in azimuthal position an appropriate bias signal must be injected into the phase detector. The starting value for the length of the lines will be injected into the counter from the slow programmer memory.

It is anticipated that the vertical instability limit can be made to exceed 10^{13} protons in orbit. This probably will require the use of a favorable betatron tune profile in the magnet as well as the use of the improved version of the damping system.

Acknowledgement

The authors wish to gratefully acknowledge the valuable assistance of Tat Khoe of Argonne National Laboratory for the theoretical treatment of the problem and Roger Otte and Charles Pruett of Midwestern Universities Research Association for contributions in engineering and data taking and interpretation. The authors wish also to express appreciation of the diligence and expert workmanship of Anthony Donaldson in constructing and testing the equipment described in this paper.

REFERENCES

(1) M. Barton, (Brookhaven National Laboratory): private communication.

(2) L. G. Lewis: Computer Control of High Energy Accelerators, Particle Accelerator Conf. Washington 1965; IEEE Trans. on Nuclear Science NS 12, n. 3, 1 (1965).

EXPERIMENTS ON BEAM SURVIVAL AND NONLINEAR RESONANCES IN THE CERN PS

Y. Baconnier, B. de Raad and Ch. Steinbach

CERN, Geneva (Switzerland)

1) INTRODUCTION

After the observation of beam instabilities in the Stanford 500 MeV electron storage rings (1), questions have also been raised concerning the stability of the beams in the 28 GeV proton storage rings (ISR) (2) which will be built at CERN. In particular the effects due to the interaction of the two stacked beams have been the subject of much discussion. At Stanford it has e.g. been observed that a weak beam in one ring would undergo a vertical amplitude increase if it intersected with a strong beam in the other ring. It has been suggested that this might be caused by the repeated passage of the particles of the weak beam through the nonlinear field created by the ribbon-shaped strong beam. The present analytical theory of nonlinear perturbations of betatron oscillations (3) contains approximations which make it of limited value for the strong nonlinear perturbations and large number of revolutions which are of interest for storage rings. Therefore computer calculations (4) have been made on the basis of the nonlinear model mentioned above. These calculations showed no increase in betatron oscillation amplitude in one million intersections with a perturbation strength corresponding to the ISR. However, this corresponds to only 0.4 seconds in the ISR, which have 8 crossing points. Moreover the validity of such computations is limited by the extent to which the simplified mathematical model represents an actual machine. It was decided therefore, to perform an experiment on the CERN 25 GeV proton synchrotron (CPS) which would give better information about the effects of nonlinear perturbations. The CPS is a convenient machine for this purpose since its parameters are similar to those of the ISR and since it has sextupole and octupole lenses whose polarity and excitation can be chosen at will. Unfortunately the vacuum in the CPS is a few times 10^{-6} mm Hg and therefore the duration of such experiments is limited to times ranging from 5 to 10 min.

Since little information is available about higher order resonances due to constructional tolerances, stray fields, etc. we have also made some measurements on the third and fourth order resonances in the CPS.

2) CHOICE OF NONLINEAR LENSES AND Q-VALUES

The simplest, empirical way to formulate our problem is as follows: Each time when a proton passes through a nonlinear field region, it receives a kick. Could it be, that the succession of nonlinear kicks leads to a random build-up of the amplitude at Q-values where the oscillation should be perfectly stable according to the currently used approximative nonlinear theory? From this point of view the exact polarity of the nonlinear lenses should be of little importance. However, it could also be that the nonlinear lenses produce an amplitude increase at normally stable Q-values, that is somehow related to the strength of the nonlinear stopbans that they create. It is therefore also of interest to calculate the azimuthal harmonics of the nonlinear perturbation which give stopbands in the neighbourhood of the working point.

The ISR will have horizontal and vertical Q values in the vicinity of 8.75 and have 8 crossing points, but the magnet lattice has a superperiodicity 4. Therefore the azimuthal harmonics of the nonlinear intersection region fields will also be multiples of 4. Since the beams in the ISR cross at an angle of 15° in the horizontal plane, it follows from symmetry arguments, that only the vertical oscillations are affected by the intersection region fields and we can restrict ourselves to a discussion of the onedimensional vertical motion. If one expands the horizontal component of the magnetic field B_x of a ribbon shaped beam as a power series of z, it will contain only odd powers of z, so that the lowest nonlinear terms correspond to an octupole field. Therefore we can expect stopbands at values of Q_v given by

$$4\,Q_v = 32,\ 36,\ 40 \text{ etc.} \qquad [1a]$$

In the particular geometry of the ISR superperiods the width of these three stopbands is respectively 2.5, 7.9 and 0 times the width of the same stopband which vould be produced by a single crossing point If there is a vertical centering error, so that the beam centre lines do not intersect, the effective octupole field has random fluctuations from one intersection to the next and this will produce stopbands at all Q-values given by

$$4\,Q_v = \text{integer} \qquad [1b]$$

including $Q_v = 8.75$ which is inside the working diamond. However with moderately careful centering this stopband can be kept small compared to those given by eq. [1a].

It looked interesting to experiment not only with the CPS octupoles, but also with the sextupoles, since one might expect that any effect which would show up with octupoles could be a property of nonlinear fields in general and would therefore also be found with the sextupoles. Moreover, the value of $\int B\,d\,l$ at 2 cm from the axis is about twice as large in the sextupoles as in the octupoles. Finally, if the beams are not properly centered, the intersection region fields also have a sextupole component, which produces a stopband at $Q_v = 8\,^2/_3$.

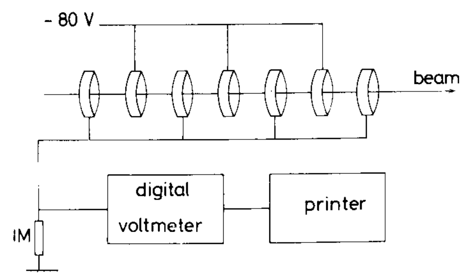

Fig. 1 - Instrumentation to measure the beam intensity.

We have used the sextupoles and octupoles in horizontally defocusing sections of the CPS, since these have the largest effect on the vertical motion. Moreover the beam survival curves are most sensitive to a vertical amplitude increase, since the vertical aperture is about half the horizontal aperture and most of the beam is lost by multiple scattering on the top and bottom wall. The CPS has 10 mid-D sextupoles and octupoles, but the sextupole and octupole in one straight section had been removed to make place for a secondary beam. The procedure was then, to excite the sextupoles or octupoles with various polarities and to measure the decrease of beam intensity versus time at constant magnetic field. The existing nonlinear theory allows us, for a given arrangement of nonlinear lenses to calculate the stopbands. Inside these stopbands the oscillations are unstable, and in their vicinity the amplitude is also increased with a corresponding decrease in beam life in our experiment. We therefore decided to choose Q-values which were reasonably far, say about 0.1, away from the known stopbands so that according to the present theory nothing should happen, and then to measure if there was any difference in beam life with the nonlinear lenses on or off. For simplicity we always chose $Q_H \approx Q_V$. The sextupoles were connected in the following two ways:

harmonic / polarity	18th	19th	20th	$Q_H \approx Q_V$
+ + + + 0 — — — — 0	0	6.1	1	6.20
+ + + + + + + + + 0	1	1	9	6.20

The strengths of the "harmonic" h listed in the Table are the ratio between the amplitude of a given harmonic of the perturbation due to the particular configuration of sextupoles listed in the Table and the amplitude of the same harmonic that would be produced by a single sextupole. The field distribution in the CPS sextupoles is such that they cannot excite the resonance $3\,Q_V = h$, but they do excite the resonance $2\,Q_V + Q_H = h$ and this should also lead to a vertical amplitude increase.

For the octupoles the corresponding Table is:

harmonic / polarity	24th	25th	26th	$Q_H \approx Q_V$
+ — + — + — + — + 0	1	9	1	6.39
+ + + + + + + + + 0	1	1	1	6.29

The octupoles can excite the resonance $4\,Q_V = h$.

3) MEASUREMENT OF BEAM SURVIVAL CURVES

All measurements were made at a proton momentum of 11 GeV/c, corresponding to the maximum current that the CPS power supply can maintain indefinitely. With the alternator voltage adjusted for the normal dB/dt during acceleration there is a large voltage ripple during the long flat top, corresponding to a field modulation with an amplitude of about $B_{\sim}/B_{dc} = 5 \times 10^{-4}$. Some trial runs were made with a 4 times lower alternator voltage in order to reduce the ripple on the long flat top, but this required rather laborious adjustments and gave less stable operation. All measurements presented here were therefore made with the normal dB/dt and the magnet ripple given above. During the first minute of the long flat top the magnet current varied by about 1%, but thereafter it was stable to within 0.1%. To hold the beam in the centre of the vacuum chamber we kept the r.f. on. The maximum amplitude of the phase oscillations under these conditions was $\Delta p/p = 5 \times 10^{-4}$.

The average pressure around the ring, measured with ionisation gauges in the pumping ports, was 2.7×10^{-6} mm Hg. The average pressure in the vacuum chamber is estimated to be about $0{,}7 \times 10^{-6}$ mm Hg higher and was therefore 3.4×10^{-6} mm Hg. The composition of the gas is not well known, but some measurements made by A Monnier indicate that its average atomic number is roughly equal to that of nitrogen.

The circulating beam intensity was measured with an ionisation chamber consisting of 7 coaxial rings surrounding the beam and connected as shown in Fig. 1. This assembly had a length of 30 cm adn its collector current was 4×10^{-8} A for 10^{11} circulating protons at a pressure of 3.4×10^{-6} mm Hg, in good agreement with calculations.

The maximum current that can be held for periods of about 10 minutes in the CPS nonlinear lenses is 30 A. The strength of the sextupoles, which have a length of 0.33 m, is then $d^2B/dx^2 = 22\,T/m^2$. The corresponding figures for the octupoles are 0.43 m and $d^3B/dx^3 = 1230\,T/m^3$.

Figure 2 shows some beam survival curves with all CPS octupoles connected in series with the same polarity and with a current of respectively + 30 A, 0 A and — 30 A. The working point was adjusted with the CPS quadrupoles to $Q_H = Q_V = 6.29$. The measurements were repeated several times and the differences between successive curves with the same octupole current were about the same as between the three curves shown in Fig. 2. We therefore conclude that the difference between these three curves is not significant. When the circulating beam intensity reaches about 7×10^9 protons, the signal induced in the pickup station becomes too low so that the phase lock system stops to operate correctly

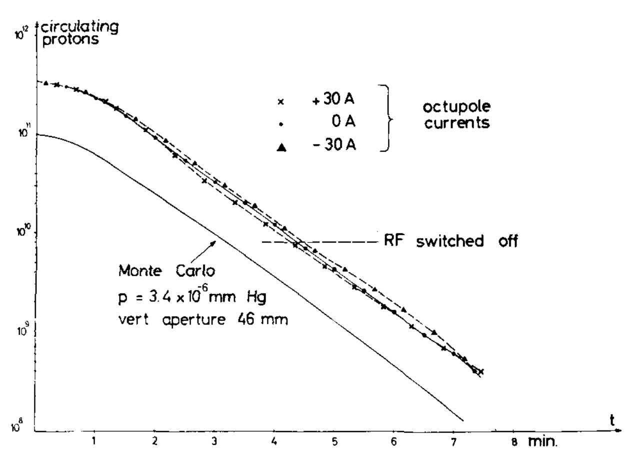

Fig. 2 - Experimental and theorical beam survival curves with all octupoles in series.

and the beam is lost. Therefore the r.f. system was switched off at about 8×10^9 protons. The beam then debunches but continues to circulate for about 3 minutes, loosing energy at the rate of about 0.16% per minute. After 7 minutes the surviving beam intensity becomes too low to measure. Since the slope of the curves in Fig. 2 after the r.f. was switched off is the same as before, we conclude that the observed beam loss is not related to the r.f. system. It is also interesting to note that the reading of the ionisation chamber is the same for a bunched and unbunched beam.

The effective vertical half aperture at 11 GeV/c during these runs was 23 mm. It was measured by A Colombo, who produced vertical bumps in the closed orbit at various places around the machine circumference with vertical kickers. The figure of 23 mm is the amplitude of the smallest bump that led to complete beam loss. Figure 2 also shows a beam survival curve calculated with a Monte Carlo computer program written by W. Richter, and which takes into account nuclear interactions and multiple scattering in the gas. From the excellent agreement between calculated and measured curves we conclude that the rate of beam loss in the CPS is entirely consistent with beam-gas interactions alone and that there is no indication of any other cause of beam loss. The Monte Carlo calculation shows, that 23% of the protons is lost by nuclear interactions with the gas, 72% is scattered into the top and bottom walls, and 5% into the side walls. Therefore our measurements are quite sensitive to the vertical aperture, or an increase in vertical amplitude. The Monte Carlo calculations also show, that 1 mm change in vertical half aperture changes the number of protons surviving after 7 minutes by a factor 1.6.

We have made several other runs with all the other sextupole and octupole connections and corresponding Q-values given in the two Tables of section 2, always, alternating + 30 A, 0 A, — 30 A but in no case was there any evidence, that the beam loss was increased by exciting the nonlinear lenses.

4) IMPLICATIONS FOR THE ISR

Let us assume, that the vertical density distribution in the ISR beam is a gaussian

$$i(z) = \frac{I}{\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}}$$

where I is the total beam current. We call the beam width w, and if $w \gg \sigma$, the octupole component of the intersection field is

$$2\left(\frac{\partial^3 B_x}{\partial z^3}\right)_{z=0} = \frac{\mu_0 I \sqrt{2}}{\sigma^3 w \sqrt{\pi}} = 1.2 \times 10^4 \text{ T/m}^3$$

for $I = 20$ A, $\sigma = 3$ mm and $w = 60$ mm. We consider $2\partial^3 B_x/\partial z^3$ in order to account for the electric field, which adds to the magnetic field. The length of the crossing region in the ISR is 0.23 m and the energy 28 GeV. The octupoles in the CPS are at places where the vertical amplitude function β has a maximum value of 22 m, whereas in the ISR crossing regions β has a minimum value of 13 m. The influence of an octupole on the orbit parameters should be roughly proportional to β^2. Scaling all these figures we find that the effective octupole strength in the CPS corresponds to about 2 times the octupole field in the ISR crossing regions. Since in the CPS experiment we could not detect any effect due to the nonlinear lenses during 7 minutes, we conclude that there should be no significant beam blow up due to nonlinear effects for at least 7 minutes in the ISR. One could also remark, that the vertical amplitude of the protons in the CPS, when they hit the wall, is about 5 times larger than in normal ISR operation. Since the field strength in an octupole is proportional to the third power of the distance from the axis the nonlinear kicks are about 2 orders of magnitude larger in the CPS experiment than in the ISR. Although this gives some extra confidence to the conclusions mentioned above, it is not clear if and how this would allow us, to extend the time scale of our predictions for the ISR.

5) NONLINEAR RESONANCES DUE TO MAGNET IMPERFECTIONS

The magnet was excited with a flat top of 8 sec duration at a proton momentum of 11 GeV/c. The r.f. was kept on and the field ripple was the same as in the beam survival measurements. When tuning the machine to $Q_H = 6\,^1/_3$, almost the complete beam was lost in 2 sec. For $Q_v = 6\frac{1}{4}$ and $Q_H = 6\frac{1}{4}$ the beam loss during 8 sec was 40% and 15% respectively. The apparent total width of all these stopbands was about 0,01 in Q but this figure is probably determined by phase oscillations and magnet ripple since the theoretical stopband width is much smaller.

To obtain a quantitative measurement of e.g. the 19th harmonic of d^2B_z/dx^2 which should cause the beam loss at $Q_H = 6^1/_3$ we excited a single mid-F sextupole with a current of a few A and measured the beam loss at various sextupole currents. It was observed that there was an optimum sextupole current at which the beam loss was a minimum. After some more experimenting it was found that, in fact, by using two sextupoles in adjacent F and D straight sections the beam loss could be suppressed completely. The value of the 19th harmonic of d^2B_z/dx^2 determined in this way corresponds to 2.7 A, or $d^2B_z/dx^2 = 1.9\ T/m^2$ in a single mid-F sextupole.

The same procedure was followed for $Q_v = 6.25$. In this case only partial reduction of the beam loss could be achieved with the available two octupoles, but from a measurement of beam loss versus octupole current we estimate that the 25th harmonic of d^3B_x/dz^3 corresponds to about 2A or $d^3B_x/dz^3 = 82\ T/m^3$ in a single octupole in a mid-D (vertically focussing) straight section.

To compare these results with theory one can e.g. assume, that due to constructional tolerances there are random block to block fluctuations in the field difference between two points x = 0 and $x = x_1$, and that these differences can be represented by random fluctuations of d^2B/dx^2 or d^3B/dx^3 from block to blok i.e.

$$\delta\left(\frac{d^2B}{dx^2}\right) = \frac{2\delta\,[B(x) - B(o)]}{x^2} \qquad [4]$$

and

$$\delta\left(\frac{d^3B}{dx^3}\right) = \frac{6\delta\,[B(x) - B(o)]}{x^3} \qquad [5]$$

The CPS magnet consists of 10^3 blocks. Assuming on r.m.s. fluctuation

$$\frac{\delta\,[B(x) - B(o)]}{B(o)} = 10^{-4} \qquad [6]$$

at x = 5 cm we find that the expectation values of the relevant harmonics correspond to $d^2B/dx^2 = 1.6\ T/m^2$ in one sextupole and $d^3B/dx^3 = 70\ T/m^3$ in one octupole in a focusing section.

These figures are very close to those found experimentally. However, we feel that the careful way in which the magnet blocks were constructed should have resulted in somewhat smaller fluctuations than those given by eq. [6]. On the other hand it is likely, that other perturbations, like remanent fields in all CPS correcting elements and from septum extraction magnets for secondary beams and other stray fields in the straight sections give important contributions to the nonlinear resonances. Therefore a more detailed comparison of theory and experiment is not meaningful without extensive measurements of these other perturbations.

6) CONCLUSIONS

In this experiment the influence of the nonlinear field of one ISR beam on the other was simulated by the nonlinear lenses of the CPS. The beam survival curves can be completely explained by nuclear interactions and multiple scattering with the residual gas and show no evidence of amplitude increase due to nonlinear perturbations for times up to 7 min. The strengths of the 3rd and 4th order resonances due to magnet imperfections in the CPS roughly agree with theoretical predictions.

Acknowledgement

We are grateful to many members of the MPS Division for their efforts to achieve stable machine operation under the special conditions of this experiment. We thank M. Georgijevic for modifications to the CPS power supply, E. Schulte and J. Jamsek for changes in the r.f. system, J. Guillet and J. Gruber for operating the CPS lenses and G. Azzoni for assistance with all the measurements.

REFERENCES

(1) B. Gittelman: Particle Accelerator Conf. Washington, 1965, p. 1033.
(2) K. Johnsen et al.: Design Study for Intersecting Storage Rings, CERN Report AR/Int. SG/64-9 (1964).
(3) A. Schoch: CERN Report 57-21 (1957); This paper also gives an extensive list of references to other work on nonlinear effects.
(4) E. Courant: Particle Accelerator Conf. Washington, 1965, p. 550.