{h}$ remain massless because there are no doublets in H and $\overline{H}$ for them to mate with—thus the name "missing partner mechanism".

At first glance one might worry that the same problem arises here as in the ordinary $SU(5)$ case discussed previously. The multiplets $\mathbf{5}'$ and $\overline{\mathbf{5}}'$ there played the same role as the multiplets H and $\overline{H}$ here. And we saw that one could not give mass to the doublets in $\mathbf{5}'$ and $\overline{\mathbf{5}}'$ without reintroducing the dangerous proton decay amplitude. This leads to the question whether there is not an analogous difficulty in giving mass to some of the components of H and $\overline{H}$, and specifically to the $(\mathbf{3}, \mathbf{2}, 1/3) + (\mathbf{1}, \mathbf{1}, 0) + (\overline{\mathbf{3}}, \mathbf{2}, -1/3) + (\mathbf{1}, \mathbf{1}, 0)$, since here also an explicit mass term $M\overline{H}H$ would reintroduce the problem of proton decay. The beautiful answer is that these "**other**" components of H and $\overline{H}$ do not have to get mass. Indeed, they *must not* get mass, because they are the Goldstone modes that get eaten when $SU(5) \otimes U(1)$ breaks to $SU(3) \otimes SU(2) \otimes U(1)$. In other words, the fact that $SU(5) \otimes U(1)$ breaks to the Standard Model group guarantees that there is no mass connecting H and $\overline{H}$ and therefore guarantees the absence of the $d = 5$ proton decay amplitude.

4.2.2 SO(10)

Now let us see why embedding flipped $SU(5)$ in $SO(10)$ in four dimensions destroys the beautiful missing partner solution to the doublet-triplet splitting and proton decay problems that we have just reviewed.

In $SO(10)$ the simplest possibility is that the terms $h\,H\,H + \overline{h}\,\overline{H}\,\overline{H}$ come from the terms $\mathbf{10}\,\mathbf{16}\,\mathbf{16} + \mathbf{10}\,\overline{\mathbf{16}}\,\overline{\mathbf{16}}$, where $\mathbf{10} = \overline{h} + h$, $\mathbf{16} = H + \overline{h}' + \mathbf{1}^5$, and $\overline{\mathbf{16}} = \overline{H} + h' + \mathbf{1}^{-5}$. Here $h' = \mathbf{5}^3$ and $\overline{h}' = \overline{\mathbf{5}}^{-3}$. The problem is that the doublet partners that were missing from H and $\overline{H}$ are now present in $\overline{h}'$ and h'.

The terms $\mathbf{10}\,\mathbf{16}\,\mathbf{16} + \mathbf{10}\,\overline{\mathbf{16}}\,\overline{\mathbf{16}}$ contain not only $h\,H\,\langle H\rangle + \overline{h}\,\overline{H}\,\langle\overline{H}\rangle$ but also $\overline{h}\,\overline{h}'\,\langle H\rangle + h\,h'\,\langle\overline{H}\rangle$. These latter terms mate the doublet Higgs in h and $\overline{h}$ with those in $\overline{h}'$ and h', destroying the solution of the doublet-triplet splitting problem.

A possible remedy to this difficulty suggests itself. One can have h and $\overline{h}$ come from different $\mathbf{10}$s of $SO(10)$. Let us examine what happens in this case, since it will be directly relevant to what we shall do in five dimensions later. Suppose there are two vector Higgs representations, denoted $\mathbf{10}_1$ and $\mathbf{10}_2$, with couplings $\mathbf{10}_1\,\mathbf{16}\,\mathbf{16} + \mathbf{10}_2\,\overline{\mathbf{16}}\,\overline{\mathbf{16}}$. We write $\mathbf{10}_1 = h_1 + \overline{h}_1$ and $\mathbf{10}_2 = h_2 + \overline{h}_2$. Suppose that the two light doublets of the MSSM lie in h_1 and $\overline{h}_2$; then the triplet partners of these light doublets will obtain mass from the terms $h_1 H\langle H\rangle + \overline{h}_2\overline{H}\langle\overline{H}\rangle$. The terms that give superlarge mass to doublets, and which correspond to those we found troubling before, are $\overline{h}_1\overline{h}'\langle H\rangle + h_2 h'\langle\overline{H}\rangle$. These do *not* give superlarge mass to the MSSM doublets, but to the doublets in $\overline{h}_1$ and h_2. Thus, we would appear to have satisfactory doublet-triplet splitting with no dangerous $d = 5$ proton decay, just as in flipped $SU(5)$.

However, this is not so, for the question arises how the triplets in $\overline{h}_1$ and h_2 are to acquire superheavy mass. It would seem that the only way is through a mass term connecting them. But that would have to come from a term $M\overline{h}_1 h_2$, which in turn comes from $M\mathbf{10}_1\mathbf{10}_2$, and this would also give $Mh_1\overline{h}_2$ and thus superlarge mass to the MSSM doublets.

We see, then, that the missing partner mechanism does not work in four-dimensional $SO(10)$ theories. However, we shall see that it can work in five-dimensional $SO(10)$ theories. The crucial difference will be that orbifold breaking of $SO(10)$ can split the $SO(10)$ Higgs representations. In particular, in the example we just looked at the troublesome triplets in $\overline{h}_1$ and h_2 can be given Kaluza-Klein masses by the orbifold compactification while leaving the MSSM doublets in h_1 and $\overline{h}_2$ light.

4.3 An $SO(10)$ model in five dimensions

We now present an $SO(10)$ supersymmetric model in five dimensions compactified on an $S^1/(Z_2 \times Z_2')$ orbifold that yields a realistic supersymmetric flipped

$SU(5)$ model in four dimensions. The breaking of $SU(5) \otimes U(1)$ down to the Standard Model gauge group, the doublet-triplet splitting, and the solution to the problem of $d = 5$ proton-decay operators will all be as in conventional four-dimensional flipped $SU(5)$ models. Moreover, there will be distinctive flipped $SU(5)$ relationships among gaugino masses. However, the gauge couplings will be unified (with some threshold corrections, that can be argued to be small [87]) because of the underlying five-dimensional $SO(10)$ symmetry. And the Yukawa couplings of the quarks and leptons can have relationships that are similar to what is found in ordinary $SU(5)$ and $SO(10)$ models rather than in flipped $SU(5)$.

As already elaborated in Refs. [86, 197, 198, 199, 87, 200, 201], the fifth dimension, being the circle with coordinate y and circumference $2\pi R$, is compactified through the reflection $y \to -y$ under Z_2 and $y' \to -y'$ under Z_2' where $y' = y + \pi R/2$. This identification procedure leaves two fixed points O and O' of Z_2 and Z_2' respectively and reduces the physical region to the interval $y \in [-\pi R/2, 0]$. Point O at $y = 0$ is the "visible brane" while point O' at $y' = 0$ is the "hidden brane". This orbifolding procedure is shown schematically in Fig. 4.2. The compactification scale $1/R \equiv M_C$ is assumed to be close to the scale at which the gauge couplings unify, i.e. the GUT scale $M_{\rm GUT} \sim 10^{16}\,\mathrm{GeV}$.

The generic bulk field $\phi(x^\mu, y)$, where $\mu = 0, 1, 2, 3$, has definite parity assignment under $Z_2 \times Z_2'$ symmetry. Taking $P = \pm 1$ to be parity eigenvalue of the field $\phi(x^\mu, y)$ under Z_2 transformation and $P' = \pm 1$ to be parity eigenvalue under the Z_2' transformation, a field with $(P, P') = (\pm, \pm)$ can be denoted $\phi^{PP'}(x^\mu, y) = \phi^{\pm\pm}(x^\mu, y)$. The Fourier series expansion of the fields $\phi^{\pm\pm}(x^\mu, y)$

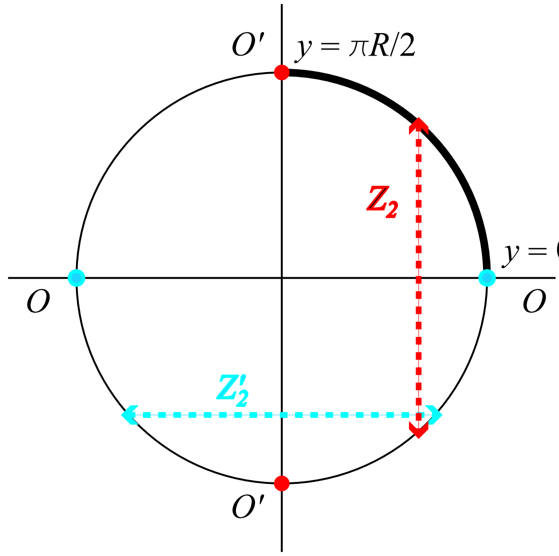


Figure 4.2: Compactification of S^1 space under $Z_2 \times Z_2'$ symmetry transformations. The reflections under Z_2 and Z_2' identify the fixed points O' and O respectively.

yields

$$\phi^{++}(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n0}}\pi R}} \sum_{n=0}^{\infty} \phi^{++(2n)}(x^\mu) \cos\frac{2ny}{R}, \tag{4.2a}$$

$$\phi^{+-}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi^{+-(2n+1)}(x^\mu) \cos\frac{(2n+1)y}{R}, \tag{4.2b}$$

$$\phi^{-+}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi^{-+(2n+1)}(x^\mu) \sin\frac{(2n+1)y}{R}, \tag{4.2c}$$

$$\phi^{--}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi^{--(2n+2)}(x^\mu) \sin\frac{(2n+2)y}{R}. \tag{4.2d}$$

The profile of the wave function in the fifth dimension is shown in Fig. 4.3.

In the effective theory in four dimensions all the fields in Eqs. (4.2) have masses of order M_C except the Kaluza-Klein zero mode $\phi^{++(0)}$ of $\phi^{++}(x^\mu, y)$, which remains massless. Moreover, fields $\phi^{-\pm}(x^\mu, y)$ vanish on the visible brane and fields $\phi^{\pm-}(x^\mu, y)$ vanish on the hidden brane.

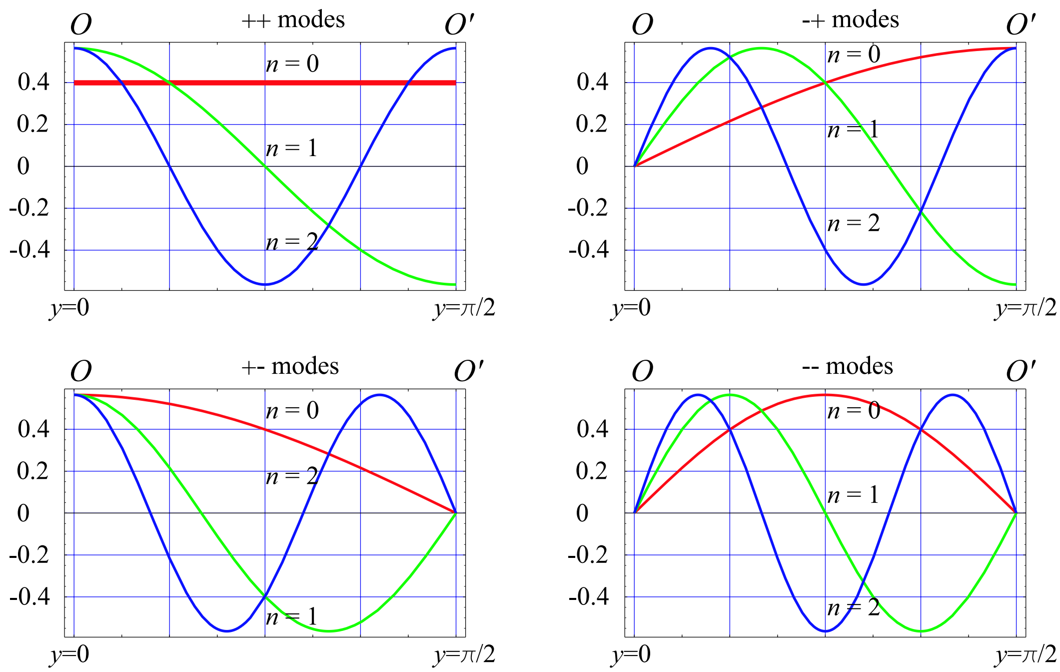


Figure 4.3: A profile of the wave functions of the first three modes of the Kaluza-Klein tower of states for every possible parity assignment. A flat line in the plot of $++$ states represents the profile of the massless $n = 0$ mode. All other states are massive with their mass being quantized in units of $1/R$. Here, we take $R = 1$.

In our model, we assume that gauge fields and gauge-non-singlet Higgs fields exist in the five-dimensional bulk, while the quark and lepton fields and certain gauge-singlet Higgs fields exist only on the visible brane at O. The gauge fields in the bulk are of course in a vector supermultiplet of 5d supersymmetry that is an adjoint representation of $SO(10)$. We will denote it by $\mathbf{45}_g$, where the subscript 'g' stands for 'gauge'. The gauge-non-singlet Higgs fields in the bulk are in hypermultiplets of 5d supersymmetry and consist of two tens of $SO(10)$, denoted $\mathbf{10}_{1H}$ and $\mathbf{10}_{2H}$, and a spinor-antispinor pair of $SO(10)$ denoted $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$. The subscript 'H' indicates a Higgs field.

The vector supermultiplet $\mathbf{45}_g$ decomposes into a vector multiplet V and a chiral multiplet Σ of $\mathcal{N}=1$ supersymmetry in four dimensions. Each hypermultiplet splits into two left-handed chiral multiplets Φ and Φ^c, having opposite gauge quantum numbers. As shown in Section 1.4.3, under the $SU(5)\otimes U(1)$ subgroup the $SO(10)$ representations decompose as follows: $\mathbf{45}\rightarrow\mathbf{24}^0+\mathbf{10}^{-4}+\overline{\mathbf{10}}^4+\mathbf{1}^0$; $\mathbf{10}\rightarrow\mathbf{5}^{-2}+\overline{\mathbf{5}}^2$; $\mathbf{16}\rightarrow\mathbf{10}^1+\overline{\mathbf{5}}^{-3}+\mathbf{1}^5$; and $\overline{\mathbf{16}}\rightarrow\overline{\mathbf{10}}^{-1}+\mathbf{5}^3+\mathbf{1}^{-5}$. With these facts in mind we shall now discuss the transformation of the various fields under the $Z_2\times Z_2'$ parity transformations.

The first Z_2 symmetry (the one we denote as unprimed) is used to break supersymmetry to $\mathcal{N}=1$ in four-dimensions. [$\mathcal{N}=1$ in five dimensions is equivalent to $\mathcal{N}=2$ in four dimensions; so we are breaking half the supersymmetries.] To do this we assume that under Z_2 the V is even, Σ is odd, Φ are even, and Φ^c are odd. The Z_2' is used to break $SO(10)$ down to $SU(5)\otimes U(1)$. The $\mathbf{24}^0$ and $\mathbf{1}^0$ of V are taken to be even under Z_2', while the $\mathbf{10}^{-4}$ and $\overline{\mathbf{10}}^4$ are taken to be odd. In $\mathbf{10}_{1H}$ the $\mathbf{5}^{-2}$ are taken to be even and the $\overline{\mathbf{5}}^2$ odd, whereas in $\mathbf{10}_{2H}$ the parities are taken to be the reverse, $\mathbf{5}^{-2}$ odd and $\overline{\mathbf{5}}^2$ even. All told we have

$$\mathbf{45}_g = V^{++}_{\mathbf{24}^0}+V^{++}_{\mathbf{1}^0}+V^{+-}_{\mathbf{10}^{-4}}+V^{+-}_{\overline{\mathbf{10}}^4}+\Sigma^{--}_{\mathbf{24}^0}+\Sigma^{--}_{\mathbf{1}^0}+\Sigma^{-+}_{\mathbf{10}^{-4}}+\Sigma^{-+}_{\overline{\mathbf{10}}^4}, \quad (4.3a)$$

$$\mathbf{10}_{1H} = \Phi^{++}_{\mathbf{5}_1^{-2}}+\Phi^{+-}_{\overline{\mathbf{5}}_1^{2}}+\Phi^{c--}_{\overline{\mathbf{5}}_1^{2}}+\Phi^{c-+}_{\mathbf{5}_1^{-2}}, \quad (4.3b)$$

$$\mathbf{10}_{2H} = \Phi^{+-}_{\mathbf{5}_2^{-2}}+\Phi^{++}_{\overline{\mathbf{5}}_2^{2}}+\Phi^{c-+}_{\overline{\mathbf{5}}_2^{2}}+\Phi^{c--}_{\mathbf{5}_2^{-2}}, \quad (4.3c)$$

$$\mathbf{16}_H = \Phi^{++}_{\mathbf{10}^1}+\Phi^{+-}_{\overline{\mathbf{5}}^{-3}}+\Phi^{+-}_{\mathbf{1}^5}+\Phi^{c--}_{\overline{\mathbf{10}}^{-1}}+\Phi^{c-+}_{\mathbf{5}^3}+\Phi^{c-+}_{\mathbf{1}^{-5}}, \quad (4.3d)$$

$$\overline{\mathbf{16}}_H = \Phi^{++}_{\overline{\mathbf{10}}^{-1}}+\Phi^{+-}_{\mathbf{5}^3}+\Phi^{+-}_{\mathbf{1}^{-5}}+\Phi^{c--}_{\mathbf{10}^1}+\Phi^{c-+}_{\overline{\mathbf{5}}^{-3}}+\Phi^{c-+}_{\mathbf{1}^5}, \quad (4.3e)$$

where the transformation properties of the fields under $SU(5)\otimes U(1)$ are indicated with the subscript. Massless zero modes of the Kaluza-Klein towers exist only for fields with $Z_2\times Z_2'$ parity $++$. This includes $\Phi^{++}_{\mathbf{5}_1^{-2}}$, $\Phi^{++}_{\overline{\mathbf{5}}_2^{2}}$, $\Phi^{++}_{\mathbf{10}^1}$, and $\Phi^{++}_{\overline{\mathbf{10}}^{-1}}$. We will call the zero modes of these components h_1, $\overline{h}_2$, H, and $\overline{H}$, respectively, using the same notation we used in the last section. The h_1 and $\overline{h}_2$ contain the two Higgs doublets of the MSSM and their colored partners.

To justify these parity assignments at the more formal level, we first specify the action of the bulk fields [210] applicable in our five-dimensional theory. Its form is

$$\begin{aligned} S_5^{\rm bulk} = \int \mathrm{d}^5x \Big\{ & \frac{1}{4kg_5^2} \mathrm{Tr}\Big[W^\alpha W_\alpha \big|_{\theta\theta} + \mathrm{H.c.} \Big] \\ & + \frac{1}{kg_5^2} \mathrm{Tr}\Big[\big((\sqrt{2}\partial_5 + \overline{\Sigma}) \mathrm{e}^{-V} (-\sqrt{2}\partial_5 + \Sigma) \mathrm{e}^{V} + \partial_5 \mathrm{e}^{-V} \partial_5 \mathrm{e}^{V} \big) \Big] \Big|_{\theta\theta\overline{\theta}\overline{\theta}} \\ & + \Big[\Phi_i^c \mathrm{e}^V \overline{\Phi}_i^c + \overline{\Phi}_i \mathrm{e}^{-V} \Phi_i \Big] \Big|_{\theta\theta\overline{\theta}\overline{\theta}} + \Big[\Phi_i^c (\partial_5 - \frac{1}{\sqrt{2}} \Sigma) \Phi_i \big|_{\theta\theta} + \mathrm{H.c.} \Big] \Big\}, \end{aligned} \tag{4.4}$$

where index i goes through all chiral hypermultiplets of the bulk (in our case $i = 1, \ldots, 4$). The second and the last line clearly indicate that the bulk field Σ pairs up with the differential operator ∂_5. Thus, the field Σ *must* be odd under $y \to -y$ reflection since under same reflection $\partial_5 \to -\partial_5$. The last term also shows that Φ_i^c and Φ_i always have opposite parities. More generally, the action in Eq. (4.4) is invariant under $y \to -y$ reflection if the superfields transform as

$$V^a(x^\mu, -y) T_a = V^a(x^\mu, y) P T_a P, \tag{4.5a}$$

$$\Sigma^a(x^\mu, -y) T_a = -\Sigma^a(x^\mu, y) P T_a P, \tag{4.5b}$$

$$\Phi_i(x^\mu, -y) = \pm P \Phi_i(x^\mu, -y), \tag{4.5c}$$

$$\Phi_i^c(x^\mu, -y) = \mp P^T \Phi_i^c(x^\mu, -y), \tag{4.5d}$$

where $V = V^a T_a$, $\Sigma = \Sigma^a T_a$, and $P = P^{-1}$ is the parity operator. The T_a are the generators of $SO(10)$ in the appropriate representation with normalization $\mathrm{Tr}[T_a T_b] = k\delta_{ab}$ in the adjoint. The replacement $y \to y'$ and $P \to P'$ in Eqs. (4.5) specifies the transformation of the superfields under $y' \to -y'$ reflection. To implement the particular parity assignment of our model, as given in Eqs. (4.3), we define P and P' through their action on the fundamental, the **10**, of $SO(10)$ as follows: we associate $P \equiv \sigma_0 \otimes I$ ($P' \equiv \sigma_2 \otimes I$) with the Z_2 (Z_2'). Here, I and σ_0 are 5×5 and 2×2 identity matrices and σ_2 is the usual Pauli matrix.

Having done with the parity assignment for the bulk fields we can turn our attention towards the brane physics. On the brane at O we put the three families of quarks and leptons. Since the gauge symmetry on this brane is $SO(10)$, these are contained in three chiral superfields that are spinors of $SO(10)$, which we denote $\mathbf{16}_i$, where $i = 1, 2, 3$ is the family index. Later for various reasons we shall introduce some gauge-singlet superfields on the brane at O, but let us first discuss the interactions of the fields introduced up to this point.

The Z_2 parity of fields in the $\mathbf{16}_i$ must be positive. The Z_2' parity, determined by the transformation properties of the gauge fields in Eqs. (4.3), is $\mathbf{16} \to \mathbf{10}^{1\pm} + \overline{\mathbf{5}}^{-3\mp} + \mathbf{1}^{5\mp}$, where $\mathbf{10}_i = (Q, \bar{d}, \bar{\nu})_i$, $\overline{\mathbf{5}}_i = (\bar{u}, L)_i$, and $\mathbf{1}_i = \bar{e}_i$, in accord with our discussion in Section 1.4.4. The action for the coupling of the matter fields, residing on the visible brane, with the Higgs fields, coming from the bulk, is

$$\begin{aligned} S_5^{\text{matter}} = &\int \mathrm{d}^5x \, \frac{1}{2} \left[\delta(y) + \delta(y - \pi R)\right] (2\pi R)^{1/2} \, \lambda_{ij}^d \mathbf{16}_i \mathbf{16}_j \mathbf{10}_{1H}\big|_{\theta\theta} \\ &+ \int \mathrm{d}^5x \, \frac{1}{2} \left[\delta(y) - \delta(y - \pi R)\right] (2\pi R)^{1/2} \, \lambda_{ij}^u \mathbf{16}_i \mathbf{16}_j \mathbf{10}_{2H}\big|_{\theta\theta} + \text{H.c.}, \end{aligned}$$

where λ_{ij}^u and λ_{ij}^d are Yukawa couplings. The normalization factor $(2\pi R)^{1/2}$ is inserted by hand to make them dimensionless. [Note that the mass dimension of the bulk superfield is $3/2$, the dimension of the four-dimensional brane superfield is 1, and the dimension of θ is $1/2$.] What we really expect in a realistic scenario is to have the suppression of the Yukawa couplings by a factor of $1/(M_* R)^{1/2}$ since the Higgs field wave function spreads throughout the extra-dimension. The mass M_* is an ultraviolet cutoff that specifies the scale at which new physics (eg. other dimensions beyond five, strings) become important. We take M_* to be close to M_{GUT} but, of course, somewhat larger. In general, if the Yukawa term involves n ($n = 1, 2, 3$) bulk fields the overall normalization factor is $1/(M_* R)^{n/2}$. [Note that the mass dimension of the Yukawa coupling is $-n/2$ where n is the number of the bulk fields.] This has suggested the idea to use the geography (the brane or the bulk localization) of the matter fields and the Higgs fields to achieve the hierarchy of the quark and lepton masses in the orbifold models [88]. The main problem with this idea is that it tends to ruin one of the most significant features of the GUT—the unification of matter—if one places the matter fields in the bulk. For example, it would take two different $\overline{\mathbf{16}}$s in the bulk to reproduce the content of one family on the brane. This shortcoming is the main reason why we do not pursue this idea here.

If we now integrate Eq. (4.6) over the fifth dimension using Eqs. (4.2), keeping only the terms that involve the Yukawa couplings of the MSSM Higgs doublets and their triplet partners, we obtain the following Lagrangian in four dimensions

$$\mathcal{L}_4^{\text{matter}} = \sum_{n=0}^{\infty} \sqrt{\frac{2}{2^{\delta_{n0}}}} \Big\{ \lambda_{ij}^d \Big[Q_i \bar{d}_j \overline{d}_{1H}^{(2n)} + L_i \bar{e}_j \overline{d}_{1H}^{(2n)} + \frac{1}{2} Q_i Q_j t_{1H}^{(2n)} + \bar{u}_i \bar{e}_j t_{1H}^{(2n)} \Big] + \lambda_{ij}^u \Big[Q_i \bar{u}_j d_{2H}^{(2n)} + L_i \bar{\nu}_j d_{2H}^{(2n)} + Q_i L_j \overline{t}_{2H}^{(2n)} + \bar{u}_i \bar{e}_j \overline{t}_{2H}^{(2n)} \Big] \Big\} \Big|_{\theta\theta} + \text{H.c.}$$

where $\overline{d}_{1H}^{(2n)}$ and $t_{1H}^{(2n)}$ are the doublet and triplet contained in $\Phi^{++}_{\mathbf{5}_1^{-2}}$ (whose zero mode is h_1) and $d_{2H}^{(2n)}$ and $\overline{t}_{2H}^{(2n)}$ are the doublet and triplet contained in $\Phi^{++}_{\overline{\mathbf{5}}_2^{2}}$ (whose zero mode is $\overline{h}_2$). All the remaining terms coming from Eq. (4.6) are found by the replacement $\lambda_{ij}^u \leftrightarrow \lambda_{ij}^d$, $(1H) \leftrightarrow (2H)$, $(2n) \to (2n+1)$, and $\delta_{n0} \to 1$ in Eq. (4.6).

This represents a minimal set of Yukawa terms, and would lead to the following relations among the quark and lepton mass matrices: $L = D \propto \lambda^d$ and $N = U \propto \lambda^u$, with λ^u and λ^d being completely independent symmetric matrices. This is different from the relations that arise with a minimal set of Yukawa terms in four-dimensional models based on $SO(10)$ or flipped $SU(5)$. In four-dimensional flipped $SU(5)$, the minimal Yukawa terms give $N = U^{\mathrm{T}}$, where these matrices are not predicted to be symmetric, and no relation for L and D. In four-dimensional $SO(10)$, as we saw in Section 1.4.3, the minimal Yukawa terms give $L = D \propto N = U$, with these matrices predicted to be symmetric.

The Higgs fields, though defined in the bulk, will also couple to each other on the branes. We assume that the Higgs coupling on the visible brane is of the form

$$\begin{aligned} S_5^{\text{Higgs}} = & \int \mathrm{d}^5x \, \frac{1}{2} \left[\delta(y) + \delta(y - \pi R)\right] (2\pi R)^{3/2} \, \mathbf{10}_{1H} \mathbf{16}_H \mathbf{16}_H \big|_{\theta\theta} \\ & + \int \mathrm{d}^5x \, \frac{1}{2} \left[\delta(y) - \delta(y - \pi R)\right] (2\pi R)^{3/2} \, \mathbf{10}_{2H} \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H \big|_{\theta\theta} + \text{H.c..} \end{aligned} \tag{4.6}$$

There could also be terms of the form $\mathbf{10}_{iH}\mathbf{10}_{jH}$, which would directly produce a GUT-scale μ term and destroy the gauge hierarchy. These must be forbidden by a symmetry. This is not a novel requirement introduced by the fact that there are extra dimensions. Terms that would directly produce a GUT-scale μ term must also be forbidden in four-dimensional unified theories. For example, in four-dimensional $SU(5)$ theories as well as four-dimensional flipped $SU(5)$ theories, there are Higgs multiplets in $\overline{\mathbf{5}}$ and $\mathbf{5}$, and these must be prevented from obtaining a superheavy mass term together. Similarly, in four-dimensional $SO(10)$ theories the light Higgs doublets are typically in a $\mathbf{10}$ of Higgs, which must be prevented from acquiring a superheavy self-mass term [211]. The same problem arises also in GUTs in higher dimensions. Generally, some symmetry must be imposed to protect the gauge hierarchy from such dangerous terms. We shall assume here that there is a $U(1)'$ of the Peccei-Quinn type under which the quark and lepton spinors $\mathbf{16}_i$ have charge $+1$, the Higgs fields $\mathbf{10}_{1H}$ and $\mathbf{10}_{2H}$ have charge -2, and the Higgs fields $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ have charge $+1$. This approach of using a vector-like symmetry to prevent a large direct μ term is used in Ref. [206]. A drawback of using that method here, as will be seen later, is that to generate large Majorana mass terms for the neutrinos without too large a μ term being generated by higher-dimension operators, it will be necessary to assume a hierarchy of 10^{-4} between the $U(1)'$ breaking scale and M_{GUT}.

Another way of suppressing direct GUT-scale μ terms is by means of a continuous $U(1)_R$ symmetry as in Ref. [87]. In that paper it was found that μ and μB parameters of the order of the weak scale could be generated, without any fine-tuning, through the Giudice-Masiero mechanism [212]. We do not pursue other approaches such as that here.

The most general effective action of our theory should also include brane-localized kinetic terms for the modes of the bulk fields that have non-vanishing wave function on the branes. Since the symmetry that survives on the hidden brane differs from the symmetry that governs the interactions on the visible brane and in the bulk, one might worry that the hidden-brane kinetic terms with arbitrary coefficients for the gauge fields would spoil the gauge coupling unification, and that the hidden-brane kinetic terms for the Higgs fields could affect the mass matrix prediction that stems from Eq. (4.6).

As it turns out, the gauge kinetic terms on the hidden brane do not spoil the gauge coupling unification if the volume of the extra dimension is large enough [87]. In that case the arbitrary coefficients of the gauge kinetic terms on the hidden and the visible brane get diluted and their contribution to the gauge couplings of the Standard Model can be neglected. The dominant contribution comes from the universal coefficient that belongs to the gauge kinetic term in the bulk obeying the full symmetry of the theory.

The hidden brane kinetic terms for the Higgs fields do not affect the mass relations $L = D \propto \lambda^d$ and $N = U \propto \lambda^u$. These hidden-brane terms violate $SO(10)$ but respect $SU(5) \otimes U(1)$, and so will have the effect of changing the relative normalization of the $\overline{\mathbf{5}}$ and $\mathbf{5}$ of Higgs that are inside the same $\mathbf{10}$ of $SO(10)$. However, the $\mathbf{5}$ of Higgs and the $\overline{\mathbf{5}}$ of Higgs that contribute to quark and lepton masses in this model come from different $\mathbf{10}$'s of Higgs anyway. The former comes from $\mathbf{10}_{1H}$, while the latter comes from $\mathbf{10}_{2H}$. While the matrices λ^u and λ^d will be differently affected by the hidden-brane kinetic terms, the predictions that $L = D \propto \lambda^d$ and $N = U \propto \lambda^u$ are not affected by that. The essential point is that these predictions depend only on the $SU(5)$ that is respected by the hidden-brane kinetic terms and not on the full $SO(10)$.

As noted earlier, the only massless modes of the Higgs fields are $h_1 \subset \Phi^{++}_{\mathbf{5}_1^{-2}} \subset \mathbf{10}_{1H}$, $\overline{h}_2 \subset \Phi^{++}_{\overline{\mathbf{5}}_2^{2}} \subset \mathbf{10}_{2H}$, $H \subset \Phi^{++}_{\mathbf{10}^1} \subset \mathbf{16}_H$, and $\overline{H} \subset \Phi^{++}_{\overline{\mathbf{10}}^{-1}} \subset \overline{\mathbf{16}}_H$. Therefore, after integrating over the fifth dimension, the terms in Eq. (4.6) yield in the superpotential of the low-energy effective theory the terms $h_1\, H\, H + \overline{h}_2\, \overline{H}\, \overline{H}$. These are just the same terms that are present in conventional four-dimensional flipped $SU(5)$ models to do the doublet-triplet splitting.

We assume that the H and $\overline{H}$ acquire superlarge vacuum expectation values that break $SU(5) \otimes U(1)$ down to the Standard Model group. The tree-level scalar potential generated by the terms $h_1 H H + \overline{h}_2 \overline{H}\overline{H}$ is flat in this direction. However, as noted in [85], this flatness can be lifted by radiative effects after supersymmetry is broken. It is also possible that additional terms in the Higgs superpotential on the visible brane can lead to a tree-level superpotential that produces the required symmetry breaking, as we shall see later.

Besides breaking the gauge symmetry from $SU(5) \otimes U(1)$ down to $SU(3) \otimes SU(2) \otimes U(1)$, the vacuum expectation values of the fields $H \subset \mathbf{16}_H$ and $\overline{H} \subset \overline{\mathbf{16}}_H$ allow masses for the right-handed neutrinos. Such masses come from effective operators of the form $\mathbf{16}_i \mathbf{16}_j \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H$. However, this product of fields has charge $+4$ under the symmetry $U(1)'$. Consequently, this symmetry must be spontaneously broken. It must be broken in such a way as to permit sufficiently large right-handed neutrino masses without at the same time allowing too large a μ parameter (which is the coefficient of the term $\mathbf{10}_{1H}\mathbf{10}_{2H}$). This can be done in the following way (which we do not claim to be unique). Suppose that there are fields S and $\overline{S}$ living on the brane at O that are singlets under $SO(10)$ and that have $U(1)'$ charges $+1$ and -1 respectively. In the superpotential on the brane at O there can be terms of the form $(\overline{S}S - M^2)X$, where $M = \epsilon M_{\rm GUT}$, with $\epsilon \ll 1$. These terms force $\langle S \rangle = \langle \overline{S} \rangle = M$. Let us suppose that on the brane at O there are, in addition to the quark and lepton families in $\mathbf{16}_i$, some leptons $\mathbf{1}_i$ ($i = 1, 2, 3$) that are $SO(10)$ singlets but have charge

-1 under $U(1)'$. Then the following terms in the superpotential at O are possible: $C_{ij}\mathbf{16}_i\mathbf{1}_j\overline{\mathbf{16}}_H\overline{S}/M_* + F_{ij}\mathbf{1}_i\mathbf{1}_jS^2/M_*$, where the dimensionless coefficients C_{ij} and F_{ij} are assumed to be of order one. These terms give a mass matrix for the neutrinos that has the form

$$(\nu_i \quad \bar{\nu}_i \quad \mathbf{1}_i)\begin{pmatrix} 0 & (N)_{ij} & 0 \\ (N)_{ji} & 0 & C_{ij}\epsilon\overline{M} \\ 0 & C_{ji}\epsilon\overline{M} & F_{ij}\epsilon^2\overline{M} \end{pmatrix}\begin{pmatrix} \nu_j \\ \bar{\nu}_j \\ \mathbf{1}_j \end{pmatrix}, \tag{4.7}$$

where $\overline{M} \equiv M_{\rm GUT}^2/M_*$. [Note that we have taken $\langle \mathbf{16}_H \rangle = M_{GUT}$.] It is clear that the six superheavy neutrinos have masses of order $\epsilon\overline{M}$, whereas the three light (left-handed) neutrinos have masses of order $N^2/\overline{M}$. [See discussion on the double see-saw mechanism after the Eq. (1.51).] Taking the largest neutrino mass m_3 to be about 6×10^{-2} eV, as suggested by the atmospheric neutrino data, and its Dirac mass to be $m_c \cong 174\,\text{GeV}$, as suggested by the relation $N = U$ (which would hold in a minimal $SO(10)$ model), one has that $\overline{M} \sim 10^{15}\,\text{GeV}$. This accords well with the assumption that M_* is slightly larger than the GUT scale $M_{\rm GUT} \sim 10^{16}\,\text{GeV}$.

The reason that we have assumed that the parameter $\epsilon \equiv \langle S \rangle / M_{\rm GUT}$ is much smaller than one is that it suppresses certain dangerous operators. For example, $U(1)'$ allows the effective operator $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_k\mathbf{16}_\ell\overline{S}^4/M_*^5$. This gives a $d = 5$ proton decay operator with coefficient of order $\epsilon^4(1/M_*)$. Sufficient suppression of proton decay requires that $\epsilon \sim 10^{-3}$ to 10^{-4}. Similarly, $U(1)'$ allows the operator $\mathbf{10}_{1H}\mathbf{10}_{2H}S^4/M_*^3$. This gives a μ parameter for the MSSM doublet Higgs fields that is of order $\epsilon^4 M_*$. Requiring that this be no larger than the weak scale requires that ϵ be less than about 3×10^{-4}. This corresponds to right-handed neutrino masses of order 3×10^{11} GeV. Such intermediate mass scales for M_R are good from the point of view of leptogenesis [213].

The same singlet Higgs field S can play a role in generating the vacuum expectation value for the spinor Higgs fields $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$. Such VEVs, as we have already noted, can arise due to radiative effects after SUSY breaking. But they can also arise at tree level from a term in the superpotential on the brane at O of the form $(\lambda\overline{\mathbf{16}}_H\mathbf{16}_H - S^2)Y$, where Y is a singlet superfield with $U(1)'$ charge of -2, and $\lambda \sim \epsilon^2$. Note that the F-terms of the fields $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ force $\langle Y \rangle = 0$, meaning that there is no mass term linking $\overline{\mathbf{16}}_H$ to $\mathbf{16}_H$ and thus $\overline{H}$ to H. The $U(1)'$ charge assignments allow the higher dimensional term $\overline{S}^2\overline{\mathbf{16}}_H\mathbf{16}_H/M_*$.

This will shift the VEV of Y, but the F-terms of the fields $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ still enforce the condition that there is no mass term linking $\overline{\mathbf{16}}_H$ to $\mathbf{16}_H$.

Let us now examine the doublet-triplet splitting and proton decay problems. The terms $h_1 H\langle H\rangle + \overline{h}_2\overline{H}\langle\overline{H}\rangle$ will couple the triplets in h_1 and $\overline{h}_2$ to those in H and $\overline{H}$. The doublets in h_1 and $\overline{h}_2$ remain light and are the two doublets of the MSSM. There is no problem with $d = 5$ proton decay, because the triplet partners of the MSSM Higgs doublets are not connected to each other. The triplets in h_1 and H have no mass terms with the triplets in $\overline{h}_2$ and $\overline{H}$. Moreover, there are no unwanted light states contained in the Higgs multiplets $\mathbf{10}_{1H}$, $\mathbf{10}_{2H}$, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$. In the zero modes (h_1, $\overline{h}_2$, H, and $\overline{H}$), the doublets remain light, the triplets become superheavy by coupling to the VEVs of H and $\overline{H}$, and the other gauge-non-singlet fields get eaten by the Higgs mechanism when $SU(5) \otimes U(1)$ breaks to the Standard Model group. All the non-zero modes, of course, have superheavy Kaluza-Klein masses. This is the crucial difference with four-dimensional theories in which flipped $SU(5)$ is embedded in $SO(10)$. In four dimensions, as we saw in the last section, the $SO(10)$ Higgs multiplets $\mathbf{10}_{1H}$ and $\mathbf{10}_{2H}$ when decomposed under $SU(5) \otimes U(1)$ contain not only h_1 and $\overline{h}_2$ but also $\overline{h}_1$ and h_2; and these multiplets have triplets that cannot be given mass without destroying the gauge hierarchy. Here, however, these extra pieces are all made heavy by the orbifold compactification, since they do not have parity $++$. Thus it is the fact that the unification of $SU(5) \otimes U(1)$ into $SO(10)$ occurs only in higher dimensions that allows the missing partner mechanism to be implemented.

We have seen that with what may be called the minimal Yukawa couplings $\mathbf{16}_i\mathbf{16}_j(\lambda^d_{ij}\mathbf{10}_{1H} + \lambda^u_{ij}\mathbf{10}_{2H})$ this model gives distinctive relations among mass matrices that are different from those that result in four-dimensional models with minimal Yukawa couplings in either $SO(10)$ or flipped $SU(5)$. In particular, $L = D$, and $N = U$, with all these matrices being symmetric. This does give the desired relation $m_b = m_\tau$ at the unification scale, a result of the fact that flipped $SU(5)$ is embedded in $SO(10)$. However, this minimal set of Yukawa terms is clearly not enough to give a realistic model of quark and lepton masses.

Recently it has been found that realistic and simple models of quark and lepton masses can be constructed using the "lopsided" mass matrices [73, 76, 77, 78]. The essential feature of such models is that the mass matrices of the down quarks and charged leptons are highly asymmetric and that $L \sim D^{\mathrm{T}}$. Such a relationship between L and D^{T} is typical of models with an ordinary $SU(5)$, not flipped $SU(5)$. However, as we shall now see, because the flipped $SU(5)$ is here embedded in $SO(10)$ at the five-dimensional level, it is possible to obtain such a lopsided structure.

Suppose that one introduces on the visible brane not only spinors of quarks and leptons, but $SO(10)$ vectors as well. And suppose that there is in the bulk a spinor Higgs field $\mathbf{16}'_H$ that has a weak-doublet component that contributes to the breaking of the electroweak symmetry. Then a diagram like that shown in Fig. 4.4(a) may be possible. When decomposed under the $SU(5) \otimes U(1)$ subgroup, this diagram contains the two diagrams shown in Figs. 4.4(b) and 4.4(c). It is easy to see that these give contributions to L and D that are asymmetric and that are transposes of each other, just as needed to build "lopsided" models. The reason for this is that the diagram in Fig. 4.4(a) directly depends only on the GUT-scale breaking done by the $\mathbf{16}_H$ and not on that coming from orbifold compactification. The point is that the $\mathbf{16}_H$ VEV by itself would only break $SO(10)$ down to the

Georgi-Glashow $SU(5)$. [It is the orbifold compactification that breaks $SO(10)$ to the flipped $SU(5) \otimes U(1)$ group.] That is why this diagram leads to the kind of mass contributions that one expects from ordinary Georgi-Glashow $SU(5)$. This reasoning also shows that in order to introduce into the mass matrices contributions that break Georgi-Glashow $SU(5)$ it is necessary that the mass-splittings produced by the orbifold compactification be involved. For example, by mixing quarks and leptons that are on the visible brane with quarks and leptons in the bulk, it should be possible to break the (bad) minimal $SU(5)$ relations $m_s = m_\mu$ and $m_d = m_e$.

4.4 Gaugino mediated supersymmetry breaking

In this section we address the issue of how to break $\mathcal{N} = 1$ supersymmetry of our model below the compactification scale M_C. As it turns out, the solution allows the construction of viable SUSY breaking model that can easily satisfy present experimental constraints.

It is well known that the models with visible and hidden branes separated by extra dimension(s) naturally accommodate breaking of supersymmetry via gaugino mediation [214, 215]. The basic idea behind gaugino mediation in the models based on the orbifold compactification is as follows. The source of the SUSY breaking is localized at the hidden brane. It couples directly to the gauginos at that brane providing them with nonzero masses. If the gauge symmetry at the hidden brane is reduced with respect to the bulk gauge symmetry this coupling induces non-universal gaugino masses. For example, if the bulk symmetry is $SO(10)$ and hidden brane symmetry is flipped $SU(5)$ one obtains $M_3 = M_2 \neq M_1$. Here, M_1, M_2, and M_3 represent gaugino masses of the MSSM.

Following in the footsteps of [206], we take the source of the SUSY breaking to be a flipped $SU(5)$ singlet chiral superfield Z localized on the hidden brane with the VEV

$$\langle Z \rangle = \theta^2 F_Z. \tag{4.8}$$

The gaugino masses originate from the non-renormalizable operators at the hidden brane of the form

$$\mathcal{L}_5^Z = \frac{1}{2}[\delta(y-\pi R/2)+\delta(y+\pi R/2)]\Big(\lambda_5' \frac{Z}{M_*^2} W^{i\alpha}W_\alpha^i\big|_{\theta\theta} + \lambda_1' \frac{Z}{M_*^2} W^{\alpha}W_\alpha\big|_{\theta\theta} + \text{H.c.}\Big),$$

where the first and the second term under the integral represent the $SU(5)$ and $U(1)$ part of the gauge group respectively. Corresponding gaugino masses generated in this way are

$$M_{SU(5)} = \frac{\lambda_5' F_Z M_c}{M_*^2}, \qquad M_{U(1)} = \frac{\lambda_1' F_Z M_c}{M_*^2}, \tag{4.9}$$

which translates into the following MSSM gaugino masses (we normalize the generators of $SO(10)$ demanding that $k = 1/2$)

$$\frac{M_1}{g_1^2} = \frac{1}{25}\frac{M_{SU(5)}}{g_{SU(5)}^2} + \frac{24}{25}\frac{M_{U(1)}}{g_{U(1)}^2}, \qquad M_2 = M_{SU(5)}, \qquad M_3 = M_{SU(5)}. \tag{4.10}$$

Here $g_{SU(5)}$, and $g_{U(1)}$ are gauge coupling constants of the $SU(5)$ and $U(1)$ gauge groups respectively, while g_1 represents the $U(1)_Y$ gauge coupling constant of the Standard Model (normalized as in GUTs). The relations of Eq. (4.10), which is valid at the compactification scale M_C, show that the gaugino mass M_1 can in general be completely different from the mass $M_2 = M_3$ due to their different origins. Namely, the mass M_1 is dominated by the $U(1)$ sector of the theory as oppose to the masses M_2 and M_3 that have their origin in the $SU(5)$ part of the theory. We will later see that this feature of non-universality of gaugino masses allows the construction of the theory of SUSY breaking that leads to the realistic mass spectrum.

At this point we note that the natural scale for $\sqrt{F_Z}$ is the cutoff scale M_*. [For the reasons that have to do with gauge coupling unification we take ($M_C \sim 10^{16}\,\text{GeV}$) $<$ ($M_{\text{GUT}} = 2\times 10^{16}\,\text{GeV}$) $<$ ($M_* \sim 10 M_C$) [206].] This implies that masses in Eq. (4.9) are close to the compactification scale M_C if the dimensionless coefficients λ_1' and λ_5' are taken to be of order one. To obtain SUSY breaking masses that are in the TeV range we need to decrease the value of F_Z in a way that does not involve any fine-tuning. To do that we propose to use the shining mechanism [216, 217] which can reduce the natural scale of F_Z by an exponential factor.

The shining mechanism requires the existence of a source J that is localized on the visible brane and a massive hypermultiplet in the bulk. The hypermultiplet of mass m is taken to be a gauge singlet and has couplings with both the source and the superfield Z. These couplings can be arranged in a manner that leaves the superfield Z with the nonzero F-term $F_Z \sim J\exp(-\pi mR/2)$ after the massive hypermultiplet is integrated out [217]. If the mass m is taken to be of order M_* the $\sqrt{F_Z}$ will be of order 10^{12} GeV which gives desired TeV scale masses for gauginos in Eq. (4.9).

The matter fields in our model reside on the visible brane. Thus, due to the spatial separation between the branes the soft SUSY breaking scalar masses and trilinear couplings are negligible at the compactification scale. This is good because the number of the soft SUSY breaking parameters one has to consider is reduced with respect to the usual set.

There are two additional positive features of the gaugino mediated supersymmetry breaking models with the non-universal gaugino masses. Firstly, the renormalization group running of scalar masses and trilinear couplings between M_C and electroweak scale is significantly affected by gaugino masses but these contributions, being flavor blind, do not cause any disastrous flavor violating effects. Secondly, non-universality opens up the possibility for the deviation from the experimentally disfavored prediction of the models with universal gaugino mass of stau being the lightest supersymmetric particle (LSP). [The last statement holds for $M_C < M_{\rm GUT}$ which is exactly the case we have.]

The class of models with non-universal gaugino mediated supersymmetry breaking has been studied in more details by Baer *et al.* [218] (see also [219, 220]). Their numerical study of the allowed region of SUSY parameter space shows that viable models with acceptable mass spectrum and neutral LSP particle can be obtained. The study includes the case of completely independent M_3, M_2, and M_1, as well as the case where M_1 is a definite linear combination (determined by group theory) of M_2 and M_3. [The former case can be seen as a consequence of orbifold reduction of $SU(5)$ down to the Standard Model group on the hidden brane as in Ref. [87] and the latter one follows from the reduction of $SO(10)$ down to the Pati-Salam group as in Ref. [206].] We have an intermediate scenario where M_1 is independent of M_2 and M_3 which are made equal due to the $SU(5)$ part of the flipped $SU(5)$. [This possibility was considered in Ref. [204] in the context of a six-dimensional $SO(10)$ model.]

It is not difficult to adapt the analysis of Baer *et al.* to our model to conclude that for large enough M_1 (i.e. $|M_1| > |M_2|, M_2 = M_3$) at the compactification scale M_C a viable region of parameter space opens up regardless of $\tan\beta$ value yielding realistic mass spectrum with the LSP being wino-like or a mixture of higgsino and bino. An example of this behavior is shown in Fig. 4.5.

At the end we observe that if we had the case of $SO(10)$ being reduced on the hidden brane to the Georgi-Glashow $SU(5)$ with an extra $U(1)$ symmetry we would not only be prevented from using the simple form of the missing partner mechanism but would also obtain universal gaugino masses $M_1 = M_2 = M_3$.

4.5 Conclusions

We have seen that by embedding a four-dimensional flipped $SU(5)$ model into a five-dimensional $SO(10)$ model the advantages of flipped $SU(5)$ can be maintained while avoiding its well-known drawbacks. The two main drawbacks are the loss of unification of gauge couplings and the loss of the possibility of relating down quark masses to charged lepton masses, and therefore of obtaining desirable predictions such as $m_b = m_\tau$ and realistic quark and lepton mass schemes such as those based on "lopsided" mass matrices. By embedding $SU(5) \otimes U(1)$ in $SO(10)$, the unification of gauge couplings is restored. There are corrections to this unification, coming for example from gauge kinetic terms on the hidden brane; however, these have been argued to be small [87]. The embedding in $SO(10)$ also yields relationships between the charged lepton and down quark mass matrices. We have also found that interesting patterns of quark and lepton masses are possible that are different from those encountered in four-dimensional grand unified theories, for example $L = D \neq N = U$.

Embedding flipped $SU(5)$ in $SO(10)$ in four dimensions is well known to destroy the missing partner mechanism for doublet-triplet splitting, which is one of the most elegant features of flipped $SU(5)$. However, when the unification in $SO(10)$ takes place in higher dimensions and the breaking to $SU(5) \otimes U(1)$ is achieved through orbifold compactification, then the missing partner mechanism can still operate, as we have shown. One of the advantages of the missing partner mechanism in flipped $SU(5)$ is that it kills the dangerous $d = 5$ proton decay operators that plague supersymmetric grand unified theories.

Thus in extra dimensions it is possible to have the best of both worlds, the best of $SO(10)$ combined with the best of flipped $SU(5)$. One of the distinctive predictions of the flipped $SU(5)$ scheme that we have presented is that the gaugino masses will have the pattern $M_3 = M_2 \neq M_1$. The fact that M_1 is independent of M_2 and M_3 allows a much larger viable region of parameter space for the MSSM.

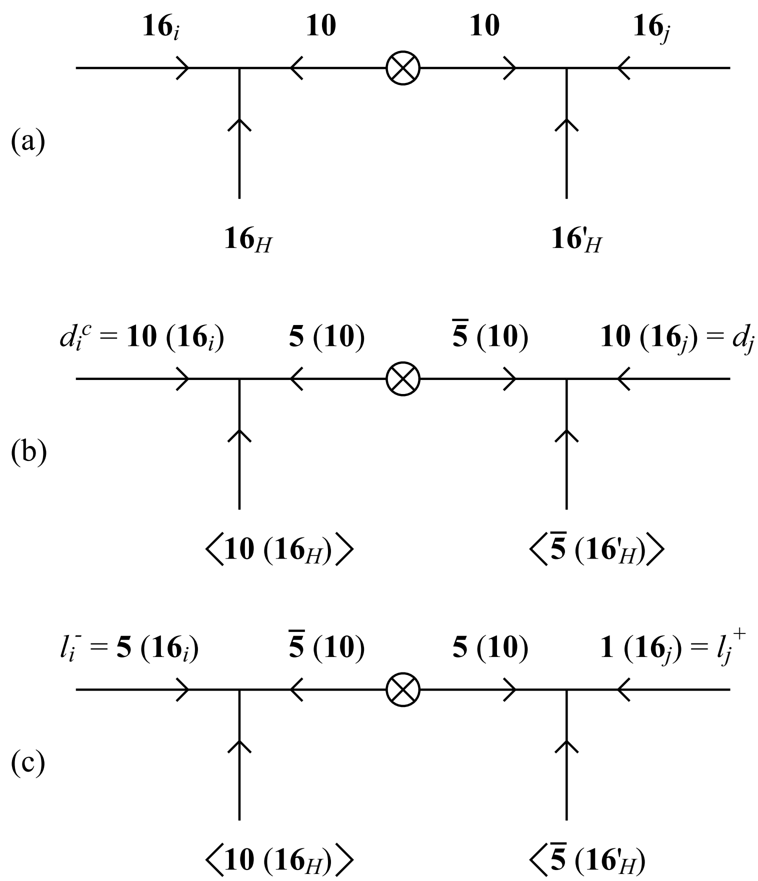


Figure 4.4: (a) A diagram that can give operators producing "lopsided" contributions to D and L. (b) A term in its $SU(5) \otimes U(1)$ decomposition that contributes to D. (c) A term in its $SU(5) \otimes U(1)$ decomposition that contributes to L.

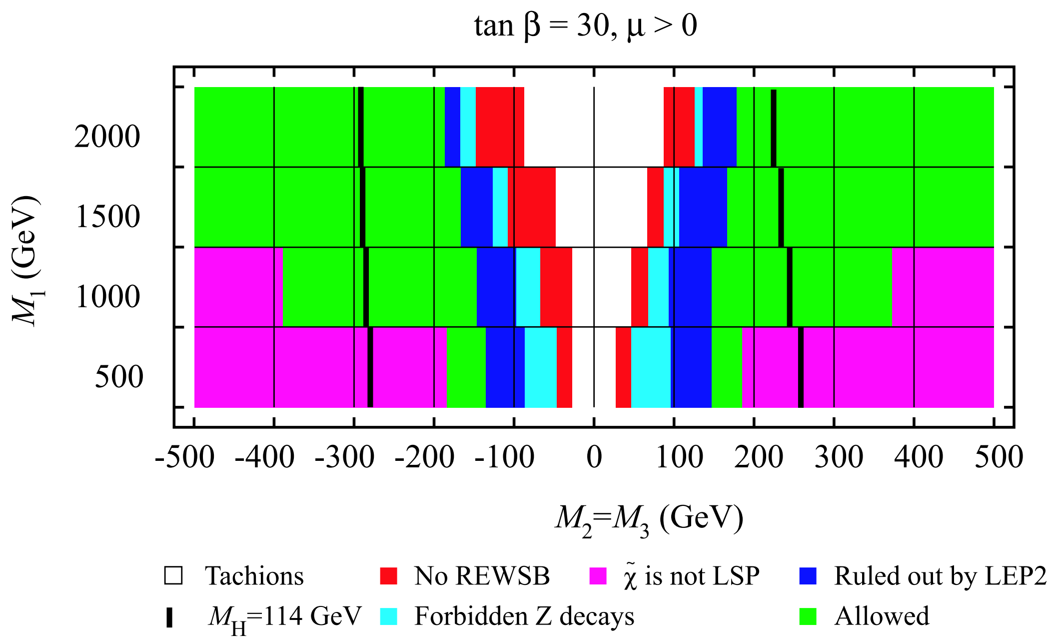


Figure 4.5: This diagram represents the results of numerical analysis of Baer *et al.* [218] for the case of gaugino mediated SUSY breaking scenario in the flipped $SU(5)$ setting ($M_2 = M_3 \neq M_1$) for $\tan\beta = 30$ and $\mu > 0$. The allowed region in M_1 vs. $M_2 = M_3$ plane is shown in green. The excluded regions are white (due to presence of tachyonic particles in mass spectrum), red (due to lack of radiative breakdown of EW symmetry), light blue (due to LEP constraint), dark blue (due to LEP2 constraint), and magenta (due to the fact that charged particle is LSP). Vertical black line is where $M_H = 114\,\text{GeV}$. For a full discussion of numerical methods and assumptions used in the analysis see Ref. [218].

Chapter 5

KALUZA-KLEIN UNIFICATION IN A FIVE-DIMENSIONAL MODEL

5.1 Introduction

The main motivation for SUSY, besides its ability to stabilize the Higgs mass against the radiative corrections, is the way it steers the gauge couplings, within the MSSM, towards the unification at the GUT scale. Assuming this is not an accident but a signal for a new physics we are prompted not only to embrace the MSSM but to incorporate it into the grand unified theory where the gauge unification represents a genuine *prediction* of the framework. It turns out, however, that it is very problematic to build both realistic and simple SUSY GUT scheme and still preserve the gauge unification. For example, the simplest of all such schemes, the minimal Georgi-Glashow $SU(5)$, is already ruled out by the experimental limits on proton decay [221, 222, 223]. The crux of the problem is that the exact gauge unification requires threshold corrections. But to create these corrections one needs certain fields, responsible for the proton decay, to be too light compared to the existing experimental constraints. This problem was not so serious in the past since the low-energy values of the gauge couplings were not known well enough, leaving a lot of room for maneuvering. The situation has changed after the electroweak precision measurements and the improvements in measurements of the strong coupling constant. The error bars have simply become sufficiently small to prevent the exact unification without the help of the troublesome threshold corrections. So, the question of whether we can achieve the gauge unification in accord with the low-energy measurements in a natural manner within SUSY GUTs is something we have to address.

Among the fields that can improve on the gauge unification, via threshold corrections, are the familiar colored Higgsinos. These are the fields, as we saw in Chapter 4, that are responsible for a $d = 5$ proton-decay operator. Therefore, one wants them light enough to generate the corrections but heavy enough to avoid violation of the experimental limits on proton lifetime. The idea of Kawamura [86, 197, 198] to use a five-dimensional theory seems perfectly suited to accommodate both of these requirements. But, one might expect naively that the exact gauge unification is impossible due to the threshold corrections that originate from the towers of Kaluza-Klein modes. This naive expectation turns out to be wrong. The five-dimensional theory, being non-renormalizable, *must* have a cutoff (M_*). Therefore, the number of Kaluza-Klein modes that contribute is finite. This also makes the threshold corrections finite and calculable so that the exact unification cannot be excluded *a priori*.

This chapter is going to be devoted to the issues pertaining to the gauge coupling unification in the five-dimensional setting. We show that it is possible to achieve the unification using an $\mathcal{N} = 1$ supersymmetric $SO(10)$ model on an $S^1/(Z_2 \times Z_2')$ orbifold. The orbifold has two inequivalent fixed points, O and O', identified by the action of $(Z_2 \times Z_2')$ twisting. On the point O there will be an $SO(10)$ gauge symmetry while on the point O' there will be a flipped $SU(5)$ gauge symmetries. Both symmetries will be the leftovers of a bigger, $SO(10)$, bulk symmetry. The bulk contains, besides the vector supermultiplet, a pair of chiral hypermutiplets: $\mathbf{10}_{1H}$ and $\mathbf{10}_{2H}$. They give the Higgs fields of the MSSM: $\overline{\mathbf{2}}$ and $\mathbf{2}$. The orbifolding procedure also reduces the amount of the supersymmetry from $\mathcal{N} = 1$ in five dimensions to $\mathcal{N} = 1$ in four dimensions. To obtain the low-energy phenomenology of the SM group $\mathcal{H}$ we break flipped $SU(5)$ on the O' brane by implementing the missing partner mechanism. This time, in contrast to the model presented in Chapter 4, we do the breaking with the chiral superfields that reside on the O' brane.

It should be stressed that there are already two models in the literature that provide the gauge coupling unification in the five-dimensional $S^1/(Z_2 \times Z_2')$ setting.

- The first one is an $SU(5)$ model of Hall and Nomura [87, 88]. In their model, the orbifolding leaves an $SU(5)$ gauge symmetry on one brane and the SM gauge symmetry on the other. In addition, the orbifolding accomplishes the doublet-triplet splitting by assigning the odd parity to the triplet fields. [The common feature for both models is the placement of the multiplets that contain the MSSM Higgs fields and the gauge sector in the bulk.] There is no need for any extra Higgs breaking except for the usual electroweak one. For gauge coupling unification not to be spoiled by arbitrary non-universal contributions coming from the brane with the SM gauge symmetry they have to invoke two requirements: (i) the couplings at the cutoff scale M_* *must* enter a strong coupling regime; (ii) the dimension(s) of the bulk *must* be large enough (when expressed in terms of the compactification scale M_C). We adopt their requirements in our model, too.

- The second model is an $SO(10)$ model of Dermíšek and Mafi [206]. Their model is described in some length in Chapter 4. Here, we just outline the features that are relevant for comparison with our work. Since the breaking of $SO(10)$ down to $\mathcal{H}$ demands the reduction of the group rank, the authors use an extra Higgs breaking. In the original version of Dermíšek and Mafi [206] the breaking of $SO(10)$ down to $SU(5)$ takes place on the visible brane. The low-energy signature of the SM gauge group is then due to the intersection of the Pati-Salam and $SU(5)$. The subsequent analysis of the second model by Kim and Raby [224] demonstrated the feasibility of the gauge unification. The breaking, in their case, takes place on a Pati-Salam brane affecting only the gauge sector of the theory. [The orbifolding has already projected out the triplet partners by assigning them odd parity.] We adopt and extend their method of analysis to demonstrate the successful unification in our case. The reason behind the extension is that, in our case, the extra Higgs breaking affects not only the gauge sector but also the Higgs sector. Namely, the breaking is what makes the triplets heavy via missing partner mechanism. This, as it turns out, has profound consequences on the RGE running of the gauge couplings as we demonstrate later.

In Section 5.2 we introduce our model and specify the mass spectrum of all the fields. We rely heavily on the material already covered in Chapter 4 to avoid being repetitious. We then proceed with the discussion on the gauge coupling RGE running in five-dimensional orbifold setting in Section 5.3. This is where our two main results, the relevant beta coefficients and their RGE numerical analysis, are presented. Finally, we briefly conclude in Section 5.4.

5.2 An SO(10) model

We present an $SO(10)$ supersymmetric model in five dimensions compactified on an $S^1/(Z_2 \times Z_2')$ orbifold. The orbifold is created after the fifth dimension, being the circle S^1 of radius R, gets compactified through the reflection $y \to -y$ under Z_2 and $y' \to -y'$ under Z_2', where $y' = y + \pi R/2$. As seen from Fig. 4.2, there are two fixed points, O and O', that bound the physical space $y \in [0, \pi R/2]$ of the bulk. The point O is referred to as the "visible brane" while point O' at $y' = 0$ is referred to as the "hidden brane".

We assume that the bulk contains an $\mathcal{N}=1$ vector supermultiplet, a $\mathbf{45}_g$ of $SO(10)$, and two chiral hypermultiplets, $\mathbf{10}_{1H}+\mathbf{10}_{2H}$. The vector supermultiplet decomposes into a vector multiplet V, which contains the gauge bosons A_μ and corresponding gauginos, and a chiral multiplet Σ of $\mathcal{N}=1$ supersymmetry in four dimensions. Each hypermultiplet splits into two left-handed chiral multiplets Φ and Φ^c, having opposite gauge quantum numbers. To reduce $\mathcal{N}=1$ supersymmetry in five dimensions to $\mathcal{N}=1$ supersymmetry in four dimensions we use the parity assignment under Z_2. To reduce the gauge symmetry from $SO(10)$ down to flipped $SU(5)\otimes U(1)$ on the hidden brane we use the parity assignment under Z_2' t . Using the same notation as in the previous chapter the bulk content and the parity assignments are

$$\mathbf{45}_g = V^{++}_{\mathbf{24}^0} + V^{++}_{\mathbf{1}^0} + V^{+-}_{\mathbf{10}^{-4}} + V^{+-}_{\overline{\mathbf{10}}^4} + \Sigma^{--}_{\mathbf{24}^0} + \Sigma^{--}_{\mathbf{1}^0} + \Sigma^{-+}_{\mathbf{10}^{-4}} + \Sigma^{-+}_{\overline{\mathbf{10}}^4}, \tag{5.1a}$$

$$\mathbf{10}_{1H} = \Phi^{++}_{\mathbf{5}_1^{-2}} + \Phi^{+-}_{\overline{\mathbf{5}}_1^{2}} + \Phi^{c--}_{\overline{\mathbf{5}}_1^{2}} + \Phi^{c-+}_{\mathbf{5}_1^{-2}}, \tag{5.1b}$$

$$\mathbf{10}_{2H} = \Phi^{+-}_{\mathbf{5}_2^{-2}} + \Phi^{++}_{\overline{\mathbf{5}}_2^{2}} + \Phi^{c-+}_{\overline{\mathbf{5}}_2^{2}} + \Phi^{c--}_{\mathbf{5}_2^{-2}}. \tag{5.1c}$$

Only the fields with the $++$ parity contain Kaluza-Klein zero mode fields ($n=0$) that have no effective four-dimensional mass. [See Fourier series expansion in Eqs. (4.2).] The masses of all other modes become quantized in units of $1/R \equiv M_C$, where M_C is the compactification scale. For example, all $+-$ and $-+$ parity states are actually the Kaluza-Klein towers of states with masses $M_C, 3M_C, \ldots, (2n+1)M_C, \ldots$, where n is the mode number.

We want to have the low-energy phenomenology that is described by the SM group $\mathcal{H}$. But, at this point, the brane O feels the $SO(10)$ gauge symmetry while the brane O' feels the flipped $SU(5)$ gauge symmetry. One could introduce a pair of Higgses in the bulk, the $\mathbf{16}_H$ and the $\overline{\mathbf{16}}_H$, and use the parity assignment to project out all the states except a pair $\mathbf{10}^1_H + \overline{\mathbf{10}}^{-1}_H$ that is needed for the missing partner mechanism on the hidden brane. This time, however, we pursue slightly different direction. Namely, noting that the minimal set of Higgses that breaks flipped $SU(5)$

Table 5.1: The decomposition of the three lowest lying representations of $SO(10)$ under the flipped $SU(5)$ group and the Standard Model gauge group.

$SO(10)$	$SU(5)\otimes U(1)$	$SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$
$\mathbf{45}$	$\mathbf{24}^0$	$(\mathbf{1},\mathbf{1},0)\oplus(\mathbf{1},\mathbf{3},0)\oplus(\mathbf{3},\mathbf{2},1/3)\oplus(\overline{\mathbf{3}},\mathbf{2},-1/3)\oplus(\mathbf{8},\mathbf{1},0)$
	$\mathbf{10}^{-4}$	$(\mathbf{1},\mathbf{1},-2)\oplus(\overline{\mathbf{3}},\mathbf{1},-4/3)\oplus(\mathbf{3},\mathbf{2},-5/3)$
	$\overline{\mathbf{10}}^4$	$(\mathbf{1},\mathbf{1},2)\oplus(\mathbf{3},\mathbf{1},4/3)\oplus(\overline{\mathbf{3}},\overline{\mathbf{2}},5/3)$
	$\mathbf{1}^0$	$(\mathbf{1},\mathbf{1},0)$
$\mathbf{16}$	$\mathbf{1}^5$	$(\mathbf{1},\mathbf{1},2)$
	$\overline{\mathbf{5}}^{-3}$	$(\mathbf{1},\overline{\mathbf{2}},-1)\oplus(\overline{\mathbf{3}},\mathbf{1},-4/3)$
	$\mathbf{10}^1$	$(\mathbf{1},\mathbf{1},0)\oplus(\overline{\mathbf{3}},\mathbf{1},2/3)\oplus(\mathbf{3},\mathbf{2},1/3)$
$\mathbf{10}$	$\mathbf{5}^{-2}$	$(\mathbf{1},\overline{\mathbf{2}},-1)\oplus(\mathbf{3},\mathbf{1},-2/3)$
	$\overline{\mathbf{5}}^2$	$(\mathbf{1},\mathbf{2},1)\oplus(\overline{\mathbf{3}},\mathbf{1},2/3)$

down to $\mathcal{H}$ is a pair of Higgs fields, $\mathbf{10}^1_H+\overline{\mathbf{10}}^{-1}_H$, we posit their existence on the *hidden* brane. With these fields in place we specify the following brane localized entry of the superpotential:

$$\kappa\Big[\delta\big(y-\frac{\pi R}{2}\big)+\delta\big(y-\frac{3\pi R}{2}\big)\Big]\Big[\Phi^{++}_{\mathbf{5}_1^{-2}}\,\mathbf{10}^1_H\,\mathbf{10}^1_H+\Phi^{++}_{\overline{\mathbf{5}}_2^{2}}\,\overline{\mathbf{10}}^{-1}_H\,\overline{\mathbf{10}}^{-1}_H\Big], \qquad (5.2)$$

where κ represents the Yukawa coupling with the mass dimension -1/2. Clearly, by giving very large VEVs to the $(\mathbf{1},\mathbf{1},0)$ components of $\mathbf{10}^1_H$ and $\overline{\mathbf{10}}^{-1}_H$, we allow the triplet partners of the doublets in $\Phi^{++}_{\mathbf{5}_1^{-2}}$ and $\Phi^{++}_{\overline{\mathbf{5}}_2^{2}}$ to get large masses through the mating with the triplets of $\mathbf{10}^1_H$ and $\overline{\mathbf{10}}^{-1}_H$ without disturbing the lightness of the doublets. This is schematically depicted in Eq. (4.1). Moreover, the symmetry breaking makes the states $(\mathbf{1},\mathbf{1},0)$, $(\mathbf{3},\mathbf{2},1/3)$, and $(\overline{\mathbf{3}},\mathbf{2},-1/3)$ from $V^{++}_{\mathbf{24}^0}$ and $V^{++}_{\mathbf{1}^0}$ of $\mathbf{45}_g$ absorb the corresponding components of the brane Higgses to become massive, leaving unbroken $\mathcal{H}$ gauge symmetry behind. [See Table 5.1 for the decomposition of $SO(10)$ down to $\mathcal{H}$ via flipped $SU(5)$.]

In the discussion from the previous paragraph, we have glossed over a fact that the bulk fields are Kaluza-Klein towers of states. The explicit brane localized breaking terms will disturb every state of that tower due to the change of the boundary conditions. Since we want to do an RGE analysis we need to determine the position, i.e. the mass, of every state after the disturbance has taken place. This is what we do next.

5.2.1 Mass Spectrum of the Gauge Fields

The five-dimensional theory is non-renormalizable. Therefore, we expect the theory to have a cutoff scale M_* where some new physics comes into play (e.g. other dimensions beyond five, strings). We take the VEVs of the symmetry breaking Higgs fields to be of the order of this cutoff: $\langle(\mathbf{1},\mathbf{1},0)\rangle \equiv M \sim M_*$. Then the Lagrangian involving the gauge fields gets additional contribution [225, 224]

$$\mathcal{L} \subset \frac{1}{2}\Big[\delta\big(y-\frac{\pi R}{2}\big)+\delta\big(y-\frac{3\pi R}{2}\big)\Big]g_5^2 M^2 A_\mu^{\hat{a}} A^{\hat{a}\mu}, \tag{5.3}$$

where g_5^2 represents the gauge coupling of the five-dimensional theory and $\hat{a}$ is an $SO(10)$ group index that goes through all the gauge fields associated with the broken ++ parity generators we mentioned in Section 5.2. [The five-dimensional gauge coupling g_5^2 has mass dimension -1.] The equations of motion for the "broken" gauge bosons are

$$-\partial_y^2 A_\mu^{\hat{a}}(x,y)+\Big[\delta\big(y-\frac{\pi R}{2}\big)+\delta\big(y-\frac{3\pi R}{2}\big)\Big]g_5^2 M^2 A_\mu^{\hat{a}}(x,y)=(M_n^A)^2 A_\mu^{\hat{a}}(x,y), \tag{5.4}$$

where M_n^A represents the effective Kaluza-Klein mass in four dimensions of the nth mode. It is defined via Klein-Gordon equation $\big[\partial_\nu\partial^\nu+(M_n^A)^2\big]A_\mu^{\hat{a}}(x,y)=0$. The second term on the left side of Eq. (5.4) is responsible for the deviation from the usual mass spectrum of the ++ parity fields ($M_n^A=0, 2M_C, \ldots, 2nM_C, \ldots$). It reminds us of the delta function-type potential in the ordinary Schrödinger's equation. The role of this term is thus to repel the bulk field wave function away from the brane. In the language of the effective four-dimensional theory this means that even the zero mode ($n = 0$) of the gauge bosons becomes massive. Taking the following ansatz for the five-dimensional gauge field on the segment $y \in [0, \pi R/2]$:

$$A^{\hat{a}}_{\mu}(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} N_n A^{\hat{a}(n)}_{\mu}(x) \cos M^A_n y, \tag{5.5}$$

the eigenvalue equation for the effective mass, due to the nontrivial boundary condition at the hidden brane, takes the form [225]

$$\tan \frac{M^A_n \pi R}{2} = \frac{g_5^2 M^2}{2 M^A_n}. \tag{5.6}$$

The normalization constant for the ++ parity bulk fields also changes from $1/\sqrt{2^{\delta_{n0}}}$ to $N_n = \left[1 + M_C g_5^2 M^2 \cos^2 \frac{M^A_n \pi R}{2} / (4\pi (M^A_n)^2)\right]^{-1/2}$. The plot of the modified wave function profile for $n = 1$ is given in Fig. 5.1. [We excluded the normalization constants for simplicity.]

There are two interesting approximations that we can consider: $g_5^2 M^2 \gg M^A_n$ and $g_5^2 M^2 \ll M^A_n$. The former one generates the following approximate solution of the eigenvalue equation for the mass spectrum

$$M^A_n \simeq (2n+1) M_C \big[1 - \varepsilon + \varepsilon^2\big], \tag{5.7}$$

while the latter one yields

$$M^A_0 \simeq 2 M_C \sqrt{\frac{1}{\pi^2 \varepsilon}}, \text{ and } M^A_{n\neq 0} \simeq 2n M_C \big[1 + \frac{1}{\pi^2 \varepsilon n^2}\big], \tag{5.8}$$

where we define $\varepsilon \equiv (4 M_C)/(\pi g_5^2 M^2)$. The two approximations generate qualitatively different mass spectra. Therefore, it is very important to determine which one is applicable to our scenario. Assuming that all the couplings of the theory enter the strong regime at the cutoff M_* we can use the result of the naive dimensional analysis [226] in higher dimensional theories that suggests $g_5^2 \simeq 24\pi^3/M_*$ and $M \simeq M_*/(4\pi)$, which gives $g_5^2 M^2 \simeq 3/2\pi M_* > M_* \gg M^A_n$. We thus choose the former approximation. Following the work of Kim and Raby [224], we introduce the

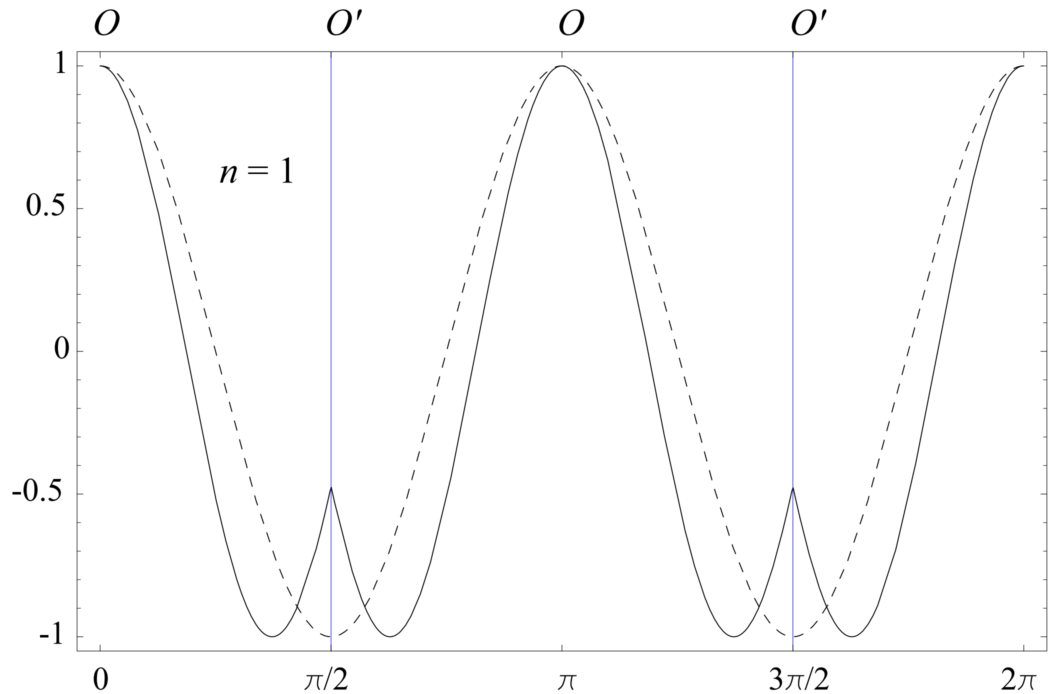


Figure 5.1: A plot of an $n = 1$ mode of the bulk field wave function profile in the fifth dimension. The dashed line represents undisturbed profile given by $\cos 2ny$. The solid line represents the profile after the perturbation due to the boundary condition is accounted for. The radius R is taken to be 1.

parameter $\zeta = 2N\varepsilon$, where $2N = M_*/M_C$ and $\zeta \simeq 8/(3\pi^2) \simeq 0.27$, to rewrite the approximate mass spectrum of the broken gauge bosons as

$$M_n^A \simeq M_C\big(2n + 1 - \frac{n}{N}\zeta\big). \tag{5.9}$$

One interesting feature to note is that the boundary condition in Eq. (5.6) is not absolute. In our case, the broken $++$ parity fields start off with the mass spectrum that mimics the spectrum of the $+-$ and $-+$ parity fields but then gradually merges with the spectrum of undisturbed $++$ and $--$ parity bulk fields as one moves up the Kaluza-Klein tower of states. One should also keep in mind that the supersymmetry ensures the same fate for the chiral partners Σ of the vector fields

V. Namely, the mass spectrum of the fields in Σ^{--} are shifted in the same manner as the states in the V^{++} that are made massive through the brane gauge breaking. With that said, we turn to the consideration of the Higgs field mass spectrum.

5.2.2 Mass Spectrum of the Higgs Fields

The missing partner mechanism affects *only* the color triplets of the bulk states with $++$ and $--$ parities. To determine their mass spectrum we concentrate on the masses of the color Higgsinos. Supersymmetry then ensures the same mass spectrum for their bosonic partners. Moreover, since there are two separate color triplet sectors as indicated by the vertical line in Eq. (4.1), we treat only one of them. The other sector will have the same mass spectrum as long as both sectors share the *same* dimensionful coupling κ. We assume this to be the case. Note that the bulk states with the $+-$ and $-+$ parities, i.e. the odd states, do not get affected by the brane breaking.

To make the discussion as transparent as possible we adopt the following notation for the triplet Higgsinos: $H_C \in \Phi^{++}_{\mathbf{5}_1^{-2}}$, $H_C^c \in \Phi^{c--}_{\overline{\mathbf{5}}_1^{2}}$, and $H_{\overline{C}_H} \in \mathbf{10}^1_H$. Their equations of motion, derived from the brane coupling term in Eq. (5.2) and the bulk action of Eq. (4.4), read [227]

$$\mathrm{i}\bar{\sigma}^\mu \partial_\mu H_{\overline{C}_H} - \kappa M \overline{H}_C|_{y=(\pi R/2,\, 3\pi R/2)} = 0, \tag{5.10}$$

$$\mathrm{i}\bar{\sigma}^\mu \partial_\mu H_C - \partial_y \overline{H}^c_C - \kappa M \overline{H}_{\overline{C}_H}\big(\delta(y-\pi R/2) + \delta(y - 3\pi R/2)\big) = 0, \tag{5.11}$$

$$\mathrm{i}\bar{\sigma}^\mu \partial_\mu H^c_C + \partial_y \overline{H}_C = 0. \tag{5.12}$$

These equations are satisfied by the following ansatz for the five-dimensional Higgsino fields on the segment $y \in [0, \pi R/2]$

$$H_C(x,y) = \frac{1}{\sqrt{\pi R}} \sum_n N_n^H h_1^{(n)}(x) \cos M_n^{H_C} y, \tag{5.13}$$

$$H^c_C(x,y) = \frac{1}{\sqrt{\pi R}} \sum_n N_n^H h_2^{(n)}(x) \sin M_n^{H_C} y, \tag{5.14}$$

and the Higgsino field localized on the hidden brane

$$H_{\overline{C}_H}(x) = \frac{1}{\sqrt{\pi R}} \sum_n N_n^H \frac{\kappa M}{M_n^{H_C}} h_2^{(n)}(x) \cos \frac{M_n^{H_C} \pi R}{2}. \tag{5.15}$$

Here, the eigenvalue equation for the effective mass, due to the nontrivial boundary condition at the hidden brane, takes the form

$$\tan \frac{M_n^{H_C} \pi R}{2} = \frac{\kappa^2 M^2}{2 M_n^{H_C}}, \tag{5.16}$$

where we define the effective KK mass via a pair of Weyl equations: $\mathrm{i}\bar{\sigma}^\mu \partial_\mu h_1^{(n)} = M_n^{H_C} \overline{h}_2^{(n)}$ and $\mathrm{i}\bar{\sigma}^\mu \partial_\mu h_2^{(n)} = M_n^{H_C} \overline{h}_1^{(n)}$.

The naive dimensional analysis [226] in the strong coupling regime yields $\kappa \simeq (24\pi^3/M_*)^{1/2}$, which implies that $\kappa^2 M^2 (\simeq g_5^2 M^2) \gg M_n^{H_C}$. In this limit, the mass spectrum of the Higgsino triplets looks, in form, exactly the same as the mass spectrum of the broken gauge fields. Namely, the mass eigenvalues of Eq. (5.16) are

$$M_n^{H_C} \simeq M_C \big(2n + 1 - \frac{n}{N} \zeta\big), \tag{5.17}$$

where we assume that $\kappa^2 M^2 = g_5^2 M^2$ for simplicity. For completeness, the normalization constant $N_n^{H_C}$ is

$$N_n^{H_C} = \left(1 + \frac{M_C \kappa^2 M^2}{\pi (M_n^{H_C})^2} \cos^2 \frac{M_n^{H_C} \pi R}{2}\right)^{-1/2}. \tag{5.18}$$

In the case of the color Higgsinos there is a mixing between the bulk and the brane fields. It is the role of the brane field $H_{\overline{C}_H}$ to give the mass to the zero mode component of H_C. As described in Ref. [227], the Weyl spinors, $h_1^{(n)}$ and $h_2^{(n)}$, pair up at every Kaluza-Klein level to obtain the Dirac mass. The remaining states in the $\mathbf{10}_H^1$ of Higgs get absorbed by the broken gauge bosons and completely disappear as far as the running is concerned. We show the mass spectrum of one part of the Higgs sector in Fig. 5.2. The other part looks exactly the same. Since this concludes the discussion on the mass spectrum of both the gauge and the Higgs fields we turn our attention towards the RGE analysis.

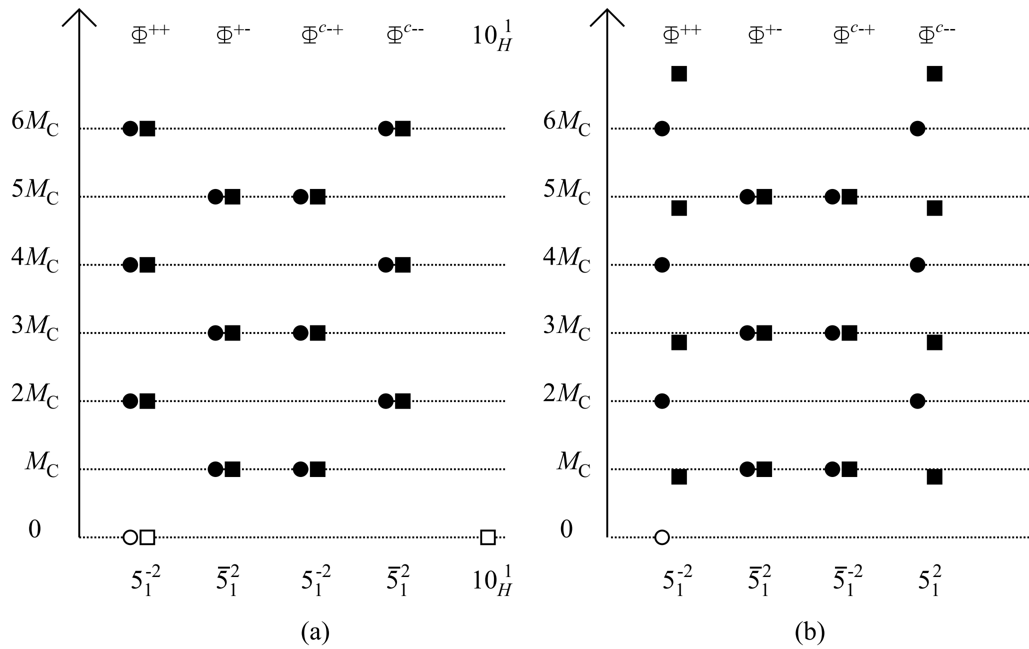


Figure 5.2: (a) A mass spectrum of the Kaluza-Klein towers of the Higgs sector after the compactification, but before the brane localized breaking. (b) The mass spectrum after the brane localized breaking. The circles represent the doublets and the squares represent the triplets.

5.3 Kaluza-Klein unification

The running of the gauge couplings in our model is the same as the running in the usual four-dimensional theory as long as we stay below the compactification scale M_C. But, once we venture over M_C, the running is affected by the towers of Kaluza-Klein states until we reach the cutoff scale M_*, which we define as the scale where effective gauge couplings merge. Since there are numerous states in the KK towers one might expect that the analysis of the threshold effects on the gauge coupling running from M_C to M_* is very difficult even at a one-loop level. This, however, is not the case as we show next.

Let us, for concreteness, limit our discussion to the five-dimensional theory that is based on the simple gauge group $\mathcal{F}$. The main simplification originates from the observation that the compactification procedure forces all the states that make up a single representation of $\mathcal{F}$ to appear within the interval $[2nM_C, 2(n+1)M_C]$ for every $n \neq 0$. [This statement is true regardless of the type of the additional brane boundary conditions we discussed in the previous two sections.] These states obviously contribute in an $\mathcal{F}$ invariant way to the running of all the gauge coupling constants after we go over $2(n+1)M_C$. Thus, the contribution of the nth Kaluza-Klein level that enters at $2nM_C$ drops out of the running of the difference of the gauge couplings after we reach $2(n+1)M_C$. In view of this fact we are motivated to pursue the differential running, i.e. the running of the difference of the gauge couplings. The previous observation also implies that the beta coefficients reset themselves to the values of the familiar coefficients of the Standard Model group $\mathcal{H}$ every time we go over another $2(n+1)M_C$ scale.

Nontrivial boundary conditions distort the spectrum of Kaluza-Klein masses. In our case, the members of the nth mode emerge at $2nM_C$, $(2n+1-\frac{n}{N}\zeta)M_C$, $(2n+1)M_C$, and $(2n+2)M_C$ energy levels. We have already concluded that from $2nM_C$ to $(2n+1-\frac{n}{N}\zeta)M_C$ the beta coefficients *must* be the coefficients of the SM group $\mathcal{H}$. We call this region I. Region II is the region from $(2n+1-\frac{n}{N}\zeta)M_C$ to $(2n+1)M_C$, while region III stretches from $(2n+1)M_C$ to $(2n+2)M_C$ for $n \neq 0$. The notation here and in what follows is exactly the same as the notation of Kim and Raby [224]. Note that we do not mention the matter fields at any point. The reason is that the matter fields of one family contribute equally to the running of the gauge couplings regardless of their origin, i.e. whether they are located in the bulk or on the brane.

As shown by Kim and Raby [224], if the compactification breaks $\mathcal{F}$ to $\mathcal{G}$ and, then, the brane breaking reduces $\mathcal{G}$ to the SM group $\mathcal{H}$, the beta coefficients[§] of the

[§] We use the notation $b \equiv (b_1, b_2, b_3)$, where b_1, b_2, and b_3 are the coefficients gauge sector are:

$$
\begin{aligned}
b^{\mathrm{I}}_{\text{gauge}} &= b^{\mathcal{H}}(V);\\
b^{\mathrm{II}}_{\text{gauge}} &= b^{\mathcal{H}}(V) + b^{\mathcal{G}/\mathcal{H}}(V) + b^{\mathcal{G}/\mathcal{H}}(\Sigma) = b^{\mathcal{G}}(V) + b^{\mathcal{G}}(\Sigma) - b^{\mathcal{H}}(\Sigma);\\
b^{\mathrm{III}}_{\text{gauge}} &= b^{\mathcal{H}}(V) + b^{\mathcal{G}/\mathcal{H}}(V) + b^{\mathcal{G}/\mathcal{H}}(\Sigma) + b^{\mathcal{F}/\mathcal{G}}(V) + b^{\mathcal{F}/\mathcal{G}}(\Sigma) = -b^{\mathcal{H}}(\Sigma).
\end{aligned} \tag{5.19}
$$

Here, we use the fact that $b^{\mathcal{F}}(V) \equiv b^{\mathcal{H}}(V) + b^{\mathcal{G}/\mathcal{H}}(V) + b^{\mathcal{F}/\mathcal{G}}(V)$ is an $\mathcal{F}$ invariant coefficient that drops out from the running of the differences of the gauge couplings. The same statement holds for $b^{\mathcal{F}}(\Sigma) \equiv b^{\mathcal{H}}(\Sigma) + b^{\mathcal{G}/\mathcal{H}}(\Sigma) + b^{\mathcal{F}/\mathcal{G}}(\Sigma)$ coefficient. $\mathcal{G}/\mathcal{H}$ and $\mathcal{F}/\mathcal{G}$ represent the appropriate coset-spaces (e.q. states that are in $\mathcal{G} \supset \mathcal{H}$ but *not* in $\mathcal{H}$ belong to $\mathcal{G}/\mathcal{H}$). Note that we always have $b(\Sigma) = -b(V)/3$ since Σ is the chiral superfield and V is the vector superfield . In our case $\mathcal{F}$ corresponds to $SO(10)$ and $\mathcal{G}$ corresponds to the flipped $SU(5)$ group.

Before we consider the beta coefficient of the Higgs sector we note the following: the beta coefficients of the *two* supersymmetric Higgs doublets (triplets) are $b(\mathbf{2}) \equiv (3/5, 1, 0)$ ($b(\mathbf{3}) \equiv (2/5, 0, 1)$). Therefore, the sum of their contributions does not affect the differential running and can be freely discarded. Moreover, as far as the differential running is concerned, we can write $b(\mathbf{2}) = -b(\mathbf{3}) = (0, 2/5, -3/5)$, where we subtract the overall constant to make $b_1 = 0$. This we do with all the other beta coefficients in what follows. Recalling that there are two Higgs sectors we can write:

$$
\begin{aligned}
b^{\mathrm{I}}_{\text{Higgs}} &= b(\mathbf{2});\\
b^{\mathrm{II}}_{\text{Higgs}} &= b(\mathbf{2}) + 2b(\mathbf{3}) = b(\mathbf{3});\\
b^{\mathrm{III}}_{\text{Higgs}} &= b(\mathbf{3}) + 2b(\mathbf{2}) + 2b(\mathbf{3}) = b(\mathbf{3}).
\end{aligned} \tag{5.20}
$$

Finally, we are ready to analyze the running at one-loop level. The relevant RGEs and all the definitions are taken from Kim and Raby [224]. We present them

associated with the gauge couplings of $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ respectively.

here for completeness of this work. The one-loop RGEs for the gauge couplings in the effective four-dimensional theory are

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_*)} + \left[b_i^{\mathcal{H}}(V) + b_i^{\mathcal{H}}(\mathbf{2}) + b_{\text{matter}}^{\mathcal{H}}\right] \ln \frac{M_C}{\mu} + \Delta_i^{\text{Higgs}} + \Delta_i^{\text{gauge}}, \tag{5.21}$$

where Δ's describe the appropriate threshold corrections of the Kaluza-Klein modes from M_C to M_*. They are given by

$$\Delta \equiv \; b^{\text{eff}} \ln \frac{M_*}{M_C} = b^{\text{I}} A_{\text{I}} + b^{\text{II}} A_{\text{II}} + b^{\text{III}} A_{\text{III}}, \tag{5.22}$$

with

$$A_{\text{I}} = \sum_{n=1}^{N-1} \ln \frac{2n+1-\frac{n}{N}\zeta}{2n}, \tag{5.23a}$$

$$A_{\text{II}} = \sum_{n=1}^{N-1} \ln \frac{2n+1}{2n+1-\frac{n}{N}\zeta}, \tag{5.23b}$$

$$A_{\text{III}} = \sum_{n=1}^{N} \ln \frac{2n}{2n-1}. \tag{5.23c}$$

Obviously, A_{I}, A_{II} and A_{III} allow us to sum over the threshold corrections from the corresponding regions.

Taking the large N limit, where $2N = M_*/M_C$, and using the approximation $\ln(1+x) = x + \cdots$, Kim and Raby obtained

$$A_{\text{I}} = \frac{1}{2}\ln 2N - \frac{1}{2}\ln\frac{\pi}{2} - \frac{\zeta}{2} + \mathcal{O}(\frac{1}{N}), \tag{5.24a}$$

$$A_{\text{II}} = \frac{\zeta}{2} + \mathcal{O}(\frac{1}{N}), \tag{5.24b}$$

$$A_{\text{III}} = \frac{1}{2}\ln 2N + \frac{1}{2}\ln\frac{\pi}{2}. \tag{5.24c}$$

This gives the following expression for the threshold corrections of the gauge and the Higgs sector:

$$\Delta = \frac{1}{2}(b^{\text{III}} + b^{I}) \ln \frac{M_*}{M_C} + \frac{1}{2}(b^{\text{III}} - b^{\text{I}}) \ln \frac{\pi}{2} + \frac{1}{2}(b^{\text{II}} - b^{\text{I}})\zeta. \tag{5.25}$$

Looking back at Eqs. (5.19) and (5.20) we have

$$\Delta^{\text{gauge}} = \frac{2}{3}b^{\mathcal{H}}(V)\ln\frac{M_*}{M_C} - \frac{1}{3}b^{\mathcal{H}}(V)\ln\frac{\pi}{2} + \frac{1}{3}\big[b^{\mathcal{G}}(V) - b^{\mathcal{H}}(V)\big]\zeta, \quad (5.26a)$$

$$\Delta^{\text{Higgs}} = -b(\mathbf{2})\ln\frac{\pi}{2} - b(\mathbf{2})\zeta. \quad (5.26b)$$

We are ready to evaluate the threshold corrections. Since $b^{\mathcal{H}}(V)$ represents the beta coefficients of the gauge sector of the MSSM we have $b^{\mathcal{H}}(V) = (0, -6, -9)$. On the other hand, $b^{\mathcal{G}}(V)$ represents the beta coefficients of the gauge sector of the supersymmetric flipped $SU(5)$: $\mathbf{24}^0 + \mathbf{1}^0$. Therefore, $b^{\mathcal{G}}(V) = (-3/5, -15, -15) \equiv (0, -72/5, -72/5)$, where we again subtract the overall constant contribution to make b_1 coefficient equal to zero. Using these results we find:

$$\Delta^{\text{gauge}} = \Big(0, -4\ln\frac{M_*}{M_C} + 2\ln\frac{\pi}{2} - \frac{14}{5}\zeta, -6\ln\frac{M_*}{M_C} + 3\ln\frac{\pi}{2} - \frac{9}{5}\zeta\Big), \quad (5.27a)$$

$$\Delta^{\text{Higgs}} = \Big(0, -\frac{2}{5}\ln\frac{\pi}{2} - \frac{2}{5}\zeta, \frac{3}{5}\ln\frac{\pi}{2} + \frac{3}{5}\zeta\Big). \quad (5.27b)$$

Our goal is to find the values of M_C and M_* that allow the exact unification, at least at one-loop level, of the gauge coupling constants at the scale M_*. To be able to do that we first recall the situation we have in the usual four-dimensional SUSY GUT. There we define M_{GUT} to be the scale where $\alpha_1(M_{\text{GUT}}) = \alpha_2(M_{\text{GUT}}) \equiv \bar{\alpha}_{\text{GUT}}$ with the running given by

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_i(M_{\text{GUT}})} + \big[b_i^{\mathcal{H}}(V) + b_i^{\mathcal{H}}(\mathbf{2}) + b^{\mathcal{H}}_{\text{matter}}\big]\ln\frac{M_{\text{GUT}}}{\mu}. \quad (5.28)$$

If we ask how far off from $\bar{\alpha}_{\text{GUT}}$ the coupling $\alpha_3(M_{\text{GUT}})$ is, and parameterize the degree of nonunification via $\epsilon_3 = \big(2\pi/\alpha_3(M_{\text{GUT}}) - 2\pi/\bar{\alpha}_{\text{GUT}}\big)$, we obtain $5 \leq \epsilon_3 \leq 6$ depending on the exact spectrum of SUSY particles. We show one example of differential running in Fig. 5.3. This example takes into the account not only the one-loop but the two-loop effects on the running of the gauge couplings. We also assume that the superpartners have masses of the order of m_t, and take the lower experimental limit $\tan\beta = 3$ [47].

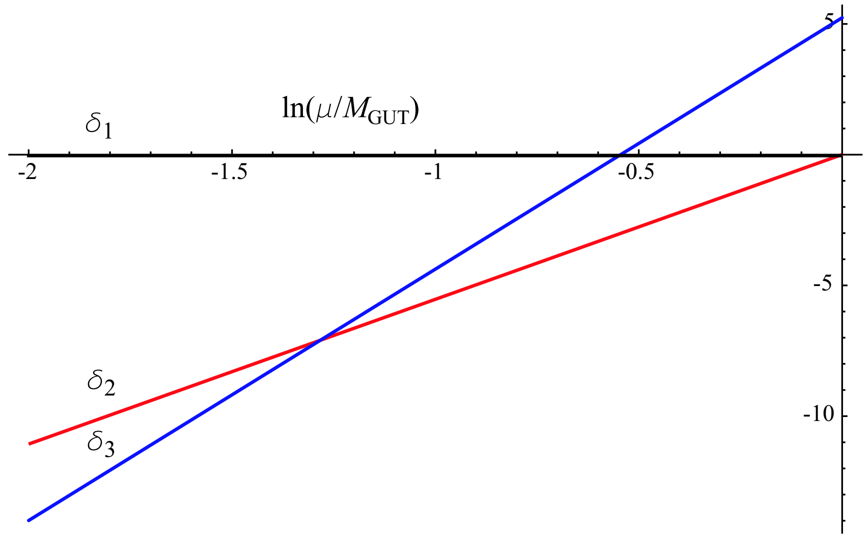


Figure 5.3: A plot of the differential running $\delta_i(\mu) = 2\pi(1/\alpha_i(\mu) - 1/\alpha_1(\mu))$ versus $\ln(\mu/M_{\rm GUT})$, where $M_{\rm GUT} = 2.37 \times 10^{16}$ GeV.

In the five-dimensional setting the deviation from the usual running starts at M_C scale. Therefore, at M_C, the left hand sides of Eqs. (5.21) and (5.28) *must* be the same. Thus, we have that

$$\begin{aligned} \delta_2(M_C) &= \left[b_2^{\mathcal{H}}(V) + b_2^{\mathcal{H}}(\mathbf{2})\right] \ln \frac{M_{\rm GUT}}{M_C} \\ &= \Delta_2^{\rm gauge} + \Delta_2^{\rm Higgs}, \\ \delta_3(M_C) &= \left(2\pi/\alpha_3(M_{\rm GUT}) - 2\pi/\bar{\alpha}_{\rm GUT}\right) \\ &\quad + \left[b_3^{\mathcal{H}}(V) + b_3^{\mathcal{H}}(\mathbf{2})\right] \ln \frac{M_{\rm GUT}}{M_C} \\ &= \Delta_3^{\rm gauge} + \Delta_3^{\rm Higgs}. \end{aligned} \tag{5.29}$$

Solving these equations yields

$$M_C \approx 5.5 \times 10^{14}\,\text{GeV}, \text{ and } M_* \approx 1.0 \times 10^{17}\,\text{GeV}, \tag{5.30}$$

where we use the same value of ϵ_3 as is used by Kim and Raby [224] ($\epsilon_3 \simeq 6$) and we take $M_{\rm GUT} = 3 \times 10^{16}$ GeV. These values imply that $N = 90$, justifying the large N

approximations. In view of our results the following picture emerges. The effective theory below the compactification scale looks exactly the same as the usual MSSM theory. Then, once we go above M_C, there emerge the towers of the Kaluza-Klein states that change the behavior of the gauge running through the set of small but numerous threshold corrections. The theory finally yields the gauge unification at $M_* > M_{\rm GUT}$ where all the couplings of the theory enter the strong regime. At that point the five-dimensional theory must be embedded into more fundamental physical picture.

We should note that our result is not very sensitive to the exact value of the small parameter ζ. On the other hand, the values of M_C and M_* depend very strongly on the value of ϵ_3. We have taken $\epsilon_3 \simeq 6$ to be able to compare our results with the analysis of Kim and Raby [224]. This value, coming from the RGE propagation of the experimental value of $\alpha_3(m_Z) = 0.118 \pm 0.003$ from the m_Z scale to the GUT scale, could be reduced by a factor of two or three in near future. Namely, the latest analysis of Erler and Langacker as presented in [229] suggests the new value to be $\alpha_3(m_Z) = 0.1221^{+0.0026}_{-0.0023}$. This would have a large impact on our result since $\epsilon_3 \simeq 3$ would imply $N = 2$, making the whole KK unification picture questionable. The model of Kim and Raby [224] might be in better shape since $\epsilon_3 \simeq 3$ in their case suggests $N = 27$.

This chapter, as promised, has been devoted to the analysis of the gauge unification. This means that there are many questions left unanswered. For example, one might ask what mechanism breaks four-dimensional $\mathcal{N} = 1$ supersymmetry. Or, how the Higgs fields responsible for the missing partner mechanism get their VEVs. Our intention was not to answer the questions like these but to demonstrate the possibility of the five-dimensional Kaluza-Klein unification and this we did.

Our result for M_C and M_* is very similar to the result obtained by Kim and

Raby [224]. This is due to the fact that the biggest correction to the standard four-dimensional running in both cases comes from the first term in Eq. (5.26a). Since this term involves the beta coefficients of the SM gauge group the leading corrections must be the same for all the schemes with the realistic low-energy signature. The main difference between the two models in the gauge sector is generated by the beta coefficients $b^{\mathcal{G}}(V)$ of the gauge group on the hidden brane. In our case the hidden brane has the flipped $SU(5)$ group with $b^{\mathcal{G}}(V) = (0, -72/5, -72/5)$, while in the case of Kim and Raby the hidden brane harbors PS gauge group with $b^{\mathcal{G}}(V) = (0, 12/5, -18/5)$. The main difference in the Higgs sector stems from the fact that there is no distinction between the region I and region II in Kim and Raby case since the additional boundary conditions do not affect the Higgs sector at all. Therefore, the second term in Eq. (5.26b) is absent in their case. It is interesting to note that the difference between the two models is in the terms that are proportional to the small parameter ζ. Therefore, the limit $\zeta \to 0$ gives the same result in both cases. In that limit we obtain $M_C \approx 3.2 \times 10^{14}$ GeV, and $M_* \approx 2.2 \times 10^{17}$ GeV. Interestingly enough, the same limit reproduces the results of the analysis on the gauge coupling unification of the five-dimensional $SU(5)$ model.

Even though the exact unification of the gauge couplings in the four-dimensional flipped $SU(5)$ cannot be excluded [228], one can never justify the charge quantization and the hypercharge assignment without embedding it into $SO(10)$. In our case this is not an issue. As long as the matter fields are placed in the bulk or on the visible brane we guarantee the charge quantization. [Of course, if the matter comes from the bulk multiplets we might lose the unification of quarks and leptons of one family.] The only *ad hoc* feature of our model is the existence of the Higgses on the hidden brane. It is difficult to justify their $U(1)$ charges unless they originate from the **16** of $SO(10)$. We argue that their $U(1)$ charges are what one expects from the fields of flipped $SU(5)$ and that they provide the anomaly cancellation on the hidden brane. The model can still produce interesting mass matrix patterns $L = D$ and $N = U$ that we discussed in Chapter 4.

5.4 Conclusion

We have presented an $SO(10)$ model in five dimensions. The model, admittedly not complete, has served to demonstrate that the exact unification of the gauge couplings is possible even in the higher dimensional setting. The corrections to the usual four-dimensional running have been due to the Kaluza-Klein towers of states. We have shown that despite the large amount of these states the corrections for the MSSM running can be unambiguously and systematically evaluated. Demanding the exact unification, the compactification scale is deduced to be $M_C \approx 5.5 \times 10^{14}$ GeV with the cutoff of the theory at $M_* \approx 1.0 \times 10^{17}$ GeV. Therefore, the five-dimensional theory exists in a rather large energy region before one needs to replace it with the more fundamental one.

The usual problems of SUSY GUTs, such as the doublet-triplet splitting problem, have been solved in a natural way. For example, the presence of the flipped $SU(5)$ symmetry on the hidden brane has allowed us to implement the missing partner mechanism. At the same time the presence of the $SO(10)$ symmetry on the visible brane still allows one to obtain desirable predictions for the quark and lepton masses such as $m_b = m_\tau$.

The model yields the low-energy signature of the MSSM. In addition, it allows for the justification of the charge quantization as long as the matter lives on the visible brane or the bulk. On the other hand, the unification of the quarks and the leptons is possible only if the matter resides on the visible brane.

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