STANDARD AND NON-STANDARD WEAK INTERACTIONS

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Thesis for the Degree of
Doctor of Philosophy

by

Miriam Leurer

Submitted to the Scientific Council of the
Weizmann Institute of Science, Rehovot
December 1985

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This work was carried out under the supervision of Professor

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To Udi and Maymon

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ABSTRACT

This work consists of three independent chapters, all of them dealing with the weak interactions.

The subject of the first chapter is left-right symmetric theories. The two main versions of these theories are discussed and compared. In addition, the $K - \bar{K}$ mixing term is analysed: It has been known for several years now that in a left-right symmetric model there are new contributions to the mixing of the kaons. However, the importance of these contributions was not clear: Though (the absolute value of) every contribution by itself is large — it could in principle happen that the various contributions will cancel each other, leaving therefore no significant effect. We show that in the most appealing left right symmetric model — the new contributions add up ***constructively***. Consequently, we may derive reliable bounds on the mass of the right-handed gauge boson and the average mass of the (unavoidable) physical Higgs scalars. We also found that the new contributions are proportional to a new CP violating phase. This phase could serve as an alternative source for CP violation if the Kobayashi-Maskawa phase fails to account for the observed ϵ value. While all previous treatments of the $K - \bar{K}$ system were limited to the *minimal* model, we succeed to show that our results hold also in the general case of nonminimal models.

The second chapter deals with the possibility that W and Z are composite. Three experimental tests are discussed: (i) Universality — if W is composite then its coupling to the fermions are expected to deviate from universality. Since such deviations were

not yet seen — we derive a lower bound on the compositeness scale. (ii) Possible enhancement of the reaction $\bar{p} + p \longrightarrow Z^0 + \gamma + any$ — we show that if Z^0 is composite then the cross section for the process $\bar{p} + p \longrightarrow Z^0 + \gamma + any$ might be considerably enhanced and this effect could be measured at CERN and Fermilab. (iii) The $ee\gamma$ events of the 1983 run in CERN — we show that in contradiction to suggestions made in several papers, these events may not be explained by a composite-Z decaying through a scalar.

In the last chapter we discuss the quark mixing angles. We suggest a new parametrization to the mixing matrix. The new parameters have simple physical meaning and they are simply and conveniently related to measurable quantities. We use this parametrization to repeat the analysis of the potential problem the standard model might have with the ϵ-parameter. The results of the analysis are very conveniently expressed in terms of bounds on the new parameters. In this chapter we also discuss the Fritzsch mass matrices and show that, presently, they are consistent with the minimal standard model.

1 Preface

Fourteen years ago, it was proved [1] that the Glashow-Weinberg-Salam (GWS) model [2] is *theoretically* self-consistent. Since then, the model, proving itself to be also in agreement with experiment, gained more and more respect, till it became known as the "standard-model".

In spite of its big success, there are still some facts which inspire people to suspect that the standard model is not the fundamental theory of the world but rather an effective low energy theory. These facts are:

(1) The Higgs-scalar becomes unnatural at energies around 1 TeV. This unpleasant feature of the scalar would have been cured if there is some underlying physics whose scale is 1 TeV or less. Technicolor theories (for example) have suggested a solution to this "scalar-problem".

(2) Parity and charge-conjugation are explicitly broken by the weak interactions of the standard-model. This feature of the model might be considered as somewhat "unaesthetic", and it leads to speculations about some underlying physics in which parity and charge conjugation are only *spontaneously* broken. The simplest extensions of the standard model which incorporate spontaneous breakdown of P and C are the "left-right-symmetric models". They are the subject of the first chapter of this work.

(3) The standard model has many fundamental particles and many free parameters. The "proliferation" of particles and parameters might hint that the standard model is not fundamental. An even stronger hint in this direction we get from the *pattern* which is observed in the spectrum: The particles are falling into three "generations", with in-

creasing masses. The mixings between the generations seem also to have some definite pattern. The standard model has no means for explaining the proliferation of particles and parameters, nor may it provide explanation for their pattern. The way is therefore open for imagining underlying theories which might explain these observations. Some of the candidate underlying theories are horizontal models, grand-unified theories and composite models. In the second chapter we discuss some of the aspects of composite models. The subject of the third (and last) chapter is the quark mixing angles and their pattern.

(4) In the last few years a possible experimental difficulty for the standard model have been discussed: It was observed that if the t-quark mass is relatively low and if b-decay rates are sufficiently slow then the model fails to explain the observed value of the CP violating parameters ϵ and ϵ'. In the framework of the third chapter we analyze this potential problem in detail. The problem is also discussed in the first chapter where we show that left-right symmetric theories have an additional CP violating phase whose contribution to ϵ is independent of the t-quark mass or b-decay rates.

(5) An important fact is our IGNORANCE: The standard model of weak interactions have been tested mainly in the low energy regime, while the "real test" is at the typical scale of the model, namely, 100 GeV. This fact used to encourage the "composite-W" people (and also supporters of other low-energy-nonstandard-models). Recently, experiment have penetrated the higher energy domain, and the W and Z have been observed and their mass measured. However this results are still preliminary and we are as yet quite ignorant of the physics of 100 GeV (for example: The mass ratio of the W and

Z is not yet accurately measured, the width of W and Z is not yet known, the Higgs particle was not observed). The composite-W models do therefore still compete with the standard model. The second chapter of this work deals with the possibility that the W and Z are composite and $SU(2)_W$ is a *global* symmetry.

(6) Nowadays the term "standard-model" refer not only to the electro-weak interactions of GWS but also to the QCD model of strong interactions. There are several problems of the model which are strongly connected to the strong interactions, like: The strong CP problem, chiral symmetry breaking and the $\Delta I = \frac{1}{2}$ rule. These problems will not be discussed here.

In this work we deal with some of the above mentioned problems of the weak interactions of the standard model. In the first chapter left-right symmetric models are discussed. The subject of the second chapter is composite W and Z and in the last chapter we discuss the quark mixing angles in the framework of the (minimal) standard model.

Every one of the three chapters is independent of the others and has its own introduction and summary. References are collected into a single list and tables, figures and appendices appear in the end.

Left-Right Symmetric Theories

1.1 Introduction

The standard model [2] though successful in all experimental tests has, from an "aesthetic" point of view, several unattractive features. One of them is the explicit breakdown of parity (P), charge conjugation (C) and CP. The left-right symmetric (LRS) models are the simplest extensions of the standard model in which parity or charge conjugation are restored [3] . In an LRS theory the Lagrangian is invariant under the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)$ and under P or C. The discrete symmetry (P or C) relates the coupling constants g_L, g_R of $SU(2)_L$, $SU(2)_R$ to each other so that $g_L = g_R$. At a high energy scale the discrete symmetry breaks down spontaneously together with the gauge symmetry, which breaks to the standard $SU(2) \times U(1)$. From this scale downward the left-right symmetric theory mimics the standard model, except for small corrections.

These small corrections are, at present, our only tool for studies of the "hidden right handed sector" of the theory. It turns out that the most important corrections are strongly dependent on the right handed quark mixing angles. Therefore, it is necessary to have some understanding of these mixings.

In 1977 Beg et al. [4] introduced the following assumption: The right handed mixing angles are equal to the left handed ones. The meaning of this assumption is that parity is conserved in the quark mass sector, i.e., the information about the spontaneous breaking of parity does not reach the quark mass matrices. Beg et al. called their model "manifest" since parity was manifested in the low lying quark spectrum. The manifest LRS model was quite popular and important calculations were done in its framework.

However we found the "manifest" model unsatisfactory for two reasons:(i) The assumption that P is unbroken in the quark mass matrices is unjustified (as we will show). (ii) In a manifest model the discrete symmetries C and CP are *explicitly* broken. Since the main motivation for LRS models is "aesthetics" — we find that models which are symmetric under both P and C are more appealing.

We thus suggest an alternative point of view to the mixing angles in LRS theories: Instead of concentrating on parity — consider charge conjugation. It turns out that in an LRS model with C-conserving Lagrangian the (tree-level) quark mass matrices are necessarily C-invariant, i.e., the information about the spontaneous breakdown of C may never reach the quark masses. Thus, the right handed mixings in such models are always related, through C, to the left handed ones (exact relations will be given later).

In section 1.2 we introduce the LRS theories in more detail with emphasis on the manifest and C-invariant models. (We call the C-invariant models "CCC" - Charge Conjugation Conserving). The $K - \bar{K}$ mixing parameter, M_{12}, will be discussed in section 1.3. Four years ago Beall, Bander and Soni [5] showed that M_{12} is very sensitive to effects of the right-handed currents. They were therefore able to derive from M_{12}

a strong lower bound on the mass of the right handed W. The computation of Beall et al. was carried out in the framework of *two-generation manifest* LRS model. Many authors [6–11] have since discussed M_{12} in various LRS models, pointing out important contributions which were neglected in the original work. We here collect all these contributions and complete them in order to get M_{12} for a CCC model. We show that $M_{12}(CCC)$ has an especially simple structure which enables us to derive interesting conclusions on phenomenology of CCC models.

1.2 Some Features of Left-Right Symmetric Models

In this section we describe the particle content of an LRS theory (subsection 1.2.1) and the transformation rules of the particles under the discrete symmetries P,C and CP (subsection 1.2.2). We then briefly review the breakdown of the gauge symmetries and give the masses of the W-bosons in terms of the parameters of the theory (subsection 1.2.3). Finally, in subsection 1.2.4 we discuss the mass matrices of the fermions and introduce the manifest and CCC models.

1.2.1 The Particle Content of a Left-Right Symmetric Model

The gauge particles of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are the three W_L's, the three W_R's and the vector-boson which couples to (B-L). After symmetry breaking the neutral vector-bosons mix to form the photon, the Z and the Z'. The charged bosons mix slightly and form the mass eigenstates W_1 and W_2

The fermions are the quarks, q, and the leptons, l. The left handed quarks, $q_L^{(0)}$, and the left handed leptons, $l_L^{(0)}$, are grouped into doublets of $SU(2)_L$ and are singlets under $SU(2)_R$. The right handed quarks, $q_R^{(0)}$, and leptons, $l_R^{(0)}$, are grouped into doublets of $SU(2)_R$ and are singlets of $SU(2)_L$. (The superscript $^{(0)}$ stands for *interaction* eigenstates, as opposed to *mass* eigenstates.)

The Higgs spectrum of an LRS model is not unique and depends on the specific model. However, every LRS model must contain at least one complex scalar Φ in the $(\frac{1}{2},\frac{1}{2}^*)_0$ representation of $SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$. This scalar is necessary because it is the only one which may provide the charged fermions (i.e. the quarks and the e, μ, τ) with nonzero mass. We need at least one *complex* Φ because a single *real* Φ gives the u-quarks and the d-quarks equal masses. Φ 's in $(\frac{1}{2},\frac{1}{2}^*)_0$ representation do not exhaust the Higgs spectrum. This is because $< \Phi >$ breaks $SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$ into $U(1) \times U(1)$ while the gauge symmetry should be broken into a single $U(1)$. There exist several suggestions for the additional Higgs fields (see e.g. [3] versus [12]). We will restrict ourselves to the class of models in which the additional Higgs fields are Δ_L and Δ_R in the $(1,0)_2$ and $(0,1)_2$ representations. The advantages of these models are that (i) They may provide a natural explanation for the smallness of the left-handed neutrino mass [12] and (ii) all the Higgs fields, Φ, Δ_L, Δ_R may be formed from fermion bilinears (i.e. $\Phi \sim q_L\overline{q_R}$ or $l_L\overline{l_R}$, $\Delta_L \sim \overline{l_L l_L}$, $\Delta_R \sim \overline{l_R l_R}$). Consequently, this Higgs spectrum is favorable from the point of view of composite models [13] .

1.2.2 Parity and Charge Conjugation in LRS Models

The P and C transformation rules are summarized in the following table:

under parity	under charge conjugation
$(W_L^{\pm})^{\mu} \leftrightarrow (W_R^{\pm})_{\mu}$	$(W_L^{\pm})^{\mu} \leftrightarrow -(W_R^{\mp})^{\mu}$
$(W_L^0)^{\mu} \leftrightarrow (W_R^0)_{\mu}$	$(W_L^0)^{\mu} \leftrightarrow -(W_R^0)^{\mu}$
$V^{\mu} \leftrightarrow V_{\mu}$	$V^{\mu} \leftrightarrow -V^{\mu}$
$q_L^{(0)} \leftrightarrow q_R^{(0)}$	$q_L^{(0)} \leftrightarrow i\sigma_2 (q_R^{(0)})^*$
$l_L^{(0)} \leftrightarrow l_R^{(0)}$	$l_L^{(0)} \leftrightarrow i\sigma_2 (l_R^{(0)})^*$
$\Delta_L^{++} \leftrightarrow \Delta_R^{++}$	$\Delta_L^{++} \leftrightarrow \Delta_R^{--}$
$\Delta_L^{+} \leftrightarrow \Delta_R^{+}$	$\Delta_L^{+} \leftrightarrow \Delta_R^{-}$
$\Delta_L^{0} \leftrightarrow \Delta_R^{0}$	$\Delta_L^{0} \leftrightarrow \Delta_R^{0}$
$\Phi \leftrightarrow \Phi^{+}$	$\Phi \leftrightarrow \Phi^{t}$

where $q_{L(R)}^{(0)}$ are the left (right) handed quark doublets; σ_2 is a Pauli matrix in the two-dimensional space of the Weyl spinors; $\Delta_{L(R)}^{++}$, $\Delta_{L(R)}^{+}$ and $\Delta_{L(R)}^{0}$ are the charge 2, 1 and 0 components of $\Delta_{L(R)}$; Φ is denoted as a 2×2 matrix:

$$\Phi = \begin{pmatrix} \Phi_1^0 & \Phi_1^+ \\ \Phi_2^- & \Phi_2^0 \end{pmatrix}$$

We note that Φ and $\tau_2 \Phi^* \tau_2$ have the same properties under the gauge group. Therefore the C and P transformation rules of Φ are not unique: Under parity Φ could transform to $\eta_P \Phi^+$ or to $\eta_P(\tau_2 \Phi^* \tau_2)^+$. ($\eta_P$ is a phase, called the "intrinsic parity" of Φ.) Similarly, under charge conjugation Φ could transform to $\eta_C \Phi^t$ or to $\eta_C(\tau_2 \Phi^* \tau_2)^t$. In order to choose the transformation rules for Φ we employed two (different and unrelated) criteria:

(i) Φ should transform like a fermion bilinear, i.e., like $q_L \overline{q_R}$ or $l_L \overline{l_R}$. This requirement is in the spirit of composite models [13] .

(ii) Consider a *minimal*, P and C invariant LRS model. (a "minimal" model has a minimal Higgs spectrum: A single Φ and only one pair of Δ fields). Every choice of the P and C transformation rules for Φ leads (through the requirement for P and C invariance) to constraints on the Yukawa couplings and hence to constraints on the mass matrices. We demand that these constraints will not lead to nonrealistic mass spectrum. (for example — we do not allow identical mass spectrum in the u and d sectors).

It turns out that every one of the two criteria singles out the transformation rules which are given in the table above.

1.2.3 Breakdown of Gauge Invariance and the Masses of the Gauge Bosons

We first discuss the case of the minimal model. The vacuum expectation values (VEV's) of the Higgs fields are:

$$< \Delta_L >= \begin{pmatrix} 0 \\ 0 \\ V_L \end{pmatrix} \quad , \quad < \Delta_R >= \begin{pmatrix} 0 \\ 0 \\ V_R \end{pmatrix} \quad , \quad < \Phi >= \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \tag{1.1}$$

These VEV's break the gauge symmetry and give the gauge bosons masses. The mass matrix of the charged gauge bosons is:

$$m^2(W^\pm) = \frac{1}{2}g^2 \begin{pmatrix} 2 \mid V_L \mid^2 + \mid k \mid^2 + \mid k' \mid^2 & -2k'k^* \\ -2kk'^* & 2 \mid V_R \mid^2 + \mid k \mid^2 + \mid k' \mid^2 \end{pmatrix} \tag{1.2}$$

Since the right-handed W should be much higher in energy than the left-handed W we find that:

$$\mid V_R \mid \gg \mid k \mid , \mid k' \mid , \mid V_L \mid$$

In order to secure the preservation of the Weinberg mass relation:

$$m(W) = m(Z) cos\theta_W$$

we must also require

$$\sqrt{\mid k \mid^2 + \mid k' \mid^2} \gg \mid V_L \mid$$

We denote the W mass eigenstates by W_1, W_2. Their masses are:

$$m^2(W_1) \cong \frac{1}{2}g^2(\mid k \mid^2 + \mid k' \mid^2)$$

$$m^2(W_2) \cong g^2 \mid V_R \mid^2$$

We denote the mass ratio $m^2(W_1)/m^2(W_2)$ by β. W_1, W_2 are equal to W_L, W_R up to small mixings:

$$W_1 \sim W_L + \xi W_R$$

$$W_2 \sim -\xi^* W_L + W_R$$

where: $\xi = \frac{kk'^*}{|V_R|^2}$

Note that $|\xi|$ is smaller than β:

$$|\xi| = \frac{|kk'^*|}{|V_R|^2} \leq \frac{1}{2}\frac{|k|^2 + |k'|^2}{|V_R|^2} \cong \beta$$

Several years ago it was believed that β could be as large as 0.1 [4] . The bound $|\xi| \leq \beta$ seemed thus insufficient since an analysis of nonleptonic K-decays gave a much stronger bound [14]: $|\xi| \leq 0.004$. It was therefore customary to assume that $|k'/k| \ll 1$ and so:

$$|\xi| = \frac{|kk'^*|}{|V_R|^2} \sim \left|\frac{k'}{k}\right| \beta \ll \beta$$

However, as we will show, β is probably not larger than 10^{-4} and consequently the bound

$$|\xi| \leq \beta$$

is satisfactory. *We therefore do not assume that* $|k'/k| \ll 1$.

The generalization to the case of a nonminimal model, where we have several Δ_L's, Δ_R's and Φ's, is straight-forward. All the equations above should be modified by the replacements:

$$| V_L(R) |^2 \longrightarrow \sum_a | V_{L(R)}^{(a)} |^2$$

$$| k |^2 + | k' |^2 \longrightarrow \sum_i (| k^{(i)} |^2 + | k'^{(i)} |^2)$$

$$kk'^* \longrightarrow \sum_i k^{(i)} k'^{(i)*}$$

where a is an index which goes over the various $\Delta_{L(R)}$ fields and i is an index for the various Φ fields.

1.2.4 Quark Mass Matrices and Introduction to the "Manifest" and "CCC" Models

This subsection is organized as follows: We first discuss the Yukawa couplings and examine the constraints imposed on them by P and C invariance. We then study $< \Phi >$ and find out which of the discrete symmetries P, C, CP is broken by this VEV. Then we introduce the "manifest" and "CCC" models and discuss the properties of the quark mass matrices in the two models. Finally, we give a short comparison of the manifest and C-conserving models. All this is done in the framework of a minimal model. The generalization to the nonminimal case is given in the end.

The Yukawa Couplings

The Yukawa interaction is:

$$L_{Yukawa} = \overline{q_L^{(0)}} A \Phi q_R^{(0)} + \overline{q_L^{(0)}} B \tau_2 \Phi^* \tau_2 q_R^{(0)} + h.c. \tag{1.3}$$

In (1.3) $q_L^{(0)}$, $q_R^{(0)}$ carry a generation index. A, B are matrices in generation space, and τ_2 is the Pauli matrix acting in the $SU(2)_L$ or $SU(2)_R$ space.

It is straightforward to show that if the Lagrangian is P-invariant then the matrices A, B are hermitian, if the Lagrangian is C-invariant then A, B are symmetric and CP invariance implies that A, B are real.

The VEV of Φ

Though the symmetry between Left and Right breaks at a very high scale (at $m(W_R)$), the information about this breaking reaches the quark mass matrices only at much lower energies (at $m(W_L)$): To tree level this information arrives only through the VEV of Φ. We should therefore check which of the Left-Right symmetries is broken by Φ. The transformation rules for Φ clearly imply that $< \Phi >$ never break C. $< \Phi >$ breaks P and CP if k, k' are not real. We note here that by an $SU(2)_L \times SU(2)_R$ gauge transformation we can always make $\frac{k'}{k}$ real. Thus, as a matter of fact, $< \Phi >$ breaks P and CP when $k \cdot k'$ is not real.

The Manifest Model

In a manifest model the Lagrangian is P invariant. P is spontaneously broken by Δ_L and Δ_R. $< \Phi >$ is *assumed* to be real. Thus in a manifest model the information about parity-breaking does not reach the quark mass matrices. These matrices are:

$$\begin{aligned} M^u &= kA + k'^* B \\ M^d &= k' A + k^* B \end{aligned} \tag{1.4}$$

In a manifest model A, B are hermitian and k, k' are real. Thus M^u, M^d are also hermitian. As we show in Appendix A, the hermiticity of M^u, M^d implies:

$$C_R = F^u(\pm) C_L F^d(\pm) \tag{1.5}$$

where C_L, C_R are the left-handed and right-handed mixing matrices; $F^u(\pm)$ and $F^d(\pm)$ are diagonal unitary matrices with eigenvalues ± 1.

Relation (1.5) is the low energy "manifestation" of the symmetry between Left and Right. This "manifestation" made the "manifest" model convenient to treat and thus very popular. However, we find that this model suffers from a serious drawback: The manifestation of parity in the quark mass matrices does not *result* from the model but is rather *assumed.* Equation (1.5) stems from the *assumption* that $< \Phi >$ does not break parity. This assumption is unjustified, since $< \Phi >$ couples to Δ_L, Δ_R and $< \Delta_L >$, $< \Delta_R >$ break parity. Moreover, $< \Delta_L >$, $< \Delta_R >$ break parity at a scale much higher than $< \Phi >$.

Note also that in a manifest model C (and CP) must be explicitly broken: Suppose C would have been conserved in the manifest Lagrangian. Then, in the quark mass matrices both P and C would have been conserved (since by assumption $< \Phi >$ conserves P and it always conserves C). We then end up with CP invariant mass matrices. This result is clearly incompatible with experiment. We therefore conclude that in the manifest model C and CP are explicitly broken.

The CCC Models

In a C-conserving LRS model C is spontaneously broken by $< \Delta_L >$, $< \Delta_R >$. However, the information about C breaking never reaches the tree level quark mass matrices, since $< \Phi >$ may not break C. The Yukawa couplings A, B are symmetric and thus, the mass matrices M^u, M^d are also symmetric. As we show in Appendix A the symmetry of M^u, M^d implies:

$$C_R = F^u C_L^* (F^d)^* \tag{1.6}$$

where F^u, F^d are diagonal unitary matrices. It is actually possible to find particular phase conventions in which the relation between C_L and C_R is even simpler:

$$C_R = C_L^*$$

The relation (1.6) is the low energy manifestation of a symmetry between left and right. This manifested symmetry is not parity but charge conjugation.

We distinguish two kinds of CCC models which we call CCC_1 and CCC_2: The CCC_1 models have a higher degree of symmetry: The CCC_1 Lagrangian is invariant under P and C (and under CP). We note that for these models we must assume that $< \Phi >$ *does break P* (otherwise the mass matrices are CP invariant). This assumption (which we find to be perfectly reasonable), is the *opposite* to the manifest model assumption. The CCC_2 models have C-invariant Lagrangian but P is explicitly broken. From the point of view of "aesthetics" these models are no better than the "manifest" model, however CCC_2 models are interesting as possible effective theories of grand unified models [15]. Consider for example an SO(10) theory. One of the possible chains of spontaneous symmetry breaking is:

$$SO(10) \to \ldots \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1) \to \ldots \to SU(3)_C \times SU(2)_L \times U(1)$$

The $SU(2)_L$ and $SU(2)_R$ are related to each other through a discrete symmetry which is included in SO(10). This discrete symmetry may actually be identified as charge conjugation. As for parity — this symmetry is not included in the SO(10) group and may even be explicitly violated in the Lagrangian.(Actually, in order to account for the observed baryon asymmetry in the universe it is preferable that parity is explicitly violated [15]). We therefore find that the low lying effective LRS theory of SO(10) is necessarily C-invariant but it may be P-violating.

We favor the CCC_1 models because of their nice symmetric feature. In our papers the term "CCC model" refers actually only to the CCC_1 models. In table 1 we summarize the properties of the manifest and CCC models. Note that with respect to the quark mass matrices C plays a more important role than P: If the Lagrangian is C invariant then *automatically* C will leave its traces in the mass matrices; if there is P invariance then *only under special and unjustified assumption*, P manifests itself in the quark mass matrices. We thus find CCC models to be more attractive than the "manifest" model.

1.3 $K^0 - \bar{K}^0$ Mixing in Left Right Symmetric Theories

The various contributions to the $K - \bar{K}$ mixing term M_{12}, were computed in the last five years in the framework of different LRS models. Here we present the leading contributions to M_{12} in the manifest and CCC models and discuss the relative importance of each contribution. It turns out to be most convenient to express M_{12} in terms of M_{12}(standard-model). We therefore start with a review of M_{12} in the standard model and then turn to the discussion of M_{12}(manifest) and M_{12}(CCC).

1.3.1 M_{12} in the Minimal Standard Model

two-generation-case:

The original computation of M_{12} was carried out by Gaillard and Lee [16] . They considered the Feynman diagrams of figure 1.1:

The relevant interactions are:

$$\frac{g}{\sqrt{2}}\bar{U}\gamma_\mu\frac{1}{2}(1-\gamma_5)CD(W^+)^\mu + h.c. \tag{1.7}$$

where g is the weak-interaction coupling constant

U are the physical (mass eigenstates) up quarks: $U = \binom{u}{c}$

D are the physical (mass eigenstates) down quarks: $D = \binom{d}{s}$

C is the Cabibbo mixing matrix.

In the course of the computation the following approximations were introduced:

(i) External momenta are neglected in the internal line propagators.

(ii) Terms of second order in the small quantities $(\frac{m_c}{m(W)})^2$, $(\frac{m_u}{m_c})^2$ are neglected (m_u, m_c are the masses of the u, c quarks and m(W) is the mass of the W-boson). The result is presented in terms of an effective interaction Lagrangian:

$$L^{int}_{eff} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2\left(\frac{m_c}{m(W)}\right)^2 \bar{d}_L\gamma^\mu s_L\bar{d}_L\gamma_\mu s_L \tag{1.8}$$

L^{int}_{eff} is related to M_{12} through:

$$-\langle K^\circ \mid L^{int}_{eff} \mid \bar{K}^\circ\rangle = (M^2)_{12} = 2m_K M_{12} \tag{1.9}$$

where m_K is the kaon mass.

In the computation of the matrix element $-\langle K^\circ \mid L^{int}_{eff} \mid \bar{K}^\circ\rangle$ we actually take into account the nonperturbative QCD effects which bind the s and $\bar{d}$ quarks into a K°. In order to estimate this matrix element Gaillard and Lee introduced the so called "vacuum insertion approximation": In this approximation one sums up all four Fiertz transformations of the operator $\bar{d}_L\gamma^\mu s_L\bar{d}_L\gamma_\mu s_L$ and then, for every one of the four operators, only the contribution of one intermediate state, the vacuum, is taken into account (the contribution of the vacuum is computed through PCAC relations). Though the vacuum insertion approximation seem to be very crude it turns out that it does give a correct order of magnitude estimate. We will therefore use this approximation through all the following discussion. (For arguments which justify this approximation for LRS models see [5],[7].) Using the vacuum insertion approximation one gets:

$$\langle K^\circ \mid \bar{d}_L\gamma^\mu s_L\bar{d}_L\gamma_\mu s_L \mid \bar{K}^\circ\rangle = \frac{2}{3}f_K^2 m_K^2 \qquad (1.10)$$

where f_K is the K decay constant. Equations (1.8), (1.9) and (1.10) imply:

$$M_{12} = \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}\left(cos\theta_c sin\theta_c\right)^2\left(\frac{m_c}{m(W)}\right)^2\frac{2}{3}f_K^2 m_K \qquad (1.11)$$

M_{12} is related to Δm_K, the K-mass difference, through:

$$\Delta m_K = 2Re\ M_{12} \qquad (1.12)$$

Substituting the experimental values for G_F ($\sim 1.1\cdot 10^{-5}\ GeV^{-2}$), $\alpha(1/137)$, $sin^2\theta_W$($\sim$ 0.22), $sin\theta_c$($\sim$ 0.22), m_c($\sim$ 1.5 GeV), m(W) ($\sim$ 82 GeV) and f_K($\sim$ 165 MeV) we find that the minimal two-generation standard model predicts: $\Delta m_K/m_K \sim 0.57\cdot 10^{-14}$.

This result is in reasonably good agreement with the experimental value: $\Delta m_K/m_K \sim 0.71 \cdot 10^{-14}$.

Note that $\Delta m_K/m_K$ is extremely small. The success of the Gaillard-Lee computation in giving the correct tiny M_{12} is due to the GIM mechanism: this mechanism is responsible for three suppression factors in (1.7) . These are: α (which appears in addition to G_F because the process is of *fourth* order in the weak interaction), $(cos\theta_c sin\theta_c)^2$ and $(m_c/m(W))^2$.

We point out that there is a kind of inter-relation between the last two suppression factors: when we will consider the three-generation case there will be new terms. In such a new term the factor $(m_c/m(W))^2$ may, e.g., be replaced by $(m_t/m(W))^2$ (which is much larger than $(m_c/m(W))^2$). However, in this case the mixing factor $(cos\theta_c sin\theta_c)^2$ will be replaced by the mixing factor of the t-quark (which is much smaller than $(cos\theta_c sin\theta_c)^2$). The net result will be that the new additional term will not be larger than the original Gaillard-Lee term.

The computation of equation (1.8) was carried out in the 't Hooft-Feynman gauge and thus, the contribution of the unphysical charged Higgs should have also been taken into account. The diagrams involving the neutral Higgs are shown in figure 1.2 and the relevant interaction terms are:

$$\frac{g}{\sqrt{2}}\frac{1}{m(W)}\bar{U}\left[\hat{M}^u C\frac{1}{2}(1-\gamma_5) - C\hat{M}^d\frac{1}{2}(1+\gamma_5)\right]D\phi^{(+)} + h.c. \qquad (1.13)$$

where $\hat{M}^u, \hat{M}^d$ are the diagonal mass matrices and $\phi^{(+)}$ is the unphysical Higgs. Note that the unphysical Higgs coupling to quarks is suppressed (relative to the W-coupling) by $m_q/m(W)$ (m_q is the mass of one of the quark fields of the corresponding vertex). Therefore the contribution of diagram 1.2(c) to M_{12} is $O\left((m_q/m(W))^4\right)$ and it is clearly negligible in the two-generation case. As for diagrams 1.2(a) and 1.2(b): the Higgs coupling introduces one factor of $(m_q/m(W))^2$. It turns out that the *left-handed* character of the W-coupling together with *GIM mechanism* introduces another factor of $(m_q/m(W))^2$. Thus the whole contribution of the unphysical Higgs particles is $O\left((m_q/m(W))^4\right)$ and may be neglected. Obviously, if a *right-handed* W is taken into account or if the three generation case is considered, the unphysical Higgs contribution should be reconsidered.

We denote M_{12} of Gaillard and Lee by $M_{12}(G-L)$. Since $2M_{12}(G-L)$ is so successful in estimating Δm_K we will often approximate Δm_K by $2M_{12}(G-L)$.

Finally, we add a remark on QCD corrections: According to [17] short range corrections do not introduce significant effects. [1] [2] In the following we will not take into account these effects. We expect such an approximation to be valid within a factor of ~ 2.

[1] For a discussion of long range corrections see [18].

[2] Short range QCD corrections in LRS models are mentioned in [5] and discussed in great detail in a recent paper by Ecker and Grimus [19].

Three Generation Case

In order to discuss the three generation case — we make use of a specific parametrization of the generalized Cabibbo mixing matrix which we introduce in the third chapter of this thesis:

$$C = \begin{pmatrix} c_{1,2}c_{1,3} & s_{1,2}c_{1,3} & s_{1,3}e^{-i\delta} \\ -s_{1,2}c_{2,3} - c_{1,2}s_{2,3}s_{1,3}e^{i\delta} & c_{1,2}c_{2,3} - s_{1,2}s_{2,3}s_{1,3}e^{i\delta} & s_{2,3}c_{1,3} \\ s_{1,2}s_{2,3} - c_{1,2}c_{2,3}s_{1,3}e^{i\delta} & -c_{1,2}s_{2,3} - s_{1,2}c_{2,3}s_{1,3}e^{i\delta} & c_{1,2}c_{1,3} \end{pmatrix}$$

where: $s_{i,j} = sin\theta_{i,j}$, $c_{i,j} = cos\theta_{i,j}$, and all $\theta_{i,j}$ are between 0 and $\pi/2$. $\theta_{i,j}$ is the mixing angle of the ith and jth generations.

$\theta_{1,2}$ is actually the Cabibbo angle θ_c. The angles $\theta_{2,3}$ and $\theta_{1,3}$ are determined from b-decay. The present experimental situation [20–22] implies [23–24] that $s_{2,3} < 0.065$ (since $\tau_b \sim 1\ ps$) and $s_{1,3} < 0.0087$ (since $\tau_b \sim 1ps$ and $R(b \to u) < 0.04$). For our purposes we may approximate (this approximation was originally proposed by L. Wolfenstein [25]):

$$C \cong \begin{pmatrix} 1 & s_{1,2} & s_{1,3}e^{-i\delta} \\ -s_{1,2} & 1 & s_{2,3} \\ s_{1,2}s_{2,3} - s_{1,3}e^{i\delta} & -s_{2,3} & 1 \end{pmatrix}$$

the computation of M_{12} in the three-generation case involves reconsideration of all the diagrams of figures 1.1,1.2. The final result is [26]:

$$
\begin{aligned}
M_{12} =& \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}\frac{2}{3}f_K^2 m_K \\
&\{s_{1,2}^2 x_c + \\
&+ 2s_{1,2}s_{2,3}(s_{1,2}s_{2,3} - s_{1,3}e^{-i\delta}) \cdot x_c \left[ln\left(\frac{x_t}{x_c}\right) + \frac{3}{4}\frac{x_t}{1-x_t}\left(\left(\frac{x_t}{1-x_t}\right) ln\left(\frac{1}{x_t}\right) - 1\right)\right] \\
&+ s_{2,3}^2(s_{1,3}s_{2,3} - s_{1,3}e^{-i\delta})^2 x_t \left[1 - \frac{3}{4}\frac{x_t(1+x_t)}{(1-x_t)^2} + \frac{3}{2}\frac{x_t^2}{(1-x_t)^3} ln\left(\frac{1}{x_t}\right)\right]\Big\}
\end{aligned}
\tag{1.14}
$$

where x_c is $(m_c/m(W))^2$ and x_t is $(m_t/m(W))^2$.

The first term in the curly bracket is the Gaillard-Lee term, the other terms arise from the presence of the third generation.

While in the two generation case M_{12} is real — in the case of three generations an imaginary, CP violating part, appears. We now discuss the real and imaginary parts of M_{12}:

ReM_{12}: It is straightforward to verify that the contribution of the third generation to the real part of M_{12} (or: to Δm_K) is not significant: if m_t is around 45 GeV [27] then the strong constraints on $s_{2,3}, s_{1,3}$ imply that the t-quark contribution is not larger than ~15%.

ImM_{12}: Contrary to the real part of M_{12} — the imaginary part (in the case of the *standard* model) is totally due to the presence of the third generation [28]. ImM_{12} is related to the CP-violating parameter ϵ through:

$$\epsilon = e^{i\frac{\pi}{4}}\left(\frac{ImM_{12}}{\sqrt{2}\Delta m_K} + \frac{ImA_o}{\sqrt{2}ReA_o}\right) \tag{1.15}$$

where A_0 is the K-decay amplitude to two pions coupled to zero-isospin. In the minimal standard model it is possible to show that, to first approximation:

$$\epsilon \sim e^{i\frac{\pi}{4}} \left(\frac{ImM_{12}}{\sqrt{2}\Delta m_K} \right) \tag{1.16}$$

(1.14) and (1.16) imply that ϵ is proportional to $s_{2,3}s_{1,3}sin\delta$ and that ϵ gets larger values as m_t increases. We thus find that if further measurements of b-decay will give stronger bounds on $s_{2,3}, s_{1,3}$ and if m_t is indeed around 45 GeV (or less) then ImM_{12} may turn out to be too small [3] , i.e., ImM_{12} will provide an ϵ value which is substantially smaller than the experimentally observed ϵ. We will return to this point when we discuss ϵ in the LRS model.

1.3.2 M_{12} in the Minimal LRS Model

Two-Generation Case:

The contributions to M_{12} which we take into account are of lowest order in the weak interaction and of zero or first order in β. These contributions include the W-W box diagram, the unphysical Higgs contribution, the tree diagrams of the neutral physical Higgs particles and the box diagrams involving the charged physical Higgs.

(i) The W-W box diagram [5]:

This contribution involves the diagrams of figure 1.3. The corresponding interactions

[3] For a more detailed discussion see section 3.3.

are:

$$\frac{g}{\sqrt{2}}\left(\bar{U}\gamma_\mu\frac{1}{2}(1-\gamma_5)C_L D(W_L^+)^\mu + \bar{U}\gamma_\mu\frac{1}{2}(1+\gamma_5)C_R D(W_R^+)^\mu\right) + h.c. \tag{1.17}$$

were C_R is given by (1.5) for the manifest model and by (1.6) for the CCC model. We note here that the $W_L - W_R$ mixing may be safely neglected. It turns out that all contributions to M_{12} which involve ξ are *either* of second order in ξ (and consequently of second order in β) *or* they are suppressed both by a power of ξ and by one or two powers of $m_s/m(W)$, $m_d/m(W)$. We will therefore completely ignore ξ through all our discussion of M_{12}. Ignoring ξ, we identify the diagram 1.3(a) with diagram 1.1, and the first term of the interaction (1.17) with (1.7) .

We find [5]:

$$M_{12}^{(W-W)}(\text{manifest}) = \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2 \cdot \left(\frac{m_c}{m(W_1)}\right)^2 \frac{1}{m_K}$$
$$\left[\langle K^\circ \mid (\bar{d}_L\gamma_\mu s_L)^2 \mid \bar{K}^\circ\rangle \mp \beta\cdot 2\cdot 4(ln\left(\frac{m(W_1)}{m_c}\right)^2 - 1)\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle\right] \tag{1.18}$$

where the $\mp$ sign is determined by the relative sign of the two phases in $F^d(\pm)$.

For the CCC model we get [29]:

$$M_{12}^{(W-W)}(\text{CCC}) = \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2 \cdot \left(\frac{m_c}{m(W_1)}\right)^2 \frac{1}{m_K}$$
$$\left[\langle K^\circ \mid (\bar{d}_L\gamma_\mu s_L)^2 \mid \bar{K}^\circ\rangle - \beta e^{i\gamma}\cdot 2\cdot 4(ln\left(\frac{m(W_1)}{m_c}\right)^2 - 1)\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle\right] \tag{1.19}$$

where γ is the difference between the two phases in F^d.

The first term in the square brackets is the usual Gaillard-Lee term. The second term arises from the contributions of diagrams 1.3(b), 1.3(c). As expected, this contribution is suppressed by one power of β. However, there are also several enhancement factors:(i) A factor of 2 since two diagrams are contributing. (ii) a factor of $4(ln(m(W_1)/m_c)^2 - 1) \sim 28$. This factor arises from the loop integration. (iii) An additional enhancement from the matrix element:

$$\frac{1}{m_K}\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle = \frac{1}{2}\left[\left(\frac{m_K}{m_s + m_d}\right)^2 + \frac{1}{6}\right] f_K^2 m_K \qquad (1.20)$$

Substituting $m_K = 498\ MeV$, $m_s = 150\ MeV$, $m_d = 7\ MeV$ we find that the matrix element of (11) is enhanced by a factor of ~7.6 relative to (3). Altogether the enhancement factor of the second term amounts to ~430. Equations (1.18) and (1.19) therefore imply:

$$M_{12}^{W-W}(\text{manifest}) = M_{12}(G-L)(1 \pm 430\beta) \qquad (1.21)$$

$$M_{12}^{W-W}(\text{CCC}) = M_{12}(G-L)\cdot(1 - 430\beta e^{i\gamma}) \qquad (1.22)$$

We wish to point out the following interesting point: We mentioned above that the success of the Gaillard-Lee computation in achieving the correct tiny M_{12} is due to the GIM mechanism: GIM is responsible for the suppression factors α, $(cos\theta_c sin\theta_c)^2$ and $(m_c/m(W))^2$. We see that the $W_1 - W_2$ box diagrams are also suppressed by these three factors. However, the detailed computation shows that the origin of the $(m_c/m(W))^2$ factor in this case is not the GIM mechanism. The $(m_c/m(W))^2$ factor comes from the propagators of the internal quarks and it is due to the fact that every internal quark line couples at one end to a left-handed vertex and at the other end to a right-handed vertex.

(ii) The unphysical Higgs contribution

There are two unphysical charged Higgs fields ϕ_1^+, ϕ_2^+ which are to be "eaten up" by W_1^+, W_2^+, respectively. All contributions of ϕ_2^+ to M_{12} are of second order in β: the propagator of ϕ_2^+ introduces one β-factor and its coupling to the fermions introduces another β. Therefore we will not take ϕ_2^+ into consideration. As for ϕ_1^+ — we may identify it with the unphysical Higgs ϕ^+ of the standard model. Its interactions with the fermions are (up to negligible corrections) given in (1.14) . The diagrams involving ϕ_1^+ are described in figure 1.4. The diagrams 1.4(a)–1.4(c) are identical to 1.2(a)–1.2(c). Therefore, as discussed above, we may neglect their contribution in the two-generation case. The contribution of diagrams 1.4(d) and 1.4(e) was computed by Mohapatra, Senjanovic and Tran [10]. According to them it amounts to:

$$\begin{aligned} M_{12}^{(\phi)}(\text{manifest}) &= \mp\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2\cdot\left(\frac{m_c}{m(W_1)}\right)^2\cdot\beta e^{i\gamma}\cdot 2ln(\frac{1}{\beta}) \\ &\cdot\frac{1}{m_K}\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle \\ &= \mp M_{12}(G-L)\beta\cdot 15\cdot ln(\frac{1}{\beta}) \end{aligned} \tag{1.23}$$

$$\begin{aligned} M_{12}^{(\phi)}(CCC) &= -\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2\cdot\left(\frac{m_c}{m(W_1)}\right)^2\cdot\beta e^{i\gamma}\cdot 2ln(\frac{1}{\beta}) \\ &\cdot\frac{1}{m_K}\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle \\ &= -M_{12}(G-L)\beta e^{i\gamma}\cdot 15\cdot ln(\frac{1}{\beta}) \end{aligned} \tag{1.24}$$

(iii). The neutral Higgs contribution:

In a minimal LRS model there are four neutral complex scalars (Δ_L^0, Δ_R^0, Φ_1^0, Φ_2^0) or eight real neutral scalars. Out of these eight scalars — two are unphysical, their couplings to the quarks are diagonal and consequently they do not contribute to M_{12}.

The other six scalars are all physical. Four of the physical scalars may not contribute to M_{12}: Three do not couple to quarks at all (they originate from the Δ-fields) and the fourth couples diagonally to quarks (this is the neutral physical Higgs of the standard model). The last two Higgs fields we combine to a single complex scalar, H^0. The coupling of H^0 to the d-quarks is:

$$\frac{g}{\sqrt{2}}\frac{1}{m(W_1)}Q_H\bar{D}_L\left[-C_L^{+}\hat{M}^u C_R+\frac{2kk'^*}{|k|^2+|k'|^2}\hat{M}^d\right]D_R H^o + h.c. \tag{1.25}$$

where $Q_H = \frac{|k|^2+|k'|^2}{|k|^2-|k'|^2}$ (Note that Q_H is well defined: $|k| \neq |k'|$ since otherwise the mass spectrum in the u-quark sector becomes equal to the mass spectrum in the d-sector. Clearly: $Q_H \geq 1$. We assume that Q_H is O(1).

The natural value for the mass of H^0 is around $m(W_2)$ (for a discussion of this point see [11]). Therefore we will consider H^0 contributions only to first order in $\beta_{H^0} = m^2(W_1)/m^2(H^0)$. H^0 contributes to M_{12} through a *second* order tree diagram (see figure 1.5). For simplicity we assume that the masses of the two real components of H^0 are equal. Then, we get [10] [11]:

$$\begin{aligned}
&M_{12}^{(H^0)}(\text{manifest}) = \\
&= \mp\frac{1}{2}\frac{G_F}{\sqrt{2}}(cos\theta_c sin\theta_c)^2\cdot\left(\frac{m_c}{m(W_1)}\right)^2\beta_{H^0}\cdot 4Q_H^2\frac{1}{m_K}\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle \\
&= \mp M_{12}(G-L)\left(\beta_{H^0}\cdot Q_H^2\cdot\frac{16\pi sin^2\theta_W}{\alpha}\cdot\frac{3}{4}\left[\left(\frac{m_K}{m_s+m_d}\right)^2+\frac{1}{6}\right]\right) \\
&= \mp M_{12}(G-L)\left(\beta_{H^0}\cdot Q_H^2\cdot 11,600\right)
\end{aligned} \tag{1.26}$$

$$
\begin{aligned}
&M_{12}^{(H^0)}(\mathrm{CCC}) = \\
&= -\frac{1}{2}\frac{G_F}{\sqrt{2}}(cos\theta_c sin\theta_c)^2 \cdot \left(\frac{m_c}{m(W_1)}\right)^2 \beta_{H^0} \cdot e^{i\gamma} \cdot 4Q_H^2 \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle \\
&= -M_{12}(G-L) \left(\beta_{H^0} \cdot e^{i\gamma} \cdot Q_H^2 \cdot \frac{16\pi sin^2\theta_W}{\alpha} \cdot \frac{3}{4}\left[\left(\frac{m_K}{m_s + m_d}\right)^2 + \frac{1}{6} \right] \right) \\
&= -M_{12}(G-L) \left(\beta_{H^0} \cdot e^{i\gamma} \cdot Q_H^2 \cdot 11{,}600 \right)
\end{aligned}
\tag{1.27}
$$

The large enhancement factor of 11,600 arises because the H^0-contribution is of *second* order in the weak interaction. We note that if the two real components of H^0 are not equal, then β_{H^0} should be replaced by a number which is between β_{H_1} and β_{H_2} (where H_1, H_2 are the two components of H^0).

(iv) The charged physical Higgs contribution:

In a minimal LRS model there exist four singly-charged Higgs fields (Δ_L^+, Δ_R^+, Φ_1^+, Φ_2^+). Out of these four — two are unphysical and their contribution was discussed above. Two Higgs fields are physical. One of them does not couple to quarks at all and therefore may not contribute to M_{12}. The other charged Higgs we denote by H^+. Its couplings to fermions are:

$$
\begin{aligned}
\frac{g}{\sqrt{2}}\frac{1}{m(W_1)}Q_H \cdot \Big\{ &\bar{U}_L \left[-\hat{M}^u C_R + \frac{2kk'^*}{|k|^2+|k'|^2} C_L \hat{M}^d \right] D_R + \\
&+\bar{U}_R \left[-\frac{2kk'^*}{|k|^2+|k'|^2} \hat{M}^u C_L + C_R \hat{M}^d \right] D_L \Big\} H^+ + h.c.
\end{aligned}
\tag{1.28}
$$

H^+ contributes to M_{12} through the box diagrams of figure 1.6. Its contribution [10] is given by:

$$
\begin{aligned}
&M_{12}^{(H^+)}(\text{manifest}) = \\
&= \mp\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2\cdot\left(\frac{m_c}{m(W_1)}\right)^2\beta_{H^+}\cdot 2\cdot ln\left(\frac{1}{\beta_{H^+}}\right)Q_H^2\cdot \\
&\qquad\qquad \frac{1}{m_K}\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle = \\
&= \mp M_{12}(G-L)\cdot\beta_{H^+}\cdot 15 ln\left(\frac{1}{\beta_{H^+}}\right)Q_H^2
\end{aligned}
\tag{1.29}
$$

$$
\begin{aligned}
&M_{12}^{(H^+)}(\text{CCC}) = \\
&= -\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}(cos\theta_c sin\theta_c)^2\cdot\left(\frac{m_c}{m(W_1)}\right)^2\beta_{H^+}\cdot e^{i\gamma}\cdot 2\cdot ln\left(\frac{1}{\beta_{H^+}}\right)Q_H^2\cdot \\
&\qquad\qquad \frac{1}{m_K}\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ\rangle = \\
&= -M_{12}(G-L)\cdot\beta_{H^+}\cdot e^{i\gamma} 15 ln\left(\frac{1}{\beta_{H^+}}\right)Q_H^2
\end{aligned}
\tag{1.30}
$$

where $\beta_{H^+} = m^2(W_1)/m^2(H^+)$

For simplicity we will assume from now on that $\beta_{H^\circ} = \beta_{H^+} = \beta_H$.

Discussion of M_{12} in Minimal LRS Theories:

We collect all the contributions to M_{12} and get:

$$M_{12}(\text{manifest}) = M_{12}(G-L) \cdot \left\{1 \mp \left[430\beta + 15\beta ln\left(\frac{1}{\beta}\right) + Q_H^2 \cdot \left(11,600\beta_H + 15\beta_H ln(\frac{1}{\beta_H})\right)\right]\right\} \tag{1.31}$$

$$M_{12}(\text{CCC}) = M_{12}(G-L) \cdot \left\{1 - e^{i\gamma}\left[430\beta + 15\beta ln\left(\frac{1}{\beta}\right) + Q_H^2 \cdot \left(11,600\beta_H + 15\beta_H ln(\frac{1}{\beta_H})\right)\right]\right\} \tag{1.32}$$

The first term in the curly brackets is the old Gaillard Lee term. Note that all the other new contributions have the the same phase and thus they all add up *constructively*. (this fact was pointed out by Ecker et al [19] and independently by us [30].)

We denote $M_{12}(manifest)$ and $M_{12}(CCC)$ together by $M_{12}(LRS)$. The phenomenology of the $K - \bar{K}$ system tells us that M_{12}(LRS) is almost real and that $|M_{12}(LRS)| \sim \frac{1}{2}\Delta m_K$. As mentioned above, $M_{12}(G-L)$ is also of the order of $\frac{1}{2}\Delta m_K$. Thus, we find that the absolute value of the sum of all new contributions to M_{12}(LRS) is of the order of $\frac{1}{2}\Delta m_K$ or less:

$$M_{12}(G-L) \cdot \left| \left(430\beta + 15\beta ln\left(\frac{1}{\beta}\right) + Q_H^2 \cdot \left(11,600\beta_H + 15\beta_H ln\left(\frac{1}{\beta_H}\right)\right)\right) \right| \leq \leq \frac{1}{2}\Delta m_K \sim M_{12}(G-L) \tag{1.33}$$

(1.33) readily implies [5], [29] :

$430\beta \leq 1$ or: $m(W_2) \geq 1.7\ TeV$

and [19], [30]:

$11,600\beta_H \leq 1$ or: $m(H) \geq 8.8\ TeV$

These results hold for both the manifest and CCC models.

Consider now CP violation: Clearly in the two generation case CP is not violated in M_{12}(manifest) but it is broken in $M_{12}(CCC)$. This is due to the phases of F^u and F^d. For the ϵ parameter in the CCC model we use the approximation:

$$\epsilon \sim e^{i\frac{\pi}{4}} \frac{ImM_{12}}{\sqrt{2}\Delta m_K}$$

(some arguments which justify this approximation were given by Chang in [8]). We find [8–9], [19], [29–30] :

$$\begin{aligned}\epsilon \sim e^{i\frac{\pi}{4}} \Bigg\{ -sin\gamma \Bigg[430\beta + 15\beta ln\left(\frac{1}{\beta}\right) + \\ + Q_H^2 \left(11,600\beta_H + 15\beta_H ln\left(\frac{1}{\beta_H}\right)\right)\Bigg] \cdot \Bigg\} \frac{M_{12}(G-L)}{\sqrt{2}\Delta m_K} \cong \\ \cong e^{i\frac{\pi}{4}} \Bigg\{ -\frac{1}{2\sqrt{2}} sin\gamma \Bigg[430\beta + 15\beta ln\left(\frac{1}{\beta}\right) + Q_H^2\left(11,600\beta_H + 15\beta_H ln\left(\frac{1}{\beta_H}\right)\right)\Bigg]\Bigg\}\end{aligned} \tag{1.34}$$

three-generation case

For convenience we will concentrate on the case of the CCC model. In the end we will comment on the case of the manifest model.

We introduce the following notation

$$\lambda_i = (C_L)^*_{i,d}(C_L)_{i,s}$$

$$\lambda'_i = (C_L)^*_{i,d}(C_L)^*_{i,s}e^{i\alpha_i}$$

where i = u,c,d and α_i is the i'th phase in the diagonal matrix F^u.

Following Mohapatra, Senjanović and Tran [10] we denote

$$I_1(x_i,x_j) = \frac{x_i ln(\frac{1}{x_i})}{(x_i-x_j)(1-x_i)} + (i \leftrightarrow j)$$

$$I_2(x_i,x_j,\beta) = -\frac{x_i^2 ln(\frac{1}{x_i})}{(x_i-x_j)(1-x_i)} + (i \leftrightarrow j) + ln(\frac{1}{\beta})$$

$$J_1(x_i,x_j) = -\frac{1}{(1-x_i)(1-x_j)} + \left\{ \frac{x_i ln(\frac{1}{x_i})}{(x_i-x_j)(1-x_i)^2} + (i \leftrightarrow j) \right\}$$

$$J_2(x_i,x_j) = \frac{1}{(1-x_i)(1-x_j)} - \left\{ \frac{x_i^2 ln(\frac{1}{x_i})}{(x_i-x_j)(1-x_i)^2} + (i \leftrightarrow j) \right\}$$

In terms of these quantities:

$$M_{12}(\text{standard model}) \equiv M_{12}^{(W_1-W_1)} + M_{12}^{(W_1-\phi_1)} = \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}\cdot$$

$$\left\{ \sum_{i,j} \lambda_i\lambda_j \left[\left(1+\frac{x_i x_j}{4}\right) J_2(x_i,x_j) + 2x_i x_j J_1(x_i,x_j) \right] \right\} \frac{1}{m_K} \langle K^\circ \mid (\bar{d}_L\gamma_\mu s_L)^2 \mid \bar{K}^\circ \rangle \quad (a)$$

$$M_{12}^{(W_1-W_2)} + M_{12}^{(\phi_1-W_2)} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}\beta e^{i\gamma}$$

$$\cdot 2 \left\{ \sum_{i,j} \lambda_i'\lambda_j'^* \sqrt{x_i x_j} \cdot [4I_1(x_i,x_j) + I_2(x_i,x_j,\beta)] \right\} \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle =$$

$$= -\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W}\beta e^{i\gamma}.$$

$$\cdot 2 \left\{ \sum_{i,j} Re(\lambda_i'\lambda_j'^*)\sqrt{x_i x_j}[4I_1(x_i,x_j) + I_2(x_i,x_j,\beta) \right\} \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle \quad (b)$$

$$M_{12}^{(H^\circ)} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\beta_H e^{i\gamma} \cdot 4Q_H^2 \left\{ \sum_{i,j} \lambda_i'\lambda_j'^* \sqrt{x_i x_j} \right\} \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle =$$

$$= -\frac{1}{2}\frac{G_F}{\sqrt{2}}\beta_H e^{i\gamma} \cdot 4Q_H^2 \left\{ \sum_{i,j} Re(\lambda_i'\lambda_j'^*)\sqrt{x_i x_j} \right\} \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle \quad (c)$$

$$M_{12}^{(H^+)} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W} \cdot \beta_H \cdot 2Q_H^2$$

$$\left\{ e^{i\gamma} \left[\sum_{i,j} \lambda'_i \lambda'^*_j \sqrt{x_i x_j} \left(I_2(x_i, x_j, \beta_H) + x_i x_j I_1(x_i, x_j) \right) \right] \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle + \right.$$

$$(-)\frac{|2kk'^*|^2}{(|k|^2 + |k'|^2)^2} \left[\sum_{i,j} \lambda_i \lambda_j x_i x_j (\frac{1}{4} I_2(x_i, x_j, \beta_H) + I_1(x_i, x_j)) \right] \cdot$$

$$\frac{1}{m_K} \langle K^\circ \mid (\bar{d}_L \gamma^\mu s_L)^2 \mid \bar{K}^\circ \rangle$$

$$= -\frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W} \cdot \beta_H e^{i\gamma} \cdot 2Q_H^2$$

$$\left[\sum_{i,j} Re(\lambda'_i \lambda'^*_j) \sqrt{x_i x_j} (I_2(x_i, x_j, \beta_H) + x_i x_j I_1(x_i, x_j)) \right] \cdot \frac{1}{m_K} \langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle$$

$$+ \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi sin^2\theta_W} \beta_H \cdot 2 \frac{|2kk'^*|^2}{(|k|^2 - |k'|^2)^2}$$

$$\left[\sum_{i,j} \lambda_i \lambda_j x_i x_j (I_1(x_i, x_j) + \frac{1}{4} I_2(x_i, x_j, \beta_H)) \right] \cdot \frac{1}{m_K} \langle K^\circ \mid (\bar{d}_L \gamma^\mu s_L)^2 \mid \bar{K}^\circ \rangle \qquad (d)$$

(1.35)

The coefficients of $\lambda'_i \lambda'^*_j$ in equations (1.35) (b–d) are symmetric in i,j. Therefore, as a second step in these equations we replaced $\lambda'_i \lambda'^*_j$ by $Re(\lambda'_i \lambda'^*_j)$. With this replacement it becomes clear that all the contributions which arise from the right-handed W and from the physical Higgs fields have the same phase: $e^{i\gamma}$. An exception is the last term of $M_{12}^{(H^+)}$. Note that this exceptional term is very similar in form to the standard model M_{12}: It involves only the left-handed mixings (i.e., it involves only λ_i and not λ'_i), and its matrix element is identical to the matrix element of $M_{12}^{(W_1-W_1)}$. In the following we will neglect this last term of $M_{12}^{(H^+)}$ because it is suppressed by β_H and has no compensating enhancement factor. We remark that the above contributions to M_{12} were computed by various authors, starting with Beall et al [5] who gave $M_{12}^{(W_1-W_2)}$ for the two generation manifest model, through Mohapatra et al [10] who gave (with some errors) all the contributions (1.35) (a–d) for the three generation manifest model and finally us [29], [30] and Ecker et al [11], [19] who generalized all these results to the CCC models. Our calculation is the only one in which the assumption that $|k'/k|$ is small was not introduced.

Consider the sum:

$$M_{12}^{(W_1-W_2)} + M_{12}^{(\phi_1-W_2)} + M_{12}^{(H^\circ)} + M_{12}^{(H^+)}$$

The phase of every single term in the sum is $e^{i\gamma}$. Every term is suppressed relatively to $M_{12}(G-L)$ by a factor of β or β_H, and every term has a compensating enhancement factor. The enhancement factors for the two-generation case were given in equation (1.32) . As for the three-generation case: the strong experimental bounds on the t-quark mixings imply that the enhancement factors do not significantly change (for m_t around 45 GeV they change at most by a factor ~ 2). We thus conclude that our bounds on $m(W_2)$, m(H) still hold (up to a factor ~ 1.5).

We now compute ϵ in the three generation case. Denote

$$M_{12}(CCC) = M_{12}(\text{standard model}) - e^{i\gamma}(A\beta + A_H\beta_H)M_{12}(G-L) \tag{1.36}$$

where A, A_H are enhancement factors. (A is around 430 and A_H around 11,600).

$$\begin{aligned}\epsilon(CCC) &\simeq e^{i\frac{\pi}{4}}\frac{ImM_{12}(CCC)}{\sqrt{2}\Delta m_K} = \\ &= e^{i\frac{\pi}{4}}\left\{\frac{ImM_{12}\ (\text{standard}-\text{model})}{\sqrt{2}\Delta \text{m}_\text{K}} - sin\gamma(A\beta + A_H\beta_H)\frac{M_{12}(G-L)}{\sqrt{2}\Delta m_K}\right\} \\ &\sim \epsilon(\text{standard-model}) - e^{i\frac{\pi}{4}}\frac{1}{2\sqrt{2}}sin\gamma(A\beta + A_H\beta_H)\end{aligned} \tag{1.37}$$

We see that $\epsilon(CCC)$ is built up of two distinct contributions which arise from two different CP violating phases: The first is the familiar ϵ of the standard model which arises from the "Kobayashi-Maskawa" phase. This contribution is proportional to $s_{2,3}s_{1,3}sin\delta$. The second contribution is proportional to $\sin\gamma$ and depends only weakly (through A, A_H) on $s_{2,3}, s_{1,3}$.

Suppose that the experimental constraints on $s_{2,3}, s_{1,3}$ will be strengthened enough to imply:

$$|\epsilon(\text{standard model})| \ll |\epsilon(\text{experimental})|$$

then:

$$\epsilon(CCC) \sim e^{i\frac{\pi}{4}} \left\{ -\frac{1}{2\sqrt{2}} sin\gamma (A\beta + A_H\beta_H) \right\} \tag{1.38}$$

We now use (1.38) to get a new *upper* bound on $m(W_2)$:

$$|\epsilon(CCC)| \leq \frac{1}{2\sqrt{2}}(A\beta + A_H\beta_H) \lesssim \frac{1}{2\sqrt{2}}(A + A_H)\beta \sim \frac{1}{2\sqrt{2}}\ 12{,}000\beta \tag{1.39}$$

In the last step of (1.39) we assumed that β_H is $\leq \beta$ (or: $m(H) \geq m(W_2)$). (1.39) implies:

$$m(W_2) \lesssim 120\ TeV \tag{1.40}$$

Combining the *upper* bound (1.40) with the lower bound of Beall, Bander and Soni we find that (if ϵ(standard model) $\ll$ ϵ(experimental) then) the scale of the right handed currents is expected to be in between $\sim 1.7_{TeV}$ and $\sim 120_{TeV}$.

Let us now comment on M_{12}(manifest) in the three generation case: The contributions to M_{12}(manifest) are given by equations (1.35) (a)–(b) when we replace

$\sum_{i,j} \lambda'_i \lambda'^*_j$ by $\sum_{i,j} \lambda_i \lambda_j$ and $e^{i\gamma}$ by $(\mp)$ (where $(\mp)$ is determined by the relative sign between the first two phases of $F^d(\pm)$). The real part of M_{12}, or Δm_K, provides us with lower bounds on $m(W_2)$ and $m(H)$ which are essentially the same as those we get in the CCC model. The imaginary CP violating part of M_{12} arises from a single phase — the "Kobayashi-Maskawa" phase. We find that:

$$\epsilon(\text{manifest}) \propto s_{2,3} s_{1,3} sin\delta$$

Therefore the manifest model faces, in principle, the same potential trouble as the standard model does: b-decays may imply that $s_{2,3}$, $s_{1,3}$ are so small that ϵ(manifest) is considerably smaller than the measured ϵ. However, we note that the problem for the manifest model is not as acute as the problem for the standard model: to see this consider the approximate formula:

$$\epsilon \cong \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \frac{ImM_{12}}{\Delta m_K} \sim \frac{e^{i\frac{\pi}{4}}}{2\sqrt{2}} \frac{ImM_{12}}{ReM_{12}}$$

Inspection of the ratio $\frac{ImM_{12}}{ReM_{12}}$ shows that it may be considerably larger for the manifest model (than for the standard model) since the ratio of "Im" to "Re" is much larger in the additional terms $M_{12}^{(W_1-W_2)} + M_{12}^{(\phi_1-W_2)} + M_{12}^{(H^0)} + M_{12}^{(H^+)}$ than in M_{12}(standard model).

Finally, we comment on the case of N-generation CCC model. The contributions to M_{12} in this case are given in equations (21.1-21.4) where i,j go from 1 to N. Clearly, M_{12} is again of the form:

$$M_{12}(CCC, \text{N generations}) = M_{12}(\text{standard model}) - e^{i\gamma}(A\beta + A_H\beta_H)$$

We believe that the high generations should be, to a good approximation, decoupled from the low energy physics. We therefore expect A, A_H to be of the same order of magnitude as in the case of two and three generations. If this is true then all our results in the three generation case hold also for the case of N generations.

1.3.3 M_{12} in the Non-Minimal CCC Model

The difficulty involved in computing M_{12} for a non-minimal LRS model is that the Yukawa-couplings of the *physical* Higgs fields are unknown. This *technical* difficulty forced many authors to restrict their analysis to the minimal model, where the Yukawa couplings may be fully expressed in terms of the parameters k, k' and the matrices M^u, M^d. However, one does not really expect LRS theories to be minimal: LRS theories suffer from similar "diseases" as those of the standard model: LRS theories have many parameters, many particles, they become unnatural at high energies ($\sim 10M(W_2)$). The only advantage of LRS theories on the standard model is that they violate P, C and CP spontaneously and not explicitly. We therefore expect LRS theories to be merely effective low energy models of some more fundamental physics. If this is indeed the case then, probably, the Higgs spectrum is not minimal. (For example consider: (i) Composite models of quarks, leptons and scalars. If the Higgs is composite then we expect several scalar bound states to appear in the $(\frac{1}{2}, \frac{1}{2}^*)$ representation. (ii) Grand Unified Theories (GUTs): Many GUTs need more than a single Higgs in order to get realistic fermion masses.) We found [30] that for CCC theories it is possible to generalize our results to the case of nonminimal Higgs spectrum. The idea is as follows: Consider the contribution of the physical Higgs particles to M_{12}: The contribution of the charged Higgs particles is of fourth order in the weak interaction, while the contribution of the neutral Higgs is of second order. We therefore assume that to a first approximation we may ignore the contribution of the charged Higgs particles. The contribution of the neutral Higgs particles depends on their Yukawa couplings to the d-quarks. These couplings are symmetric matrices (as required by the C-invariance of the Lagrangian). We will now prove that this symmetry ensures that the neutral Higgs contribution is of the form:

$$M_{12}^{(H^0)} = -e^{i\gamma} A_H \beta_H M_{12}(G-L) \tag{1.41}$$

where A_H is a ***real positive*** enhancement factor.

Let H_1^0 be one of the (possibly many) real neutral Higgs fields of a CCC theory. We denote the Yukawa coupling of H_1^0 to the d quarks by a matrix N_1:

$$\mathcal{L}_{Yukawa}(H_1^0) = g\overline{D_L^o} N_1 D_R^o H_1^o + h.c. \tag{1.42}$$

where g is the weak interaction coupling constant (clearly we could have absorbed g in the Yukawa coupling N_1 but it turns out that the above representation is more convenient).

D_L^o, D_R^o are the interaction eigenstates and *not* the physical or mass eigenstates. N_1 (like every Yukawa coupling of a CCC model) is symmetric.

We now wish to rewrite the couplings of H_1 to quarks in terms of the ***physical*** d-quarks and in terms of a *diagonalized* matrix of Yukawa couplings:

(i) Since N_1 is symmetric — there exists a unitary matrix V_1 such that:

$$\hat{N}_1 = V_1 N_1 V_1^t \tag{1.43}$$

where $\hat{N}_1$ is diagonal and all its eigenvalues are real and non negative.

(ii) In order to get the relation between the interaction eigenstates and the mass eigenstates — we consider the nondiagonalized mass matrix:

$$\overline{D_L^o} M^d D_R^o + h.c.$$

M^d is related to the diagonalized mass matrix $\hat{M}^d$ through:

$$\hat{M}^d = U^d M^d (U^d)^t (F^d)^+ \tag{1.44}$$

where U^d is a unitary matrix. (As discussed in Appendix A, U^d and the analogous U^u are related to C_L through: $C_L = U^u (U^d)^+$).

We now rewrite (1.42) in terms of the physical (mass eigenstates) d-quarks and in terms of the diagonalized matrix $\hat{N}_1$:

$$\mathcal{L}_{Yukawa}(H_1^0) = g\bar{D}_L C_{H_1} \hat{N}_1 C_{H_1}^t (F^d)^+ D_R H_1^0 + h.c. \tag{1.45}$$

where: $C_{H_1} = U^d V_1^+$

Consider now the tree diagram of figure 1.5. Its contribution is:

$$M_{12}^{(H_1^0)} = -\frac{g^2}{4m^2(H_1)}$$
$$\left\{ \left[\sum_i (C_{H_1})_{d,i} (C_{H_1})_{s,i} n_i e^{-i\alpha_s} \right]^2 \langle K^\circ \mid \frac{1}{m_K} (\bar{d}_L s_R)^2 \mid \bar{K}^\circ \rangle \right.$$
$$+ \left[\sum_i (C_{H_1})^*_{d,i} (C_{H_1})^*_{s,i} n_i e^{i\alpha_d} \right]^2 \frac{1}{m_K} \langle K^\circ \mid (\bar{d}_R s_L)^2 \mid \bar{K}^\circ \rangle + \tag{1.46}$$
$$+ \left[\sum_i (C_{H_1})_{d,i} (C_{H_1})_{s,i} n_i e^{-i\alpha_s} \right] \left[\sum_j (C_{H_1})^*_{d,j} (C_{H_1})^*_{s,j} n_j e^{i\alpha_d} \right] \cdot$$
$$\left. \cdot \frac{1}{m_K} \left(\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle + \langle K^\circ \mid \bar{d}_R s_L \bar{d}_L s_R \mid \bar{K}^\circ \rangle \right) \right\}$$

where α_d, α_s are the first two phases of F^d and n_i are the eigenvalues of $\hat{N}_1$.

We use the vacuum insertion approximation to get:

$$\langle K^\circ \mid (\bar{d}_L s_R)^2 \mid \bar{K}^\circ \rangle = \langle K^\circ \mid (\bar{d}_R s_L)^2 \mid \bar{K}^\circ \rangle = \frac{1}{4} f_k^2 m_k^2 \left(-\frac{5}{3}\right) \left(\frac{m_K}{m_s + m_d}\right)^2$$

$$\langle K^\circ \mid \bar{d}_L s_R \bar{d}_R s_L \mid \bar{K}^\circ \rangle = \langle K^\circ \mid \bar{d}_R s_L \bar{d}_L s_R \mid \bar{K}^\circ \rangle = \frac{1}{4} f_k^2 m_k^2 \cdot 2 \left[\left(\frac{m_K}{m_s + m_d}\right)^2 + \frac{1}{6} \right] \tag{1.47}$$

we denote:

$$a \equiv \left(\sum_i (C_{H_1})_{d,i} (C_{H_1})_{s,i} n_i \right) e^{\frac{-i(\alpha_d + \alpha_s)}{2}} \tag{1.48}$$

and recall that $\gamma = \alpha_d - \alpha_s$. We then find:

$$
\begin{aligned}
M_{12}^{(H_1^\circ)} &= -\frac{g^2}{4m^2(H_1)} \cdot \frac{1}{4} f_K^2 m_K e^{i\gamma} \\
&\quad \cdot \left\{ \left[(a^2) + (a^*)^2\right] \left(-\frac{5}{3}\right) \left(\frac{m_K}{m_s + m_d}\right)^2 + 2|a|^2 \cdot 2 \left[\left(\frac{m_K}{m_s + m_d}\right)^2 + \frac{1}{6} \right] \right\} \\
&= -\frac{1}{2} \frac{G_F}{\sqrt{2}} f_K^2 m_K \beta_{H_1} e^{i\gamma} \\
&\quad \cdot \left\{ (Rea)^2 \cdot \frac{2}{3} \left[\left(\frac{m_K}{m_s + m_d}\right)^2 + 1 \right] + (Ima)^2 \cdot \frac{22}{3} \left[\left(\frac{m_K}{m_s + m_d}\right)^2 + \frac{1}{11} \right] \right\} \\
&= -\frac{1}{2} \frac{G_F}{\sqrt{2}} f_K^2 m_K e^{i\gamma} \left\{ (Rea)^2 \cdot 7.4 + (Ima)^2 \cdot 74 \right\}
\end{aligned}
\tag{1.49}
$$

We now express $M_{12}^{(H_1^0)}$ in terms of $M_{12}(G-L)$:

$$
\begin{aligned}
M_{12}^{(H_1^0)} &= -e^{i\gamma} \frac{4\pi sin^2\theta_W}{\alpha} \cdot \frac{3}{2} \cdot \frac{7.4(Rea)^2 + 74(Ima)^2}{(cos\theta_c sin\theta_c)^2 (\frac{m_c}{m(W)})^2} \beta_{H_1} M_{12}(G-L) \\
&\approx -e^{i\gamma} \cdot 570 \cdot \frac{7.4(Rea)^2 + 74(Ima)^2}{(cos\theta_c sin\theta_c)^2 (\frac{m_c}{m(W)})^2} \beta_{H_1} M_{12}(G-L) \\
&= -e^{i\gamma} A_{H_1} \beta_{H_1} M_{12}(G-L)
\end{aligned}
\tag{1.50}
$$

where:

$$
A_{H_1} = 570 \cdot \frac{7.4(Rea)^2 + 74(Ima)^2}{(cos\theta_c sin\theta_c)^2 (\frac{m_c}{m(W)})^2} \tag{1.51}
$$

In order to get an order of magnitude estimate of the enhancement factor A_{H_1} we remind the reader that in a minimal CCC model we have two real neutral physical Higgs particles. For one of them:

$$
a \sim \frac{Q_H}{2} cos\theta_c sin\theta_c \frac{m_c}{m(W)}
$$

and for the other:

$$
a \sim i\frac{Q_H}{2} cos\theta_c sin\theta_c \frac{m_c}{m(W)}
$$

We therefore expect A_{H_1} to be of the order of $10^3 - 10^4$ This completes our proof.

We conclude that in the nonminimal model

$$M_{12}(CCC) = M_{12}(\text{standard model}) - e^{i\gamma}(A\beta + \sum_i A_{H_i}\beta_{H_i})M_{12}(G-L) \tag{1.52}$$

where H_i are the neutral physical Higgs particles.

A_{H_i} are the corresponding enhancement factors (clearly A_{H_i} depends on the Yukawa coupling N_i).

We denote:

$$\sum_i A_{H_i}\beta_{H_i} = \bar{A}_H\bar{\beta}_H \tag{1.53}$$

where $\bar{\beta}_H$ is the average of the β_{H_i} and $\bar{A}_H$ is defined through (1.53) . We expect $\bar{A}_H$ to be of the order of magnitude of A_H ($\sim$ 12,000).

We now obtain:

$$M_{12}(CCC) = M_{12}(\text{standard model}) - e^{i\gamma}(A\beta + \bar{A}_H\bar{\beta}_H)\cdot M_{12}(G-L) \tag{1.54}$$

Equation (1.54) is actually identical to equation (1.36) . Therefore we may derive for the nonminimal case the same bounds on $m(W_2)$ and $m(H)$ as we derived for the minimal case. We also find the same expression for ϵ in the nonminimal model as we had in the minimal model.

1.4 Summary

We have tried to explore the importance of C-invariance in LRS models. We did this through a detailed comparison of CCC (Charge Conjugation Conserving) models with "manifest" LRS models.

We showed that in a CCC model the information about spontaneous breaking of C *may not* reach the quark mass matrices. Therefore, we found that the left handed Cabibbo mixing angles are related to the right handed ones through:

$$C_R = F^u C_L (F^d)^+$$

where F^u, F^d are diagonal unitary matrices. In the manifest model one *assumes* that the spontaneous breakdown of *parity* does not reach the quark mass matrices and then one finds:

$$C_R = F^u(\pm) C_L F^d(\pm)$$

where $F^u(\pm)$, $F^d(\pm)$ are *real* diagonal unitary matrices. However, as we showed the assumption of the manifest model is unjustified. Also, under this assumption one finds that the LRS Lagrangian must break C and CP explicitly.

We discussed the $K^\circ - \bar{K}^\circ$ mixing in the CCC model. We were able to show (using also results of previous works[5], [10]) that in a minimal CCC model one may derive from M_{12} the following lower bounds on $m(W_2)$ and $m(H)$:

$$m(W_2) \geq 1.7\ TeV$$

$$m(H) \geq 8.8\ TeV$$

Similar bounds may also be derived for a manifest model.

We showed that in a CCC model the CP violating parameter ϵ has the following form:

$$\epsilon(CCC) \approx \epsilon(\text{standard model}) - \sin\gamma \frac{1}{2\sqrt{2}}(A\beta + A_H \beta_H)$$

where A, A_H are large enhancement factors.

We discussed the possibility that ϵ(standard model) will become too small to account for the experimentally observed ϵ. In this case the CP violating phase γ may provide us with the main contribution to ϵ and we may even get an upper bound on $m(W_2)$:

$$m(W_2) \leq 120\ TeV$$

For the manifest model we found that there is no new source of CP violation. The only CP violating phase is the Kobayashi-Maskawa phase, and it always appears with the coefficient $s_{2,3}s_{1,3}$. Therefore if the rate of b-decay to u will be found to be very slow — then ϵ in the manifest model will be much smaller than the experimentally measured ϵ.

Finally, we considered the nonminimal LRS models. We believe that LRS models may at most be effective theories of some more fundamental physics. If this is true, then the Higgs spectrum of the LRS model is expected to be nonminimal. In a nonminimal model the Yukawa couplings of the physical Higgs particles are unknown and it is therefore difficult to estimate their contribution to M_{12}. We showed that in a CCC model it is possible to use the special form of the Yukawa couplings (these are symmetric matrices) in order to prove that the Higgs contribution to M_{12} is essentially the same as in the minimal case. We therefore generalize all the results of the minimal model to nonminimal CCC model.

Chapter 2

Composite Vector Bosons

2.1 Why Composite Vector Bosons?

In this introductory section we will explain how composite models of quarks and leptons motivate us to consider the possibility of composite W and Z.

Suppose that the quarks and leptons are composite. The fundamental building blocks inside the low energy fermions are then (presumably) bound together by some super strong force. We therefore expect that the quarks and leptons will undergo a short range interaction which is the residue of the new superstrong force. This residual interaction could be yet unknown but it could also be one of the already familiar forces. Of all the forces known today the only candidate to be this short range force is the weak interaction, since only the mediators of the weak force, the W and the Z, are *massive*. If the weak interactions are indeed residual interactions of a superstrong force, then the W and Z are, like the quarks and leptons, composites of the fundamental building blocks [31–34]. In this case, $SU(2)_W$ is not local but is only a global, approximate symmetry.

A similar scenario has been observed in the last twenty years for the nuclear forces: At the low energy level one sees pions, nucleons and ρ mesons, undergoing electromagnetic and nuclear interactions. For a while, the nuclear forces seemed to be the gauge interactions of SU(2) of (strong) isospin with the ρ-mesons as the corresponding vector bosons. When the higher energy domain was penetrated it was found that the nucleons and pions are composites of more fundamental particles — the quarks, which are bound together by strong color forces. Nuclear interactions were then seen to be only residual of the fundamental color-force, and the ρ mesons were found to be composed of quarks, like the nucleons. SU(2) of isospin is now known to be only a global, approximate symmetry.

In the following we will first discuss in more detail the theory of composite W and Z. We will then discuss three tests of such possible compositeness:

(i) Universality of the coupling constant of the W boson.

(ii) Special unrenormalizable effective interactions whose effect may be detected in present and near future $\bar{p}p$ colliders.

(iii) The $ee\gamma$ events of the 1983-run in CERN.

2.2 More on Composite Vector Bosons [35]

We start with the difficulties of the composite vector boson scenario: The basic difficulty in all composite models of W and Z is their mass ($m_W \sim 0.1_{TeV}$) which is considerably smaller than the compositeness scale ($\Lambda \geq 1_{TeV}$ [36]). So far no one has suggested a mechanism which would protect masses of composite spin 1 bosons.

Another problem is the small coupling constant g_W of the weak interactions. If the W's mediate the residue of a superstrong force, then their couplings are expected to be large (like ρ meson couplings). We have no solution to these two problems, however we note that they could be two aspects of only *one* (unknown) cause: Though both m_W and g_W are small their ratio is of the correct order of magnitude.

$$\frac{g_W^2}{m_W^2} \sim \frac{g_s^2}{\Lambda^2} \tag{2.1}$$

where g_s is a typical strong coupling constant ($g_s^2 \sim O(4\pi)$) and we assumed that Λ is $O(1_{TeV})$.

Keeping these basic problems of composite W and Z in mind we now describe the more successful aspects of the compositeness idea. A composite model of vector bosons has to provide its own explanation to the following successful predictions of the standard model:

(1) The couplings of W and Z to quarks and leptons are universal i.e., the coupling of W to $\overline{u^{(0)}}d^{(0)}$ is equal to its coupling to, e.g., $\bar{\nu}e$. (The superscript $^{(0)}$on u, d indicates that these are the *interaction* eigenstates).

(2) The neutral current of the fermions which couple to the Z boson is:

$$\frac{g_W}{\cos\theta_W}(j_\mu^{(0)} - \sin^2\theta_W j_\mu^{em})$$

where $j_\mu^{(0)}$ is the neutral component of the generating current of $SU(2)_L$, j_μ^{em} is the electromagnetic current and θ_W is an angle parameter (called the Weinberg angle).

We remind the reader that (1) and (2) together imply that there are no flavour changing neutral currents.

(3) $\sin\theta_W$ is related to e, g_W through:

$$\sin\theta_W = \frac{e}{g_W} \tag{2.2}$$

(4) The Weinberg mass relation:

$$m_Z = \frac{m_W}{\cos\theta_W} \tag{2.3}$$

In the standard model universality is automatic, since $SU(2)_W$ is a gauge symmetry, and (2)–(4) follow from the Higgs mechanism for spontaneous breaking of SU(2). In a composite model SU(2) is only global and it is not spontaneously broken. In order to reproduce (1)–(4) in the framework of a composite model of W and Z one proceeds in analogy to ρ meson physics: We assume that

(a) The approximate global $SU(2)_L$ symmetry of low energy physics is broken only by electromagnetic interaction (and fermion mass terms). Every term in the low energy Lagrangian which does not involve a photon (and is not a fermionic mass term) should be SU(2)-invariant. [1]

(b) The neutral W boson dominates the left handed part of the electromagnetic current of the primordial photon.

In order to see what is the use of these two assumptions let us consider the parts of the Lagrangian which include the kinetic and mass terms of the photon and the $\vec{W}$

[1] The $SU(2)$ symmetry may or may not exist at energies $\geq \Lambda$: For instance, in the Haplon model [33] $SU(2)_L$ exists also at high energies while in the Rishon model [31] it appears only at the low lying level.

bosons and their interactions with the fermions.

$$L = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{2}\lambda f_{\mu\nu}W^{(0)\mu\nu}$$
$$- m_W^2\vec{W}_\mu\vec{W}^\mu \tag{2.4}$$
$$- e a_\mu j^\mu_{em} - \vec{W}\cdot\vec{J}$$

where: a_μ is the primordial photon field.

$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$

$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu$

j^μ_{em} is the electromagnetic current

$\vec{J}$ is the current which couple to the $\vec{W}$ field.

The assumption of $SU(2)$ invariance of the Lagrangian implies that $\vec{J}$ is in the triplet representation of $SU(2)$.

The $SU(2)$ breaking character of the electromagnetic interactions is exhibited in the interaction of a_μ with the fermions and in the $W^{(0)}_\mu - a_\mu$ mixing term.

Diagonalizing the quadratic (kinetic and mass) terms of the above Lagrangian one finds:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2}W^{(+)}_{\mu\nu}W^{(-)\mu\nu}$$
$$+ \frac{1}{2}m_Z^2 Z_\mu Z^\mu + m_W^2 W^{(+)}_\mu W^{(-)\mu} \tag{2.5}$$
$$- eA_\mu j^\mu_{em} - Z_\mu J^\mu_Z - \frac{1}{\sqrt{2}}(W^{(+)}_\mu J^{(-)\mu} + h.c.)$$

where A_μ is the physical photon and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

A_μ, Z_μ are related to a_μ, $W^{(0)}_\mu$ through:

$$A_\mu = a_\mu + \lambda W^{(0)}_\mu$$
$$Z_\mu = \sqrt{1-\lambda^2}W^{(0)}_\mu \tag{2.6}$$

J_Z^μ is related to $J^{(0)^\mu}$, j_{em}^μ through:

$$J_Z^\mu = \frac{1}{\sqrt{1-\lambda^2}}(J^{(0)^\mu} - \lambda e j_{em}^\mu) \tag{2.7}$$

and $m_Z = \frac{m_W}{\sqrt{1-\lambda^2}}$

It is possible to show that the assumption of $W^{(0)}$-dominance in the left handed part of the electromagnetic current implies [35]:

(i) That

$$\lambda J^{(0)^\mu} = e j^{(0)^\mu} \tag{2.8}$$

where $j^{(0)^\mu}$ is the generating current of the neutral component of $SU(2)_L$. Using the $SU(2)_L$ symmetry we find:

$$\lambda \vec{J}^\mu = e \vec{j}^\mu \tag{2.9}$$

where $\vec{j}^\mu$ is the generating current of $SU(2)_L$. Note that equation (2.9) means universality of W and Z couplings.

(ii) e, g_W and λ are related:

$$\lambda = \frac{e}{g_W}$$

Defining $\lambda = \sin\theta_W$ we get:

$$J_Z^\mu = \frac{g_W}{\cos\theta_W}(j^{(0)^\mu} - \sin^2\theta_W j_{em}^\mu)$$
$$m_Z = \frac{m_W}{\cos\theta_W}$$

Therefore all the predictions (1)–(4) of the standard model are shared also by a composite model of W and Z when (a) and (b) are assumed.

We remark that under our assumptions it is possible to show that the WWW coupling is also equal to the universal g_W, as is the case in the standard model [35].

However, since WWW couplings were not yet measured we will not pursue this subject any further.

2.3 Testing Universality

The universality of the couplings of composite W's is not expected to be exact (in contrast to the standard $SU(2)_L \times U(1)$ gauge model). The reason for universality breaking is the heavier vector bosons which are expected to appear at energies of order Λ.

In order to explain in more detail what is involved — we will restrict ourselves to the specific kind of composite models where the global $SU(2)_L$ symmetry exists also at high energies (as high as Λ). The heavy vector bosons are then also grouped into multiplets of $SU(2)_L$ and here we consider only triplets. Denote by $\vec{W}_1$ the known low lying triplet of vector bosons and by $\vec{W}_2, \vec{W}_3, \ldots$ the heavy triplets. $\lambda_1, \lambda_2, \lambda_3, \ldots$ are their corresponding mixings with the photon and $\vec{J}_1, \vec{J}_2, \vec{J}_3, \ldots$ are their fermionic currents. The assumption of vector meson dominance should now be modified and relaxed: We do not assume that $W_1^{(0)}$ is dominating the (left-handed part of the) electromagnetic current, but that the bunch of all $W_i^{(0)}$ is dominating this current. Equations (2.8) , (2.9) are accordingly modified and we find:

$$\begin{aligned} &\sum_i \lambda_i J_i^{(0)\mu} = e j^{(0)\mu} \qquad (a) \\ &\sum_i \lambda_i \vec{J}_i^{\mu} = e \vec{j}^{\mu} \qquad (b) \end{aligned} \tag{2.10}$$

(where $\vec{j}^{\mu}$ is the generating current of $SU(2)_L$ and $j^{(0)\mu}$ is its neutral component).

Consider now the W-coupling to a certain fermion pair $\overline{f_{(a)}{}_L^{(0)}}\gamma_\mu f_{(b)}{}_L^{(0)}$ (a, b are generation indices; $f_{(a)}^{(0)}$ and $f_{(b)}^{(0)}$ are *interaction* eigenstates). Denote by $g_{i\,(a,b)}$ the coupling of W_i to this pair. Then, we find (see (2.10) (a)):

$$\sum_{i=1} \lambda_i g_{i\,(a,b)} = e\delta_{a,b} \tag{2.11}$$

or

$$g_{1\,(a,b)} = \frac{1}{\lambda_1}(e\delta_{a,b} - \sum_{i=2} \lambda_i g_{i\,(a,b)}) \tag{2.12}$$

(2.12) clearly implies that only in the case of a *single* composite W – the assumption of vector meson dominance is powerful enough to ensure the absence of Flavour Changing Neutral Currents (FCNC) and the universality of the flavour conserving part of the neutral currents. In order to proceed to the more reasonable case of several W's we should add two assumptions: (a) The assumption of horizontal symmetry: We assume that there is a horizontal quantum number h which is strictly conserved in the interactions of the fermions with the vector bosons. This means that the interaction eigenstates, $f_{(a)}^{(0)}$, carry a well defined h, and that the W's (as well as the photon and the gluon) carry h=0. We further assume that different generations carry different h-values. Under this assumption we are ensured that, (at least in the interaction basis), there are no FCNC, namely, $g_{i\,(a,b)} \propto \delta_{(a,b)}$. (b) We will assume that the deviation from g_1 universality which is due to the heavier vector bosons is small. More quantitatively, we assume:

$$\begin{aligned} \frac{\lambda_i}{\lambda_1} &\leq O(\frac{m_W}{\Lambda}) \\ \frac{g_i}{g_1} &\leq O(\frac{m_W}{\Lambda}) \end{aligned} \tag{2.13}$$

Roughly speaking, the justification to (2.13) is that the very heavy $W_2, W_3, \ldots$ should not have strong couplings to low lying physics. More specific justifications are to be found in [37–38].

Under the above assumptions we get:

$$g_{1(a,b)} = \delta_{a,b}(g_1 + O((\frac{m_W}{\Lambda})^2)) \tag{2.14}$$

where $g_1 \equiv \frac{e}{\lambda_1}$ is independent of (a,b) i.e., we find that the W_1 couplings to fermions are universal up to corrections of order $(\frac{m_W}{\Lambda})^2$.

Note that the deviation from universality implies that even though there are no FCNC in the interaction basis of the fermions — there might after all be FCNC when we transform to the physical basis of mass eigenstates. The couplings of these FCNC are proportional to the deviation of $g_{1(a,b)}$ from universality.

No effects due to deviation from universality have been seen up to now. We therefore may at present only give upper bounds on such deviations. Such upper bounds imply through (2.14) a lower bound on the compositeness scale Λ. The best bounds are derived from π-decays and from the absence of FCNC effects in the $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ systems. We compare π-decay to $e\nu$ with π-decay to $\mu\nu$, and π-decay to $\pi e\nu$ with μ-decay (to $\nu_\mu e\nu$). We also discuss the effect of FCNC on the $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings.

Comparison of π-Decays to $e\nu$ and $\mu\nu$

Experimentally:

$$R_{exp} \equiv \frac{\Gamma(\pi \longrightarrow e\nu)}{\Gamma(\pi \longrightarrow \mu\nu)} = 1.267 \cdot 10^{-4} \pm 1.8\% \tag{2.15}$$

Theoretically: If there is a single W particle and its couplings are strictly universal then [39]:

$$R_{theor} = 1.236 \cdot 10^{-4} \pm 0.3\% \tag{2.16}$$

Let us suppose now that the low lying W-particle is composite. Then, (2.16) is modified to:

$$R_{theor} = 1.236 \cdot 10^{-4}((1+\epsilon_1)^2 + \epsilon_2) \pm 0.3\% \tag{2.17}$$

where ϵ_1, ϵ_2 are small corrections: ϵ_1 is due to deviation from universality in the W_1-couplings. According to (2.14)

$$\epsilon_1 \sim O\left(\left(\frac{m_W}{\Lambda}\right)^2\right). \tag{2.18}$$

ϵ_2 arises from π-decay through $W_2, W_3, \ldots$ according to (2.13) :

$$\epsilon_2 = \frac{\sum_{i=2} \frac{g_i^2}{m_{W_i}^2}}{\frac{g_1^2}{m_{W_1}^2}} \sim O\left(\left(\frac{m_W}{\Lambda}\right)^4\right) \tag{2.19}$$

In the following we will consider only corrections of first order in $(\frac{m_W}{\Lambda})^2$, then, ϵ_1^2 and ϵ_2 will be neglected.

$$R_{theor} = 1.236 \cdot 10^{-4}(1+2\epsilon_1) \pm 0.3\% \tag{2.20}$$

The experimental value of R agrees with the theoretical value (2.16) within the error-bars. We therefore find the following bound on ϵ_1:

$$|2\epsilon_1| = \left|\frac{R_{exp} - R_{theor}}{R_{exp}}\right| \leq \frac{1.267 - 1.236}{1.267} + \sqrt{(1.8\%)^2 + (0.3\%)^2} \sim 4.3\% \tag{2.21}$$

Substituting for ϵ_1 the rough estimate:

$$|\epsilon_1| \sim (\frac{m_W}{\Lambda})^2$$

we find:

$$\Lambda \geq \sqrt{\frac{2}{4.3 \cdot 10^{-2}}} \cdot 82_{GeV} \sim 560_{GeV} \tag{2.22}$$

Comparison of π-Decay to $\pi e\nu$ with μ-Decay

We first assume the existence of a *single* W and strictly universal couplings. Measurements of μ^+-decay to $\bar{\nu}_\mu e^+ \mu_e$ and theoretical computations of this decay enable us to extract the value of W-coupling to fermions. Substituting this value in the theoretical computation for $\Gamma(\pi^+ \longrightarrow \pi^0 e^+ \nu_e)$ one finds [40]:

$$\Gamma(\pi^+ \longrightarrow \pi^0 e^+ \nu) = 0.391 \pm 0.027 \tag{2.23}$$

We compare this result with the direct measurement of π-decay to $\pi e\nu$ [41]:

$$\Gamma_{exp}(\pi^+ \longrightarrow \pi^0 e^+ \nu) = 0.403 \pm 0.003 \tag{2.24}$$

The prediction (2.23) , which is based on the assumption of g-universality agrees, within error bars, with the experimental value (2.24) .

Suppose now that W is composite and its coupling is universal only up to corrections $O((\frac{m_W}{\Lambda})^2)$. Equation (2.23) would then be modified to:

$$\Gamma(\pi^+ \longrightarrow \pi^0 e^+ \nu) \cong 0.391(1+\delta_1^2) \pm 0.027 \cong 0.391(1+2\delta_1) \pm 0.027 \tag{2.25}$$

δ_1 represents the deviation from universality and we roughly estimate:

$$\delta_1 \sim (\frac{m_W}{\Lambda})^2$$

Comparison of (2.25) and (2.23) implies:

$$2\left(\frac{m_W}{\Lambda}\right)^2 \leq \frac{0.403 - 0.391}{0.403} + \frac{\sqrt{0.027^2 + 0.003^2}}{0.403} \sim 0.1 \tag{2.26}$$

Consequently, we find:

$$\Lambda \geq 370_{GeV} \tag{2.27}$$

$K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings

In order to simplify our discussion we will consider only the first two generations. We denote the interaction eigenstsates by $u_L^{(0)}$, $d_L^{(0)}$, $c_L^{(0)}$, $s_L^{(0)}$. In terms of these states the neutral current of the Z^0-boson is flavour conserving and it is almost universal:

$$\begin{aligned} J_Z^\mu =& \frac{1}{\cos\theta_W}(J_1^{(0)\mu} - \lambda e j_{em}^\mu) \\ =& \frac{g_W}{2\cos\theta_W}\left(\begin{pmatrix}\overline{u_L^{(0)}} & \overline{c_L^{(0)}}\end{pmatrix}[(1-\frac{4}{3}\sin^2\theta_W)I + \frac{1}{2}\frac{\delta g}{g_W}\sigma_3]\gamma^\mu \begin{pmatrix}u_L^{(0)} \\ c_L^{(0)}\end{pmatrix}\right. \\ &\left. + \begin{pmatrix}\overline{d_L^{(0)}} & \overline{s_L^{(0)}}\end{pmatrix}[-(1-\frac{2}{3}\sin^2\theta_W)I - \frac{1}{2}\frac{\delta g}{g_W}\sigma_3]\gamma^\mu \begin{pmatrix}d_L^{(0)} \\ s_L^{(0)}\end{pmatrix}\right) \end{aligned} \tag{2.28}$$

where g_W is the coupling of the $\vec{W}_1$ multiplet to the fermions (averaged over the two generations); δg is the deviation from universality (the difference between the coupling of $\vec{W}_1$ to the first and to the second generation), I is the identity matrix and σ_3 the diagonal Pauli matrix, both acting in generation space.

We now wish to present the current (2.28) in terms of the mass eigenstates u_L, d_L, c_L, s_L. Using the notation of Appendix A we denote:

$$\begin{pmatrix}u_L^{(0)} \\ c_L^{(0)}\end{pmatrix} = U^{u+}\begin{pmatrix}u_L \\ c_L\end{pmatrix} \tag{2.29}$$

$$\begin{pmatrix} d_L^{(0)} \\ s_L^{(0)} \end{pmatrix} = U^{d+} \begin{pmatrix} d_L \\ s_L \end{pmatrix} \tag{2.30}$$

where U^u, U^d are unitary matrices acting in the (two dimensional) generation space.

Since u_L, c_L, d_L and s_L are defined only up to phases we are free to multiply U^{u+} and U^{d+} by phases from the right. We are also free to choose the phases of $u_L^{(0)}$, $c_L^{(0)}$, but once these phases were chosen we do not have any more freedom to choose the phases of $d_L^{(0)}$ and $s_L^{(0)}$, since the relative phase of $d_L^{(0)}$ and $u_L^{(0)}$ (and the relative phase of $s_L^{(0)}$ and $c_L^{(0)}$) is fixed (through the requirement that they are the two components of the same $SU(2)$-doublet). We therefore find that we may multiply U^{u+} from the left by arbitrary phases, but we do not have this freedom in U^{d+}.

Taking into account all the freedom we have we find that it is possible to choose the phases such that:

$$U^{u+} = \begin{pmatrix} \cos\theta^u & -\sin\theta^u \\ \sin\theta^u & \cos\theta^u \end{pmatrix}$$

$$U^{d+} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \begin{pmatrix} \cos\theta^d & -\sin\theta^d \\ \sin\theta^d & \cos\theta^d \end{pmatrix}$$

where $0 \leq \theta^u \leq \frac{\pi}{2}$, $0 \leq \theta^d \leq \frac{\pi}{2}$.

The Cabibbo mixing matrix is:

$$C = U^u U^{d+}$$

and it is straight-forward to see that the Cabibbo angle θ_c is related to the parameters θ_u, θ_d and ϕ through:

$$\sin\theta_c = |\cos\phi\sin(\theta^u - \theta^d) - i\sin\phi\sin(\theta^u + \theta^d)| \tag{2.31}$$

Subsi tuting U^{u+}, U^{d+} in (2.29) , (2.30) and then substituting (2.29) , (2.30) in (2.28) we get:

$$J_Z^\mu == \frac{g_W}{2\cos\theta_W}((\overline{u_L} \quad \overline{c_L})[(1-\frac{4}{3}\sin^2\theta_W)I + \frac{1}{2}\frac{\delta g}{g_W}(\cos(2\theta^u)\sigma_3 + \sin(2\theta^u)\sigma_1)]\gamma^\mu \begin{pmatrix} u_L \\ c_L \end{pmatrix}$$
$$+ (\overline{d_L} \quad \overline{s_L})[-(1-\frac{2}{3}\sin^2\theta_W)I - \frac{1}{2}\frac{\delta g}{g_W}(\cos(2\theta^d)\sigma_3 + \sin(2\theta^d)\sigma_1)]\gamma^\mu \begin{pmatrix} d_L \\ s_L \end{pmatrix}) \tag{2.32}$$

In (2.32) we clearly see that in the physical basis of mass eigenstates, there are flavour changing neutral currents, proportional to δg.

The effective four-Fermi flavour changing interaction is:

$$L^{eff}_{\text{FCNC}} = (\frac{\delta g}{g_W})^2 \frac{G_F}{\sqrt{2}}[(\sin\theta^u\cos\theta^u)^2(\overline{u_L}\gamma^\mu c_L)^2 + (\sin\theta^d\cos\theta^d)^2(\overline{d_L}\gamma^\mu s_L)^2] + \text{c.h.} \tag{2.33}$$

where G_F is the Fermi constant.

The contribution of L^{eff}_{FCNC} to $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings is:

$$(\frac{\Delta m_K}{m_K})_{\text{FCNC}} = (\frac{\delta g}{g_W})^2 \frac{G_F}{\sqrt{2}}(\sin\theta^d\cos\theta^d)^2 \frac{2}{3} B_K f_K^2 \tag{2.34}$$

$$(\frac{\Delta m_D}{m_D})_{\text{FCNC}} = (\frac{\delta g}{g_W})^2 \frac{G_F}{\sqrt{2}}(\sin\theta^u\cos\theta^u)^2 \frac{2}{3} B_D f_D^2 \tag{2.35}$$

where B_K, B_D are the "bag factors" of the $K - \bar{K}$ and $D - \bar{D}$ respectively and f_K, f_D are the K, D decay constants.

The contributions (2.34) , (2.35) to $\frac{\Delta m_K}{m_K}$, $\frac{\Delta m_D}{m_D}$ should be compared with the standard contributions which arise from the standard-model box diagrams:

$$(\frac{\Delta m_K}{m_K})_{\text{standard}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi\sin^2\theta_W}(\cos\theta_c\sin\theta_c)^2(\frac{m_c}{m_W})^2 \frac{2}{3} B_K f_K^2 \tag{2.36}$$

$$(\frac{\Delta m_D}{m_D})_{\text{standard}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi\sin^2\theta_W}(\cos\theta_c\sin\theta_c)^2(\frac{m_s}{m_W})^2 \frac{2}{3} B_D f_D^2 \tag{2.37}$$

where m_c, m_s are the masses of the c, s quarks respectively (and α is the fine structure constant).

It is well known that $(\frac{\Delta m_K}{m_K})_{\rm standard}$ is in good agreement with experiment. We therefore require that:

$$(\frac{\Delta m_K}{m_K})_{\rm FCNC} \leq (\frac{\Delta m_K}{m_K})_{\rm standard} \quad . \tag{2.38}$$

$\frac{\Delta m_D}{m_D}$ was not yet measured. Presently, we have only an experimental upper bound:

$$(\frac{\Delta m_D}{m_D})_{\rm experimental} \leq \frac{6.5 \cdot 10^{-10}\ {\rm MeV}}{1864\ {\rm MeV}} = 3.5 \cdot 10^{-13} \tag{2.39}$$

Substituting in (2.37) $B_D f_D^2 = 0.19$ GeV [42], $G_F = 1.1 \cdot 10^{-5}$ GeV^{-2}, $\alpha = \frac{1}{137}$, $\sin^2\theta_W = 0.22$, $\sin\theta_c = 0.22$ and $m_s = 150$ MeV, we find that the experimental bound on $\frac{\Delta m_D}{m_D}$ is about 900 times larger than the standard model estimate. We therefore require:

$$(\frac{\Delta m_D}{m_D})_{\rm FCNC} \leq 900 \cdot (\frac{\Delta m_D}{m_D})_{\rm standard} \tag{2.40}$$

The inequalities (2.38) , (2.40) together with equations (2.34) , (2.35) , (2.36) , (2.37) imply that:

$$(\frac{\delta g}{g_W})^2 (\cos\theta_d \sin\theta_d)^2 \leq (\sin\theta_c \cos\theta_c)^2 \cdot (\frac{m_c}{m_W})^2 \frac{\alpha}{4\pi \sin^2\theta_W} \tag{2.41}$$

$$(\frac{\delta g}{g_W})^2 (\cos\theta_u \sin\theta_u)^2 \leq (\sin\theta_c \cos\theta_c)^2 \cdot (\frac{m_s}{m_W})^2 \frac{\alpha}{4\pi \sin^2\theta_W} \tag{2.42}$$

We substitute in (2.42) : $m_s = \frac{1}{10} m_c$ and then take the square root of (2.41) and (2.42) . We get:

$$|\frac{\delta g}{g_W}|| \cos\theta^d \sin\theta^d| \leq \sin\theta_c \cos\theta_c \frac{m_c}{m_W} \sqrt{\frac{\alpha}{4\pi \sin^2\theta_W}} \tag{2.43}$$

$$|\frac{\delta g}{g_W}||\cos\theta^u \sin\theta^u| \leq 3\sin\theta_c\cos\theta_c\frac{m_c}{m_W}\sqrt{\frac{\alpha}{4\pi\sin^2\theta_W}} \tag{2.44}$$

(2.43) and (2.44) imply:

$$|\frac{\delta g}{g_W}||\sin(\theta^u - \theta^d)| \leq 4\sin\theta_c\cos\theta_c\frac{m_c}{m_W}\sqrt{\frac{\alpha}{4\pi\sin^2\theta_W}} \tag{2.45}$$

$$|\frac{\delta g}{g_W}||\sin(\theta^u + \theta^d)| \leq 4\sin\theta_c\cos\theta_c\frac{m_c}{m_W}\sqrt{\frac{\alpha}{4\pi\sin^2\theta_W}} \tag{2.46}$$

(2.45) , (2.46) and (2.31) imply:

$$|\frac{\delta g}{g_W}|\sin\theta_c\cos\theta_c \leq 4\sin\theta_c\cos\theta_c\frac{m_c}{m_W}\sqrt{\frac{\alpha}{4\pi\sin^2\theta_W}} \tag{2.47}$$

and therefore:

$$|\frac{\delta g}{g_W}| \leq 4\frac{m_c}{m_W}\sqrt{\frac{\alpha}{4\pi\sin^2\theta_W}} \tag{2.48}$$

Substituting for $|\frac{\delta g}{g_W}|$ the rough estimate $(\frac{m_W}{\Lambda})^2$, and using the values $m_c \sim 1.5_{\mathrm{GeV}}$, $m_W \sim 82_{\mathrm{GeV}}$, $\alpha \sim \frac{1}{137}$ and $\sin^2\theta_W \sim 0.22$ we get the following lower bound on Λ:

$$\Lambda \geq 1.2_{\mathrm{TeV}}$$

Let us summarize: In the standard model W-couplings to fermion-pairs are universal. In a composite model for vector bosons one expects deviations from universality. We estimated such deviations to be of order $(\frac{m_W}{\Lambda})^2$. At present no deviations from universality have been seen, and therefore we may only put upper bounds on $(\frac{m_W}{\Lambda})^2$ or lower bounds on Λ. We find:

(i) $\Lambda > 560_{GeV}$ from comparison of $\Gamma(\pi \longrightarrow e\nu)$ and $\Gamma(\pi \longrightarrow \mu\nu)$ (i.e., comparison of W-couplings to $\bar{e}\nu$ and $\bar{\mu}\nu$).

(ii) $\Lambda > 370_{GeV}$ from comparison of $\Gamma(\mu \longrightarrow \nu e \nu)$ and $\Gamma(\pi \longrightarrow \pi e \nu)$ (i.e., comparison of W-coupling to $\bar{\mu}\nu$ and to $\bar{u}d$).

(iii) The deviation from universality also implies that there are flavour changing neutral currents (FCNC). Since effects of FCNC were not yet seen we again may derive a lower bound on the compositeness scale. From the $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings we derive the following bound:

$$\Lambda \geq 1.2_{\rm TeV}$$

This bound is comparable to the Eichten-Lane-Peskin bound [36] on the compositeness scale of the fermions.

2.4 An experimental Test of Z^0 Compositeness in Proton Anti-Proton Collider [43]

In this chapter we will consider effective interactions of the form:

$$Z^0 \underbrace{V^0 \ldots V^0}_{(n-1)\ fields} \tag{2.49}$$

where V^0 is a photon or a gluon field. In the standard model such effective interactions are of n'th order and they arise as radiative corrections involving a fermion loop (see fig. 2.1(a)). In a composite model an additional source for the interactions (2.49) is effective terms of the form:

$$W^0 \underbrace{v^0 \ldots v^0}_{(n-1) fields} \tag{2.50}$$

where v^0 is a primordial photon or a gluon field. Substituting in (2.50) :

$$W^0_\mu = \frac{1}{\cos\theta_W} Z^0_\mu$$
$$a_\mu = A_\mu - \tan\theta_W z^0_\mu \tag{2.51}$$

we get:

$$\frac{1}{\cos\theta_W} Z^0 \underbrace{V^0 \ldots V^0}_{(n-1)\ fields} + \text{(terms involving two or more Z fields)} \tag{2.52}$$

$\frac{1}{\cos\theta_W}$ is ≈ 1 and in the following we will ignore this factor.

In a composite model the effective term (2.50) may be of n-1 order, reflecting direct couplings [44-45] of the photons and gluons to the preons inside the W^o (see fig. 2.1(b)). The strength of such a term is proportional to $< (eQ)^{n_\gamma} (gQ_S)^{n_g} >$ where n_γ, n_g are the numbers of photons and gluons in the effective term; e,g are the electromagnetic and QCD coupling constants; Q is the preon electric charge; Q_S is a preon color charge (Q_S=1,0 for color triplet, singlet); <> denotes an appropriately weighted summation on all preonic components of the W^o, depending on the detailed wave-function of the composite W^o. Since, in a composite model, the interactions of the form (2.49) are of smaller order in the gauge coupling, they may provide us with an interesting test of vector boson compositeness. However we note that:

(1) $SU(3)_{color} \times U(1)_{em}$ gauge invariance together with Lorentz invariance imply that a term of the form (2.49) appears with (at least) n derivatives. Therefore, the coefficient of this term should include a factor of the form $\frac{1}{E^{2n-4}}$ where E is a characteristic energy scale. If E is the compositeness scale Λ, then the terms (2.49) are strongly suppressed and are rendered uninteresting. Here we want to speculate on the possibility that E

is $O(m_W)$. This speculation is based on the observation that the energy scale which characterizes W and Z is m_W and not Λ: Note that (i) the mass of W is m_W. (ii) The $W^0-\gamma$ mixing λ is much larger than the $\rho^0-\gamma$ mixing. This is related to the fact that the mass scale which characterizes the W is m_W and not the much larger scale Λ [37].[2] In the following we will assume that $E \sim m_W$.

(2) If E is $O(m_W)$ then the interactions $W^0 v^0 \ldots v^0$ are part of the low energy (energy$\leq \Lambda$) Lagrangian. Therefore they are either SU(2) invariant or they involve one or more photons. Since a $W^0 v^0 \ldots v^0$ term may not be SU(2) invariant we conclude that it must involve at least one photon. In other words — all $Z^0 V^0 \ldots V^0$ effective interactions which are of interest for testing Z-compositeness — must involve at least one photon.

In a series of papers Renard has investigated the $Z^0V^0V^0$ and $Z^oV^oV^oV^o$ vertices. He pointed out [44] that the $Z^0\gamma\gamma$ vertex may produce a detectable effect in $e^+e^- \rightarrow Z^0\gamma$ scattering, and that the $Z^0\gamma\gamma\gamma$ vertex may strongly enhance the decay of $Z^0 \rightarrow \gamma\gamma\gamma$ [45]. We agree with these observations and we believe that they will provide good experimental tests in an e^+e^- collider such as LEP or SLC. Renard also considered the contribution of the vertex $Z^0\gamma gg$ to $Z^0 \rightarrow \gamma gg$ decay [45]. However, it turns out that the experimental signature of this process is relatively unclear, due to a variety of possible backgrounds. Finally, Renard analyzed the effects of the Z^0ggg vertex for the decay $Z^0 \rightarrow ggg$ and the production process $g+g \rightarrow Z^0+g$ (in a $\bar{p}p$ collider). We disagree with this part of his work since, as explained above, the unavoidable global SU(2)

[2] In the nonrelativistic bound state model [45], [46] which we use later the fact that λ is $O(1)$ implies that E is indeed $O(m_W)$ and not $O(\Lambda)$.

symmetry suppresses the effective $Z^o ggg$ coupling down to its standard model value. Consequently, the corresponding processes should not be enhanced in a composite-Z model and cannot serve as useful tests. There is, however, another process which has not been previously discussed and which appears to provide the only feasible experiment of this family during the next few years (prior to the completion of SLC or LEP). We refer to $Z^0\gamma$ production (through an effective $Z^0\gamma gg$ vertex) in a $\bar{p}p$ collider.

The relevant experimental process is:

$$\bar{p} + p \to Z^o + \gamma + anything \tag{2.53}$$

In the standard model $Z^0\gamma$ production is due to the subprocess:

$$\bar{q} + q \longrightarrow Z^0 + \gamma \tag{2.54}$$

with the diagrams of figure 2.2(a),(b).

In a composite model we have an additional contribution from the $Z^0\gamma gg$ vertex through the subprocess:

$$g + g \longrightarrow Z^0 + \gamma \tag{2.55}$$

(see diagram 2.2(d)).

The standard model process is $O(\alpha^2)$. The contribution of the effective $Z^0\gamma gg$ vertex in a composite model is $O(\alpha\alpha_s^2)$. At the relevant energies (0.5–2 TeV) the two contributions are of the same order of magnitude (i.e. α_s^2 is $O(\alpha)$). However, note that the angular distributions of the processes (2.54) and (2.55) are completely different: The standard model process involves the exchange of a light particle (u, d or s quark) in the t or u channel. The angular distribution of the photon is therefore expected to be concentrated around the beams directions. The composite model process is, in contrast, pointlike and is therefore expected to have a relatively flat angular distribution. Consequently, we hope to distinguish the effect of the $Z^0\gamma gg$ vertex from the background of the standard model. We note that the cuts in the analysis of experimental data tend to strengthen the effect of the $Z^0\gamma gg$ vertex (if it exists): It turns out that, in order for a single prompt photon to be identified, it must be sufficiently hard. For the $S\bar{p}pS$ collider at CERN and for the Fermilab $\bar{p}p$ collider the p_T^γ cutoff should be around 5–10 TeV (p_T^γ is the transverse momentum of the photon). Such a cutoff may considerably reduce the cross-section of the standard model process (2.54) (since the angular distribution is concentrated at small p_T), while the $Z^0\gamma gg$ vertex contribution to $Z^0\gamma$ production will not be strongly effected (since its angular distribution is quite flat).

In the rest of this chapter we give more detailed analysis of $Z^0\gamma$ production and show that it may indeed be an important test of Z^0 compositeness.

The contribution of the standard model diagrams to the unpolarized cross-section is:

$$\begin{aligned}\sigma^{tot}\left(\bar{q}_i + q_i \to Z^o + \gamma\right) = &\frac{4\pi\alpha^2}{3s} Q_i^2 \left(G_{Vi}^2 + G_{Ai}^2\right) \times \\ &\times \left[\left(1 - \frac{M_Z^2}{s}\right)\left(\log\frac{s}{m_i^2} - 1\right) + \frac{2M_Z^2}{s - M_Z^2}\log\frac{s}{m_i^2}\right]\end{aligned} \tag{2.56}$$

where Q_i and m_i are the electric charge and mass of the i'th quark and G_{Vi}, G_{Ai} are its vector and axial couplings to the Z-boson ($G_{Au} = -G_{Ad} = 1/(4\sin\theta_W\cos\theta_W)$;

$G_{Vi} = G_{Ai} - Q_i \tan\theta_W$). The angular distribution in the c.m.s. is

$$\frac{d\sigma}{d\Omega}(\bar{q}_i + q_i \to Z^o + \gamma) = \frac{\alpha^2}{3} Q_i^2 \left(G_{Vi}^2 + G_{Ai}^2\right) \times$$
$$\times \frac{1}{s - M_Z^2}\left[\frac{2\left(1 + \left(\frac{M_Z^2}{s}\right)^2\right)}{\sin^2\theta + 4\frac{m_i^2}{s}} - \left(1 - \frac{M_Z^2}{s}\right)^2\right] \tag{2.57}$$

In our calculations we have included only the light quark contributions (u,d,s), using m_i=0.3 GeV.

In a composite-Z^o model we encounter the following contributions to $\bar{p} + p \to Z^o + \gamma + any$:

(i) The standard model contributions (figures 2.2(a), 2.2(b)) remain essentially unchanged.

(ii) An effective $Z^o Z^o \gamma$ interaction may provide an additional contribution to the $\bar{q} + q \to Z^o + \gamma$ subprocess (figure 2.2(c)). This contribution is expected to be small [44].

(iii) The most important contribution may come from the subprocess (figure 2.2(d)):

$$g + g \to Z^o + \gamma$$

which is negligible in the standard model. We have already discussed the coefficient of the effective $Z^0\gamma gg$ vertex but we do not know its explicit form. We have therefore chosen a nonrelativistic bound state model which was previously used by Renard [45], [46]. We do so, just because we are not aware of any other simple framework. It is obvious that this model is totally inadequate in its details, but we hope that it may well serve as a crude order-of-magnitude estimate. We should probably not trust the results to better than a factor of 4 or so. Using this model, one obtains:

$$\frac{d\sigma}{d\Omega}(g+g\to Z^o+\gamma)=\frac{4\pi\alpha_S^2\alpha}{9s}\left(\frac{F_W}{M_W}\right)^2\frac{s-M_Z^2}{2s}(3+\cos^2\theta)(< Q_S^2 Q >)^2 \quad (2.58)$$

where $\frac{F_W}{M_W}=\frac{\lambda}{e}\sim 1.6$. The values of $(< Q_S^2 Q >)^2$ are usually of order 1 (e.g. $\frac{1}{2}$ in the Rishon model [31] and $\frac{3N_H}{2}$ in the Haplon model [33] with N_H hypercolors).

The transition from the subprocesses $\bar{q}+q\to Z^o+\gamma$ and $g+g\to Z^o+\gamma$ to the actual contributions to $\bar{p}+p\to Z^o+\gamma+any$ involves the quark and gluon distributions inside the proton. In the standard model:

$$\sigma^{tot}(\bar{p}+p\to Z^o+\gamma+any)=$$
$$\Sigma_i\int dx_1dx_2(D_i(x_1)D_i(x_2)+\bar{D}_i(x_1)\bar{D}_i(x_2))\sigma^{tot}(\bar{q}_i+q_i\to Z^o+\gamma) \quad (2.59)$$

$$\frac{d\sigma}{dp_T^2}(\bar{p}+p\to Z^o+\gamma+any)=$$
$$\Sigma_i\int dx_1dx_2(D_i(x_1)D_i(x_2)+\bar{D}_i(x_1)\bar{D}_i(x_2))P(\hat{s},p_T^2)\frac{d\sigma}{d\Omega}(\bar{q}_i+q_i\to Z^o+\gamma) \quad (2.60)$$

where i is the quark flavor; D_i, $\bar{D}_i$ are the distribution functions of the i'th quark in the proton and antiproton respectively; $\hat{s}$ is the squared invariant mass of the $Z^o\gamma$ system; $P(\hat{s},p_T^2)$ is given by:

$$P(\hat{s},p_T^2)=\frac{8\pi\hat{s}}{(\hat{s}-M_Z^2)\left[(\hat{s}-M_Z^2)^2-4\hat{s}p_T^2\right]^{\frac{1}{2}}} \quad (2.61)$$

similarly, the contribution of $g+g\to Z^o+\gamma$ leads to:

$$\sigma^{tot}(\bar{p}+p\to Z^o+\gamma+any)=\int dx_1dx_2D_g(x_1)D_g(x_2)\sigma^{tot}(g+g\to Z^o+\gamma) \quad (2.62)$$

$$\frac{d\sigma}{dp_T^2}(\bar{p}+p \to Z^o+\gamma+any) = \int dx_1 dx_2 D_g(x_1) D_g(x_2) P(\hat{s}, p_T^2) \frac{d\sigma}{d\Omega}(g+g \to Z^o+\gamma) \tag{2.63}$$

where D_g is the distribution function of gluons in the proton.

Among the various available phenomenological distribution functions we have chosen the ones of Baier et al. [47], using Λ=.4 GeV. We have checked the sensitivity of the results by repeating the computations with Λ=.1 GeV. Following the procedure of Brown et al. [48] in the case of $W\gamma$ production, we introduce in all cases a lower cutoff on the $Z\gamma$ invariant mass $M_{Z\gamma}^2 > 1.1\ M_Z^2$. This cutoff enables us to avoid all threshold divergences without "losing" any photons with $p_T > 5$ GeV. Computations were done for $\bar{\text{p}}$p colliders at $\sqrt{\text{s}} = 540$ GeV (CERN S$\bar{\text{p}}$pS) and $\sqrt{\text{s}}$=2000 GeV (Fermilab). The differential cross sections are shown in figure 2.3. Note that these cross-sections are correct (i.e. independent of the infra-red cutoff) only for $p_T > 5_{GeV}$.

We see that, as expected, the standard model contribution drops quickly down while the composite model contribution is relatively flat and is therefore dominating at large p_T.

The number of expected events in the two energy ranges ($\sqrt{s} = 540_{GeV}$ and $\sqrt{s} = 2000_{GeV}$), for the standard model and for a composite model, are given in table 2, assuming an integrated luminosity of $10^{37} cm^{-2}$ per year. The composite model leads to an energy dependent enhancement of one order of magnitude at CERN energies (540 GeV) and two orders of magnitude at Fermilab energies (2000 GeV).This energy dependence is due to the increased importance of gluon contributions at high energies. We note that the total cross sections for $\bar{q}+q \to Z^o+\gamma$ and $g+g \to Z^o+\gamma$ are, actually, of the same order of magnitude. Only the experimentally motivated limitation of $p_T^\gamma > 5$ GeV together with the "flatness" of the composite model p_T-dependence lead to the predicted enhancement.

There are many uncertainties in our calculation. Some of them are "technical", including the choice of a detailed distribution function, the value of Λ_{QCD}, the assumed quark masses, etc. All of these uncertainties probably contribute a factor of 4 or so which could go in either direction. Additional uncertainties come from QCD corrections to the standard model which are likely to flatten the p_T^γ distributions. Another unknown parameter is the model-dependent factor $< Q_S^2 Q >$. This quantity actually vanishes if the preons are colorless [32], [34] . However, in most other cases it is likely to be of order one. All the above uncertainties can be largely eliminated by performing additional calculations and by restricting one's attention to a specific composite scheme.

There is, however, one major uncertainty which may destroy the entire argument: We have assumed that the energy scale of the effective $Z^\circ\gamma gg$ term is $O(M_W)$, not $O(\Lambda)$. This is the case in the explicit, but inadequate, nonrelativistic scheme used in the computation. If, however, the relevant energy scale is Λ (greater than 1 TeV), the magnitude of the subprocess $g + g \to Z + \gamma$ is diminished at least by a factor $(M_W/\Lambda)^4$, and the predicted effect may disappear. In the absence of a clear understanding of the dynamics of a composite Z, and in view of the required small M_W/Λ ratio which remains unexplained, we must conclude that our ignorance allows for any energy scale between M_W and Λ. Consequently, all our calculations (as well as the earlier calculations of Renard) must be viewed as approximate upper limits of the expected effects. An experimental observation of the predicted cross section will indicate Z-compositeness. On the other hand, if the observed cross section agrees with the standard model, we still have several possibilities: (i) Z^o is not composite; (ii) Z^o is composite but the energy scale of the $Z^o\gamma gg$ effective coupling is Λ, not M_W; (iii) Z^o is composite but contains colorless preons.

We summarize: Several experimental tests of Z^o- compositeness have been proposed [45] earlier, using effective $Z^oV^oV^oV^o$ interactions ($V^o = \gamma$ or g). We add to these a new reaction which turns out to be the only feasible test at present and near-future $\bar{\text{p}}\text{p}$ colliders. All other tests of a similar nature must await e^+e^- colliders at $\sqrt{\text{s}} = M_Z$. Our process is $\bar{p} + p \rightarrow Z^o + \gamma + any$ and the expected signals are: a cross section which is substantially larger than the standard-model prediction, and a nearly-flat p_T^γ distribution. Either one of these, if observed, may serve as a strong indication for Z^o-compositeness.

2.5 On the $ee\gamma$ Events [49]

In 1983 UA2 [50] and UA1 [51] collaborations have reported the observation of three $\ell^+\ell^-\gamma$ events at invariant mass of 90 GeV. This number of events (when compared with 12 $Z^o \to e^+e^-$ events seen at the time) is larger by an order of magnitude than the standard model prediction. Though the possibility of statistical fluctuation was not yet ruled out — the other possibility, of new physics reflected in these events, has been already considered by several authors [52–55]. We shall discuss here the interpretation suggested by [53], [54]. The authors of these papers have suggested the following: Quarks, leptons and $W^\pm$, Z are all composite. In addition to these "standard" particles there is also a composite scalar X with mass 40–50 GeV. A new decay mode of the Z boson is responsible for the observed $l^+l^-\gamma$ events:

$$Z^o \to \gamma X, \quad X \to e^+e^- \tag{2.64}$$

In this section we explore our objection to the above scenario. We claim that in the framework of composite models the decay (2.64) is strongly suppressed and thus it is unlikely that this process is a source for high $\ell^+\ell^-\gamma$ rate: We first prove that the process (2.64) must break chiral symmetry; we then show that this symmetry breaking unavoidably causes strong suppression of (2.64) .

We follow references [53], [54] and consider only the $ee\gamma$ events. In composite models there is always some (approximate) chiral symmetry which is protecting the masses of the light fermions [56]. Associated with chiral symmetry there is an (approximately) conserved quantum number Y such that

$$Y(e_L^-) = -Y(e_R^+) = \alpha \tag{2.65}$$

$$Y(e_R^-) = -Y(e_L^+) = \beta \neq \alpha \tag{2.66}$$

Consider the electron coupling to the photon and the Z-boson: Since γ and Z are vectors they couple to electron-positron pairs which carry vanishing Y (i.e., they couple to $e_R^+ e_L^-$ and $e_L^+ e_R^-$). Therefore, the γ and Z must carry no Y. We may now show that chiral symmetry is broken in (2.64) : The initial state is Z^o which has Y=0. The final state is $\gamma + (e^+e^-)_{\text{coupled to scalar}}$. The photon carries no Y; the $(e^+e^-)_{\text{coupled to scalar}}$ is a linear combination of states that do carry nontrivial Y (i.e. it is a linear combination of $e_L^+ e_L^-$ $(Y = \alpha - \beta \neq 0)$ and $e_R^+ e_R^-$ $(Y = -(\alpha - \beta) \neq 0)$). Thus, Y is not conserved in the process, i.e. chiral symmetry is broken.

What are the consequences of this breaking? In order for (2.64) to account for the observed $e^+e^-\gamma$ rate the following condition should be satisfied [53] [54] :

$$\Gamma(Z^o \rightarrow \gamma X) BR(X \rightarrow e^+e^-) \sim 20_{MeV}. \tag{2.67}$$

We will now show that the breaking of chiral symmetry implies that the *lhs* of (2.67) is much smaller than 20 MeV: As mentioned above, the role of chiral symmetry in composite models is to protect the masses of the light fermions. Therefore terms in the low energy Lagrangian which do not conserve chiral symmetry are expected to be suppressed by a factor of $\frac{m_f}{\Lambda}$ where m_f is some characteristic fermion mass and Λ is the compositeness scale [56] . Since chiral symmetry is broken in the process (2.64) —

it must be violated (at least) in one of the vertices $Z^o\gamma X$, Xe^+e^-. Thus, one of these vertices should be suppressed by an $\frac{m_f}{\Lambda}$ factor:

(i) Suppose chiral symmetry is broken at $Z^o\gamma X$ vertex. As was calculated in reference [53] [54] : $\Gamma(Z^o \to \gamma X)$ with no suppression is $\lesssim$ 300 MeV; BR $(X \to e^+e^-)$ is $\lesssim \frac{1}{20}$ (this is because the decay strength of X to e^+e^- is supposed to be approximately equal to the decay strength of X to any of the other 20 light pairs of fermion-anti-fermion). If we now take into account a suppression factor of $(\frac{m_f}{\Lambda})^2$ for $\Gamma(Z^o \to \gamma X)$ we find:

$$\Gamma(Z^o \to \gamma X) BR(X \to e^+e^-) \lesssim (\frac{m_f}{\Lambda})^2 300_{MeV} \frac{1}{20} = (\frac{m_f}{\Lambda})^2 15_{MeV} \ll 20MeV$$

(ii) If chiral symmetry is broken at the Xe^+e^- vertex then this vertex is expected to be suppressed by $\frac{m_e}{\Lambda}$. Thus $\Gamma(X \to e^+e^-)$ is $\sim (\frac{m_e}{\Lambda})^2 m_x$ which is $\lesssim 10^{-2}$eV (the last bound arises because m_x is ~ 40-50 GeV [53] [54] and Λ is not smaller than 1 TeV [36]). In this case the important bound on BR $(X \to e^+e^-)$ comes from the competing decay $X \to \gamma\gamma$: W-dominance makes it reasonable to expect (see [53] and appendix B) that the $Z\gamma X$ and the $X\gamma\gamma$ vertices are identical, except for a factor of $\tan\theta_W (\sim 1/2)$. Taking the $\tan\theta_W$ factor and a phase space factor into account one gets [53] [54] :

$$\Gamma(X \to \gamma\gamma) \sim \frac{1}{10}\Gamma(Z \to \gamma X) \tag{2.68}$$

Thus:

$$BR(X \to e^+e^-) \lesssim \frac{\Gamma(X \to e^+e^-)}{\Gamma(X \to \gamma\gamma)} \sim 10\frac{\Gamma(X \to e^+e^-)}{\Gamma(Z \to \gamma X)} \tag{2.69}$$

and:

$$\Gamma(Z \to \gamma X) BR(X \to e^+e^-) \lesssim 10\Gamma(X \to e^+e^-) \sim 0.1_{eV} \ll 20_{MeV} \tag{2.70}$$

We thus see that whether chiral symmetry is broken at $Z^o\gamma X$ vertex or in Xe^+e^- vertex — its breaking implies a suppression of the decay (2.64) to such small values that this decay may no longer be responsible for the observed $ee\gamma$ rate.

We wish to remark that our considerations apply only to Z^o decay through intermediate scalar when the underlying theory is compositeness. Our "chiral symmetry argument" does not apply if the underlying theory is not compositeness or if the intermediate particle is not a scalar but a fermion or a vector (as proposed in [52], [54], [55]). However, though such alternative decays are not excluded by our arguments they encounter other difficulties [52], [54], [55] which we shall not treat here.

Concluding, we indicate that three composite-model explanations were suggested for $ee\gamma$ events: Z^o decay through a scalar [53], [54] , fermion [55], [54] , or a vector [52], [54] . We showed that the first suggestion (intermediate scalar) is unfavoured by chiral symmetry [49].

We are aware of the fact that with the accumulation of events in the 1984 run and the improvement of statistics — the $ee\gamma$ events are now in agreement with the Bremsstrahlung process predicted by the standard model. However the analysis we presented here might still be of some interest: First, it manifests the important role chiral symmetry is playing in composite models. Second, we may conclude that the Z^0 might after all be composite and there might exist a composite scalar X in the 40–50

GeV region. This scalar was not yet seen because chiral symmetry makes it undetectable at present energies.

2.6 Summary

In this part of the work we have discussed three tests of W and Z compositeness: (i) Universality — At present no deviations from universality have been seen, nor have FCNC effects been observed. Therefore we were able to put only upper bounds on deviations from universality. Under simple assumptions these were translated to lower bounds on the compositeness scale Λ. The best bound was obtained from the absence of FCNC effects in $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings:

$$\Lambda > 1.2_{\mathrm{TeV}}$$

(ii) In a composite model one expects the appearance of effective terms of the form $Z^0V^0 \ldots V^0$ where a V^0 is a photon or a gluon field. Considering the terms $Z^0V^0V^0$ and $Z^0V^0V^0V^0$ we found that the most important term for near future physics is the $Z^0\gamma gg$ vertex which may give a sizeable effect in the $\bar{p}p$ colliders of CERN and Fermilab.
(iii) We criticized the idea that the 1983 $ee\gamma$ events are due to composite Z decay through an intermediate scalar. Our objection is based on the observation that chiral symmetry is broken in this process.

Chapter 3

The Quark Mixing Matrix

3.1 Introduction

The many particles and parameters of the standard model lead us to speculate about the possibility of an underlying theory. However, such speculations are not only due to the *proliferation* of particles and their parameters, but also to the *pattern* which seems to exist in the spectrum:

(i) We see three "generations", having identical $SU(3)_C \times SU(2)_W \times U(1)_Y$ properties.

(ii) There is a mass hierarchy between the generations.

(iii) There is also a hierarchy inside each generation: the u-like quark is heavier than the d-like quark (except for the case of the first generation); the quarks are heavier than the charged lepton which is heavier than its (left-handed) neutrino.

(iv) mixings between neighbouring generations (i.e. mixing between the first and second generations or between the second and third generations) are bigger than other mixings (i.e. bigger than the mixing between the first and third generations).

(v) The mixing of the i'th and j'th generations seems to be related to mass ratios $\sqrt{\frac{m_i}{m_j}}$ where m_i, m_j are characteristic masses of the i'th, j'th generations. The most famous relation of this kind is: $sin\theta_c \approx \sqrt{\frac{m_d}{m_s}}$.

Clearly, the identification of phenomenological rules such as (i)–(v), may help us in our search for possible "underlying physics". In this chapter we make a modest step in this direction: We suggest a new parametrization to the mixing matrix. The new parameters have a simple meaning and they are simply and conveniently related to measurable quantities. Also, the pattern we recognize in the mixing matrix is simply formulated in terms of these parameters. Our parametrization is generalizable in a straight-forward manner to the case of more than three generations (in contrast to the Kobayashi-Maskawa parametrization which has no obvious generalization). We therefore hope that the parametrization we propose here will prove to be useful for derivations of new phenomenological rules or for generalizations of known rules to higher generations.

The rest of this chapter is divided into four sections: In section 3.2 we describe our parametrization and discuss its properties and advantages. In sections 3.3 and 3.4 we exemplify the usefulness of our parametrization: Section 3.3 includes an analysis of possible inconsistencies of the minimal standard model with experiment. In section 3.4 we discuss the Fritzsch mass matrices in the framework of the minimal standard model. In section 3.5 we summarize our results.

3.2 A Parametrization of the Mixing Matrix

The mixing between generations appears because the mass eigenstates are different from the eigenstates of the weak interaction. The weak charged current is:

$$j_\mu^{weak} = \overline{u_i} C_{ij} \gamma_\mu \frac{1}{2}(1-\gamma_5) d_j \tag{3.1}$$

where i, j are generation indices: $i, j = 1, 2, \ldots\ldots, N$; u_i, d_j are the (mass eigenstates) quark-fields: $u_i = u, c, t, \ldots$; $d_j = d, s, b, \ldots$ and C is the generalized Cabibbo mixing matrix.

The fermion fields u_i, d_j are defined only up to a phase. Therefore C is defined only up to a multiplication by phases, i.e. we are free to multiply C by a diagonal unitary matrix on the left and by another diagonal unitary matrix on the right.

C, as an $N \times N$ unitary matrix, has a priori N^2 parameters: $\frac{N(N-1)}{2}$ are rotation angles and $\frac{N(N+1)}{2}$ are phases. However, for the reasons stated above, $(2N-1)$ phases are unphysical. We are therefore left with $\left(\frac{(N-1)(N-2)}{2}\right)$ phases. Consider for example the case N=2: We have one rotation angle and no physical phase. The conventional parametrization for C in this case is:

$$C = \begin{pmatrix} cos\theta_c & sin\theta_c \\ -sin\theta_c & cos\theta_c \end{pmatrix} \tag{3.2}$$

where θ is between 0 and $\frac{\pi}{2}$.

Consider next the three generation case: We have three rotation angles and one physical phase. The conventional parametrization for C is the Kobayashi-Maskawa parametrization [28]:

$$C_{KM} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & -c_1s_2c_3 - c_2s_3e^{i\delta} & -c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix} \tag{3.3}$$

where $c_i = cos\theta_i$ and $s_i = sin\theta_i$ (i=1,2,3). All rotation angles θ_i can be chosen to lie between 0 and $\frac{\pi}{2}$; the phase δ is between 0 and 2π.

The θ_i of Kobayashi and Maskawa are actually the Euler angles:

$$C_{KM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

Since the Euler angles are defined only for the three dimensional rotation it is not clear how to generalize C_{KM} to N generations.

We therefore suggest the following parametrization for the mixing matrix in the N generation case [1] :

$$C_N = \Omega_{N-1,N}\Omega_{N-2,N}\ldots\Omega_{1,N}\Omega_{N-2,N-1}\ldots\Omega_{1,N-1}\ldots\Omega_{2,3}\Omega_{1,3}\Omega_{1,2} \tag{3.4}$$

[1] the same parametrization was independently suggested by Chau and Keung [57] but they have considered only the three-generation case.

where $\Omega_{i,j}$ for $i < j$ is a *complex* rotation between the i'th and and j'th generations:

$$\Omega_{i,j} = \begin{pmatrix} 1 & & & & & & & & & \\ & \ddots & & & & & & & & \\ & & 1 & & & & & & & \\ & & & cos\theta_{i,j} & 0 & \dots & 0 & sin\theta_{i,j} e^{-i\delta_{i,j}} & & \\ & & & 0 & 1 & & & 0 & & \\ & & & \vdots & & \ddots & & \vdots & & \\ & & & 0 & & & 1 & 0 & & \\ & & & -sin\theta_{i,j} e^{i\delta_{i,j}} & 0 & \dots & 0 & cos\theta_{i,j} & & \\ & & & & & & & & 1 & \\ & & & & & & & & & \ddots \\ & & & & & & & & & & 1 \end{pmatrix} \tag{3.5}$$

$\theta_{i,j}$ is a real rotation angle (and is between 0 and $\frac{\pi}{2}$); $\delta_{i,j}$ is a phase.

In this representation every rotation angle $\theta_{i,j}$ ($i < j$) appears with its corresponding phase $\delta_{i,j}$. Recall now that the number of physical phases is smaller than the number of rotation angles by $(N-1)$. Indeed, it turns out that it is possible to set the $(N-1)$ phases $\delta_{i,i+1}$ $(i = 1, \dots, N-1)$ to 0. All other phases are not restricted. From now on we will make this choice.

We denote $cos\theta_{i,j}$ by $c_{i,j}$ and $sin\theta_{i,j} e^{\delta_{i,j}}$ by $s_{i,j}$. (in this notation $sin\theta_{i,j}$ is $|s_{i,j}|$). $s_{i,j}$ is simply interpreted as the (complex) mixing between the i'th and j'th generations (in the Kobayashi Maskawa parametrization there is no simple interpretation to s_2, s_3). Note that the mixing between neighbouring generations is always real. In order to further analyze the properties of the parametrization (3.3) we write down the corresponding matrix for the $N = 3$ case:

$$C_3 = \Omega_{2,3}\Omega_{1,3}\Omega_{1,2} =$$

$$= \begin{pmatrix} c_{1,2}c_{1,3} & s_{1,2}c_{1,3} & s_{1,3} \\ -s_{1,2}c_{2,3} - c_{1,2}s_{2,3}s^*_{1,3} & c_{1,2}c_{2,3} - s_{1,2}s_{2,3}s^*_{1,3} & s_{2,3}c_{1,3} \\ s_{1,2}s_{2,3} - c_{1,2}c_{2,3}s^*_{1,3} & -c_{1,2}s_{2,3} - s_{1,2}c_{2,3}s^*_{1,3} & c_{2,3}c_{1,3} \end{pmatrix} \tag{3.6}$$

We see that the first row and the last column of C_3 have an especially simple form. This property is generalized to C_N in the following way: The first row of C_N is:

$$\left(c_{1,2}c_{1,3}\cdots c_{1,N}\right),\quad \left(s_{1,2}c_{1,3}\cdots c_{1,N}\right),\quad \left(s_{1,3}c_{1,4}\cdots c_{1,N}\right),\ldots\ldots,\left(s_{1,N-1}c_{1,N}\right),\quad s_{1,N}$$

and the last column is:

$$\begin{array}{c} s_{1,N} \\ s_{2,N}c_{1,N} \\ s_{3,N}c_{2,N}c_{1,N} \\ \vdots \\ s_{N-1,N}c_{N-2,N}\cdots c_{1,N} \\ c_{N-1,N}c_{N-2,N}\cdots c_{1,N} \end{array}$$

In the three-generation case all mixings $s_{1,2}$, $s_{2,3}$, $s_{1,3}$ are small. We assume that this is true also in the N-generation case. Then, to a first approximation, the first column and last row are:

$$\begin{array}{cccccc} 1 & s_{1,2} & s_{1,3} & \cdots & s_{1,N-1} & s_{1,N} \\ & & & & & s_{2,N} \\ & & & & & s_{3,N} \\ & & & & & \vdots \\ & & & & & \vdots \\ & & & & & s_{N-1,N} \\ & & & & & 1 \end{array}$$

i.e., to a good approximation the elements in the first and last column are simply the corresponding mixings:

$$(C_N)_{1,j} \approx s_{1,j}$$

$$(C_N)_{i,N} \approx s_{i,N}$$

We now proceed from the first row and last column to a discussion of all elements that lie above the main diagonal. We start by examining C_3: Note that in the limit of vanishing $s_{1,3}$ the upper right corner of C_3 looks like:

$$\begin{array}{cc} s_{1,2} & 0 \\ & s_{2,3} \end{array}$$

This property is generalizable to C_N in the following way: When $s_{1,N}$ vanishes, the upper-right corner becomes:

$$\begin{matrix} s_{1,N-1} & 0 \\ & s_{2,N} \end{matrix}$$

When $s_{1,N}$, $s_{1,N-1}$ and $s_{2,N}$ all vanish — this corner becomes:

$$\begin{matrix} s_{1,N-2} & 0 & 0 \\ & s_{2,N-1} & 0 \\ & & s_{3,N} \end{matrix}$$

and when $s_{i,j}$ vanishes for all i, j such that $j-i \geq k$ then the corner is:

$$\begin{matrix} s_{1,k} & 0 & \dots & 0 \\ & s_{2,k-1} & & \vdots \\ & & \ddots & 0 \\ & & & s_{N-(k-1),k} \end{matrix}$$

In the three-generation case we know that $\theta_{1,3}$ must be considerably smaller than $\theta_{1,2}$, $\theta_{2,3}$. ($\theta_{1,2}$ is actually the original Cabibbo mixing angle $\theta_c \sim 0.22$; $\theta_{2,3}$ is measured through the b-lifetime ($\theta_{2,3} \sim 0.065$) and bounds on $\theta_{1,3}$ are obtained from the bounds on b-decay-rate to the u-quark ($\theta_{1,3} \leq 0.0087$)). We generalize this property to N generations by assuming that $|s_{i,j}|$ becomes smaller as $(j-i)$ (the distance between the generations) increases. More quantitatively, we assume that $|s_{i,j}|$ is $O(\alpha^{j-i})$ where α is a small number (α is between $\sim \frac{1}{5}$ and $\sim \frac{1}{15}$). α is also the parameter which describes the hierarchy of the generation masses: $\frac{m_i}{m_j}$ is $\sim O(\alpha^{2(j-i)})$ where m_i is the typical mass scale of the i'th generation. Under the last assumption we find that, to a very good approximation (up to corrections of order α^4), the upper half of the mixing matrix is given by:

$$(C_N)_{i,j} = s_{i,j} \qquad for\ i < j \tag{3.7}$$

and the diagonal is:

$$\begin{matrix} c_{1,2} & & & & & \\ & c_{1,2}c_{2,3} & & & & \\ & & c_{2,3}c_{3,4} & & & \\ & & & \ddots & & \\ & & & & c_{N-2,N-1}c_{N-1,N-2} & \\ & & & & & c_{N-1,N} \end{matrix}$$

The elements below the diagonal are more complicated, but it is possible to show that $(C_N)_{i,j}$ is $O(\alpha^{|i-j|})$ for all i, j (i.e., also for the elements below the diagonal).

Note how convenient it is to have simple elements above the diagonal: $|(C_N)_{i,j}|^2$ for $i < j$, is, up to a phase space factor, the decay-rate of d_j to u_i. A measurement of this rate immediately gives the value of $|s_{i,j}|$, i.e., it gives the parameter $\theta_{i,j}$. For example, in the three-generation case, measurements of the rate of b-decay to c give the parameter $\theta_{2,3}$ while for the Kobayashi Maskawa parametrization the rate of b-decay to c gives only the value of a relatively complicated function of s_2, s_3 and δ: $|s_2 + s_3 e^{i\delta}|$.

we indicate that we could have chosen a parametrization in which the elements *below* the diagonal are simple i.e., we could have chosen :

$$\hat{C}_N = \Omega_{1,2}\Omega_{1,3}\Omega_{2,3}\ldots\ldots\Omega_{1,N}\Omega_{2,N}\ldots\Omega_{N-1,N} \tag{3.8}$$

Then we would have found that:

$$(\hat{C}_N)_{i,j} \approx s^*_{j,i} \qquad for\ i > j \tag{3.9}$$

and we would have got $\theta_{i,j}$ from measurements of u_i decay-rate to d_j. However, note that according to our experience with the first three generations we expect the u-like quarks of a heavy generation to be heavier than its d-like partner. Therefore, the main channel for u_i decay is to its partner d_i while u_i decay to lower generations is suppressed and hard to detect. In contrast to this situation, d_i is forced to decay to lower generations. Also d_i's are discovered and produced before the u_i's are (strangeness was known long before charm and the b-quark was discovered in 1977 while it is not yet clear if the t-quark has been really seen). We therefore find the parametrization (3.4) more useful than (3.8) .

Another advantage of the parametrization (3.4) is that it may be represented by a recursive formula in N, i.e.:

$$C_N = \Omega_{N-1,N} \dots \Omega_{1,N} \cdot \begin{pmatrix} & & & 0 \\ & C_{N-1} & & \vdots \\ & & & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix} \qquad (3.10)$$

This representation of C_N enables us to prove all the above mentioned properties of C_N by induction on N.

We conclude that our parametrization (3.4) for the mixing matrix is, in the case of three generations, more convenient than the traditionally used Kobayashi-Maskawa matrix. If a fourth generation will be discovered, our parametrization will be generalized in a simple and transparent manner.

3.3 Possible Inconsistencies of the Minimal Standard Model with Experiment

The particle content of the minimal standard model includes three fermion generations and a single Higgs. The Higgs is in the $\frac{1}{2}$ representation of $SU(2)_W$ and has four real components, three of which are "eaten up" by the massive W and Z and only the fourth is physical. The parameters of the theory are the gauge couplings, the W-mass, the fermion masses, the mixing matrix and the mass of the physical Higgs (and also the strong CP violation parameter ϑ). These parameters are determined from various measurements. Clearly, if the results of some measurements require a set of parameters which is different than the set required by other measurements — we say that the model is inconsistent with experiment.

It was pointed out several years ago [23–24] that the following measurements may put the minimal standard model into such inconsistency:

(1) t-quark mass(m_t).

(2) b-lifetime (τ_b).

(3) Branching ratio of b-decay to u-quark ($R(b \to u)$).

(4) CP violation in $K - \bar{K}$ system.

In this chapter we repeat the analysis of this possible inconsistency in terms of our parameters. As we will show, with these parameters the analysis is very simple and so is the representation of the results. In the following we give the relations between the mixing parameters and b-decays (subsection 3.3.1) and the relations between the mixings and the ϵ, ϵ' parameters (subsection 3.3.2). We then discuss the possible inconsistency of the minimal standard model with experiment (subsection 3.3.3).

3.3.1 Mixing parameters and b-decays

As explained in section 3.1, $s_{2,3}$ and $|s_{1,3}|$ are very simply related to b-decay rates:

$$\begin{aligned} |s_{1,3}|^2 &\approx 2.0 \cdot 10^{-3} R(b \to u) \cdot \frac{10^{-12}\text{psec}}{\tau_b} \qquad (a) \\ s_{2,3}^2 &\approx 4.2 \cdot 10^{-3} R(b \to c) \cdot \frac{10^{-12}\text{psec}}{\tau_b} \qquad (b) \end{aligned} \tag{3.11}$$

where $R(b \to u)$, $R(b \to c)$ are the branching ratios for b-decays to u+any, c+any respectively. (The numerical coefficients in (3.11) are phase space-factors.) b-decays to u were not yet seen. We therefore have only upper experimental bounds on $R(b \to u)$. The best bound is [22] $R(b \to u) \leq 0.04$. $R(b \to c)$ is very close to 1. Substituting $R(b \to u)$, $R(b \to c)$ in (3.11) we find:

$$\begin{aligned} |s_{1,3}|^2 &\leq 8 \cdot 10^{-5} \frac{10^{-12}\text{sec}}{\tau_b} \qquad (a) \\ s_{2,3}^2 &\approx 4.2 \cdot 10^{-3} \frac{10^{-12}\text{sec}}{\tau_b} \qquad (b) \end{aligned} \tag{3.12}$$

Measurements of b-lifetime [20] give $1_{psec} < \tau_b < 2_{psec}$. For every value of τ_b in the range 1–2 psec equation (3.12) gives us the corresponding value of $s_{2,3}$ and an upper bound on $|s_{1,3}|$.

Note that in the Kobayashi-Maskawa parametrization $s_{2,3}$ in (3.12) (b) is replaced by $|s_3 + s_2 e^{i\delta}|$. This last expression is inconvenient to deal with, since it involves three parameters.

3.3.2 The Mixing Parameters and the CP Violation Parameters ϵ and ϵ'

Introduction to ϵ and ϵ':

The parameters ϵ and ϵ' are given by:

$$\begin{aligned} \epsilon &= e^{i\frac{\pi}{4}}\left(\frac{ImM_{12}}{\sqrt{2}\Delta m_K} + \frac{ImA_0}{\sqrt{2}ReA_0}\right) \qquad (a) \\ \epsilon' &= \frac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{ReA_2}{ReA_0}\left(\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right) \qquad (b) \end{aligned} \tag{3.13}$$

where A_0, A_2 are the weak decay amplitudes of K^0 to two pions coupled to (strong) isospin I = 0,2; δ_0, δ_2 are strong interaction π-π phase shifts. M is the 2×2 mass matrix of the $K-\bar{K}$ system; Δm_K is the $K_L - K_S$ mass difference; We note that Δm_K is related to M_{12} through:

$$\Delta m_K = 2ReM_{12} \tag{3.14}$$

We will first simplify expressions (3.13) (a) and (b). In our parametrization (3.4) (and also in the Kobayashi-Maskawa parametrization) A_2 is real [2] . We may therefore simplify our expression for ϵ':

$$\epsilon' = -\frac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{A_2}{ReA_0} \frac{ImA_0}{ReA_0} \tag{3.15}$$

We will now argue that $|\frac{ImA_0}{ReA_0}| \ll |\frac{ImM_{12}}{\Delta m_k}|$ and therefore the expression for ϵ may be

[2] This is due to the fact that in the parametrizations (3.3) and (3.4) the first two elements of the first row are real.

simplified to:

$$\epsilon = e^{i\frac{\pi}{4}} \frac{ImM_{12}}{\sqrt{2}\Delta m_K} \tag{3.16}$$

We use the following facts:

(i) Gilman and Wise [58] have shown that, in the minimal standard model, $\frac{\epsilon'}{\epsilon}$ is real and positive.

(ii) Recent measurements of $\frac{\epsilon'}{\epsilon}$ give:

$$\frac{\epsilon'}{\epsilon} = -0.0046 \pm 0.0053 \pm 0.0024 \tag{3.17}$$

We now substitute in (3.17) the expressions (3.15) for ϵ' and (3.13) (a) for ϵ, and get:

$$\frac{|\frac{A_2}{ReA_0}||\frac{ImA_0}{ReA_0}|}{|\frac{ImM_{12}}{\sqrt{2}\Delta M_K} + \frac{ImA_0}{ReA_0}|} \leq -0.0046 + \sqrt{.0053^2 + .0024^2} = 0.0012 \tag{3.18}$$

$|\frac{A_2}{ReA_0}|$ is $\sim \frac{1}{20}$ (this is the $\Delta I = \frac{1}{2}$ rule). We therefore find that:

$$|\frac{ImA_0}{ReA_0}| \leq 2.4 \cdot 10^{-2} |\frac{ImM_{12}}{\sqrt{2}\Delta M_K}| \tag{3.19}$$

Consequently, the expression (3.19) is a good approximation for ϵ.

The Relation Between ϵ and the Mixing Angles

M_{12} was originally computed by Gaillard and Lee [16] for the two generation standard model. Later the computation was generalized to the case of three generations [59],

[26], QCD corrections were calculated [17] and the "vacuum insertion approximation" introduced by Gaillard and Lee was reexamined. The final result is:

$$M_{12} = \frac{1}{12\pi^2} G_F^2 f_K^2 m_K m_c^2 B \cdot (\eta_1 (C_{2,1}^* C_{2,2})^2 + \eta_2 (C_{3,1}^* C_{3,2})^2 f_2(m_t) + 2\eta_3 (C_{2,1}^* C_{2,2} C_{3,1}^* C_{3,2}) f_3(m_t)) \tag{3.20}$$

where:

$$f_2(m_t) = \frac{m_t}{m_c}^2 (1 - \frac{3}{4} \frac{x_t(1+x_t)}{(1-x_t)^2} (1 + \frac{2x_t}{1-x_t^2} \ln x_t))$$
$$f_3(m_t) = ln(\frac{m_t}{m_c}^2) - \frac{3}{4} \frac{x_t}{1-x_t} (1 + \frac{x_t}{1-x_t} \ln x_t)$$

G_F ($\sim 1.1 \cdot 10^{-5}_{GeV^{-2}}$) is the Fermi constant; f_K ($\sim 165_{MeV}$) is the K decay constant; m_K ($\sim 498_{MeV}$) is the kaon mass; m_c, m_t are the masses of the c, t quarks: m_c is[3] $\sim$ 1.5 GeV and we will assume throughout this work that $20_{GeV} \leq m_t \leq 80_{GeV}$ [4]. $x_t = (\frac{m_t}{m(W)})^2$; B, η_1, η_2, η_3 are correction factors: B is a multiplicative factor correcting for the "vacuum insertion approximation". We will assume here that B ranges between 0.37 and 1 [61]. (B=1 corresponds to the vacuum insertion approximation). η_1, η_2, η_3 stand for short range QCD effects [62] . QCD effects were not taken into account at all in the early works and all η_i were assumed to be equal to 1. Gilman and Wise [17]

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[3] We prefer this relatively high value for m_c since it leads, through (3.14) and (3.20) , to better estimates of Δm_K.

[4] The lower bound , $m_t > 20_{GeV}$ was established by Tasso Collaboration [60]. The UA1 Collaboration have announced the identification of six t-quarks with mass $m_t \sim 40 \pm 10_{GeV}$ [27]. This result is however controversial and we therefore allow m_t-values as low as 20 GeV. We arbitrarily choose to limit our attention to m_t-values that are not exceeding the W-mass.

computed η_i and found them all O(1) but smaller than 1. Using [17] we will substitute:

$$\eta_1 = 0.7$$

$$\eta_2 = 0.6$$

$$\eta_3 = 0.4$$

For the mixing matrix elements $C_{i,j}$ we will use the parametrization (3.4) . We substitute ImM_{12} in the expression (3.16) for ϵ and find:

$$\epsilon \approx e^{i\frac{\pi}{4}} 9.6 B s_{2,3} |s_{1,3}| \sin\delta_{1,3} \cdot \cdot \left[(0.4 f_3(m_t) - 0.7) s_{1,2} + 0.6 f_2(m_t) s_{2,3} (s_{1,2} s_{2,3} + |s_{1,3}| \cos\delta_{1,3})\right] \tag{3.21}$$

In the Kobayashi-Maskawa parametrization one finds:

$$\epsilon \approx e^{i\frac{\pi}{4}} 9.6 B s_1 s_2 s_3 \sin\delta \cdot \left[(0.4 f_3(m_t) - 0.7) s_1 + 0.6 f_2(m_t) s_2 (s_1 s_2 + s_1 s_3 \cos\delta)\right] \tag{3.22}$$

The expressions (3.21) and (3.22) look very similar and at first sight it is not clear what is the advantage of our parametrization. But, suppose m_t is known, τ_b is measured with a high accuracy and improved theoretical calculations enable us to determine B. If we use the parametrization (3.4) then, by substituting the value of τ_b in (3.12) (b) we get the value of $s_{2,3}$. Substituting in (3.21) the values of $s_{1,2}$ ($= \sin\theta_c$), $s_{2,3}$, m_c, m_t, B and ϵ — we find ourselves with an implicit equation in the *two* variables $s_{1,3}$ and $\delta_{1,3}$. If, however, we use the standard Kobayashi-Maskawa parametrization then, by τ_b, we get an implicit equation in the *three* variables s_2, s_3 and δ:

$$|s_3 + s_2 e^{i\delta}|^2 = 4.2 \cdot 10^{-3} \cdot \frac{10^{-12}\text{psec}}{\tau_b}$$

Substituting in (3.22) the values of $s_1 = \sin\theta_c$, m_c, m_t, B and ϵ — we get another implicit equation in s_2, s_3 and δ. We therefore end up with two implicit equations for three variables. Clearly, it is easier to solve the one equation we get for for $s_{1,3}$, $\delta_{1,3}$ than to solve the two equations we get for s_2, s_3 and δ.

3.3.3 Is the Minimal Standard Model Consistent with Experiment?

We will now check whether the measurements of m_t, τ_b, $R(b \to u)$, ϵ and ϵ' all agree with the same "set of parameters" for the minimal standard model.

We know that τ_b is between 1 and 2 psec and we will assume that m_t is between 20 and 80 GeV. For every value of τ_b and m_t in this range we may determine the value of $s_{2,3}$ through (3.12) (b). We then substitute m_t, $s_{2,3}$ and the experimental value of ϵ ($= 2.27 \cdot 10^{-3}$) in (3.21) . For every fixed value of the parameter B (we assume that B is in the range 0.37–1) the solutions to equation (3.21) constitute a line in the $\theta_{1,3} - \delta_{1,3}$ plane. (Note that the line exists only for $0° < \delta_{1,3} < 180°$. This is because the measured phase of ϵ is $\sim \frac{\pi}{4}$. Consequently, equation (3.21) implies that $\sin\delta_{1,3}$ is positive).

Consider $R(b \longrightarrow u)$. For our choice of τ_b the present experimental bound on $R(b \longrightarrow u)$ gives an upper bound on $\theta_{1,3}$ (see equation (3.12) (a)). In the $\theta_{1,3} - \delta_{1,3}$ plane we may describe this bound as a straight line.

We now compare our "ϵ-line" with the "b-decay bound". If none of the points on the ϵ-line obey the b-decay bound, we say that the standard model is inconsistent with our choice of m_t, τ_b (and B). If there are no values of m_t, τ_b (and B) with which the standard model agrees — we may say that it is inconsistent with experiment.

Consider, for example, the values $m_t = 45_{GeV}$, $\tau_b = 1.5_{psec}$ and $B = 0.4$. The corresponding ϵ-line and b-decay bound are described in fig. 3.1. The ϵ-line lies high above the b-decay bound and therefore our parameters clearly do not agree with the minimal standard model.

Straight-forward analysis of equation (3.21) shows that:

(1) The ϵ-line "goes down" as B and m_t increase (see e.g. figures 3.2, 3.3).

(2) The ϵ-line "goes down" and the b-decay bound "goes up" when τ_b decreases. (see e.g. figures 3.2,3.3).

We conclude from these observations that:

(a) For fixed values of m_t and τ_b the "lowest" ϵ-line is the line corresponding to B=1. Therefore the B=1 line gives a *lower* bound on $\theta_{1,3}$. We call this line "the ϵ bound". Allowed values of $\theta_{1,3}$ and $\delta_{1,3}$ correspond to points lying in between the ϵ bound and b-decay bound.

(b) Consider the ϵ-bound and b-decay bound for fixed value of τ_b and varying value of m_t: As m_t decreases, the ϵ-bound goes up till, at some value of m_t, it crosses the b-decay bound. For smaller values of m_t the ϵ *lower* bound is *above* the b-decay *upper* bound. Clearly, there are no $\theta_{1,3}$ and $\delta_{1,3}$ that may obey such bounds. We therefore find that these smaller values of m_t are inconsistent with the standard model. Similarly, if we fix m_t and consider the ϵ and b-decay bounds, we find that as τ_b increases, the two bounds are approaching each other (b-decay bound goes down while ϵ-bound goes up) till, at a certain value of τ_b the two lines cross each other. Clearly, larger values of τ_b are forbidden (since there are no $\theta_{1,3}$, $\delta_{1,3}$ which may satisfy the bounds corresponding to lower τ_b's). Concluding, we find that there is a line in the m_t-τ_b plane such that all points on one side of the line are consistent with the standard model while all those on the other side are not. The inconsistent points have the lower m_t-values and higher τ_b-values. The picture in the $m_t - \tau_b$ plane is described in figure 3.4.

Finally, we take into account the constraints imposed by the ϵ'- parameter. According to Gilman and Hagelin [63] $\frac{\epsilon'}{\epsilon}$ is, in the minimal standard model, given by:

$$\frac{\epsilon'}{\epsilon} = A\frac{Im(C^*_{2,1}C_{2,2})}{\sin\theta_c} \tag{3.23}$$

where A is ~ 8.4.

As mentioned above, the latest experimental results imply:

$$\frac{\epsilon'}{\epsilon} \leq 1.2\cdot 10^{-3} \tag{3.24}$$

In our notation, (3.23) and (3.24) give:

$$8.4\frac{s_{2,3}|s_{1,3}|\sin\delta_{1,3}}{0.22} \leq 1.2\cdot 10^{-3} \tag{3.25}$$

For every value of τ_b and m_t we now have:

(i) An upper bound on $\theta_{1,3}$ from b-decay,

(ii) a lower bound on $\theta_{1,3}$ through ϵ and

(iii) another upper bound through ϵ'.

In figure 3.5 we present the situation for $\tau_b = 1$ psec, $m_t = 45$ GeV. We see that the ϵ bound is far from agreement with the ϵ' bound. However we note that the situation would have improved if the constant A would have been smaller and the bound on $\frac{\epsilon'}{\epsilon}$ relaxed. Indeed, the theoretical uncertainties in the computation of A allow for A-values as small as 2 [64]. The experimental errors in the measurement of $\frac{\epsilon'}{\epsilon}$ allow us to relax the upper bound (3.25) and we do so by summing the experimental errors *linearly* (and not quadratically):

$$|\frac{\epsilon'}{\epsilon}| \leq -0.0046 + .0053 + .0024 = 3.1 \cdot 10^{-3} \tag{3.26}$$

substituting in (3.23) $A = 2$ and the modified bound (3.26) on $|\frac{\epsilon'}{\epsilon}|$ we get a weaker bound on $\theta_{1,3}$ which is described in figure 3.6. The allowed region (according to the standard model) lies now in between the ϵ-bound, the ϵ'-bound and b-decay bound.

Let us summarize: In order to find a set of parameters with which the standard model becomes consistent with present experimental results one should:

(1) Use figure 3.4 in order to choose the allowed values for τ_b and m_t.

(2) For these values of τ_b, m_t one should find the region in the $\theta_{1,3} - \delta_{1,3}$ space which is allowed by the ϵ, ϵ' and b-decay bounds. In figure 3.7 we give the allowed region corresponding to $\tau_b = 1$ psec and $m_t = 45$ GeV.

The standard model will be "in trouble" if τ_b and m_t will be measured and found to be in the "forbidden" region, or if the experimental bounds on $R(b \to u)$ and $\frac{\epsilon'}{\epsilon}$ will become so strong that they will exclude every set of parameters. [5]

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[5] "Brand new" experimental results seem to indicate that the minimal standard model is doing well: In the Tokyo conference [65] it was mentioned that the bound on $R(b \longrightarrow u)$ stated in [22] is too strong: New careful analysis of the CLEO-experiment give: $R(b \longrightarrow u) \leq .08$. In addition, new measurements of τ_b seem to indicate that this quantity is somewhat *smaller* than 1 psec [66] .

3.4 The Fritzsch Mass Matrices and the Minimal Standard Model

3.4.1 Introduction to the Fritzsch Mass Matrices [67–68]

In the late 70's the relation:

$$\sqrt{\frac{m_d}{m_s}} \sim \sin\theta_c \tag{3.27}$$

have attracted a lot of attention. Many proposals for the quark mass matrices have arose, amongst them — the Fritzsch mass matrices. Fritzsch suggested that the quark mass matrices are of the following form [67]:

$$M = \begin{pmatrix} 0 & a \\ a^* & b \end{pmatrix} \tag{3.28}$$

where a and b are hierarchical, e.g.:

$$|\frac{a}{b}| \leq O(\alpha) \tag{3.29}$$

(α is the small parameter discussed in the first section of this chapter.) It is straightforward to show that if M^u, M^d are of the form (3.28) then:

$$\sin\theta_c = |e^{i\phi}\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}| \tag{3.30}$$

where ϕ is the relative phase between the a-parameter of M^u and the a-parameter of M^d. If we choose $\phi \sim \frac{\pi}{2}$ then we get the relation (3.27) to a very good approximation. A few months after the publication of [67], Fritzsch has generalized his matrices for the three-generation case. Here we will give the generalization to N generations:

$$M = \begin{pmatrix} 0 & a_1 & & & & \\ a_1^* & 0 & a_2 & & & \\ & a_2^* & 0 & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & 0 & a_{N-1} \\ & & & & a_{N-1}^* & b \end{pmatrix} \qquad \text{b real and positive} \tag{3.31}$$

The parameters a_i, b are "hierarchical":

$$|\frac{a_i}{a_{i+1}}| \leq O(\alpha^2) \quad \text{and} \quad |\frac{a_{N-1}}{b}| \leq O(\alpha).$$

It is straight-forward to show that:

(1) The fermion masses are related to a_i, b through:

$$b \approx m_N \qquad a_i \approx \sqrt{m_i m_{i+1}} \tag{3.32}$$

("$\approx$" means equality up to corrections of order α^2). Note that the mass spectrum of (3.31) is hierarchical:

$$\frac{m_i}{m_{i+1}} \leq O(\alpha^2) \tag{3.33}$$

(2) The mixing angles of *neighbouring* generations are [69]:

$$\theta_{i,i+1} \approx |\frac{a_i^d}{m_{i+1}^d} - \frac{a_i^u}{m_{i+1}^u}| \tag{3.34}$$

where a_i^u, b^u are the parameters of M^u and m_i^u are its eigenvalues. a_i^d, b^d and m_i^d are similarly related to M^d. It is useful to rewrite equation (3.34) in the following way:

$$\theta_{i,i+1} \approx |e^{i\phi_i}\sqrt{\frac{m_i^d}{m_{i+1}^d}} - \sqrt{\frac{m_i^u}{m_{i+1}^u}}| \tag{3.35}$$

where ϕ_i is the relative phase of a_i^u and a_i^d. Note that equation (3.35) reduces for $i = 1$ to (3.30) .

One of the nice features of the Fritzsch mass matrices is that the effective matrices of the low lying generations are also of the Fritzsch type and are very simply related to the original matrix [6] . The effective matrix for the first n generations ($n < N$) is:

$$M_{eff}^{(\text{n generations})} \approx \begin{pmatrix} 0 & a_1 & & & & \\ a_1^* & 0 & a_2 & & & \\ & a_2^* & 0 & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & \ddots & 0 & a_{n-1} \\ & & & & a_{n-1}^* & m_n \end{pmatrix}$$

Another attractive feature of the Fritzsch matrices is that they seem to arise from an interesting underlying dynamics:(i) The many zeros in the matrices presumably arise from some symmetries (originally, Fritzsch suggested discrete symmetries). (ii) The hermiticity of the matrices seems to be related to some symmetry (Fritzsch suggested left-right symmetry. In Appendix C we comment on his suggestion). (iii) The masses of the lower generations seem to be *fed down* from the mass of the highest generation in a hierarchical way. Many authors have tried to suggest mechanisms which could produce such feed-down [69, 70].

[6] By the effective matrices for the $n(< N)$ low lying generations we mean: $n \times n$ matrices whose eigenvalues give the low lying spectrum and whose mixings are the mixings of the low lying generations *among themselves*.

Here we will not try to understand what is the possible underlying physics which may give rise to Fritzsch mass matrices. We will only consider these matrices in the framework of the minimal standard model.

3.4.2 The Fritzsch Mass Matrices and the Standard Model

In this subsection we discuss the following question: Is it possible that the low lying physics consists of the minimal standard model with Fritzsch mass matrices? In order to see where problems may arise — consider the number of *physical* parameters in the mass matrices: If M^u, M^d are of the "Fritzsch-form", then they have 2N (dimensionful) mass parameters ($|a_i^u|$, b^u, $|a_i^d|$, b^d) and 2(N–1) (dimensionless) phase parameters (the phases of a_i^u, a_i^d). We now show that only (N–1) of the phases are physical. Consider the quark mass term in the Lagrangian

$$L_m = \overline{U_L^{(0)}} M^u U_R^{(0)} + \overline{D_L^{(0)}} M^d D_R^{(0)} + h.c. \tag{3.36}$$

$$U_{L(R)}^{(0)} = \left\{ u_{L(R)_i}^{(0)} \right\}_{i=1}^{N} \quad ; \quad D_{L(R)}^{(0)} = \left\{ d_{L(R)_i}^{(0)} \right\}_{i=1}^{N}$$

where i is a generation index and the index $^{(0)}$ indicates that the quark fields are interaction eigenstates and not mass eigenstates (i.e. $(u_{L\ i}^{(0)}, d_{L\ i}^{(0)})$ is an $SU(2)_L$ doublet).

We are clearly free to make the following redefinitions:

$$U^{(0)}_{L(R)} \longrightarrow FU^{(0)}_{L(R)}$$

$$D^{(0)}_{L(R)} \longrightarrow FD^{(0)}_{L(R)}$$

$$M^{u,d} \longrightarrow FM^{u,d}F^{+}$$

Where F is a diagonal unitary matrix

$$F = \begin{pmatrix} e^{i\varsigma_1} & & & \\ & e^{i\varsigma_2} & & \\ & & \ddots & \\ & & & e^{i\varsigma_N} \end{pmatrix} \tag{3.37}$$

Note that under these redefinitions the mass matrices keep their Fritzsch form. The only change in these matrices is in the phases of $a_i^{u,d}$: The phase of $a_i^{u,d}$ is changed by $-\varsigma_i$. We can choose the ς_i such that all the a_i^u of the new M^u are real and positive. We then have 2N mass parameters in M^u, M^d and (N-1) phase parameters which arise from M^d alone. Concluding, we find that, if the mass matrices are of the Fritzsch form, they depend on 3N–1 physical parameters.

We now count the number of measurable parameters which M^u and M^d should provide: These are the 2N masses of the u_i and d_i quarks, the $\frac{N(N-1)}{2}$ mixing angles and the $\frac{(N-1)(N-2)}{2}$ physical phases of the generalized Cabibbo matrix. Altogether we should have (N^2+1) parameters.

For $N = 1,2$ the number of measurable parameters,(N^2+1), is equal to the number of the parameters of the Fritzsch mass matrices, $3N-1$. But for $N \geq 3$:

$$N^2 + 1 > 3N - 1$$

and we therefore find that if the mass matrices are of the "Fritzsch" type then there must be relations between measurable quantities. In particular, note that for the case of three generations we have 10 measurable quantities but (assuming Fritzsch mass matrices) only 8 of them are independent.

Consider now a minimal standard model with Fritzsch mass matrices. In the previous chapter we discussed the experimental constraints imposed on the mixing matrix parameters and the quark masses. If the mass matrices are of the Fritzsch form we have additional constraints. Is it possible to satisfy all these constraints simultaneously? In order to answer this question we proceed in the following steps:

(a) We choose values of τ_b and m_t that are "allowed" according to fig 3.4.

Our choice is: $\tau_b = 1_{psec}$, $m_t = 45_{GeV}$. From τ_b we extract the corresponding value of $s_{2,3}$ (=0.065). We now have a set of eight parameters: The six quark masses

$$m_u = 4_{MeV} \qquad m_d = 7_{MeV}$$

$$m_c = 1.5_{GeV} \qquad m_s = 150_{MeV}$$

$$m_t = 45_{GeV} \qquad m_b = 4.8_{GeV}$$

and two mixing angles

$$\theta_{1,2} = \theta_c = 0.22$$

$$\theta_{2,3} = .065$$

(b) Assuming a Fritzsch form of the mass matrices we proceed to compute the last two parameters namely, $\theta_{1,3}$ and $\delta_{1,3}$. We denote the three-generation Fritzsch mass matrices by:

$$M^u = \begin{pmatrix} 0 & a^u & 0 \\ a^{u^*} & 0 & b^u \\ 0 & b^{u^*} & c^u \end{pmatrix} \qquad M^d = \begin{pmatrix} 0 & a^d & 0 \\ a^{d^*} & 0 & b^d \\ 0 & b^{d^*} & c^d \end{pmatrix} \tag{3.38}$$

We choose the convention in which all parameters of M^u (a^u, b^u,c^u) are real and positive and denote:

$$\begin{aligned} a^d &= |a^d| e^{i\varphi_1} \\ b^d &= |b^d| e^{i\varphi_2} \end{aligned} \tag{3.39}$$

The eight parameters of M^u, M^d are related to the six quark masses and the two mixing angles $\theta_{1,2}$, $\theta_{2,3}$ through:

$$\begin{aligned} a^u &\approx \sqrt{m_u m_c} & |a_d| &\approx \sqrt{m_d m_s} \\ b^u &\approx \sqrt{m_c m_t} & |b_d| &\approx \sqrt{m_s m_b} \\ c^u &\approx m_t & c^d &\approx m_b \end{aligned} \tag{3.40}$$

$$\begin{aligned} s_{1,2} &\approx \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\varphi_1} \sqrt{\frac{m_u}{m_c}} \right| \\ s_{2,3} &\approx \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\varphi_2} \sqrt{\frac{m_c}{m_t}} \right| \end{aligned} \tag{3.41}$$

From (3.41) we find:

$$\cos\varphi_1 \approx \frac{\frac{m_d}{m_s} + \frac{m_u}{m_c} - s_{1,2}^2}{2\sqrt{\frac{m_d}{m_s}}\sqrt{\frac{m_u}{m_c}}} = 0.042(1 + O(\alpha^2))$$

$$\Longrightarrow \varphi_1 \approx \pm(87.6^\circ \pm 0.1^\circ) \tag{3.42}$$

$$\cos\varphi_2 \approx \frac{\frac{m_s}{m_b} + \frac{m_c}{m_t} - s_{2,3}^2}{2\sqrt{\frac{m_s}{m_b}}\sqrt{\frac{m_c}{m_t}}} = 0.94(1 + O(\alpha^2))$$

$$\Longrightarrow \varphi_2 \approx \pm 21^{\circ}{}^{+5^\circ}_{-8^\circ} \tag{3.43}$$

(In (3.42) , (3.43) we estimate: $O(\alpha^2) \leq (\frac{1}{5})^2 = 0.04$.) In order to get $\theta_{1,3}$ and $\delta_{1,3}$ we use results presented in a paper by Fritzsch in 1978 [68]. In this paper Fritzsch gives the exact eigenstates e_i^u, e_i^d of the mass matrices M^u, M^d (equation (3.38)). In a certain phase convention the Cabibbo mixing matrix is then given by:

$$C_{i,j} = (e_i^u, e_j^d) \tag{3.44}$$

where (e_i^u, e_j^d) means hermitian product of the vectors e_i^u, e_j^d.

$\sin\theta_{1,3}$ is $|C_{1,3}| = |(e_1^u, e_3^d)|$. Substituting the Fritzsch formula for e_1^u, e_3^d we get:

$$\begin{aligned} \sin\theta_{1,3} &\approx \left|\frac{m_s}{m_b}\sqrt{\frac{m_d}{m_b}} + e^{-i\varphi_1}\sqrt{\frac{m_u}{m_c}}\left(\sqrt{\frac{m_s}{m_b}} - e^{-i\varphi_2}\sqrt{\frac{m_c}{m_t}}\right|\right. \\ &= \left|\frac{m_s}{m_b}\sqrt{\frac{m_d}{m_b}} + e^{-i\varphi_1}\sqrt{\frac{m_u}{m_c}}e^{-i\varphi_2'}s_{2,3}\right| \end{aligned} \tag{3.45}$$

(φ_2' is defined through the second equality of (3.45)).

Substituting in (3.45) the values of the quark masses and of φ_1, φ_2 we find two solutions:

$$\sin\theta_{1,3} = 0.0045(1 + O(\alpha^2)) \quad \text{or} \quad \sin\theta_{1,3} = 0.0022(1 + O(\alpha^2))$$

The smaller solution ($\sin\theta_{1,3} \approx 0.0022$) is excluded by the ϵ-bound on $\theta_{1,3}$ (see fig. 3.6). In order to compute $\delta_{1,3}$ we use the following phase:

$$\text{phase}\left(\frac{C_{2,1}C_{3,3}}{C_{3,1}C_{2,3}}\right)$$

(Note that this phase is independent of phase convention). It is straightforward to check that in the Fritzsch convention:

$$\text{phase}\left(\frac{C_{2,1}C_{3,3}}{C_{3,1}C_{2,3}}\right) \approx \text{phase}\left(\sqrt{\frac{m_u}{m_c}}e^{i\varphi_1} - \sqrt{\frac{m_d}{m_s}}\right) \tag{3.46}$$

and in terms of our parameters (3.4) :

$$\text{phase}\left(\frac{C_{2,1}C_{3,3}}{C_{3,1}C_{2,3}}\right) \approx \text{phase}\left(e^{-i\delta_{1,3}} - \frac{s_{1,2}s_{2,3}}{s_{1,3}}\right) \tag{3.47}$$

We therefore find:

$$\text{phase}\left(e^{-i\delta_{1,3}} - \frac{s_{1,2}s_{2,3}}{s_{1,3}}\right) \approx \text{phase}\left(\sqrt{\frac{m_u}{m_c}}e^{i\varphi_1} - \sqrt{\frac{m_d}{m_s}}\right) \tag{3.48}$$

Substituting in (3.48) m_u, m_c, m_d, m_s, $s_{1,2}$, $s_{2,3}$, $s_{1,3}$ and φ_1 we get:

$$\delta_{1,3} = \pm 33^{\circ}{}^{+8^{\circ}}_{-11^{\circ}} \quad \text{or} \quad \pm 118^{\circ} \pm 3^{\circ}$$

(c) Finally, we check whether the $\theta_{1,3}$, $\delta_{1,3}$ we have computed in step (b), through the assumption of Fritzsch form of the mass matrices, are in the allowed range of the minimal standard model. We therefore look at the allowed range corresponding to $\tau_b = 1_{psec}$ and $m_t = 45_{GeV}$ and check whether any of the points:

$$(\theta_{1,3}, \delta_{1,3}) = (4.5 \pm 0.2_{\text{milirad}}, \pm 35^{\circ}{}^{+8^{\circ}}_{-11^{\circ}}) \qquad (4.5 \pm 0.2_{\text{milirad}}, \pm 118^{\circ} \pm 3^{\circ})$$

fall into this range.We first note that the standard model allows only for $\delta_{1,3}$ in the range 0°–180°. We are therefore left with two possible points which may agree with both the constraints on the minimal standard model and the constraints arising from the Fritzsch mass matrices. In figure 3.8 we describe the two points and the range of $\theta_{1,3} - \delta_{1,3}$ values allowed for the standard model. We see that the point $(\theta_{1,3}, \delta_{1,3}) = (4.5_{\text{milirad}}, 118^{\circ})$ sits inside the allowed region.

Let us summarize: Assuming a Fritzsch form for the quark mass matrices we find that $\theta_{1,3}$ and $\delta_{1,3}$ are not independent parameters but are functions of the quark masses and the mixing angles $\theta_{1,2}$ and $\theta_{2,3}$. We showed that the $\theta_{1,3}$ and $\delta_{1,3}$ values corresponding to $m_t = 45$ GeV and $\tau_b = 1$ psec are getting values consistent with the minimal standard model We therefore conclude that, (at present), a minimal standard model with Fritzsch mass matrices is consistent with experiment.

3.5 Summary

We have proposed a new parametrization for the mixing matrix. Our parameters have both a simple interpretation and a simple relation to measurable quantities. We used the new parametrization for an analysis of the present status of the standard model. Assuming that m_t is in the range 20–80 GeV and using the experimental knowledge that τ_b is between 1 and 2 psec we were able to determine the region in the m_t– τ_b plane which agrees with the standard model. For every value of m_t and τ_b which agrees with the standard model, we were then able to describe in the $\theta_{1,3} - \delta_{1,3}$ plane an allowed region. This region includes the points which are consistent with the standard model according to measurements of $R(b \longrightarrow u)$ and the CP violating parameters ϵ, ϵ'. We note that in previous literature one usually find only the first part of our analysis (namely, the allowed region in $\tau_b - m_t$ space). This is because in the traditional Kobayashi-Maskawa notation the second part of the analysis is difficult and the results are obscured by the inconvenient choice of parameters.

We have also shown that the "Fritzsch-mass-matrices" are still consistent with the minimal standard model (a similar analysis was independently carried out by Shin [71].)

We conclude that the parametrization proposed here is more convenient to use than the traditional Kobayashi-Maskawa parametrization. Also, our parametrization is generalizable in a straight-forward manner to N generations. If more generations will be discovered we believe that this parametrization should become the standard one.

APPENDIX A

In this appendix we discuss the diagonlization of quark mass matrices. We will first give the general procedure and then specialize to the manifest-model and CCC-model cases.

Diagonalization of Quark Mass Matrices — General Case

Let M be any mass matrix. In order to diagonalize it we will use the following mathematical theorem.

Theorem: There exist unitary matrices U_L, U_R such that:

(i) $U_L M U_R^+$ is diagonal and positive definite.

(ii) U_L diagonalizes MM^+ and U_R diagonalizes M^+M.

A corollary from this theorem is: The eigenmasses of M are the square roots of the eigenvalues of MM^+.

Note that the theorem and its corollary specify the masses in a unique way. However, the mixing matrices U_L and U_R are not so clearly specified: If we chose an arbitrary $\tilde{U}_L$ that diagonalizes MM^+ and an arbitrary $\tilde{U}_R$ that diagonalizes M^+M then , usually, $\tilde{U}_L M \tilde{U}_R^+$ is not diagonal and not positive-definite. However, we will now introduce the assumption that there are no two up (or two down) quarks with the same mass, i.e., the eigenvalues of MM^+ (and M^+M) are not degenerate. Under this assumption it is possible to show that the unitary matrix that diagonalizes MM^+ (or M^+M) is unique up to permutations and phases. In this last statement we mean the following: If $\tilde{U}_L$ and $\tilde{\tilde{U}}_L$ are both diagonalizing MM^+—then, there exist P_L and F_L such that:

$$\tilde{\tilde{U}}_L = F_L P_L \tilde{U}_L \tag{A.1}$$

where:

P_L is a matrix of permutations — it is a unitary matrix whose elements are 0 or 1. Note that P_L permutes the rows of $\tilde{U}_L$.

F_L is a matrix of phases — it is a diagonal unitary matrix. Note that F_L multiplies every row of $P_L U_L$ by a phase. We now conclude that if $\tilde{U}_L$ and $\tilde{U}_R$ are any two matrices that diagonalize MM^+ and M^+M respectively — then U_L and U_R are given by:

$$\begin{aligned} U_L &= F_L P_L \tilde{U}_L \\ U_R &= F_R P_R \tilde{U}_R \end{aligned} \tag{A.2}$$

We note that there are many pairs of U_L and U_R such that $U_L M U_R^+$ is diagonal and positive definite. But, once we have a certain such pair (which we denote by V_L, V_R) all other pairs are given by:

$$\begin{aligned} U_L &= FPV_L \\ U_R &= FPV_R \end{aligned}$$

where F is a matrix of phases and P a matrix of permutations.

We now turn to examine the mixing matrices in the specific cases of the manifest and CCC models.

Diagonalization in the manifest model

Consider first M^u. In the manifest model this matrix is hermitian, and thus there exists a unitary matrix $\tilde{U}_L^U$ such that $\tilde{U}_L^u M^u (\tilde{U}_L^u)^+$ is diagonal. We therefore choose

$$\tilde{U}_L^u = \tilde{U}_R^u = \tilde{U}^u \tag{A.3}$$

Then, clearly $\tilde{U}_L^u M^u (\tilde{U}_R^u)^+$ is diagonal and real, but it is not necessarily positive definite, i.e. M^u may have negative eigenvalues [7] . To correct for these possible negative eigenvalues we multiply $\tilde{U}_R^u$ by $F^u(\pm)$. $F^u(\pm)$ is a matrix of real phases (i.e., a diagonal unitary matrix whose eigenvalues are 1 or -1). Then: $\tilde{U}_L^u M^u (\tilde{U}_R^u)^+ F^u(\pm)$ is diagonal and positive definite, thus:

$$\begin{aligned} U_L^u &= \tilde{U}_L^u \\ U_R^u &= F^u(\pm)\tilde{U}_R^u = F^u(\pm)U_L^u \end{aligned} \tag{A.4}$$

Similarly, we get:

$$\begin{aligned} U_L^d &= \tilde{U}_L^d \\ U_R^d &= F^d(\pm)\tilde{U}_R^d = F^d(\pm)U_L^d \end{aligned} \tag{A.5}$$

Since $C_{L(R)} = U_{L(R)}^u (U_{L(R)}^d)^+$ (A.4) and (A.5) imply:

$$C_R = F^u(\pm) C_L F^d(\pm) \tag{A.6}$$

.

[7] this was pointed up to us by G. Ecker.

Diagonalization in the CCC Model

Consider first M^u. Choose $\tilde{U}_L^u$ to be any unitary matrix that diagnalizes $M^u(M^u)^+$. Choose now $\tilde{U}_R$ to be $(U_L^u)^*$. Such a choice is possible in a CCC model because M^u is symmetric. Consider the matrix:

$$\tilde{U}_L^u M^u (\tilde{U}_R^u)^+$$

We claim that this matrix is diagonal (though it is not necessarily real). To prove our claim we make use of the following points:

(i) $\tilde{U}_L^u M^u (\tilde{U}_R^u)^+$ is symmetric. This is because M^u is symmetric and $(\tilde{U}_R^u)^+ = (\tilde{U}_L^u)^t$.

(ii) $\tilde{U}_L^u$ and $\tilde{U}_R^U$ are up to phases and permutations U_L and U_R. This means that $\tilde{U}_L^u M^U (\tilde{U}_R^u)^+$ has the following general form: In every row (and every column) there is one and only one nonzero element. The absolute values of these elements are the masses of the u-quarks.

Suppose now that $\tilde{U}_L^u M^u (\tilde{U}_R^u)^+$ is not diagonal, i.e., there exist a nonzero element outside the diagonal. Then, by (i) this element is accompanied by another element on the other side of the diagonal and these two elements are equal. (ii) will then imply that there are two equal masses in the spectrum. This is, of course, in contradiction with our the assumption of non-degeneracy of the eigenmasses. Thus, we conclude that $\tilde{U}_L^u M^u (\tilde{U}_R^u)^+$ is indeed diagonal.

We now want to correct for the phases of the eigenvalues of $\tilde{U}_L^u M^u (\tilde{U}_R^u)^+$: Let $\tilde{F}$ be a matrix of phases that does these corrections and let $\tilde{F}^{\frac{1}{2}}$ be a matrix of phases which is the square root of $\tilde{F}$, then:

$$\tilde{F}^{\frac{1}{2}}\tilde{U}_L^u M^u (\tilde{U}_R^u)^+ \tilde{F}^{\frac{1}{2}}$$

is diagonal and positive definite. Thus:

$$\begin{aligned} U_L^u &= \tilde{F}^{\frac{1}{2}}\tilde{U}_L^u \\ U_R^u &= (\tilde{F}^{\frac{1}{2}})^+ \tilde{U}_R^u = (\tilde{F}^{\frac{1}{2}})^* (\tilde{U}_L^u))^* = (U_L^u)^* \end{aligned} \tag{A.7}$$

Similarly we get:

$$U_R^d = (U_L^d)^* \tag{A.8}$$

(A.7) and (A.8) imply:

$$C_R = C_L^* \tag{A.9}$$

We note that (A.9) holds only in a specific phase convention. To get C_L and C_R in any other phase convention, we have to multiply both C_L and C_R by the same phase matrices:

$$\begin{aligned} C_L &\longrightarrow F_1 C_L F_2^* \\ C_R &\longrightarrow F_1 C_R F_2^* \end{aligned} \tag{A.10}$$

It is easy to derive from (A.9) and (A.10) the following relation for C_L and C_R in the new phase convention:

$$C_R = F^u C_L^* (F^d)^* \tag{A.11}$$

where $F^u = (F_1)^2$ and $F^d = (F_2)^2$.

APPENDIX B

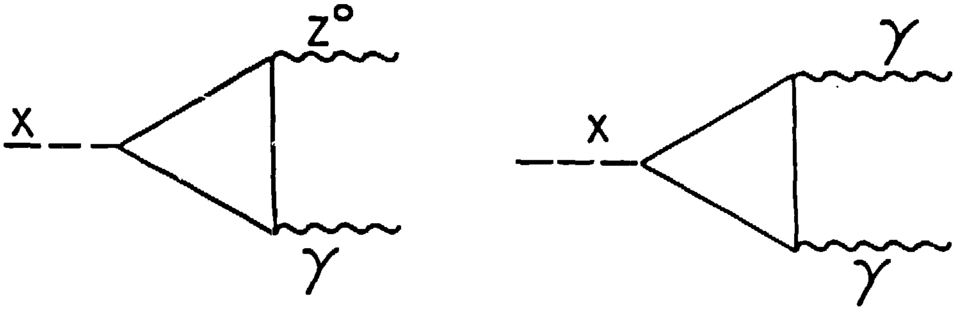
We will explain here why the $Z^o\gamma X$ and $\gamma\gamma X$vertices are expected to be related to each other by a factor $\tan\theta_W$ (or, a factor of the same order of magnitude).

Let us denote:

$$j_{em} = j^{(0)} + j_Y \qquad (B.1)$$

where $j^{(0)}$ is the generating current of the neutral component of the $SU(2)_L$ symmetry and J_Y is the generating current of weak hypercharge.

The $Z^0\gamma X$ and $\gamma\gamma X$ vertices arise from loop diagrams like:



We assume that only the $j^{(0)}$ part of the electromagnetic current is relevant to these diagrams. Under this simplifying assumption we find that the γ coupling to the particles in the loop is given by $ej^{(0)}$, while the Z coupling is:

$$\frac{g_W}{\cos\theta_W}(j^{(0)} - \sin^2\theta_W j^{(0)}) = \frac{1}{\tan\theta_W} e j^{(0)} \qquad (B.2)$$

i.e., we find that the $\gamma\gamma X$ vertex is identical to the $Z\gamma X$ vertex, except for a $\tan\theta_W$ factor.

APPENDIX C

Fritzsch suggested Left -Right Symmetry as the underlying physics which is responsible for the hermiticity of his matrices. As we discuss in Chapter 2 of the thesis, we prefer those versions of left-right symmetric theories which produce *symmetric* (and not hermitian) mass matrices. In this appendix we show that, as far as the low-energy standard-model is concerned — the hermitian and symmetric Fritzsch matrices are equivalent.

The mass matrices M^u, M^d are defined through:

$$L_{mass} = \overline{u_L^{(0)}} M^u u_R^{(0)} + \overline{d_L^{(0)}} M^d d_R^{(0)} + h.c.$$

In the standard model the right-handed fermions are singlets of $SU(2)_L$. We are therefore free to make the following transformations:

$$\begin{aligned} u_R^{(0)} &\longrightarrow U^u u_R^{(0)} \\ d_R^{(0)} &\longrightarrow U^d d_R^{(0)} \\ M^u &\longrightarrow M^u U^{u+} \\ M^d &\longrightarrow M^d U^{d+} \end{aligned} \qquad (C.1)$$

where U^u, U^d are any unitary matrices. Consider now the mass matrices of Fritzsch:

$$M^u = \begin{pmatrix} & a_1^u & & \\ a_1^{u*} & & \ddots & \\ & \ddots & & a_{N-1}^u \\ & & a_{N-1}^{u}{}^* & b^u \end{pmatrix} \qquad M^d = \begin{pmatrix} & a_1^d & & \\ a_1^{d*} & & \ddots & \\ & \ddots & & a_{N-1}^d \\ & & a_{N-1}^{u}{}^* & b^d \end{pmatrix} \qquad (C.2)$$

We will show that under transformations of the type (C.1) the hermitian matrices (C.2) become symmetric. Choose:

$$U^u = e^{i\xi^u}\mathrm{diag}(\ldots\ldots, \frac{a^u_{N-2}}{{a^u_{N-2}}^*} \frac{a^u_{N-1}}{{a^u_{N-1}}^*}, \frac{a^u_{N-1}}{{a^u_{n-1}}^*}, 1) \qquad (C.3)$$

$$U^d = e^{i\xi^d}\mathrm{diag}(\ldots\ldots, \frac{a^d_{N-2}}{{a^d_{N-2}}^*} \frac{a^d_{N-1}}{{a^d_{N-1}}^*}, \frac{a^d_{N-1}}{{a^d_{n-1}}^*}, 1) \qquad (C.4)$$

where ξ^u, ξ^d are arbitrary phases. Transforming the hermitian Fritzsch mass matrices (C.2) according to the rule (C.1) with the U's of(C.3), (C.4) we get symmetric mass matrices.

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TABLES

Table 1: Comparison of the manifest and CCC models

Table 2: predicted number of $Z^o\gamma$ events per year in p̄p colliders, assuming an integrated luminosity of 10^{37} cm^{-2}. The distribution functions used are of Baier et.al.[13] with Λ=.4 GeV. The cross sections are integrated up to p_T^γ=90 GeV. Composite model predictions include standard model contributions and the $g + g \rightarrow Z + \gamma$ contribution. The latter are computed using the model mentioned in the text, assuming $< Q_s^2 Q > = 1$. For p_T^γ >10 GeV the enhancement factors of the composite model are 20 (at 540 GeV) and 170 (at 2000 GeV). For $\Lambda = 0.1$, the corresponding enhancement factors are 7 and 40 (respectively).

Table 1

	Manifest	CCC	
		CCC$_1$	CCC$_2$
Symmetry of the Lagrangian	P	P,C,CP	C
Assumption on $<\phi>$	$<\phi>$ conserves P (unreasonable)	$<\phi>$ breaks P (reasonable)	no assumption
Symmetry manifested in the quark mass matrices	P	C	
Relation between C_L and C_R	$C_R = F^u(\pm)C_L F^d(\pm)$	$C_R = F^u C_L^* (F^d)^*$	

Table 2

c.m. energy	Standard Model		Composite-Z^o Model	
	$p_T^\gamma > 5$ GeV	$p_T^\gamma > 10$ GeV	$p_T^\gamma > 5$ GeV	$p_T^\gamma > 10$ GeV
540 GeV	33	15	390	300
2000 GeV	160	90	17000	15000

FIGURE CAPTIONS

Fig. 1.1 Lowest order diagrams contributing to $K - \bar{K}$ mixing in the minimal standard model. u_i, u_j stand (in the 2-generation case) for u, c.

Fig. 1.2 Unphysical Higgs contribution to M_{12} in the standard model.

Fig. 1.3 Contribution of W-bosons to $K - \bar{K}$ mixing in a left-right symmetric model. Only diagrams which are lowest order in α (the fine structure constant) and zero or first order in $\beta \left(= \left(\frac{m(W_1)}{m(W_2)} \right)^2 \right)$ are included.

Fig. 1.4 Unphysical Higgs contribution to M_{12} in left-right symmetric model.

Fig. 1.5 Neutral physical Higgs contribution to M_{12}.

Fig. 1.6 Charged physical Higgs contribution to M_{12}.

Fig. 2.1 Effective $Z^oV^oV^oV^o$ vertex ($V^o = \gamma$ or g) in the standard model (2.1(a)) and in a composite model (2.1(b)).

Fig. 2.2 Subprocesses contributing to $\bar{p} + p \rightarrow Z^o + \gamma + any$. In the standard model, the lowest order contributions correspond to $\bar{q} + q \rightarrow Z^o + \gamma$ (2.2(a);2.2(b)). In a composite model additional effective terms appear (2.2(c);2.2(d)). The $g + g \rightarrow Z^o + \gamma$ subprocess (figure 2.2(d)) is likely to dominate the large p_T cross-section.

Fig. 2.3 The differential cross section for $\bar{p} + p \rightarrow Z^o + \gamma + any$ due to the standard model diagrams and to the $g + g \rightarrow Z^o + \gamma$ subprocess in a composite-Z^o model. We use the cutoff $M^2_{Z\gamma} \geq 1.1\ M^2_Z$. The distribution functions are those of Baier et al.[13] with $\Lambda = 0.4$. The composite-Z^o contributions were computed using the model mentioned in the text, assuming $< Q_S^2 Q > = 1$. The c.m. energies are: 540 GeV (2.3a) and 2000 GeV (2.3b).

Fig. 3.1 b-decay (upper) bound and ϵ-line for τ_b=1.5 psec, m_t=45 GeV and B=0.4. Clearly, non of the points of the ϵ-line obeys the bound on $\theta_{1,3}$ imposed by b-decay.

Fig. 3.2 b-decay bound and ϵ-lines for τ_b=1.5 psec and various values of m_t and B. We see that as m_t and B increase, the ϵ-line "goes down".

Fig. 3.3 b-decay bound and ϵ-lines for τ_b=1 psec and various values of m_t and B. Comparing this figure with its former we see that as τ_b increases, the ϵ-line "goes down".

Fig. 3.4 Allowed and forbidden regions in τ_b–m_t plane, according to the standard model.

Fig. 3.5 b-decay (upper) bound, ϵ (lower) bound and ϵ' (upper) bound for τ_b=1 psec and m_t=45 GeV. Clearly the ϵ' bound does not agree with the ϵ bound.

Fig. 3.6 b-decay bound, ϵ bound and relaxed ϵ' bounds. The bound on ϵ' is relaxed by summing the experimental errors in the measurements of $\frac{\epsilon'}{\epsilon}$ linearly (and not quadratically). As we see in the figure the ϵ' is further relaxed by taking A value as small as possible. The relaxed ϵ' bound corresponding to $A = 2$ is compatible with the ϵ bound.

Fig. 3.7 The shaded area is the allowed region in $\theta_{1,3}$-$\delta_{1,3}$ plane for τ_b=1 psec and m_t=45 GeV, as determined by the b-decay bound, the ϵ-bound and the relaxed ϵ'-bound.

Fig. 3.8 The two "Fritzsch"-points and the allowed region in $\theta_{1,3}$-$\delta_{1,3}$ space for τ=1 psec and m_t=45 GeV.

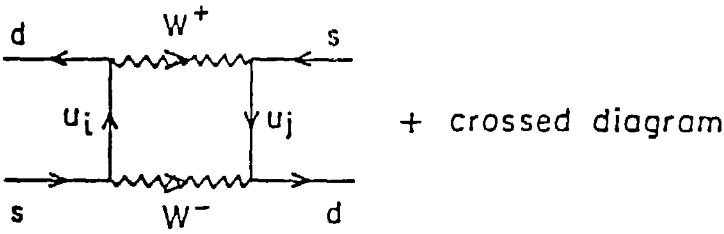


Figure 1.1

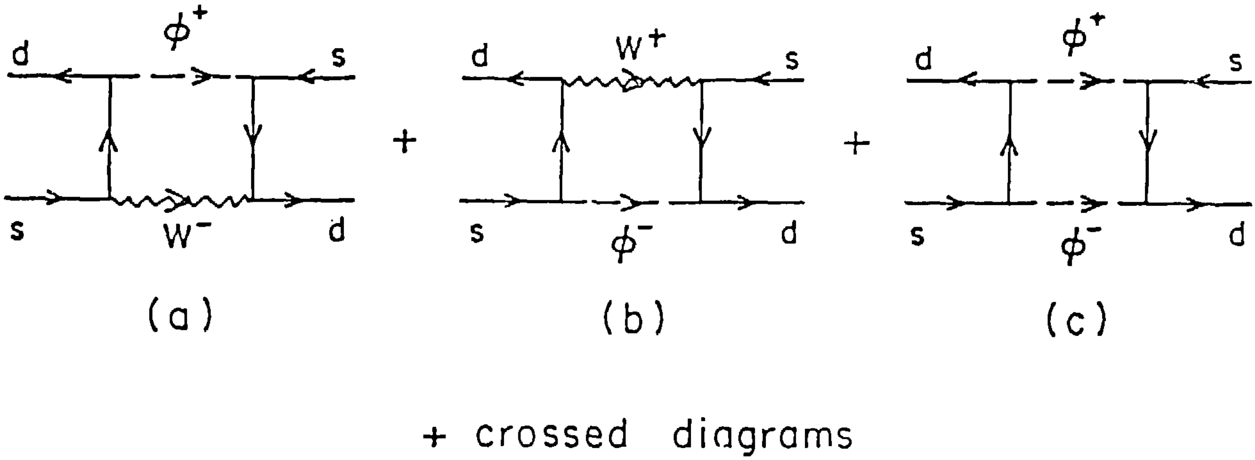


Figure 1.2

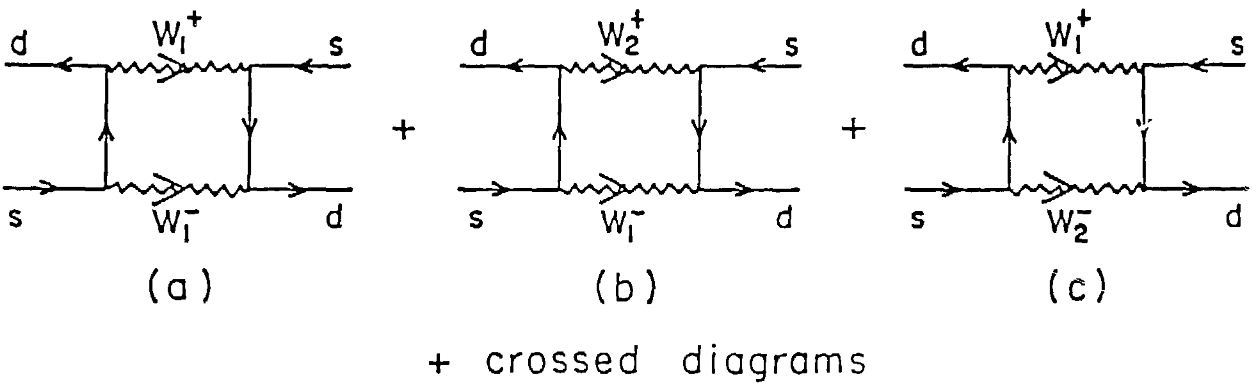


Figure 1.3

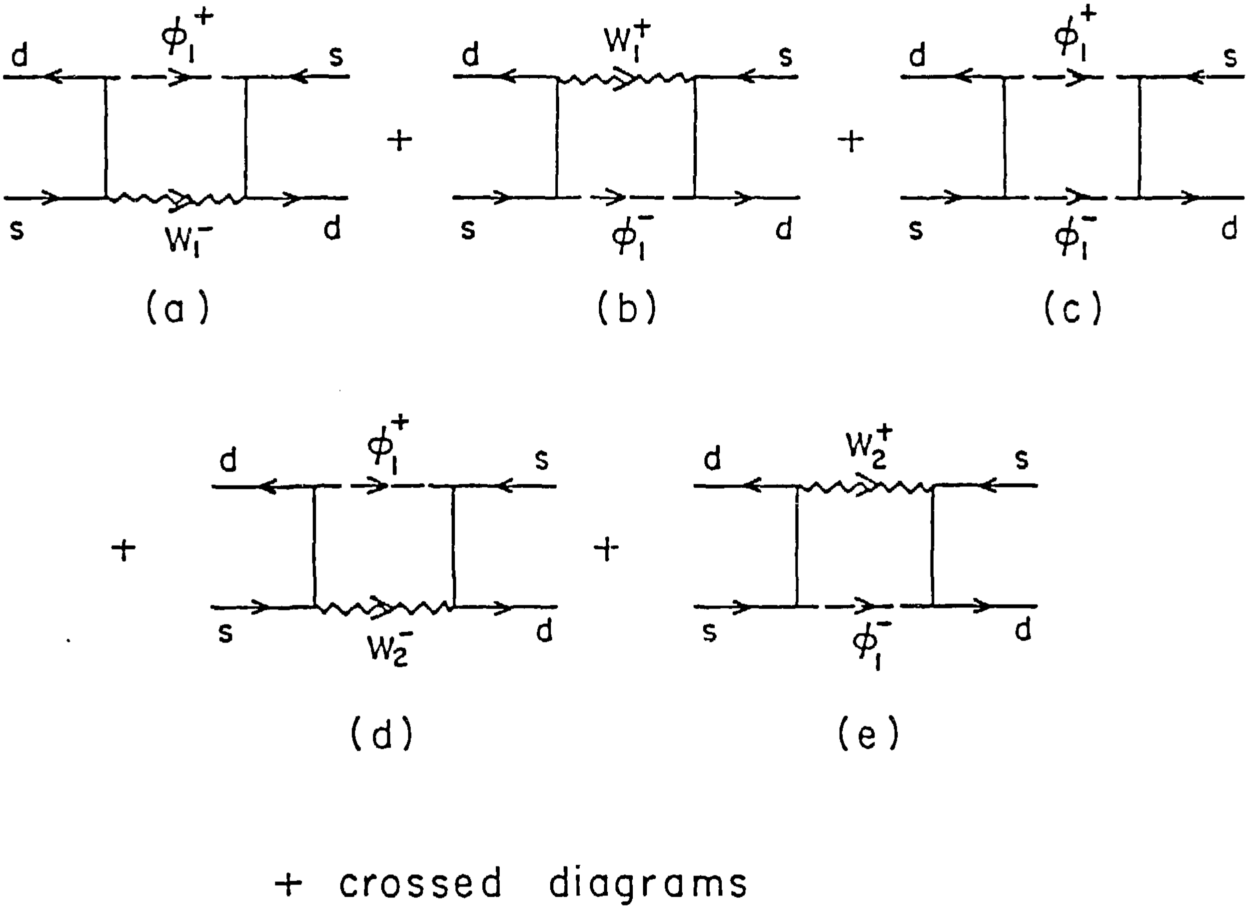


Figure 1.4

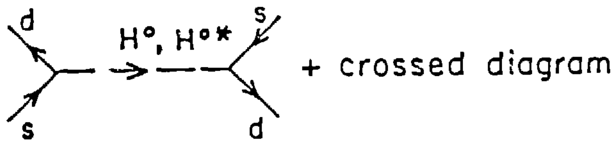


Figure 1.5

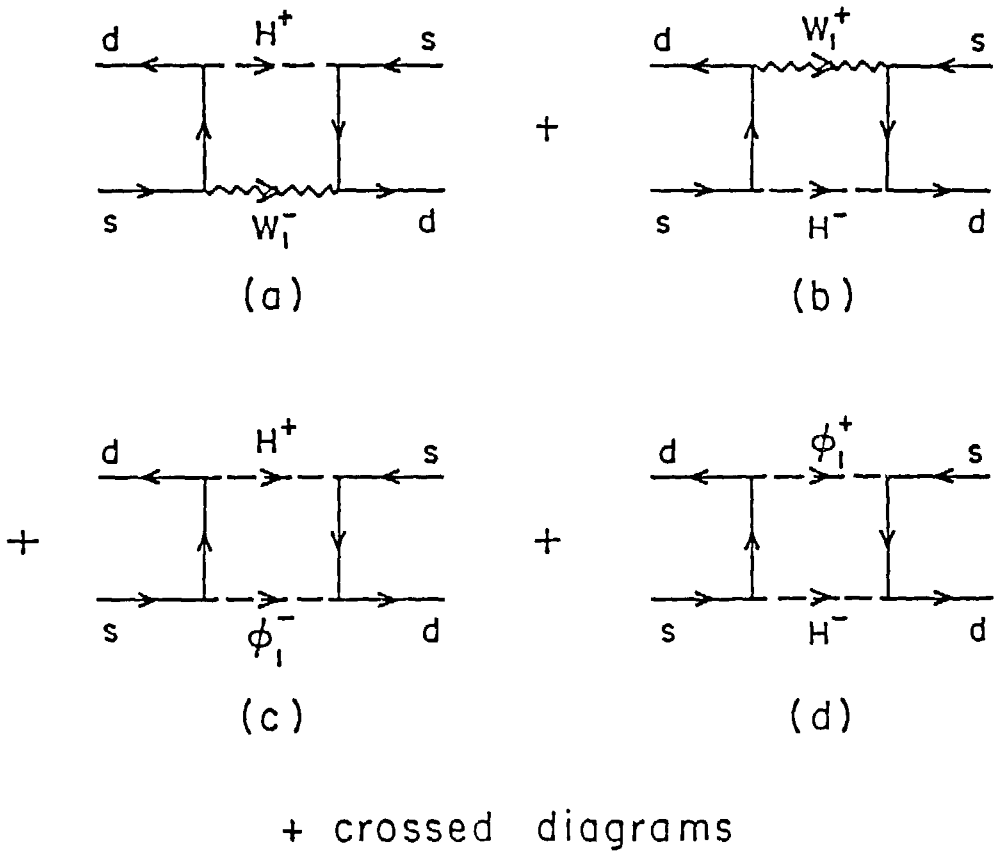


Figure 1.6

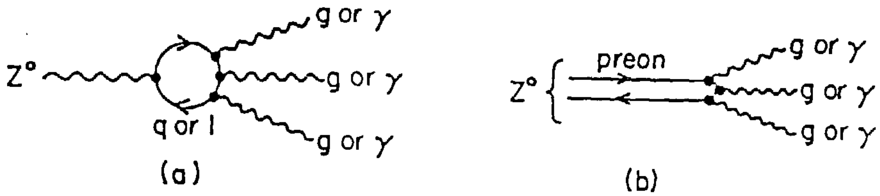


Figure 2.1

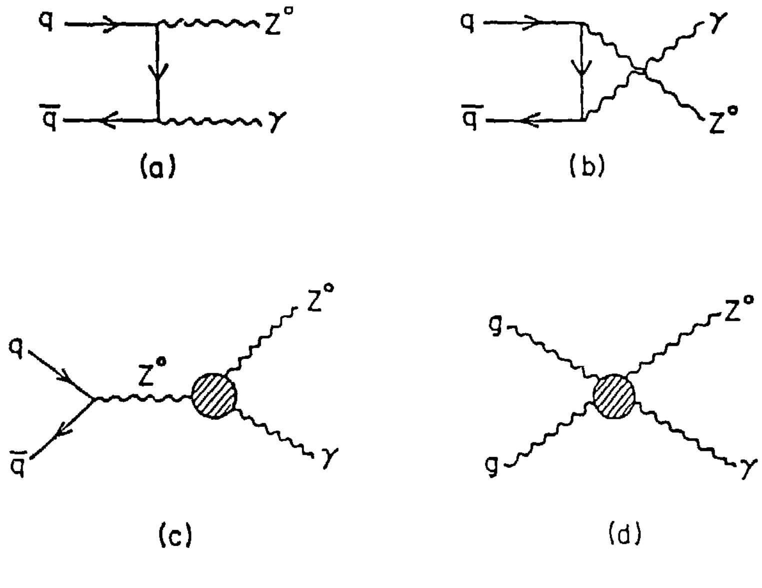
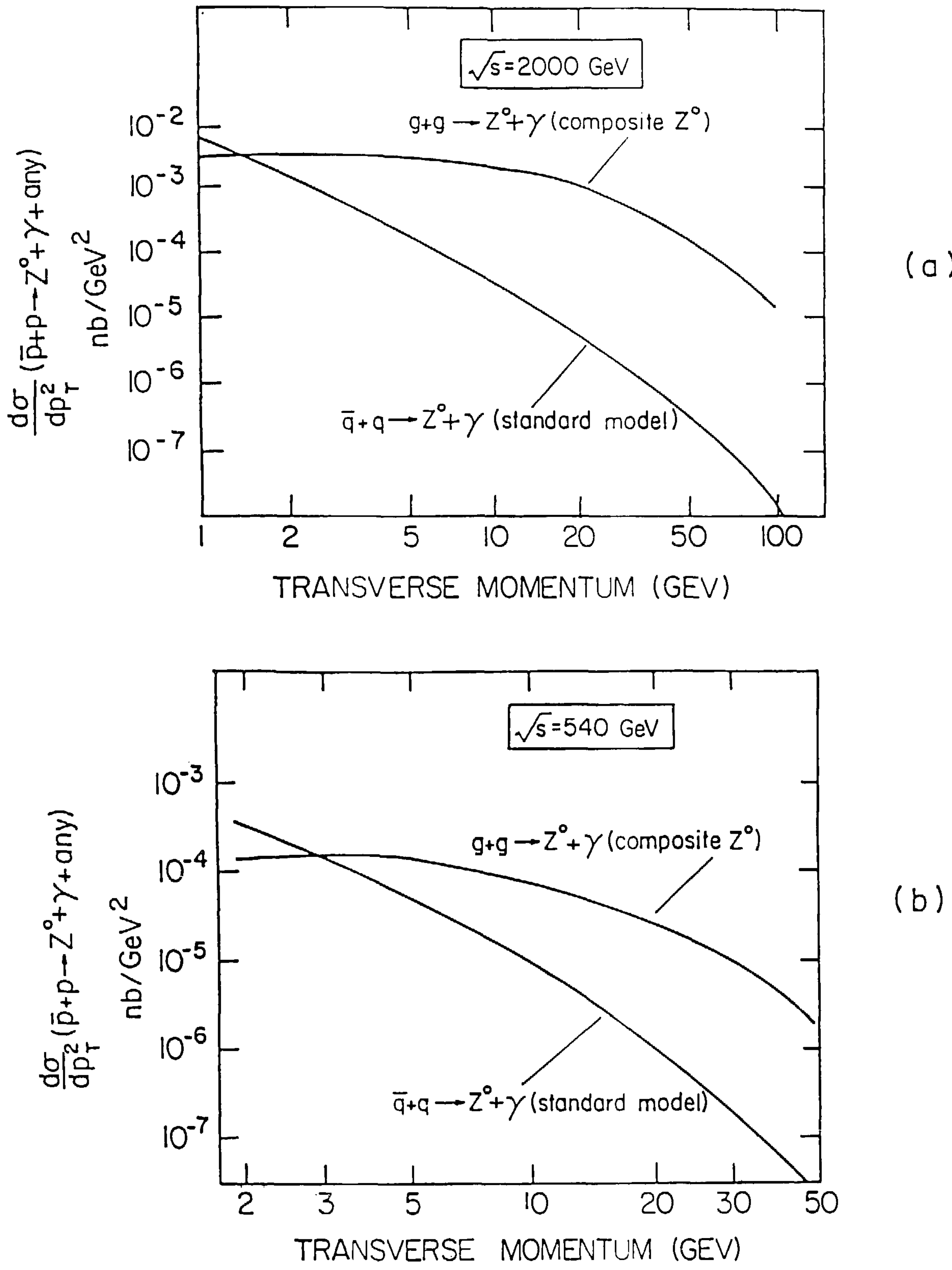
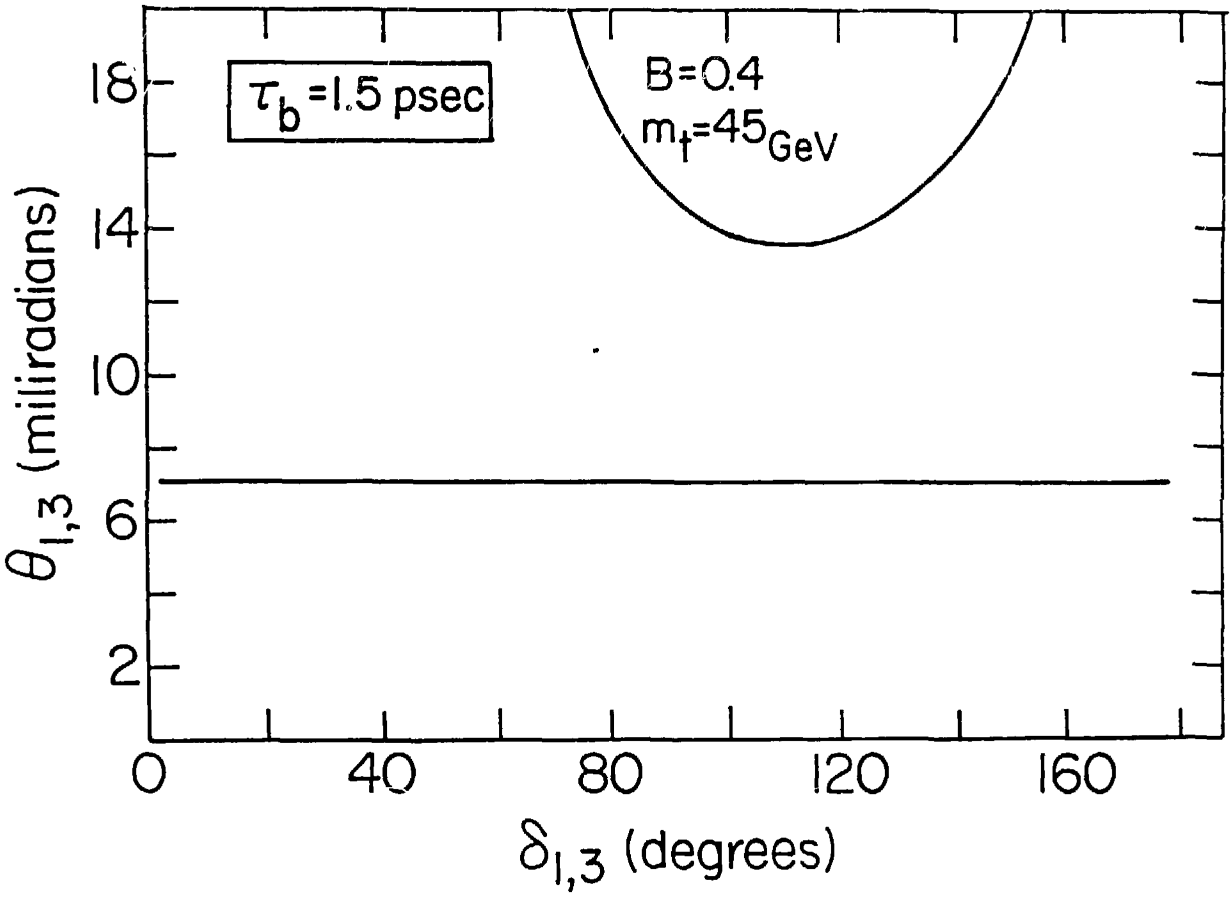


Figure 2.2



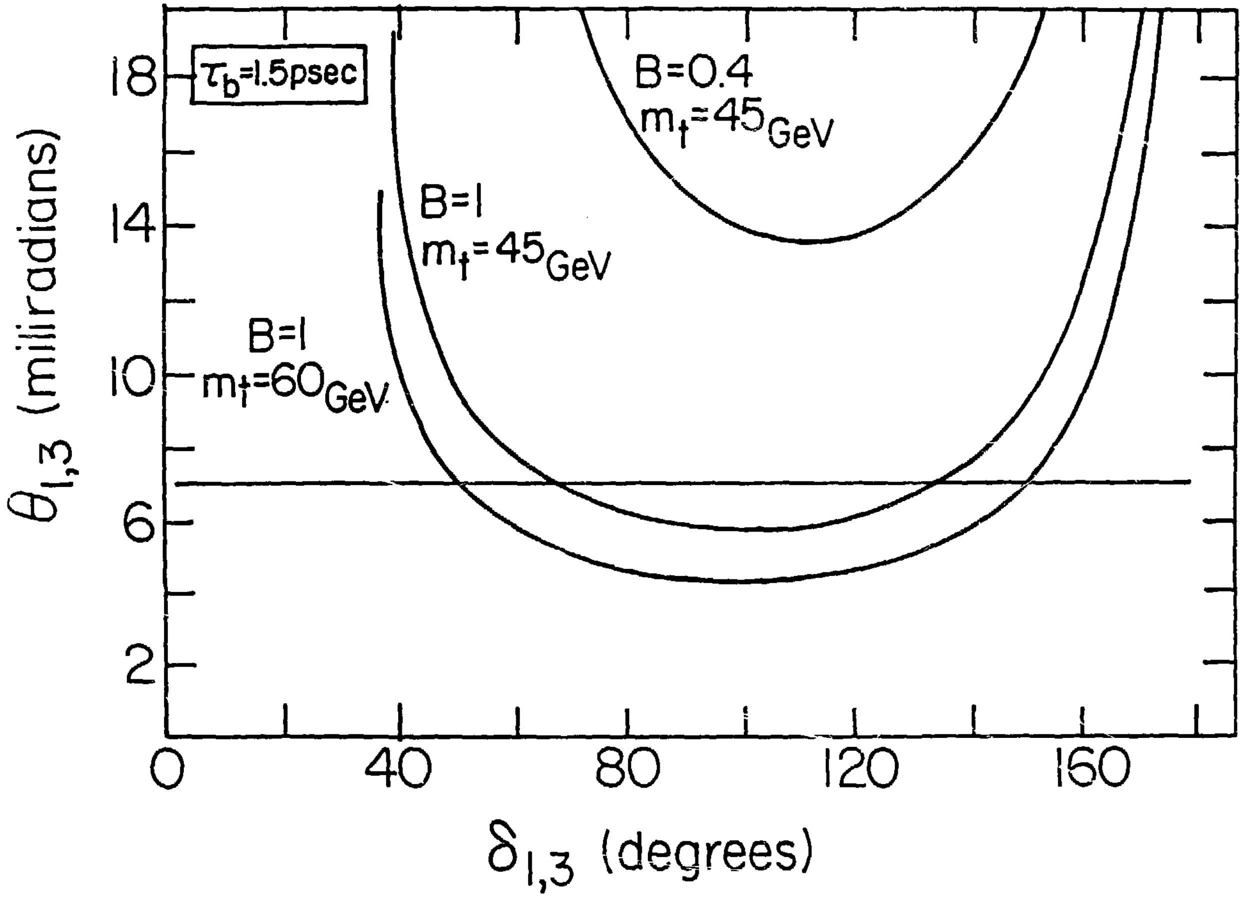
√s = 2000 GeV
g+g → Z°+γ (composite Z°)
q̄+q → Z°+γ (standard model)
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
dσ/dp_T^2 (p̄+p → Z°+γ+any) nb/GeV^2
1
2
5
10
20
50
100
TRANSVERSE MOMENTUM (GEV)
(a)
√s = 540 GeV
g+g → Z°+γ (composite Z°)
q̄+q → Z°+γ (standard model)
10^-3
10^-4
10^-5
10^-6
10^-7
dσ/dp_T^2 (p̄+p → Z°+γ+any) nb/GeV^2
2
3
5
10
20
30
50
TRANSVERSE MOMENTUM (GEV)
(b)

Figure 2.3



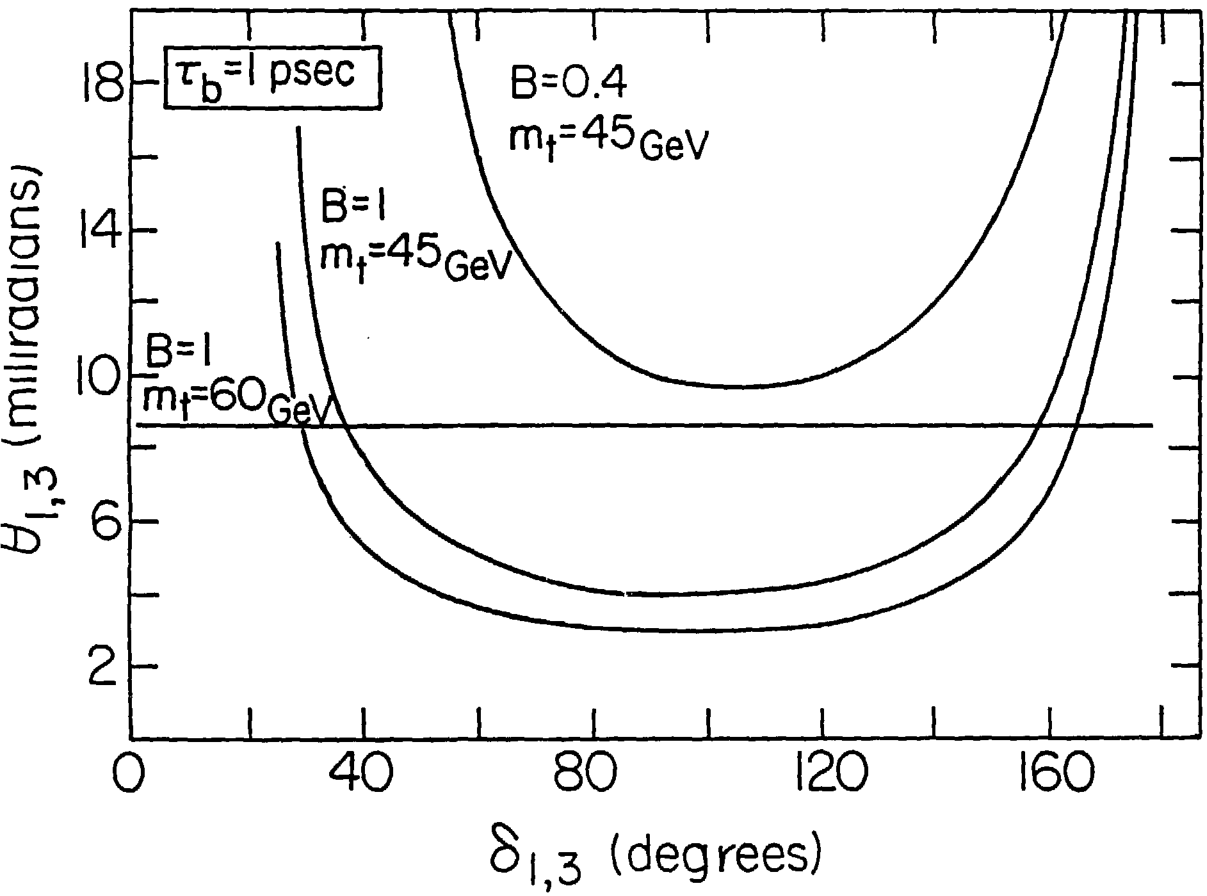
τ_b=1.5 psec
B=0.4
m_t=45GeV
18
14
10
6
2
$\theta_{1,3}$ (miliradians)
0
40
80
120
160
$\delta_{1,3}$ (degrees)

Figure 3.1



τ_b=1.5psec
B=0.4
m_t=45GeV
B=1
m_t=45GeV
B=1
m_t=60GeV
18
14
10
6
2
0
40
80
120
160
$\theta_{1,3}$ (miliradians)
$\delta_{1,3}$ (degrees)

Figure 3.2



τ_b=1 psec
B=0.4
m_t=45 GeV
B=1
m_t=45 GeV
B=1
m_t=60 GeV
18
14
10
6
2
0
40
80
120
160
$\theta_{1,3}$ (milliradians)
$\delta_{1,3}$ (degrees)

Figure 3.3

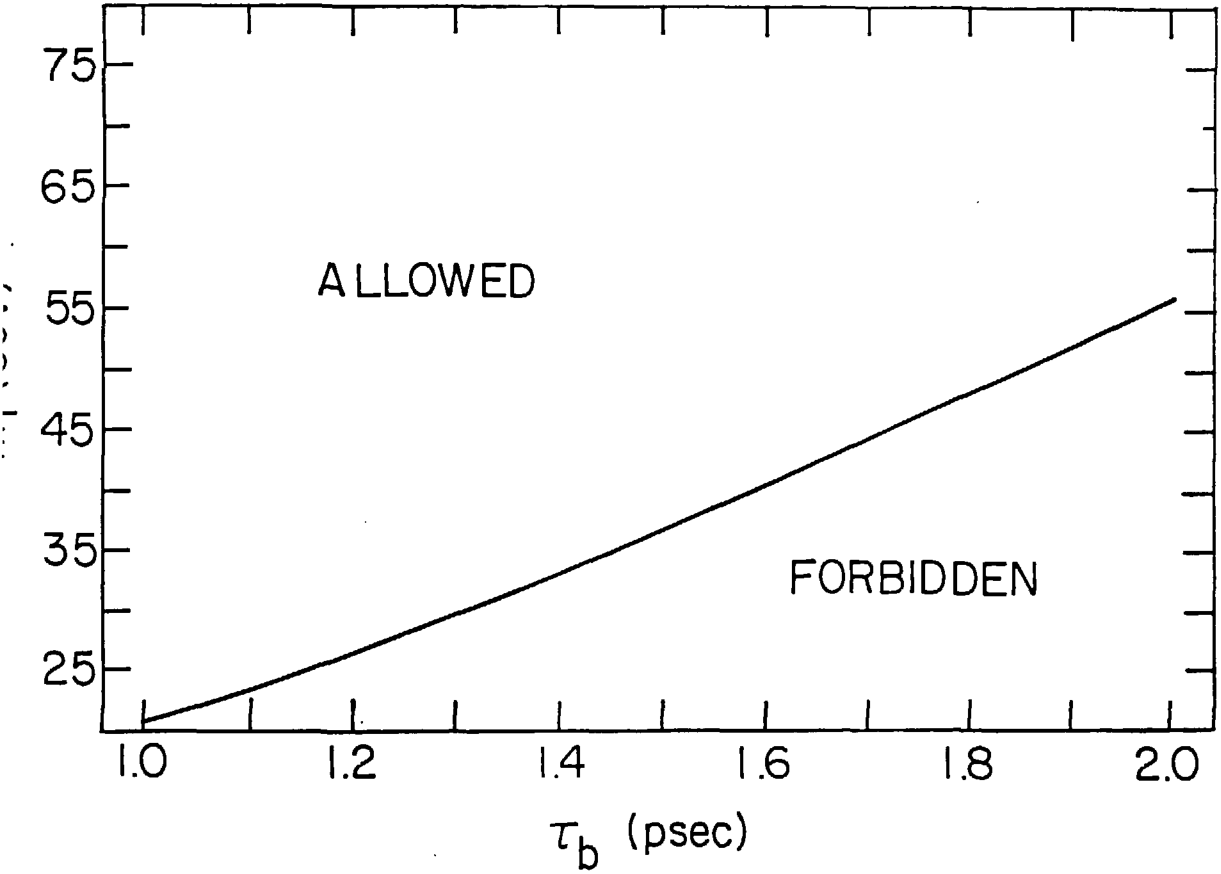
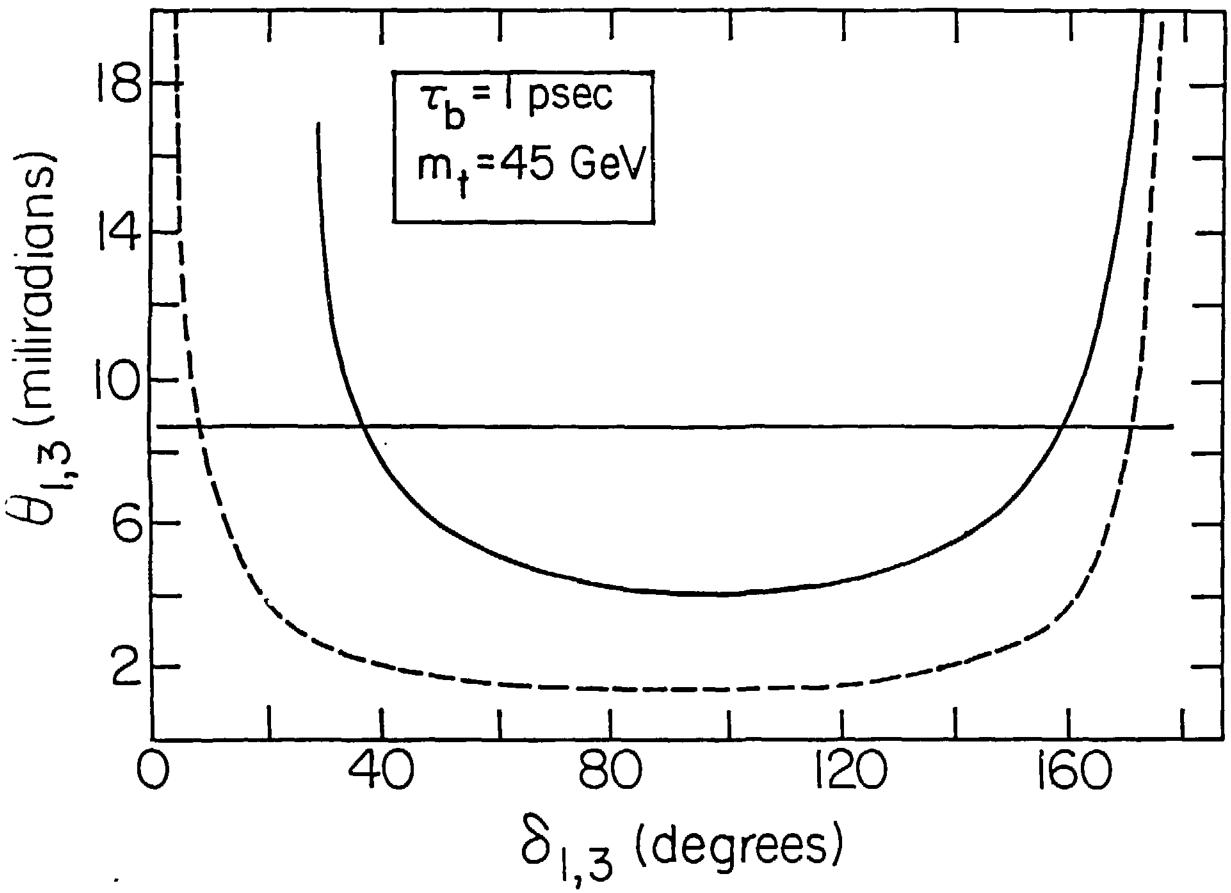
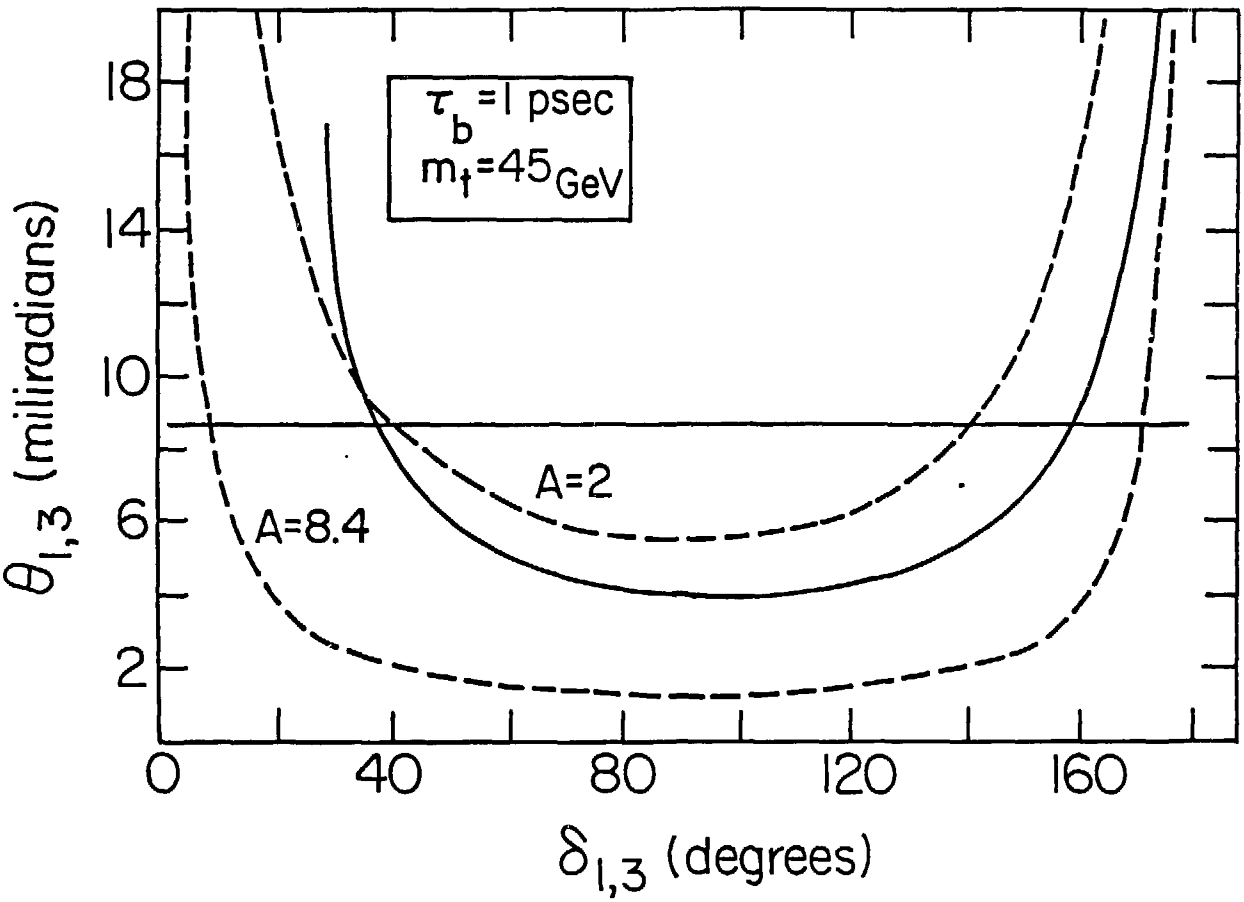


Figure 3.4



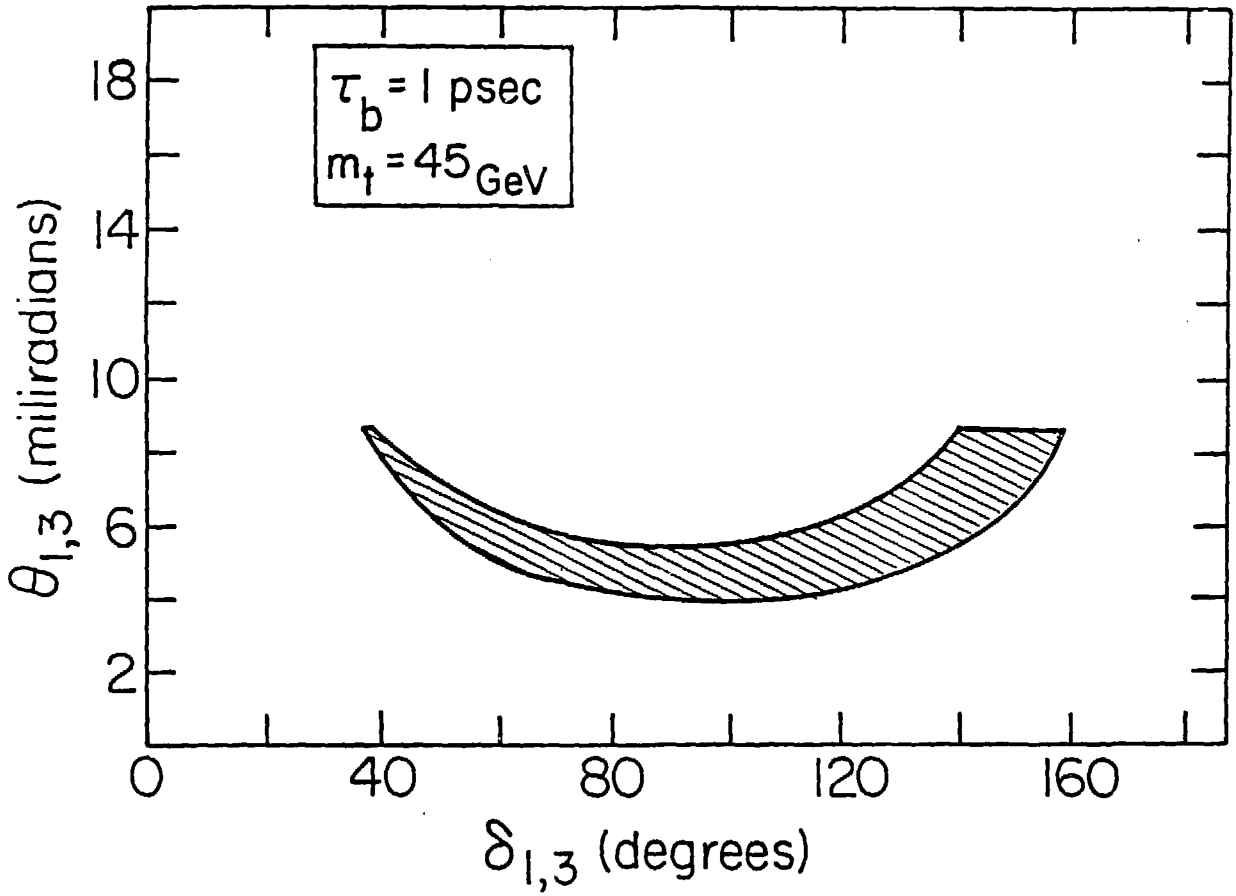
τ_b = 1 psec
m_t=45 GeV
18
14
10
6
2
$\theta_{1,3}$ (miliradians)
0
40
80
120
160
$\delta_{1,3}$ (degrees)

Figure 3.5



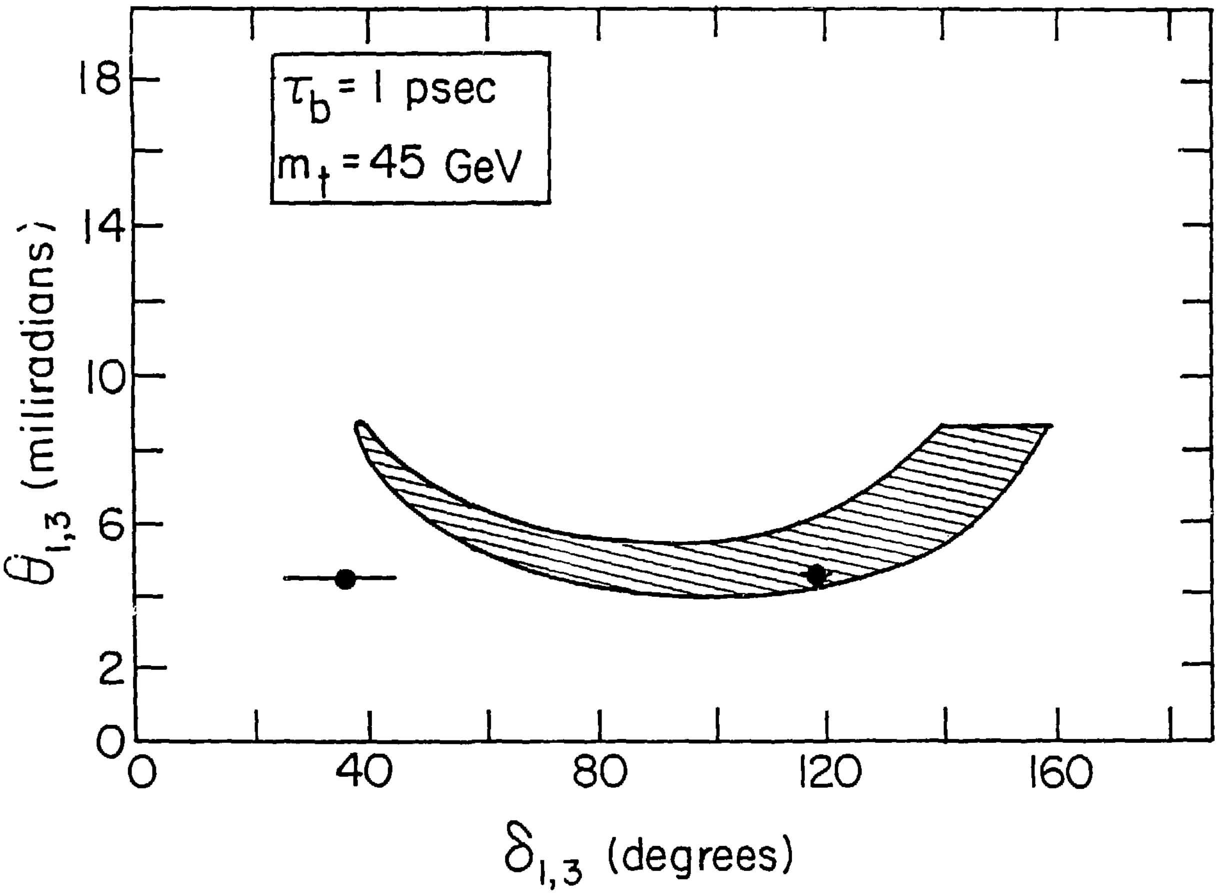
τ_b=1 psec
m_t=45 GeV
A=2
A=8.4
18
14
10
6
2
0
40
80
120
160
$\theta_{1,3}$ (miliradians)
$\delta_{1,3}$ (degrees)

Figure 3.6



τ_b = 1 psec
m_t = 45 GeV
18
14
10
6
2
$\theta_{1,3}$ (miliradians)
0
40
80
120
160
$\delta_{1,3}$ (degrees)

Figure 3.7



τ_b = 1 psec
m_t = 45 GeV
$\theta_{1,3}$ (miliradians)
$\delta_{1,3}$ (degrees)
0
2
6
10
14
18
0
40
80
120
160

Figure 3.8

List of Publications

1. "Left-Right Symmetry and the Mass Scale of a Possible Right-Handed Boson", H. Harari and M. Leurer, Nucl. Phys. B233 (1984) 221.
2. "An Experimental Test of Z^0 Compositeness in Proton Anti-Proton Colliders", M. Leurer, H. Harari and R. Barbieri, Phys. Lett. 141B (1984) 455.
3. "eeγ Events — a Comment on the Possibility of Composite Z^0 Decay through an Intermediate Scalar", M. Leurer, Phys. Lett. 144B (1984) 273.
4. "$K^0 - \bar{K}^0$ Mixing in Minimal and Non-Minimal Left-Right-Symmetric Theories", M. Leurer, Nucl. Phys., B266 (1986) 147.
5. "Recommending a Standard Choice of Cabibbo Angles and KM Phases for any Number of Generations", H. Harari and M. Leurer, Weizmann preprint WIS-86/24/May-PH.

ת ק צ י ר

העבודה בנויה משלושה פרקים בלתי תלויים שכולם דנים באינטראקציות החלשות.

נושא הפרק הראשון הוא תורות left-right סימטריות. אנו דנים בשתי הגרסאות העיקריות של תורות אלה ומשווים ביניהן. בנוסף - אנו בודקים את אבר העירבוב של $K-\bar{K}$: לפני שנים אחדות הוכח שבמודל left-right סימטרי יש תרומות חדשות לאבר העירבוב של הקאונים. מידת החשיבות של תרומות אלה לא היתה ברורה: למרות שהערך המוחלט של כל תרומה כשלעצמה הוא גדול - קימת האפשרות שכאשר נסכם את התרומות השונות הן תבטלנה זו את זו. עבור אותה גרסא של מודלים left-right סימטריים שלדעתנו היא המוצלחת יותר - אנו מוכיחים שהתרומות החדשות מתוספות זו לזו באופן בונה. כתוצאה מכך אנו מקבלים חסמים מהימנים על המסה של חלקיק הכיול הימני ועל המסה הממוצעת של חלקיקי ההיגס הפיסיקליים.

מצאנו שכל התרומות החדשות פרופורציונליות לפאזה שוברת CP חדשה. הפאזה החדשה הזו תוכל לשמש כמקור חלופי לשבירת CP , אם יתברר שהתרומה של הפאזה של Kobayashi-Maskawa לפרמטר ε - קטנה מדי.

כל העבודות על מודלים left-right סימטריים המופיעות בספרות מוגבלות לדיון במודל המינימלי. אנו הצלחנו להוכיח שכל התוצאות שלנו תופסות גם במודל הלא מינימלי.

הפרק השני עוסק באפשרות שה W וה Z מורכבים. נידונים שלושה מבחנים נסיוניים: (א) אוניברסליות - אם W הוא חלקיק מורכב אז צימודיו לפרמיונים אמורים לסטות מאוניברסליות. מכיון שסטיות מאוניברסליות טרם נראו - אנו מקבלים חסם תחתון על סקלת המורכבות. (ב) הגברה אפשרית של התהליך: $\bar{p}+p\rightarrow Z^0+\gamma+$ any - אנו מראים שאם Z^0 מורכב אז חתך הפעולה לתהליך $\bar{p}+p\rightarrow Z^0+\gamma+$ any עשוי להיות מוגבר באופן ניכר ותופעה זו ניתנת למדידה במאיצים של CERN ו- Fermilab . (ג) מאורעות ה $ee\gamma$ שהופיעו ב 1983 ב CERN - אנו מראים, שבניגוד להצעות שהופיעו בכמה מאמרים, לא ניתן להסביר מאורעות אלה על ידי תהליך של דעיכת Z מורכב דרך חלקיק דקלרי.

בפרק האחרון אנו דנים בזויות העירבוב של הקוורקים. אנו מציעים פרמטריזציה חדשה למטריצת העירבוב. לפרמטרים החדשים יש משמעות פיסיקלית פשוטה והם מתיחסים בצורה פשוטה ונוחה לגדלים מדידים. תוך שימוש בפרמטריזציה שהצענו אנו חוזרים על ניתוח הפרמטר ε במודל הסטנדרטי ודנים באפשרות ששבירת CP במודל הסטנדרטי תוכח כחלשה מכדי להסביר את הגודל הנמדד של ε. התוצאות של דיון זה מוצגות על ידי חסמים על הפרמטרים החדשים. אנו דנים גם במטריצות של פריץ' ומראים שבשלב הנוכחי הן קונסיסטנטיות עם המודל הסטנדרטי המינימלי.