# Gravitational particle creation in the early universe

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# Gravitational particle creation in the early universe

by

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I, David Rodríguez Román, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

# Abstract

Within the last decade, we have been able to observe our universe with incredibly high precision. Nevertheless, there are theoretical problems that still do not have an answer. In this thesis, we have studied the issue of the initial conditions of inflation, and we proposed a model for which they arise naturally as an attractor. It is achieved by considering the interaction of the inflaton with a thermal bath of particles, which are in equilibrium with the horizon temperature. If inflation happens at high energies, the electroweak vacuum is unstable. Therefore, we studied how fermions help make the Standard Model more stable, by calculating their gravitational creation during inflation.

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# Chapter 1

# Introduction

In the last century, research in the fields of particle physics and cosmology has led us to describe the universe, as we know it, by two great theories: the standard model of cosmology ($\Lambda$CDM), also known as Concordance Model, and the Standard Model (SM) of particle physics. The former explains the dynamical evolution of our universe as a whole, whilst the latter describes the content of the matter that we can study on Earth.

$\Lambda$CDM model is a gravitational theory for a homogeneous and isotropic universe which is exceptionally accurate with its predictions [1]. It tells us that the universe is 13.8 billion years old. Soon after the beginning of the universe, observations indicate that the universe was almost uniform with small perturbations. These tiny perturbations grew to give the cosmos its rich structure: galaxies, galaxy clusters, planets and stars. We study the first light emitted about 380 000 years after the initial singularity, which is called the Cosmic Microwave Background (CMB), to obtain information about the early universe. It was discovered in 1965 by Penzias and Wilson [2] for which they later won the Nobel prize in 1978. The CMB radiation is homogeneous and isotropic with a blackbody spectrum at a temperature of $T = 2.7K$. The anisotropies of this radiation were later studied by the COBE satellite [3] in 1992, which led to John C. Mather and George F. Smoot to gain the Nobel prize in 2006 for the discovery of the blackbody form and anisotropy of the CMB. An extensive study of these perturbations was later done by two satellites:

the American WMAP [4] in 2001 and the European PLANCK [1] in 2009. While phenomenologically ΛCDM is a very successful model, there are still various theoretical drawbacks yet to be explained, such as the nature of dark matter needed to explain the formation of cosmological structures, dark energy to solve the current accelerated expansion of the universe and an extension to justify the initial conditions of the universe; being inflation one of the most popular, which we will study later in this thesis.

The Standard Model of particle physics explains quantum-mechanically three fundamental interactions, electromagnetism, weak and strong forces (the forth is gravity described by General Relativity). They are responsible for the structure of atoms, its decay and the structure of quarks within the nuclei of the atom, respectively. The last missing particle to be discovered and complete the Standard model was the Higgs boson in 2012 [5,6], predicted in 1964 by Peter Higgs [7], Englert and Brout [8] and Guralnik, Hagen and Kibble [9], which consequently lead to the Nobel prize in 2013 to be awarded to Higgs and Englert. The Higgs boson is responsible for giving all massive SM particles a mass. Even though the SM is able to reproduce precise experimental tests, there are also some caveats. It can not explain the masses of the neutrinos, there is a hierarchy problem between the SM and gravity, and it does not point towards a successful grand unified theory (GUT), as many physicists would like.

It is of crucial importance to understand these theories in situations where they both apply, as in the early universe.In this thesis, we are going to address the issue of combining these two theories in the early universe, when the energies were so high that instead of planets and stars, the universe was filled with a primordial "soup" of particles. We will elaborate on a model of inflation, taking into account the Hawking radiation, as well as studying the behaviour of the Higgs fields in this early high energetic environment.

In the rest of this chapter, we will introduce the basis of cosmology, the issues of the early universe, and how inflation can provide a solution. It will lead us to the concept of gravitational particle creation by a dynamic background metric, and we will relate it to the anisotropies in the CMB. Finally, we will illustrate the relevance of the Higgs in the SM and its problem in the early universe.

Useful notation in this thesis: we are going to use natural units where $\hbar = c = 1$, for which the reduced Planck mass is $M_{\mathrm{P}} = \sqrt{\frac{\hbar c}{8\pi G}} = 2.4 \cdot 10^{18}\mathrm{GeV}$. We use $df/dt \equiv \dot{f}$ for time differentiation and $df/dx \equiv f'$ for the other differentiations. log is for decimal logarithm and ln for the natural one. Greek indices $\mu, \nu$... runs from 0 to 4 whereas latin indices $i, j$... go from 1 to 3.

## 1.1 Cosmology

In 1915, Einstein published his equations of motion of gravity [10]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}\,, \tag{1.1}$$

where $G$ is the gravitational constant ($G = 6.67 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$), $R_{\mu\nu}$ the Ricci tensor, $R$ the Ricci scalar and $T_{\mu\nu}$ the stress energy tensor. The left hand side of the equation is related to the metric and determines the space-time curvature of the universe, whereas the right hand side is given by the matter content of the universe. Einstein equations of motion in vacuum can be derived from the Einstein-Hilbert action by varying the action with respect to the metric $g_{\mu\nu}$

$$S_g = \int d^4x \sqrt{-g} \frac{M_{\mathrm{P}}^2 R}{2}\,, \tag{1.2}$$

where $g = det(g_{\mu\nu})$.

By taking the covariant derivative of Eq. (1.1) and using the Bianchi identity, the Einstein equations imply the continuity equation

$$\nabla_\mu T^{\mu\nu} = 0\,. \tag{1.3}$$

Modern cosmology is based on the evidence that at large scale (larger than 300 million light years [11]) the universe is homogeneous and isotropic. Different experiments have corroborated these assumptions. One is the identically observed temperature of the CMB, another is the distribution of galaxies around us measured by the 2dF Galaxy Redshift Survey [12] (see Fig. (1.1)).

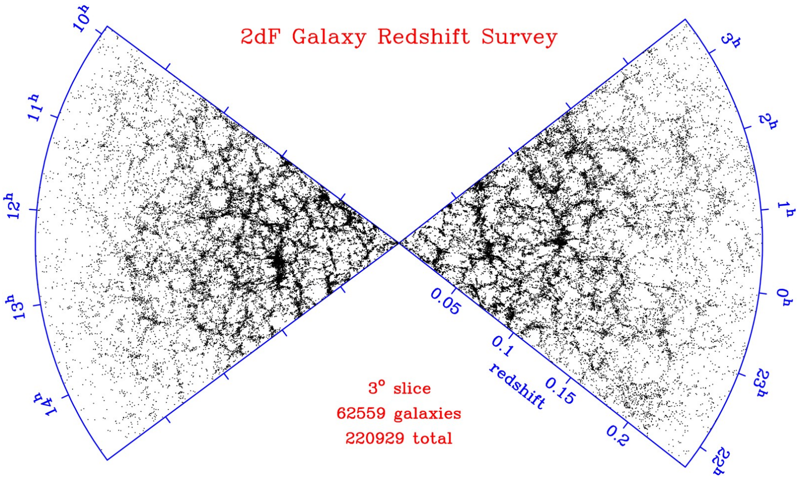


**Figure 1.1:** *Each dot is a galaxy with the Earth in the center of the map. It shows the spatial distribution of galaxies as a function of redshift from the 2dF Galaxy Redshift Survey [12]. On small scales the distribution is inhomogeneous but becomes more homogeneous on large scales.*

It points out that when averaged over large distances, the distribution of galaxies is independent of the direction (isotropic). Also to any free-falling observer, independently of his position, the universe is going to look isotropic, resulting in the universe being homogeneous as well.
Our universe is expanding with time, galaxies move away from each other as time passes. Then we can decompose space-time in slices of constant time that are homogeneous and isotropic (*foliation*). The metric for such a universe is called the Friedmann–Lemaître–Robertson–Walker (FRW) metric [13, 14]

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - Kr^2} + r^2 \left( d\theta^2 + \sin^2(\theta) d\phi^2 \right) \right) . \tag{1.4}$$

The function of time, $a(t)$, is called scale factor and determines the expansion of the universe, and $K$ is the spatial curvature. If $K = 0$ the universe is flat, $K > 0$ if it is closed and $K < 0$ for an open universe. The coordinates $x^i = \{r, \theta, \phi\}$ are called co-moving coordinates as they co-move with the expansion of the universe. Physical

coordinates are defined as $x^i_{\rm phys} = a(t)x^i$.
It is useful to define the Hubble parameter, $H \equiv \dot{a}/a$, which is the rate of expansion of the universe, as well as the number of $e$-folds of expansion, $N_e = \ln a_f/a_i$ between an initial time $a_i = a(t_i)$ and a final time $a_f = a(t_f)$; in differential form it is defined as $dN_e = d\ln a = Hdt$. Instead of using $t$ as a time coordinate, sometimes is useful to use the *conformal time*, defined as $d\eta = dt/a(t)$, which will make the metric look similar to a Minkowski space-time, $ds^2 = a(\eta)^2(-d\eta^2 + d\mathbf{x}^2)$.

The notion of the horizon is also important. Two points in space are causally connected if, in a given time interval, light can travel between both points. Since light follows null geodesics ($ds^2 = 0$), we can define a horizon as $d_p = \int dr = \int dt/a(t)$; it tells us the maximum comoving distance at which past or future events are causally connected. For past events we call it *particle horizon* and for future events we use *event horizon*.

$$d_p = \int \frac{dt}{a(t)} = \int (aH)^{-1} d\ln a \,. \tag{1.5}$$

One important extra definition is the integrand, which is the Hubble radius, $R_H = (aH)^{-1} = 1/\dot{a}$.

Given the FRW metric, we can study the left-hand side of Einstein equations. For the right-hand side, we need to study the content living in that space-time via the stress-energy tensor, $T_{\mu\nu}$. In cosmology, we study the evolution of the universe as a whole, which is composed of a vastly large number of galaxies, clouds and elementary particles ultimately. While in principle it is possible to study all of them individually, it is more convenient to treat them as a continuum and describe them as a fluid, by studying their macroscopic properties such as density and pressure.
The stress-energy density for a perfect fluid is

$$T^\mu_\nu = (\rho + p)U^\mu U_\nu + p\delta^\mu_\nu \,, \tag{1.6}$$

where $U^\mu$ is the fluid 4-velocity, $\rho$ the energy density and $p$ the isotropic pressure.

For a co-moving observer to the FRW metric, $U^\mu = (1,0,0,0)$, we can easily see that the energy density is $\rho = -T^0_0$ and the pressure is $p = T^i_i$.
The equation of state of a fluid is defined as $w = p/\rho$, which is always a number between $+1$ and $-1$. There are three types of matter that are important to mention:

1. $w = 0$ is called cold *matter*, for fluids formed of non-relativistic particles ($v \ll 1$). It is the case of dark matter and baryons.

2. $w = 1/3$ is called *radiation*, for fluids formed of relativistic particles ($v \approx 1$). It is the case of photons, neutrinos and gravitons.

3. $w = -1$ for fluids with negative pressure. It is the case for a cosmological constant and dark energy.

### 1.1.1 Friedmann equations

The Friedmann equations are the Einstein equations in the FRW metric for a universe filled with a perfect fluid

$$H^2 = \frac{\rho}{3M_\mathrm{P}^2} - \frac{K}{a^2}, \tag{1.7}$$

$$\dot{H} + H^2 = -\frac{\rho + 3p}{6M_\mathrm{P}^2}. \tag{1.8}$$

From the continuity equation (1.3) we are mostly interested in the first one with index $\nu = 0$

$$\nabla_\mu T^{\mu 0} = 0 \quad \rightarrow \quad \dot{\rho} + 3H(\rho + p) = 0\,. \tag{1.9}$$

Out of these three equations (1.7), (1.8) and (1.9), only two are independent.
For a universe filled with a fluid with equation of state $w$, the solution of the continuity equation (1.9) is $\rho \propto a^{-3(1+w)}$. Similarly we define an equation of state for the *curvature* ($K$), as a fluid with equation of state $w = -1/3$ in order to make this *fluid* redshift as the curvature term does in Eq. (1.7).
By solving the first Friedmann equation (1.7) and using the previous solution for the continuity equation (1.9), we get $a \propto t^{2/3(1+w)}$ for $w \neq -1$ and $a \propto e^{Ht}$ for $w = -1$.

See the table below for a summary of these solutions for the most important equations of state.

| $w$ | $\rho(a)$ | $a(t)$ | $a(\eta)$ |
|---|---|---|---|
| 0 | $a^{-3}$ | $t^{2/3}$ | $\eta^2$ |
| 1/3 | $a^{-4}$ | $t^{1/2}$ | $\eta$ |
| -1 | const. | $e^{Ht}$ | $-1/\eta$ |

### 1.1.2 $\Lambda$CDM

Observations constrain the current content of the universe in what is called the $\Lambda$CDM (Lambda Cold Dark Matter) model [1]. Defining the dimensionless density parameter

$$\Omega = \frac{\rho}{\rho_c}\,; \quad \rho_c = 3H_0^2 M_{\rm P}^2\,, \tag{1.10}$$

where $H_0$ is the current observed Hubble parameter $H_0 = 67.4$ km s$^{-1}$Mpc$^{-1}$ $=$ $1.4 \cdot 10^{-42}$GeV [15]. Observations indicate that [15]

$$\Omega_m = 0.31\,, \quad \Omega_\Lambda = 0.68\,, \quad \Omega_r = 9 \cdot 10^{-5}\,, \quad |\Omega_k| < 0.01\,, \tag{1.11}$$

where subscript $m$ is for matter, $\Lambda$ for dark energy, $r$ for radiation and $k$ for the curvature density parameter.

The dark energy is modelled as a cosmological constant, $w = -1$ ($\Lambda$), currently dominating the energy budget of the universe. The matter in the universe is composed by dark matter (85%) and baryons (15%). Radiation nowadays has a subdominant contribution to the energy budget of the universe and curvature is so small that can only be constrained to be smaller than 1%, therefore the universe is assumed to be flat ($K = 0$).

## 1.2 Inflation

Despite the success of describing the current state of the universe, the $\Lambda$CDM model leaves us with three important problems to solve, the *horizon, flatness and monopole*

*problems*, addressing the questions of what initial conditions seem likely for the Big Bang. Alan Guth [16] proposed the theory of inflation to solve them which later on was improved by Linde [17] and Albrecht & Steinhardt [18].
Inflation is a theory that describes the very early universe. One of the most appealing features of inflation is not that it solves these three problems, but as a consequence, it also gives us a natural mechanism to explain the density perturbations in the CMB (see Fig. (1.2)).

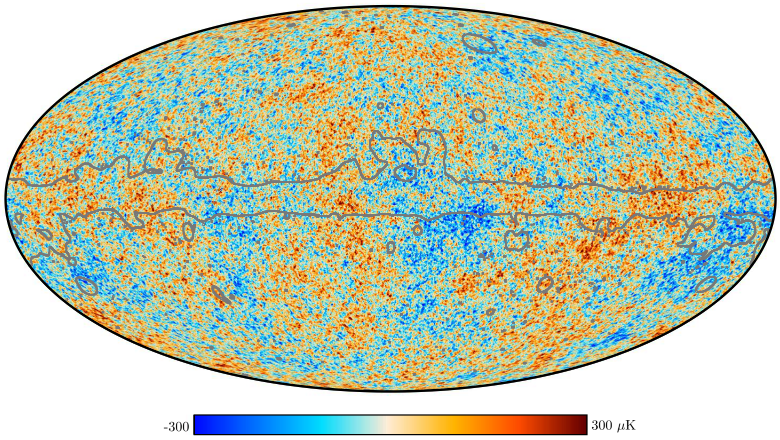


**Figure 1.2:** *The temperature of the CMB is on average $T_{CMB} = 2.7K$. In this map by Planck [1] we can see the anisotropies on top of $T_{CMB}$. The scale of this inhomogeneities is $\mathcal{O}(100)\mu K$, five orders of magnitude smaller than $T_{CMB}$, from which we can assume than the universe is homogeneous on large scales.*

In this section, firstly we will describe the three problems that inflation solves and how they can be solved, then we will explain the slow-roll dynamics of inflation and finally how we can obtain the correct spectrum of perturbations.

## 1.2.1 Three problems of the Big Bang theory

### 1.2.1.1 Horizon problem

The CMB is the first light that we can observe after the Big Bang. It was emitted at the time when electron and protons form bounded structures, Hydrogen atoms. Since then, the photons can travel freely until today. One of the most remarkable characteristics of the CMB is its isotropy. Today it is measured to be $T = 2.7K$

with perturbations of the order of $\delta T/T \sim 10^{-5}$.

If we compare the size of the particle horizon from today to the time when the CMB was emitted, $d_p(t_0) = \int_{t_{CMB}}^{t_0} \frac{dt}{a(t)}$ with the comoving particle horizon at recombination, $d_p(t_{CMB}) = \int_0^{t_{CMB}} \frac{dt}{a(t)}$, we can show that $d_p(t_0) \gg d_p(t_{CMB})$, in particular the angle subtended by the comoving horizon at recombination is $\theta = d_p(t_{CMB})/d_p(t_0) = 1.16°$. Therefore we should not observe correlations in the CMB for angles larger than 2 degrees, then how can the temperature of the CMB be $2.7K$ everywhere to a five-digit precision? This is the so-called *Horizon problem.* To solve this, we need to realise that for a universe dominated by a fluid with an equation of state $w$, the Hubble radius is $R_H \propto t^{1-\frac{2}{3(1+w)}}$. Therefore the Hubble radius grows for $w > -1/3$ and shrinks for $w < -1/3$.

The Horizon problem can be solved if we introduce a period of decreasing Hubble radius with time, which will make the particle horizon at recombination much larger. This condition is equivalent to the introduction of an accelerated expansion of the universe or to a period dominated by a fluid with equation of state $w < -1/3$. This also implies that $\epsilon \equiv -\dot{H}/H^2 < 1$.

$$\frac{d}{dt}(aH)^{-1} < 0 \quad \leftrightarrow \quad \frac{d^2a}{dt^2} > 0 \quad \leftrightarrow \quad w < -1/3 \quad \leftrightarrow \quad \epsilon = -\frac{\dot{H}}{H^2} < 1\,. \tag{1.12}$$

All of these conditions are equivalent to each other and define a period of inflation needed to solve the Horizon problem and, as we will see, they will solve the other Big Bang problems.

The next question to answer is how long does inflation need to last. The requirement is that at least our current observable horizon ($R_{H_0}$) fits inside the Hubble radius at the start of inflation, $R_{H_I} > R_{H_0}$. Now, for simplicity, we assume that the universe has always been dominated by radiation ($H \propto a^{-2}$), then the Hubble radius at the end of inflation ($R_{H_E}$) is related with today's Hubble radius as

$$R_{H_E} = \frac{a_0 H_0}{a_E H_E} R_{H_0} = \frac{a_E}{a_0} R_{H_0} \approx \frac{T_0}{T_E} R_{H_0} = 10^{-28} R_{H_0}\,, \tag{1.13}$$

where today's temperature $T_0 = 2.7K \approx 10^{-4}$eV and we have estimated a temperature at the end of inflation $T_E = 10^{15}$GeV. In the first approximation we have used the fact that the temperature redshifts like $T \propto a^{-1}$. Since during inflation $\epsilon < 1$, we can approximately say that $H_E \approx H_I$, then $R_{H_I} = \frac{a_E}{a_I} R_{H_E} = \frac{a_E}{a_I} 10^{-28} R_{H_0}$

$$\frac{a_E}{a_I} > 10^{28} \quad \leftrightarrow \quad \ln \frac{a_E}{a_I} > 64\,. \tag{1.14}$$

Therefore inflation needs to last for at least 60 $e$-folds.

#### 1.2.1.2 Flatness problem

From the Friedmann equations (1.7), we get

$$\Omega - 1 = \frac{k}{(aH)^2} = kR_H^2\,. \tag{1.15}$$

It has been measured $|1 - \Omega(a_0)| < 0.01$ (1.11). It indicates that the universe is effectively flat nowadays. If the universe is dominated by matter or radiation, the contribution from the curvature to $\Omega$ will grow with time (as $R_H$ does), meaning that the universe was even flatter before.

$$R_H^{\text{matter}} \propto t^{1/3}\,, \tag{1.16}$$

$$R_H^{\text{rad}} \propto t^{1/2}\,. \tag{1.17}$$

In particular, at the Planck time ($t_{pl} \sim 10^{-43}$ seconds), $|1 - \Omega(t_p)| < 10^{-42}$ or $|1 - \Omega(t_p)| < 10^{-62}$ respectively for a matter or radiation dominated universe, since our universe undergoes a radiation period followed by a matter-dominated era, the real number is in between these two estimates. Then to solve the flatness problem, we need to propose a theory that explains why the universe was so flat at early times. Similarly to the solution of the Horizon problem, it can be solved with a period where the Hubble radius decreases with time, so whichever configuration we had initially, inflation drives our universe towards a flat configuration. During inflation for $w = -1$

$$\Omega - 1 = kR_H^2 \propto e^{-2Ht}\,. \tag{1.18}$$

We need at least 60 $e$-folds of inflation to solve the Horizon problem, that means that for the flatness problem, whatever initial configuration of the curvature of the universe is, by the end of inflation, it is going to be smaller by a factor $10^{-56}$. We conclude that by solving the Horizon problem, we also solve the flatness problem.

#### 1.2.1.3 Monopole problem - Unwanted relics

There is another problem that is usually found in the literature, the monopole problem [19–21]. A monopole, or any other topological defect, arises when a phase transition takes place early in the history of the universe. If there is a Grand Unified Theory (GUT) unifying Standard Model (SM) gauge groups at high energies, early in the history of the universe, a phase transition will occur, and generally, we expect monopoles to be produced. These monopoles would dominate the energy density of the universe before helium synthesis [19], something that has been disproved by experiments. Since we do not observe any of these topological defects, we need a mechanism to wash them out. This principle also applies to any other unwanted relic of particles or topological defects.
The energy density is diluted as $\rho \propto a^{-3(1+w)}$. For inflation, we require $w < -1/3$, so for any unwanted relic with an equation of state larger than the one for inflation, its energy density is going to be incredibly reduced after 60 $e$-folds of inflation.

### 1.2.2 Scalar field dynamics

The scalar field that is going to drive inflation is called the inflaton ($\phi$), with action

$$S_\phi = \int d^4x\sqrt{-g}\mathcal{L}_\phi\,; \qquad \mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\,, \tag{1.19}$$

where $V(\phi)$ is the inflaton potential and $g$ the determinant of the metric $g_{\mu\nu}$. The equation of motion for the inflaton is obtained by varying the action with respect to $\phi$. For a FRW metric we get

$$\ddot{\phi} - \frac{\nabla^2\phi}{a^2} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0\,, \tag{1.20}$$

where $\nabla^2$ is the Laplacian. For consistency with the FRW symmetries we impose a homogeneous field distribution throughout the universe at the background level $\phi(t,\mathbf{x}) = \phi(t)$, then $\nabla^2\phi = 0$. To recover the Einstein equations of motion, the stress energy density is

$$T_{\mu\nu} = -2\frac{\partial\mathcal{L}_\phi}{\partial g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_\phi\,. \tag{1.21}$$

For the inflaton Lagrangian (1.19) then

$$T^\mu_\nu = \partial^\mu\phi\partial_\nu\phi - \delta^\mu_\nu\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right)\,, \tag{1.22}$$

a homogeneous scalar field can be studied as a perfect fluid, with the diagonal terms of the stress energy tensor corresponding to an energy density and pressure

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{1.23}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{1.24}$$

The Friedmann equations for the inflaton field are,

$$3H^2M_{\mathrm{P}}^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{1.25}$$

$$-2\dot{H}M_{\mathrm{P}}^2 = \dot{\phi}^2\,. \tag{1.26}$$

Inflation occurs for $w < -1/3$ (1.12). If $w = -1$, we can see that the inflaton has only potential energy, then it is just a cosmological constant, the space-time in this case is the *de-Sitter* solution to the Einstein equations. When the potential is not constant, the inflaton can acquire kinetic energy, then $-1 < w < -1/3$ and the space-time is called *quasi de-Sitter*. In a quasi de-Sitter scenario, the inflaton slowly increases the kinetic energy up to $\epsilon = 1$, point at which inflation ends. After inflation we need to transfer all the energy in the inflaton to SM particles, this epoch is called *reheating*.

#### 1.2.2.1 Slow-roll

Slow-roll parameters are defined as

$$\epsilon_{n+1} = \frac{d\ln\epsilon_n}{dN_e}; \qquad \epsilon \equiv \epsilon_0 = -\frac{d\ln H}{dN_e}\,, \tag{1.27}$$

where $N_e$ is the number of $e$-folds and $H$ is the Hubble parameter. From the inflation conditions (1.12) we know that $\epsilon < 1$ and we need to maintain it for at least 60 $e$-folds, then we impose that $\epsilon$ does not change too much with time, meaning $|\epsilon_1| < 1$.
The slow-roll approximation takes the conditions $\{\epsilon, \epsilon_1\} \ll 1$, to simplify the Friedmann equations of motion for the inflation.

$$\epsilon = 3\frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \ll 1\,, \tag{1.28}$$

$$\epsilon_1 = 2\frac{\ddot{\phi}}{H\dot{\phi}} - 2\frac{\dot{H}}{H^2} \ll 1\,. \tag{1.29}$$

The first equation tells us that the kinetic term is smaller than the potential whereas the second that the acceleration is smaller than $H\dot{\phi}$.
Thus the equations of motion for the inflaton are simplified to be

$$3H^2 M_{\mathrm{P}}^2 = V(\phi)\,, \tag{1.30}$$

$$3H\dot{\phi} = \frac{dV}{d\phi}\,. \tag{1.31}$$

Now with these equations of motion is much easier to study analytically the evolution of the inflaton. It is convenient to define a new set of slow-roll parameters as a function of the potential of the inflaton, using the definitions (1.28) and (1.29) and the approximations to the Friedmann equations (1.30) and (1.31)

$$\epsilon_V = \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'(\phi)}{V}\right)^2 , \tag{1.32}$$

$$|\eta_V| = M_{\mathrm{P}}^2 \frac{|V''(\phi)|}{V} . \tag{1.33}$$

Slow-roll inflation occurs for $\{\epsilon_V, |\eta_V|\} \ll 1$.

In the next section we are going to relate these parameters with physical observables that we can observe experimentally.

## 1.3 Perturbations in the CMB

So far, we have described the dynamics of a homogeneous and isotropic universe. From CMB observations, we know that at the time of decoupling, the universe was inhomogeneous at the level of one part in a hundred thousand. The inflation model is able to generate these deviations quite naturally. In this section, we are going to introduce how we quantify these perturbations. In Sec. 1.3.1 we will develop the theory of quantum field theory in curved space-time needed to calculate them and in Sec. 1.3.2 we will link the physical observables with the inflaton field.

The anisotropies are studied using perturbation theory, treating the background evolution of the universe homogeneous and isotropic and on top of that small perturbations.

$$\rho(t, \mathbf{x}) = \bar{\rho}(t) + \delta\rho(t, \mathbf{x}) . \tag{1.34}$$

Due to the symmetries of a spatially flat, homogeneous and isotropic universe, we can decompose the metric perturbations in scalar, vector and tensor perturbations, defined by their helicities [22]. At a linear level, each perturbation evolves independently of the others, and we can study them separately.

For inflation, we are only interested in scalar and tensor perturbations, since vectors perturbations are not sourced by inflation [22]. A crucial feature of this decomposition is its non-uniqueness, which means we need to choose a gauge and all of our physical observables need to remain gauge invariant. Otherwise we could find coordinate frames in which we create fictitious perturbations or even remove real perturbations. Tensor fluctuations are gauge invariant but scalar perturbations are not [22].

For scalars perturbations, we are only going to be interested in the curvature perturbation on uniform-density hypersurfaces, using the spatially flat gauge [22], it is defined as,

$$\zeta \equiv -\frac{H}{\dot{\bar{\rho}}}\delta\rho\,. \tag{1.35}$$

The key feature of this quantity is that it does not evolve on super-horizon scales ($k \ll aH$), for adiabatic matter perturbations. Therefore, it relates the perturbations produced during inflation with the temperature fluctuations that we observe in the CMB radiation.

$$\dot{\zeta} = 0 \quad \text{for} \quad k \ll aH\,. \tag{1.36}$$

Primordial scalar perturbations are studied by the power spectrum of $\zeta$, obtained from the 2-point correlator function

$$\langle \zeta_{\mathbf{k}}\zeta_{\mathbf{k'}} \rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k'})P_\zeta(k)\,; \qquad \Delta_s^2 \equiv \Delta_\zeta^2 = \frac{k^3}{2\pi^2}P_\zeta(k)\,. \tag{1.37}$$

The power spectrum has a scale dependence defined by the scale spectral index (also called *tilt*)

$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k}\,. \tag{1.38}$$

If the power spectrum is scale invariant, then $n_s = 1$.

It can be approximated as

$$\Delta_s^2(k) = A_s(k_*)\left(\frac{k}{k_*}\right)^{n_s(k)-1}\,, \tag{1.39}$$

where $k_*$ is a pivot scale, an arbitrary reference value for the k modes. From CMB observations [1], we know that the scalar power spectrum 60 $e$-folds before the end of inflation has to be

$$\ln\left(10^{10}A_s\right) = 3.047 \pm 0.014\,. \tag{1.40}$$

Tensor perturbations have two polarizations ($h^+$,$h^\times$). The power spectrum of tensor perturbations is the addition of both

$$\Delta_t^2 = 2\Delta_h^2 = A_t(k_*)\left(\frac{k}{k_*}\right)^{n_t}\,. \tag{1.41}$$

From observations we can put bounds on the ratio of tensor to scalar perturbations

$$r = \frac{A_t}{A_s}\,. \tag{1.42}$$

In Fig. (1.3) we can see the current bounds on the tilt of the power spectrum of primordial fluctuations and the tensor to scalar ratio. We know that $n_s < 1$ but for the tensor to scalar ratio we only have a bound since there is still no detection of primordial gravitational waves. In the next section, we are going to study scalar fields in curved space-time and show how can we relate $n_s - 1$ and $r$ to the inflaton field.

### 1.3.1 Quantum field theory in curved space-time

We are going to study a spectator field. It is a field with a sub-dominant contribution to the total energy density. It means that the backreaction of the field to the metric is negligible. Therefore it is possible to quantise the field in curved space-time. We will show how it leads to the concept of particle creation by gravitational fields and also the production of primordial fluctuations during inflation.

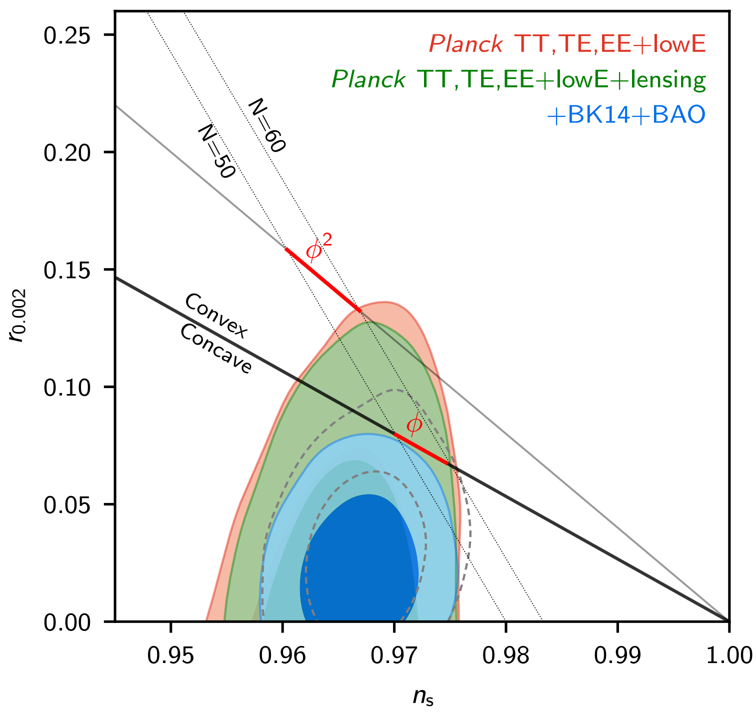


**Figure 1.3:** *Plot by Planck [23] showing the current bound on the tensor to scalar ratio for a pivot scale of $k^* = 0.002 Mpc^{-1}$, $r < 0.1$ and the spectral tilt $n_s - 1 = 0.97$. Current measurements disfavour a monomial inflaton potential with exponent larger than one for a duration in between 50 to 60 e-folds. It also favours a concave inflaton potential.*

The total action of the system is

$$S = S_g + S_m = \int d^4x \sqrt{-g} \frac{M_{\rm P}^2 R}{2} + \int d^4x \sqrt{-g} \mathcal{L}_m \,. \tag{1.43}$$

The gravity action is not quantised, and the equations of motion for an FRW metric are the Friedmann equations.

For a field $\phi(x)$, the canonical momenta is $\pi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi)}$ and the canonical commutation relations hold the same as in Minkowski space-time. For a boson it would be

$$[\phi_a(t,\mathbf{x}), \phi_b(t,\mathbf{x'})] = 0\,, \tag{1.44}$$

$$[\pi_a(t,\mathbf{x}), \pi_b(t,\mathbf{x'})] = 0\,, \tag{1.45}$$

$$[\phi_a(t,\mathbf{x}), \pi_b(t,\mathbf{x'})] = i\delta_{a,b}\delta(\mathbf{x}-\mathbf{x'})\,, \tag{1.46}$$

for fermions this is different and we will study them in chapter 3.
The Lagrangian density for a general real massive scalar field ($\phi$) is [24, 25]

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\xi R\phi^2\,, \tag{1.47}$$

where $\xi$ is the non-minimal coupling of $\phi$ to gravity and $R$ the Ricci scalar. There are two important values for $\xi$: the field is minimally coupled to gravity for $\xi = 0$ and is conformally coupled for $\xi = -1/6$ (in four dimensions). In general this term modifies the equation of motion of gravity, but for our scenario, where the field is sub-dominant ($\xi\phi^2 \ll M_{\rm P}^2$) it will not.
The corresponding equation of motion is the Klein-Gordon equation

$$(-\Box + m^2 - \xi R)\phi = 0\,. \tag{1.48}$$

The inner product of two solutions to the equation of motion is defined as

$$(\phi_1,\phi_2) = i\int d^3x\sqrt{|g|}\phi_1^*(x)\overleftrightarrow{\nabla}_0\phi_2(x); \qquad \overleftrightarrow{\nabla}_\mu = \overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu\,. \tag{1.49}$$

To solve the Klein-Gordon we decomposed the field in Fourier modes with an orthonormal basis $\{f_i, f_i^*\}$, where $\{f_i\}$ is a complete set of positive norm solutions and $\{f_i^*\}$ is a complete set of negative norm solutions. Both are solutions to the wave equation, we can write generically the solution to the Klein-Gordon equation as

$$\phi = \sum_k (a_\mathbf{k} f_\mathbf{k} + a_\mathbf{k}^\dagger f_\mathbf{k}^*)\,, \tag{1.50}$$

where $a_{\mathbf{k}}$ and $a^{\dagger}_{\mathbf{k}}$ are the annihilation and creation operator, respectively, as defined in Minkowski space-time. According to the canonical quantization (1.44) they need to satisfy

$$[a_{\mathbf{k}}, a^{\dagger}_{\mathbf{k'}}] = \delta_{\mathbf{k},\mathbf{k'}}\,; \qquad [a_{\mathbf{k}}, a^{\dagger}_{\mathbf{k}}] = 0\,. \tag{1.51}$$

This expansion defines a vacuum state $|0\rangle$ such that $a_{\mathbf{k}}|0\rangle = 0$ for any $\mathbf{k}$.
The functions $f_i$ are the basis of the Fourier decomposition

$$f_{\mathbf{k}}(x) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}}y(t)\,. \tag{1.52}$$

In flat spacetime we define the basis function $y(t)$ as the set of solutions with a positive frequency, which, independently of the Lorentz frame, let us define the same and unique Minkowski vacuum state. In general relativity this choice is not unique and the basis functions are not independent of our frame. But our vacuum state ($|0\rangle$) does not change with time; this is the Heisenberg Picture. Then since our basis functions change with time but our state does not, this implies that there is a time dependence of the annihilation and creation operators. Therefore, in general relativity, we can not uniquely define a vacuum solution. This characteristic of non-uniqueness is essential for the concept of particle creation since we can not identify a vacuum state with the notion of particle content. Now we will present an example where we explicitly show how this leads to the concept of particle creation.

Let us consider a space-time which is asymptotically flat (Minkowski) in the past and in the future in which we do not have a problem with the definition of a vacuum state. We will denote the basis of the solution to the Klein-Gordon in the asymptotic past as $f_i$, and $F_i$ for the future. It is commonly called the "in-region" for the past and the "out-region" for the future. These two set of solutions form an orthonormal basis, which satisfy

$$(f_i, f_j) = (F_i, F_j) = \delta_{ij}\,, \tag{1.53}$$

$$(f_i^*, f_j^*) = (F_i^*, F_j^*) = -\delta_{ij}\,, \tag{1.54}$$

$$(f_i, f_j^*) = (F_i, F_j^*) = 0\,. \tag{1.55}$$

Then we are able to express one basis as a linear combination of the other, such that

$$f_j = \sum_k (\alpha_{jk} F_k + \beta_{jk} F_k^*)\,, \tag{1.56}$$

where $\alpha$ and $\beta$ are called Bogoliubov coefficients.
The orthogonality conditions give us the relation

$$\sum_k \alpha_{ik}\alpha_{jk}^* - \beta_{ik}\beta_{jk}^* = \delta_{ij}\,, \tag{1.57}$$

$$\sum_k \alpha_{ik}\alpha_{jk} - \beta_{ik}\beta_{jk} = 0\,. \tag{1.58}$$

The solution to the Klein-Gordon equation can be expressed in either basis

$$\phi = \sum_i (a_i f_i + a_i^\dagger f_i^*) = \sum_i (b_i F_i + b_i^\dagger F_i^*)\,, \tag{1.59}$$

where $a, a^\dagger$ are the annihilation and creation operators in the "in-region" and where $b, b^\dagger$ are the annihilation and creation operators in the "out-region". In the same way that we have related the "in" and "out" functions, we can also relate the creation and annihilation operators.
The key concept here is that we have two set of annihilation and creation operators, therefore there are two vacuum solutions, $a_i|0\rangle_{in} = 0$ and $b_i|0\rangle_{out} = 0$. This implies the physical phenomena of particle creation by a gravitational field. A vacuum state is defined initially in the "in-region" with the operators $a$, then at late times, the state has not changed $|0\rangle_{in}$ (Heisenberg picture) but the solution of a vacuum state at that time has changed and is defined by the operators $b$. They measure the

number of particles in a state, thus the number of particles measured at late time of an initial vacuum state is

$$N_k =_{in} \langle 0|b_k^* b_k|0\rangle_{in} = \sum_i |\beta_{ik}|^2 \,. \tag{1.60}$$

For concreteness if we study a scalar field as before (1.48) in a FRW metric using conformal time, the Fourier decomposition is $f_{\mathbf{k}}(x) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}}\frac{h(\eta)}{a(\eta)}$, and we need to solve

$$\frac{d^2h(\eta)}{d\eta^2} + \left(k^2 + a^2(\eta)\left(m^2 - \left(\xi + \frac{1}{6}R(\eta)\right)\right)\right) = 0\,, \tag{1.61}$$

the inner product give us the norm for $f_{\mathbf{k}}$, translating in to the Wronskian

$$h(\eta)h'^*(\eta) - h^*(\eta)h'(\eta) = i\,. \tag{1.62}$$

The second boundary condition is set by our choice of vacuum, either $a_i|0\rangle_{in} = 0$ or $b_i|0\rangle_{out} = 0$. Thus the vacuum is defined as a solution with a positive norm. From (1.61) the frequency is $w^2 = k^2 + a^2(\eta)\left(m^2 - \left(\xi + \frac{1}{6}R(\eta)\right)\right)$ and the initial conditions to the equation (1.61) at a given time ($\eta_0$) are,

$$|h(\eta_0)|^2 = \frac{1}{2w}\,; \qquad h'(\eta_0) = -iwh(\eta_0)\,. \tag{1.63}$$

### 1.3.2 Particle production in de-Sitter/inflaton perturbations

The idea of this subsection is to link the calculation of the inflaton power spectrum with the particle production that we just derived. The power spectrum is calculated with the gauge invariant quantity

$$\zeta = -\frac{H}{\dot{\bar{\rho}}}\delta\rho = -H\frac{\delta\phi}{\dot{\bar{\phi}}}\,, \tag{1.64}$$

where in the last equality we have used the fact that during inflation $V \ll \dot{\phi}^2/2$, then $\delta\rho = V'\delta\phi$ and also $3H\dot{\phi} = -V'$.
The inflaton field is decomposed as we did for the perturbations

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})\,. \tag{1.65}$$

Then from the equation of motion of the inflaton (1.20), we obtain

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V'(\bar{\phi}) + \delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{\nabla^2\delta\phi}{a^2} + V''\delta\phi = 0\,, \tag{1.66}$$

where $V$ includes the mass of the field and its coupling, also including the non-minimal coupling.
We want to express the quantities $\Delta_s^2$, $n_s - 1$ and $r$ as a function of the inflaton potential and/or the slow-roll parameters.
For the perturbations of the inflaton $\delta\phi$, we study its Fourier transform

$$\delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + \frac{k^2}{a^2}\delta\phi_{\mathbf{k}} + V''\delta\phi_{\mathbf{k}} = 0\,. \tag{1.67}$$

The modes outside the horizon are frozen (1.36), therefore we need to study them when they are inside the horizon (sub-horizon), $k \gg aH$. Once the horizon shrinks to the size of the perturbation $k = aH$ (horizon crossing), they freeze and remain unchanged until they re-enter the horizon after inflation has ended when the horizon is growing. From the slow-roll approximation, we know that $\eta_V \ll 1$, which implies $V'' \ll V/M_{\rm P}^2$ and using the first slow-roll condition, $\epsilon_V \ll 1$, we obtain $V'' \ll H^2$, then for sub-horizon modes during inflation $\frac{k^2}{a^2} \gg V''$.
The inflaton perturbations are quantized

$$\delta\phi = \int \frac{d^3k}{(2\pi)^{3/2}}\left(\delta\phi_{\mathbf{k}} a_{\mathbf{k}} e^{i\mathbf{kx}} + \delta\phi^*_{\mathbf{k}} a^\dagger_{\mathbf{k}} e^{-i\mathbf{kx}}\right), \tag{1.68}$$

and satisfy the canonical commutation relations that we have showed already, $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k'}}] = (2\pi)^3\delta(\mathbf{k} - \mathbf{k'})$. We use the definition $\delta\phi_{\mathbf{k}} \equiv v_k(\eta)/a$, where $\eta$ is the conformal time and $R = 6a''/a$, to get

$$v_k''(\eta) + (k^2 - \frac{a''}{a})v_k(\eta) = 0\,. \tag{1.69}$$

From here we can see how for super-horizon modes ($k \ll aH$), the solution is homogeneous and $v \propto a$ making $\delta\phi$ constant, this is why the perturbations "freeze" on super horizon scales and we are just interested in them inside the horizon.
During inflation the universe evolves as a de-Sitter space-time, then $a''/a = 2/\eta^2$, which we solve by requiring orthonormality (1.62) and being a vacuum state initially (1.63), i.e. at $\eta \to -\infty$

$$v_k^{\rm ini}(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}\,, \tag{1.70}$$

the full solution for $v_k$ is

$$v_k(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}(1 - \frac{i}{k\eta})\,, \tag{1.71}$$

now we can calculate the value for the two point function

$$\begin{aligned}\langle \delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k'}}\rangle &= \langle 0|\delta\phi_{\mathbf{k}}\delta\phi_{\mathbf{k'}}|0\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k'})\frac{|v_k(\eta)|^2}{a^2} \\ &= (2\pi)^3\delta(\mathbf{k}+\mathbf{k'})\frac{H^2}{2k^3}(1+\frac{k^2}{a^2H^2}) = (2\pi)^3\delta(\mathbf{k}+\mathbf{k'})\frac{k^3}{2\pi^2}\Delta^2_{\delta\phi}\,. \end{aligned} \tag{1.72}$$

On super horizon scales and at horizon crossing, the value of the perturbation is the same

$$\Delta^2_{\delta\phi} = \left(\frac{H}{2\pi}\right)^2. \tag{1.73}$$

Then the power spectrum for scalar perturbations is (1.64)

$$\Delta^2_s(k) = \frac{H^2}{\dot{\phi}^2}\left(\frac{H}{2\pi}\right)^2 = \frac{1}{8\pi^2}\frac{H^2}{M_{\rm P}^2}\frac{1}{\epsilon}\,, \tag{1.74}$$

where all the quantities are evaluated at horizon crossing.

Tensor perturbations ($h_{ij}$) are obtained by expanding the Einstein Hilbert action to second order. The action for the graviton is the same than for a massless scalar particle but twice, accounting for the two polarizations of the graviton $(\times, +)$. Then the power spectrum of the tensor perturbations is double the one for scalar perturbations. Tensor perturbations are dimensionless, so to be able to do this relation, we need to identify the scalar perturbation $h^s = \frac{2}{M_{\rm P}}\delta\phi$, where $s = \times, +$ is the polarization of the perturbations

$$\Delta_t^2(k) = 2\Delta_h^2(k) = 2\left(\frac{2}{M_{\rm P}}\,\frac{H}{2\pi}\right)^2 , \tag{1.75}$$

evaluated at horizon crossing ($k = aH$).
From here we can easily calculate the tensor to scalar ratio

$$r = \Delta_t^2/\Delta_s^2 = 16\epsilon \,. \tag{1.76}$$

From CMB observations we have measured $\Delta_s^2 = 2.2 \cdot 10^{-9}$, therefore observations of $r$ would tell us the energy scale of inflation ($H$).
From the scalar power spectrum (1.74) we can calculate its scale dependence ($n_s - 1$); since it is evaluated at horizon crossing, $k = aH \rightarrow \ln(k) = N_e + \ln(H)$, we get

$$n_s - 1 \equiv \frac{d\ln\Delta_s^2}{d\ln k} = -2\epsilon - \epsilon_1 = 2\eta_V - 6\epsilon_V \,. \tag{1.77}$$

### 1.3.3 Stochastic motion of fields

We have shown how the modes inside and outside the horizon behave differently; in particular, outside the horizon, they are frozen. We can use this to split the field mode function between modes sub and super horizon. By doing this we are going to show how the sub horizon modes act as a stochastic noise term in the equation of motion of the averaged super horizon modes [26, 27]. This is true for any field, it does not matter if it is sub-dominant or not.

$$\phi(t,\mathbf{x}) = \bar{\phi}(t,\mathbf{x}) + \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\theta(k-\epsilon aH)\Big(a_{\mathbf{k}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + a^{\dagger}_{\mathbf{k}}\phi^{*}_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}\Big)\,. \tag{1.78}$$

The function $\theta$ is the Heaviside step function, it ensures that short wavelength modes are inside the integral and the long wavelength modes are included in $\bar{\phi}$. Also $\bar{\phi}$ is the average over distances larger than the horizon since it includes all the modes smaller than $k < \epsilon aH$, where $\epsilon$ is just a number smaller than one. Then the volume over which we averaged the field is for distances larger than the Hubble horizon. Due to this split of wave modes, we can say that any two points closer than a Hubble horizon have the same value of $\bar{\phi}$, this is called *coarse-graining*.
From the equation of motion of the field $\phi$ with potential $V(\phi)$ (1.20) by using the decomposition (1.78) during inflation, $\ddot{\phi} \ll 3H\dot{\phi}$ (this holds for long and short wavelengths), we obtain

$$\dot{\bar{\phi}}(t,\mathbf{x}) = -\frac{V'(\bar{\phi})}{3H} + f(t,\mathbf{x})\,, \tag{1.79}$$

where

$$f(t,\mathbf{x}) = \epsilon aH^2 \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\delta(k-\epsilon aH)\Big(a_{\mathbf{k}}\phi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + a^{\dagger}_{\mathbf{k}}\phi^{*}_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}\Big)\,, \tag{1.80}$$

here $\delta(x)$ is the Dirac delta, which comes from the time differentiation of the previous Heaviside step function.
From Eq. (1.79) we can see that the field is going to evolve with time following a classical trajectory given by $-\frac{V'(\bar{\phi})}{3H}$ which is called a drift and a random contribution from the stochastic noise $f(t,\mathbf{x})$. This random contribution is going to be a collection of quantum 'kicks', and by the Central Limit Theorem the effect after a small time step is Gaussian. The mean of a Gaussian distribution is zero and the variance is given by

$$\langle f(t,\mathbf{x})^2\rangle = \frac{H^3}{(2\pi)^2}\,. \tag{1.81}$$

The missing key now to identify the equation of motion of the scalar field $\phi$ with the Langevin equation is the fact that, formally, $\bar{\phi}$ remains a quantum operator. This can be handled by using an auxiliary classical stochastic scalar field ($\varphi$) with the same expectation values for all the observables.

$$\frac{d\varphi}{dN_e} = \frac{-V'}{3H^2} + \frac{H}{2\pi}\xi\,, \tag{1.82}$$

where $\xi$ is a Gaussian noise term with vanishing mean and unit variance. This is the Langevin equation, it is better expressed as a function of the number of $e$-folds instead of the cosmic time $t$ for situations where inflation does not occur in perfect de-Sitter and $H$ changes slowly with time. Since in the Itô interpretation, the corresponding Fokker-Planck equation would be different [28]. From the stochastic differential equation we obtain a Partial Differential Equation (PDE) for the Probability Distribution Function (PDF) of the scalar field, the Fokker-Planck equation for the probability distribution function $P(\bar{\phi} = \varphi, N_e)$ is given by,

$$\frac{\partial P(\varphi, N_e)}{\partial N_e} = \frac{\partial}{\partial \varphi}\Big(\frac{V'(\varphi)}{3H^2}P(\varphi, N_e)\Big) + \frac{H^2}{8\pi^2}\frac{\partial^2}{\partial \varphi^2}(P(\varphi, N_e))\,, \tag{1.83}$$

which has a stationary solution for perfect de-Sitter space-time,

$$P_{static}(\varphi) \propto e^{-\frac{8\pi^2 V(\varphi)}{3H^4}}\,, \tag{1.84}$$

which needs to be normalized.
The value of the variance in the static limit from this calculation and the two point function from the gravitational particle creation of the previous section give us the very same result [29].

We have shown how a scalar field is disturbed during inflation; it makes sense now to study the only scalar field that we have in the SM and what consequences may these effects have.

## 1.4 Standard Model of particle physics

The Standard Model (SM) of particle physics is the theory which describes three of the fundamental forces (electromagnetic, weak and strong). It describes the interactions between quarks and leptons, divided in three families.

| | Families | | | Interactions | |
|---|---|---|---|---|---|
| | 1st | 2nd | 3rd | gauge bosons | scalar boson |
| quarks | u | c | t | g | H |
| | d | s | b | $\gamma$ | |
| leptons | e | $\mu$ | $\tau$ | Z | |
| | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | $W^\pm$ | |

The Gauge symmetry of the Standard Model is $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$; the term $SU_C(3)$ corresponds to the strong interaction, formed by quarks and gluons; $SU_L(2) \otimes U_Y(1)$ represents the electroweak (EW) interaction, possessed by leptons, quarks and gauge bosons $\gamma$, $Z$ and $W^\pm$. $SU_L(2) \otimes U_Y(1)$ is spontaneously broken, leaving only the electromagnetic subgroup $U_{em}(1)$ unbroken.

The SM Lagrangian corresponds to the matter content of the universe in Eq. (1.43). It has been tested to high precision in the Large Hadron Collider (LHC) with energies up to 10 TeV [30].

### 1.4.1 Higgs and the interactions with other fields

The fundamental field in the SM that gives particles their mass is the Higgs field, by their interaction, after a spontaneous symmetry breaking, SM particles acquire a mass (except for the photon which is massless).

To spontaneous break $SU(2)$ we need two complex Higgs fields to do it, they form the Higgs doublet $H = \binom{H_1}{H_2}$, in the unitary gauge we can choose $H_1 = 0$ and $H_2 = \frac{1}{\sqrt{2}}(v + h)$ real, where $v$ is the Higgs field vacuum expectation value (vev) and $h$ are the oscillations around the vev called Higgs bosons. The Higgs potential has a Mexican-hat shape

$$V(H) = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2; \qquad \mu^2 > 0\,, \tag{1.85}$$

where $\lambda$ is the Higgs self-interaction term and $\mu = 88\text{GeV}$ is a parameter of the Higgs field related to its mass. Because of the negative sign in front of $\mu^2$, it is not a mass term, sometimes it is called a tachyonic mass. The minimum of the potential is not at the origin but at $v$

$$v \equiv \sqrt{\frac{\mu^2}{\lambda}}\,, \tag{1.86}$$

which is the Higgs vev. From muon decay [31] we know its value $v = 1/\sqrt{\sqrt{2}G_F} \approx 246\text{GeV}$, where $G_F \sim 10^{-5}\text{GeV}^{-2}$ is the Fermi coupling constant. Then we can calculate the value of the self-interaction $\lambda = \mu^2/v^2 = 0.12$. The mass for the Higgs boson is obtained expanding the potential around the vev

$$m_h = \sqrt{2\mu^2} = 125\text{GeV}\,. \tag{1.87}$$

The full SM Lagrangian includes all the quarks and leptons, but in this thesis, we are going to focus mostly on the interaction of the Higgs field with one fermion, avoiding the complexities of quark mixing. In the SM, the fermion that affects the most the Higgs field is the top quark, the interaction between the Higgs field and other fermions is sub-dominant. The interaction term in the Lagrangian for a system with the Higgs field and the top quarks is

$$\mathcal{L}_{\text{Higgs \& top}} = 3y_t\frac{v+h}{\sqrt{2}}\bar{\psi}\psi\,, \tag{1.88}$$

$y_t$ is the top Yukawa coupling, it is the dimensionless parameter that parametrises the interaction between the Higgs and the tops ($\psi$). There are three tops quarks, because of the colour charge. The mass of the top quarks is [31]

$$m_t = y_t\frac{v}{\sqrt{2}} = 173\text{GeV}\,. \tag{1.89}$$

Then the top Yukawa coupling value is $y_t = 0.93$.

### 1.4.2 Quantum corrections and instability of the EW vacuum

The dimensionless parameters in the SM Lagrangian $(\lambda, y_t)$ depend on the energy scale at which they are measured. The physical constants that we can observe are the subtraction of two infinitely large quantities, the bare physical constant and the counter term, which subtracts the divergent part of the former, this process is called *Renormalisation* [32]. The counter terms cancel the divergence up to an energy scale, cut off. This leads us to realise that the values of the physical constant changes depending on the energy scale, this is calculated via the beta functions, in particular for the standard model Higgs to one-loop order [33]

$$\beta_\lambda = \frac{d\lambda(\mu)}{d\ln(\mu)} = \frac{1}{(4\pi)^2}(24\lambda^2 + 12\lambda y_t^2 - 6y_t^4)\,, \tag{1.90}$$

$$\beta_{y_t} = \frac{dy_t(\mu)}{d\ln(\mu)} = \frac{1}{(4\pi)^2}(\tfrac{9}{2}y_t^3)\,, \tag{1.91}$$

$\mu$ is not the mass but the energy scale. There is one crucial feature that appears even at one-loop order, the negative term from the top Yukawa coupling in the running of the Higgs self-interaction. At very high energies, it will make $\beta_\lambda$ negative, meaning that $\lambda$ would decrease with the scale and at some energy, it would be zero and then take negative values. This is the instability of the electroweak (EW) vacuum. If $\lambda < 0$ the Higgs potential is unbounded and has no minima.

Fig. (1.4) shows the running of $\lambda$ with the scale. The LHC has measured $\lambda = 0.12$ and $y_t = 0.93$ at a energy scale $\mu = 100$ GeV, these are the initial conditions for solving the Renormalization Group Equations (RGE) (1.90) and (1.91); in the plot we can see how small variations of the top mass and/or $\alpha_s$ (the coupling strength of the Strong Interaction between quarks and gluons) within the experimental error bars can change the instability scale.

The beta functions (1.90,1.91) are obtained from the SM effective potential to one-loop order. It is the addition of the classical potential $(V^{(0)})$, and other terms that arise from quantum fluctuations around the classical field value $(V^{(1)})$. For the SM

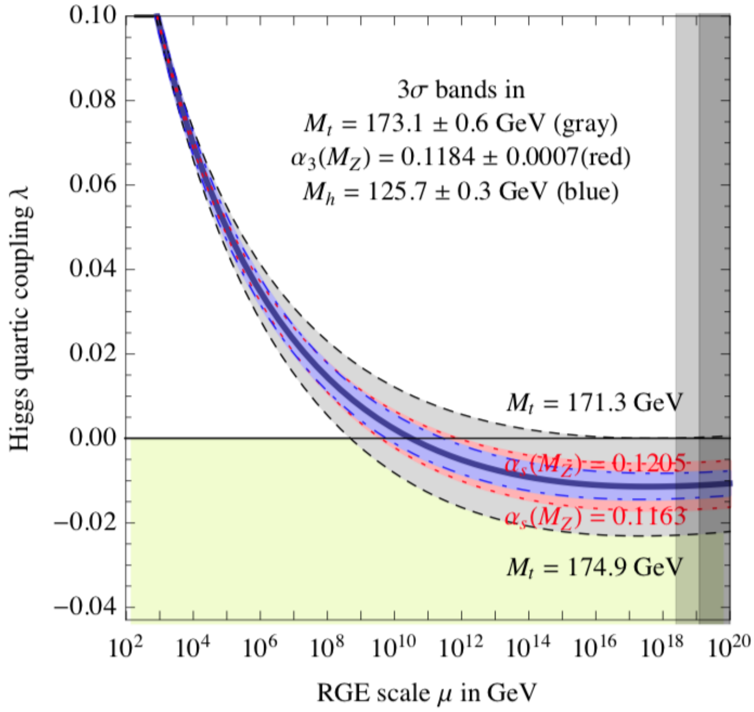


**Figure 1.4:** *Running of the Higgs self-interaction coupling from [34]. The instability scale is at about $10^{10}$ GeV for SM central values. It shows how much depends on the mass of the top quark, the Higgs mass and $\alpha_s$.*

Higgs without including interactions with other particles is [33]

$$V = V^{(0)} + V^{(1)} = -\frac{1}{2}m_h^2 h^2 + \frac{\lambda}{4}h^4 + \frac{1}{64\pi^2}M^4\left(\log\frac{|M^2|}{\mu^2} - \frac{3}{2}\right)\ ; \quad M^2 = 3\lambda h^2 - m_h^2\,. \tag{1.92}$$

The scale $\mu$ is chosen to make the logarithmic divergences that arise from quantum fluctuations in the effective potential small. In Eq. (1.92) it is clear that if $\mu = h$ that would be the case. For the full SM effective potential, there is a contribution like $V^{(1)}$ from each SM particle with $M^2$ proportional to the mass of the particle. Since all the SM particles acquire a mass proportional to the Higgs vev, the scale $\mu$ in flat space-time is commonly chosen $\mu = h$ [35].

In a curved space-time background, there is another scale in the problem, gravity. In this case, because there is a non-minimal coupling between the Higgs and gravity, the logarithmic divergences depend on an effective mass, which is related to the SM masses as before and the Ricci scalar, $m^2 \to m^2 - \xi R$. In this case, in order to keep the divergences as small as possible, it is better to choose the scale [33]

$$\mu^2 = h^2 + H^2 \,, \tag{1.93}$$

where $H$ is the Hubble parameter, since $R = 3(1 - 3w)H^2$. We have not included order one factors in front of $H$ since for our purposes they will not influence the calculation, and there is not a general consensus in the literature [33, 36–38].
It is important to notice that this choice does not rely on having a non-minimal coupling, since this is another interaction term in our theory, and it depends on the scale $\mu$ as well. So even though we can choose $\xi = 0$ at one scale, because its beta function $\beta_\xi \propto (\xi + 1/6)$, it will be different from zero at a different scale. Which means that this term is always present.

The instability of the EW vacuum is particularly important during inflation since the Higgs vev is obtained using the Langevin equation Eq. (1.79). Even if the Higgs field is initially at the origin, in one $e$-fold, it will move $H/2\pi$. As a very crude approximation, if the scale of inflation is larger than the scale of the Higgs instability, the Higgs may end up in the regime where $\lambda < 0$, rendering our vacuum unstable and with no clear answer for why we are not in the unstable region of the Higgs potential nowadays. It will be discussed more in detail in chapter 3.

These corrections to the potential have been studied in empty space. There are situations (as we will study in this thesis) where the field is interacting with many particles and technically is too complicated to calculate a many-particle scattering reaction. If we are interested in averaged quantities over a long period of time, we can represent a bath of particles by its temperature $T$. There is a loss of information about the microscopic dynamics but allows us to work in a thermal background. In this case, there is also a contribution to the effective potential from the thermal background [39]

$$V = V^{(0)}_{T=0} + V^{(1)}_{T=0} + V^{(1)}_{T\neq 0}\,; \qquad V^{(1)}_{T\neq 0} = \frac{1}{2} g^2 T^2 h^2\,. \tag{1.94}$$

The effective interaction between a thermal bath and the Higgs or any other scalar field is an effective thermal mass $m_{\text{eff}} = g^2 T^2$, where $g$ is an adimensional parameter that mimics the interaction between the background and the scalar field.
Since this is a positive mass term, it can help stabilise the EW vacuum [35].

## 1.5 Thesis outline

In this chapter, we have given an overview of the inflation theory, the problems that solve, and how the perturbations are generated. To explain it, we have developed the mechanism of gravitational particle creation for scalar fields. At the end of the section we have introduced the standard model Higgs field and its running, which combined with its quantum fluctuations during inflation, it leads us to the problem of the stability of the electroweak vacuum. The rest of the thesis is going to be structured as follows:

- In chapter 2, we consider the effect of the Gibbons-Hawking radiation on the inflaton in the situation where it is coupled to a large number of spectator fields. We argue that this will lead to two significant effects - a thermal contribution to the potential and a gradual change in parameters in the Lagrangian, which results from thermodynamic and energy conservation arguments. We present a scenario of hilltop inflation, where the field starts trapped at the origin, before slowly experiencing a phase transition during which the field, extremely slowly, moves towards its zero temperature expectation value. We show that it is possible to obtain enough e-folds of expansion as well as the correct spectrum of perturbations without hugely fine-tuned parameters in the potential (albeit with many spectator fields). We also comment on how initial conditions for inflation can arise naturally in this situation. This chapter is based on the work published in [40].

- In chapter 3, we study the (Brout-Englert-)Higgs quartic coupling which becomes negative at high energies rendering our current electroweak vacuum metastable, but with an instability timescale much longer than the age of the current universe. During cosmological inflation, unless there is a non-minimal coupling to gravity, the Higgs field is pushed away from the origin of its potential due to quantum fluctuations. It is therefore a mystery how we have remained in our current vacuum if we went through such a period of inflation. In this chapter, we study the effect of top quarks created gravitationally during inflation and their effect upon the Higgs potential using only general relativity with minimal couplings and Standard Model particle physics. We show how the evolution of the Higgs field during inflation is modified concluding that this effect is non-negligible for scales of inflation close to or larger than the stability scale but small for scales where the Higgs is stable. Also, we briefly discuss the effect of other fermions on the Higgs instability. This chapter is based on work published in [41].

# Chapter 2

# Horizon Feedback Inflation

## 2.1 Introduction

Cosmological inflation, as we have explained in Sec. 1.2, is the leading paradigm which explains the horizon, flatness and defect problem of the extremely successful FLRW hot Big Bang model as well as explaining the source of the initial density perturbations observed to exist in the CMB [42] (see Sec. 1.3). Furthermore, the exponential expansion seems to have an elegant explanation in field theory as being sourced by the potential energy of a field which rolls slowly down to its minimum, and its kinetic energy is being redshifted by this rapid expansion [17, 18, 43]. It is also challenging to think up alternatives to inflation which are natural [44], even with significant modifications of general relativity and those which do exist often create the wrong spectrum of perturbations [45].

Unfortunately, there are several problems with the standard inflationary scenario [46]. The most difficult problem is (arguably) the fact that for inflation to start in the first place, one needs to find a Hubble patch in the early universe across the entirety of which the potential energy dominates the kinetic energy of the field and last long enough [47]. Another way of putting this is that to solve the horizon problem, one creates another one at earlier times. Other problem is that in order for the kinetic energy to be redshifted by the expansion for long enough to explain the horizon and flatness problems, either the inflaton field has to be transplanckian during inflation or the shape of the potential below the Planck scale has to be extremely flat, in other words, the extent in the scalar field direction has to be much larger than its height in the energy direction.

Cosmological inflation usually requires a period of de-Sitter or at least quasi de-Sitter expansion in order to obtain the many $e$-folds required to solve the horizon problem. The Gibbons-Hawking temperature associated with the cosmological horizon in de Sitter space [48]

$$T_H = \frac{H}{2\pi}\,, \tag{2.1}$$

plays a central role in inflation as it acts as the source of quantum fluctuations in the inflaton field, which source density perturbations (Eq. (1.73)). This thermal radiation is analogous to the thermal population observed by an accelerating observer, i.e. Unruh radiation [49] and closely resembles the Hawking radiation which surrounds a black hole [50]. In the case of a black hole, the thermal Hawking radiation escapes to infinity and energy is conserved only by postulating that the mass of the black hole is correspondingly released - a hypothesis which cannot be proved in semi-classical quantum gravity without including back-reaction but which does fit coherently into the theory of black hole thermodynamics [51]. Black hole evaporation has an interpretation of resulting from a flux of particles with negative mass going into the black hole. A similar physical interpretation for de Sitter radiation is possible, where the Cosmological Constant is reduced from the addition of negative vacuum (zero-point) energy to the overall energy density of de Sitter [52].

In a first principle approach, this can be achieved in a prescription where the energy-momentum tensor sourcing gravity is coarse-grained to include only the observable degrees of freedom, i.e. those inside the cosmological horizon [39, 52–54].

Including a source of continuous particle production in de Sitter space is well-known to bring about a qualitative change in the system's behaviour: when given enough time exponential expansion will cease, since the subdominant component being produced start having a non-negligible contribution to the equations of motion, implying that de Sitter space with particle production is not stable. In addition to thermal radiation from the horizon a de Sitter instability has been speculated to result from a variety other underlying mechanisms ranging from quantum gravity and infrared divergences in propagators to environment induced decoherence and the second law of thermodynamics [52–74, 74–77]. Despite all the work about the decay of the cosmological constant, in the seminal paper of Gibbons and Hawking [48] where the radiation of a cosmological horizon was studied in the first place, it is mentioned that “the cosmological event horizon is stable” by using thermodynamic arguments, although in the Padmanabhan work [78] by using also thermodynamics arguments in de-Sitter, the opposite is claimed. One of the main key points in the discussion of the stability, from a quantum field theory point of view, has to do with the IR modes during inflation, which grows exponentially outside of the horizon for a light field. There is a discussion in the community about how to deal with them, since we can only access to the information inside our horizon. A recent paper by Moreau and Serreau [79, 80] claimed that the IR divergence of a scalar field in de-Sitter can be sorted out by imposing a infrared cutoff that ignores all the super horizon modes as the universe expands. Although, in the famous paper by Starobinsy [26], where the stochastic motion of a coarse grained field (expected value averaged over the horizon) is studied, it was showed that the main contribution comes from these IR modes and that it is only divergent in the case of a massless field (a more rigorous quantum field theory approach also shows the same [28]). A purely massless field should be protected by symmetries, otherwise a mass can be generated by renormalization [81] and prevent the divergence. What was showed in [52] is that even a massless conformally coupled field, can acquire a vev inside the horizon which can affect the dynamics in the long term [53, 54]. The equation of state for these fields is given by the relation between the energy density and the pressure (both obtained from the renormalized stress-energy tensor) inside the horizon, otherwise the decay of the cosmological constant could be avoided by having, for example, a field with a non-standard equation of state. There are also other authors who advocate for the stability of de-Sitter [65, 66, 82] by using a different vaccum state rather than the Bunch Davies for a massless field since it is no longer de Sitter invariant [70, 83]

Despite all the different works in this subject, we support the idea of the instability of de Sitter due to the effect of the Hawking radiation inside the horizon generating a thermal bath of particles in equilibrium with the horizon temperature.

A de Sitter instability unavoidably leads to a dynamical cosmological constant and multiple studies have looked at the possibility that particle production could gradually reduce the value of the cosmological constant from a phenomenological point of view [84–90], see [91,92] for observational investigations. It has been argued that since particle production from gravitational fields seems to be an irreversible process, this reduction is inevitable from a thermodynamical perspective [71,93–96]. Normally this effect is minimal and would not be a practical way of ensuring a small cosmological constant. In particular, the rate of decrease would not get rid of the cosmological constant fast enough in the late universe to explain today's cosmic acceleration [54].

It is interesting to consider the situation where the cosmological constant is not identified as a geometric term in the Einstein field equation but is rather the energy density of a field which is located at a stable point in its potential where $dV/d\varphi = 0$. In this situation, the same logic would imply that terms in the Lagrangian that set the scale of the potential would also decay over time due to the gravitational production of particles. For example for the potential

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - \varphi_0^2\right)^2 \tag{2.2}$$

and for a field resting at the metastable origin $\varphi = 0$, the particle production associated with the space-time curvature arising from the non-zero potential energy at the origin should have the effect of making $\varphi_0$ decrease with time, as we will see in Sec. 2.2 and in particular in Eq. (2.9).

Another implication of the Gibbons-Hawking temperature would be the possibility that it might create non-negligible finite temperature contributions to the potential energy of scalar fields (Eq. (1.94)) which are evolving in the quasi de Sitter background.

In thermal inflation, a thermal sector with the equation of state of radiation and coupled to the inflaton keeps the field trapped at the origin for a few $e$-folds until the thermal radiation has been redshifted away, decreasing its temperature [97]. However, if the origin of the thermal radiation is the Gibbons-Hawking temperature associated with the horizon, then its temperature will be constant and will not decrease over time if there is no corresponding change in the vacuum energy. It is clear however that in this situation only a radiation bath with a large number of degrees of freedom, all of which are coupled to the scalar field, would be able to keep the field trapped at the origin (Eq. (2.12)). Also once located and stabilised there due to this horizon temperature, the field would essentially be stuck at the origin for all time. A constant $T_H$ would also violate the continuity equation [98].

If however $\varphi_0$ changes over time due to the production of radiation, one can imagine a situation where $\varphi_0$ and consequently the energy density at the origin, the rate of expansion of the universe and the temperature of the thermal radiation all decrease with time. The stable point at the origin then eventually becomes tachyonic, and the field becomes free to roll away from $\varphi = 0$, setting the initial conditions for inflation (see also [99] for a similar idea in a different context). We argue that this could happen and that thermal effects would subsequently slow the phase transition sufficiently such that successful inflation can take place.

In section 2.2, we will go through the equations of this scenario in more detail and study the dynamics of the field and how it might produce enough $e$-folds of expansion. Then in section 2.3, we will study the inflationary predictions, namely the perturbations and the spectral tilt, as well as comparing our analytic estimates to a numerical treatment.

## 2.2 Phase transition with decaying vacuum energy

Let us examine the situation with the potential (2.2) in the presence of a thermal sector characterized by the Gibbons-Haking temperature (2.1) in more detail. We start by writing down the Friedmann equations

$$\begin{cases} 3H^2M_{\mathrm{P}}^2 & = T_{00} \equiv \rho \\ -(3H^2 + 2\dot{H})M_{\mathrm{P}}^2 & = T_{ii}/a^2 \equiv p \end{cases} . \tag{2.3}$$

Our model will consist of a scalar field $\varphi$ and importantly $N$ conformal fields that couple to $\varphi$ and are in thermal equilibrium with the horizon as described by (2.1). This will lead to an additional temperature component for the energy and pressure densities

$$\rho = \frac{\dot{\varphi}^2}{2} + V(\varphi, T_H) + N\frac{\pi^2}{30}T_H^4 \,; \quad p = \frac{\dot{\varphi}^2}{2} - V(\varphi, T_H) + \frac{N}{3}\frac{\pi^2}{30}T_H^4 \,, \tag{2.4}$$

and in addition to (2.2) the potential contains a contribution from the thermalised conformal fields

$$V(\varphi, T_H) = \frac{\lambda}{4}\left(\varphi^2 - \varphi_0^2\right)^2 + \frac{1}{2}Ng^2T_H^2\varphi^2 \,. \tag{2.5}$$

We will throughout work in the approximation where the energy density of the thermal component is subdominant to that of the potential and in particular to the vacuum energy piece

$$\frac{\lambda}{4}\varphi_0^4 \equiv \rho_\Lambda \approx 3H^2M_{\mathrm{P}}^2 \gg N\frac{\pi^2}{30}T_H^4 \,. \tag{2.6}$$

This condition will turn out to be easily satisfied for a large parameter range i.e.

$$1440\pi^2\left(\frac{M_{\mathrm{P}}}{H}\right)^2 \gg N \,. \tag{2.7}$$

From (2.3) we get the dynamical Friedman equation of motion

$$-2\dot{H}M_{\mathrm{P}}^2 = \dot{\varphi}^2 + \frac{4}{3}N\frac{\pi^2}{30}T_H^4 \,, \tag{2.8}$$

from which it is quite apparent that a thermal sector with the Gibbons-Hawking temperature is inconsistent with strict de Sitter space ($\dot{H} = 0$), but will lead to $\dot{H} < 0$. Furthermore, it is easy to see that the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0\,, \tag{2.9}$$

can only be satisfied if $\varphi_0$ is not strictly constant but still provides the source for the continuous particle creation required for maintaining $T_H$ in the conformal fields, despite the dilution from the expansion of space. This may be understood from the situation when $\varphi = 0$ giving $\rho \sim \rho_\Lambda$, but $\rho + p \sim T_H^4$. However, as we will see, we will not be relying on this feature of the current scenario in the current work other than to set initial conditions, the dynamics of $\varphi_0$ will be irrelevant by the time we come to calculate observables, i.e. $\varphi_0 \sim$ const. during 60 $e$-folds. Also, we have deemed more natural to keep the usual interpretation of a strict coupling constant for the other constants of our theory, $\lambda$ and $g$, and only allow $\varphi_0$ change with time. Since the continuity equation (2.9) provides only one constraint, in principle, one could consistently allow the other parameters to vary, at least from a purely phenomenological point of view.

From (2.8) we may conclude that the change in $H$ due to the thermal sector is very gradual: the first Hubble slow roll parameter for $\varphi = 0$ and $\dot{\varphi} = 0$ reads

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{NH^2}{720\pi^2 M_\mathrm{P}^2}\,, \tag{2.10}$$

and if initially $N \ll (720\pi^2 M_\mathrm{P}/H_{init})^2$ then $\epsilon \ll 1$ is clearly satisfied. For the parameters in the example situation that we will present later $\epsilon \simeq 10^{-7}$. The evolution of the Hubble expansion rate as a function of the number of $e$-folds is then given by

$$\frac{H(N_e)}{H_\mathrm{SB}} = 1/\sqrt{1 + N_e \frac{NH_\mathrm{SB}^2}{360\pi^2 M_\mathrm{P}^2}}; \qquad H_\mathrm{SB} = \frac{4\pi^2\sqrt{12\lambda}}{Ng^2} M_\mathrm{P}\,, \tag{2.11}$$

where $H_\mathrm{SB}$ is the value of the Hubble expansion at symmetry breaking. (The number

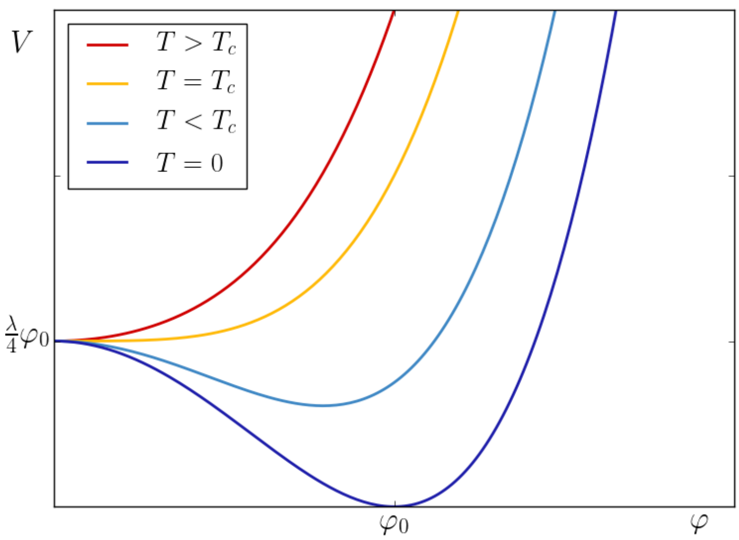


**Figure 2.1:** *Initially for a high temperature the field is located at the origin in the symmetric phase. As the temperature drops a new minimum is generated and the symmetry is broken.*

of $e$-folds $N_e$ are obtained simply by integrating $dN_e = Hdt$.)

However, the Friedman equations also show for $\varphi = 0$ and Eq. (2.7) that $\varphi_0 \propto \sqrt{HM_{\rm P}}$ will decrease slower than $T_H \propto H$. Therefore, the effective mass squared for $\varphi$

$$m^2 \equiv Ng^2T_H^2 - \lambda\varphi_0^2\,, \tag{2.12}$$

will eventually cross over to negative values, even if initially $Ng^2T_H^2 \gg \lambda\varphi_0^2$. Simply put, at some point the system will undergo a phase transition at $m = 0$ ($H_{\rm SB}$). This phase transition in contrast to [97] is extremely gradual. As we will show, it can take several thousands of $e$-folds to complete. This is due to the special nature of the thermal radiation as given by $T_H$: the thermal bath is continuously replenished by the decay of $\varphi_0$ and hence does not dilute in the usual fashion.

After the phase transition, $\varphi$ will start rolling to the new minimum and plays the role of the inflaton in the usual sense. The evolution of $\varphi$ can be characterized with (2.12) to consist of three regions, $m^2 \gtrsim H^2$, $|m^2| \lesssim H^2$ and $m^2 \lesssim -H^2$.

### 2.2.1 $m^2 \gtrsim H^2$: gradual decay of $\rho_\Lambda$

If the inflaton is close to the origin of the potential initially, then eventually it will end up rolling down to the minimum after symmetry breaking. The initial conditions for the rolling down of the potential are set dynamically, as we will see later, but they rely on us starting in a symmetric phase. For that we need the temperature term in the potential to be large enough, but, at the same time, this thermal contribution is only different from zero if the universe is inflating. This is always possible if the inflaton starts at the origin, but we are going to study first how far away from the origin the inflaton can be and always end up relaxing to the origin. Once the field starts rolling down, the horizon is formed and the temperature correction to the potential lifts the field. We will study if from that position the field can roll down to the origin and still remain in a de-Sitter phase.

We study this by checking that $\epsilon < 1$. For the initial situation where the horizon has not been formed yet, the field rolls down from the origin to the minimum with a mexican hat potential, then

$$\epsilon_{T=0} = \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'(\varphi)}{V(\varphi)}\right)^2 = \frac{M_{\mathrm{P}}^2}{2}\left(\frac{4\varphi}{\varphi^2-\varphi_0^2}\right)^2 . \tag{2.13}$$

For values smaller than $\varphi_0$, the maximum value for the field is $\varphi_{\max}^{T=0}/M_{\mathrm{P}} = \sqrt{(\varphi_0/M_{\mathrm{P}})^2 - 2\sqrt{2}\sqrt{(\varphi_0/M_{\mathrm{P}})^2+2}+4}$, if it is higher then $\epsilon_{T=0} > 1$. The inflaton will form an horizon after one $e$-fold of expansion while rolling down and after that, the field is lifted because of the temperature term.

In a de-Sitter phase with the inflaton dominating the energy density of the universe (over the spectator fields), we can simplify the first Friedman equation to obtain an effective potential for the inflaton that only depends on its value

$$3H^2M_{\mathrm{P}}^2 = \frac{\lambda}{4}\left(\varphi^2-\varphi_0^2\right)^2 + \frac{1}{2}Ng^2\left(\frac{H}{2\pi}\right)^2\varphi^2 . \tag{2.14}$$

Solving for $H^2$ and substituting into the temperature correction term, we get

$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2-\varphi_0^2\right)^2 + \frac{1}{2}Ng^2T_H^2\varphi^2 \tag{2.15}$$

$$= \frac{\lambda}{4}\left(\varphi^2-\varphi_0^2\right)^2 \frac{1}{1-\frac{Ng^2\varphi^2}{24\pi^2M_{\rm P}^2}}\,. \tag{2.16}$$

There is a critical value for the inflaton $\varphi_{\rm crit} = \sqrt{\frac{24\pi^2M_{\rm P}^2}{Ng^2}}$, at this value, the Hubble parameter diverges, this means that in this configuration, the field can not acquire larger values without having a quantum gravity backreaction, thus we will focus only on values below $\varphi_{\rm crit}$ during this phase. Also, we are interested in studying the evolution of this system starting in a symmetric phase, i.e. $m^2 \gtrsim H^2$. Studying the potential in eq. (2.15) and requiring it to be convex in its whole domain, we obtain the condition $\frac{Ng^2\varphi_0^2}{48\pi^2M_{\rm P}^2} \gg 1$. This ensures that wherever the field starts, it will roll down to the origin as long as the kinetic energy is not large enough to stop the de-Sitter expansion. Another key consequence is that $\varphi_{\rm crit} < \varphi_0$, therefore the field can not roll down from higher values than $\varphi_0$ as it is otherwise commonly studied in the literature for a polynomial inflaton potential [22].

In this situation the field will now roll down to the origin rather than to $\varphi_0$. From the definition $\epsilon = -\dot{H}/H^2$ and using the second Friedman equation, we know that there are two contributions: one from the rolling of the field and another from the Hawking temperature. Since we are studying when $\epsilon = 1$ and we know from eq. (2.10) that the contribution from the temperature is small, we only need to focus on the contribution from the kinetic energy of the inflaton to $\dot{H}$. Using the potential (2.15)

$$\epsilon = \frac{Ng^2}{48\pi^2}\left(\frac{4\sqrt{\frac{Ng^2\varphi^2}{24\pi^2M_{\rm P}^2}}}{\frac{Ng^2\varphi^2}{24\pi^2M_{\rm P}^2} - \frac{Ng^2\varphi_0^2}{24\pi^2M_{\rm P}^2}} + \frac{2\sqrt{\frac{Ng^2\varphi^2}{24\pi^2M_{\rm P}^2}}}{1-\frac{Ng^2\varphi^2}{24\pi^2M_{\rm P}^2}}\right)^2 . \tag{2.17}$$

As we are interested in the situations where the field ends in the origin, the most constrained situation comes from the largest possible values of $\varphi_0$, because the potential becomes stiffer and the kinetic energy can be comparable to the potential one. From (2.17), we take the limit for $\varphi_0 \to \infty$ and making $\epsilon = 1$ we can obtain what is the maximum value of $\varphi$ that will lead to inflation. To simplify more the solution, we also consider that the number of fields is large, $Ng^2 \gg 24\pi^2$,

$$\varphi_{\max} = \frac{1}{\sqrt{2}} \frac{24\pi^2}{Ng^2} M_{\rm P} \,, \tag{2.18}$$

and for the situations where we are close to the symmetry breaking this maximum value can be extended up to

$$\varphi_{\max,\rm SB} = 2^{1/6} \left( \frac{24\pi^2}{Ng^2} \right)^{2/3} M_{\rm P} \,, \tag{2.19}$$

which is obtained by taking $\varphi_0 \to (\varphi_0)_{\rm SB}$. We compare this value with $H_{\rm SB}$ to confirm that quantum fluctuations of that order will not be able to spoil the de-Sitter evolution

$$\varphi_{\max,\rm SB}/H_{\rm SB} = \frac{2^{1/6}}{\sqrt{\lambda/3}} \left( \frac{Ng^2}{24\pi^2} \right)^{1/3} \,. \tag{2.20}$$

To conclude the initial conditions study, the inflaton will end up at the origin in the symmetric phase if it starts with a value smaller than the minimum of the set of these three values: $\left(\varphi_{\max}, \varphi_{\max,\rm SB}, \varphi_{\max}^{T=0}\right)$, which for the values chosen later in Sec. 2.3 it gives us $(1.08, 14.9, 1.02) \cdot 10^{-3} M_{\rm P}$ respectively, and we confirm that $\varphi_{\max,\rm SB}/H_{\rm SB} = 500 \gg 1$.

So far we have only studied the temporal part of the field. The spatial part will allow inflation to occur as long as there is a Hubble patch which supports inflation. This patch will grow, since the gradient energy within this Hubble patch will be diluted. We can quantify the spatial inhomogeneities and impose a maximum bound for them, although for an accurate description we will need to evolve the initial configuration in the lattice, as done in [100,101]. As a first order of approximation, we can estimate them to be [100]

$$\rho_{grad} < V \quad \to \quad a \times L_{ph} > \frac{2\pi}{\sqrt{6}} \frac{\Delta\varphi}{H M_{\rm P}} \,, \tag{2.21}$$

where $L_{ph}$ is the physical size of the patch and $\Delta\varphi$ the spatial variations within. For subplanckian perturbations we only need an initial spatial homogeneous Hubble patch with the proper homogeneous value across the Hubble radius to start inflation.

When the effective mass of the field (2.12) is very large and positive the minimum of the potential is at $\varphi = 0$. In this situation the Friedmann equations are Eqns. (2.6) and (2.8) with $\dot{\varphi} = 0$, which leads to

$$H = \frac{H_{init}}{\left(\frac{NH_{init}^3}{240\pi^2 M_{\rm P}^2}t + 1\right)^{1/3}} \quad \overset{t\to\infty}{\longrightarrow} \quad \left(\frac{240\pi^2}{N}\right)^{1/3} (M_{\rm P}^2/t)^{1/3}\,, \tag{2.22}$$

which is consistent with the continuity equation

$$\dot{\rho}_\Lambda = -3H(\rho + p) = -N\frac{4\pi^3}{15}T_H^5\,, \tag{2.23}$$

Interestingly, the late time limit of (2.22) is independent of the initial Hubble rate indicating that the case $m^2 \gtrsim H^2$ exhibits late time attractor evolution, which is independent of initial conditions; after a sufficiently long time with $m^2 \gtrsim H^2$ the system will always relax to a configuration with $H \sim (M_{\rm P}^2/t)^{1/3}$ and $\varphi = \dot{\varphi} = 0$. For the remaining analysis we will choose the attractor configuration as our initial condition.

What is apparent from this section is that when $m^2 \gtrsim H^2$ the system has fairly unremarkable behaviour: the field sits put in its vacuum state and the vacuum energy $\rho_\Lambda$ gradually decays. The quantum fluctuations around the mean $(\varphi)$ will be denoted as $\phi$. Similarly as we did for light fields in Sec. 1.3.2, the equation of motion for the perturbations $\phi_{\mathbf{k}} = v_k(\eta)/a$ give us

$$v_k''(\eta) + \left(k^2 + m^2a^2 - \frac{a''}{a}\right)v_k(\eta) = 0\,, \tag{2.24}$$

which in de-Sitter has the solution

$$v_k(\eta) = \frac{-1}{2}\sqrt{\frac{\pi}{a(\eta)H}}\mathrm{H}_\nu^{(1)}\left(\frac{k}{a(\eta)H}\right); \qquad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}\,, \tag{2.25}$$

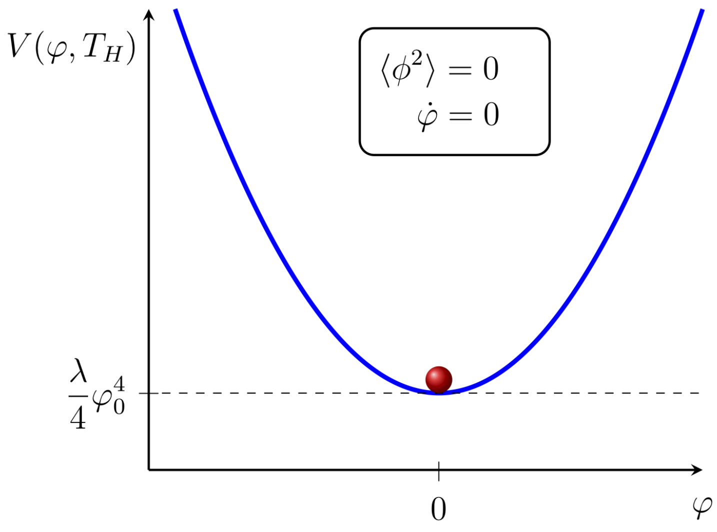


**Figure 2.2:** *In the unbroken phase the field is at rest at $\varphi = 0$ with practically no quantum fluctuations, $\langle \phi^2 \rangle \sim 0$. The suppression of the quantum fluctuations is caused by the large effective mass making the field heavy with respect to the background curvature.*

for $\nu = 3/2$ reduces to Eq. (1.71), but for heavy masses as is the case here, the fluctuations are exponentially suppressed [1].

$$\langle \phi^2 \rangle \sim 0 \,. \tag{2.26}$$

The case $m^2 \gtrsim H^2$ is illustrated in Fig. (2.2).

As discussed after (2.12), eventually, the effective mass will vanish and the system will undergo a phase transition leading to interesting dynamics for $\varphi$.

### 2.2.2 $|m^2| \lesssim H^2$: large quantum fluctuations

The phase transition happens when the effective mass of the field (2.12) vanishes at the origin ($m = 0$). The value of the Hubble parameter at and soon after the symmetry breaking transition is approximately given by the first Friedmann

[1] The asymptotic form of the Hankel function $\mathrm{H}_\nu^{(1)}(z) \sim \sqrt{\frac{2}{\pi z}} \exp i\,(z - \nu\pi/2 - \pi/4)$ show how for heavy fields $\nu = im/H$ there is an exponential suppression.

equation from (2.6) as

$$H_{\rm SB} \simeq \sqrt{\frac{\lambda}{12}}\frac{(\varphi_0)^2_{\rm SB}}{M_{\rm P}}\,, \tag{2.27}$$

where $(\varphi_0)^2_{\rm SB}$ corresponds to the time of symmetry breaking. In this phase, Eq. (2.11) is still a good approximation because we can neglect both the thermal and kinetic contributions to the first Friedman equation and the kinetic term in the dynamical Friedman equation (2.8) so long as $\dot\varphi^2 \ll NT_H^4 \ll V$. We can see that $NT_H^4 \ll V$ is fulfilled at the time of symmetry breaking if the number of fields $N \gg \lambda/g^4$, which is obtained via (2.6) and (2.12). The second condition $\dot\varphi^2 \ll NT_H^4$ holds classically since at symmetry breaking $\dot\varphi = 0$; but as it is shown later due to the quantum fluctuations $\dot\varphi \neq 0$. As will be discussed more in section 2.2.3 (see Eq. (2.35)) when the quantum fluctuations dominate over the classical motion one may write $|\dot\varphi| \sim H^2/(2\pi)$ which immediately implies $\dot\varphi^2 \ll NT_H^4$ for large $N$ ($N \gg 4\pi^2$). Also, the value of the field $\varphi$ during this epoch is not large enough to affect the dynamics of $H$, since from Eq. (2.27) and (2.11), $(\varphi_0)_{\rm SB} = \sqrt{48\pi^2/Ng^2}M_{\rm P}$ which is orders of magnitude larger than $\varphi \sim H$.

However, importantly, $T_H$ and hence $m$ change very gradually and therefore, immediately after symmetry breaking, we expect a long epoch of huge quantum fluctuations before the rolling becomes dominant as illustrated in Fig. (2.3). The behaviour and magnitude of these fluctuations may be analytically solved via the stochastic formalism [26,102] (Sec. 1.3.3). This epoch of large quantum fluctuations is quite important in our model since, as we will further discuss in section 2.2.3, when the $T_H$ has dropped enough, the classical rolling will take over, and the initial condition will be dynamically set by the period of quantum jumping. We note that during this epoch where $|m^2| \lesssim H^2$ we would expect the large density perturbations to lead to the formation of domain walls, but since we expect to obtain many more than 60 $e$-folds of inflation after this period, we expect these evils to be swept outside the horizon (in the example of Fig. (2.5), symmetry breaking happens hundreds of $e$-folds before we obtained the correct spectrum of perturbations from inflation).

In order to obtain a qualitative picture of the system's behaviour as the rep-

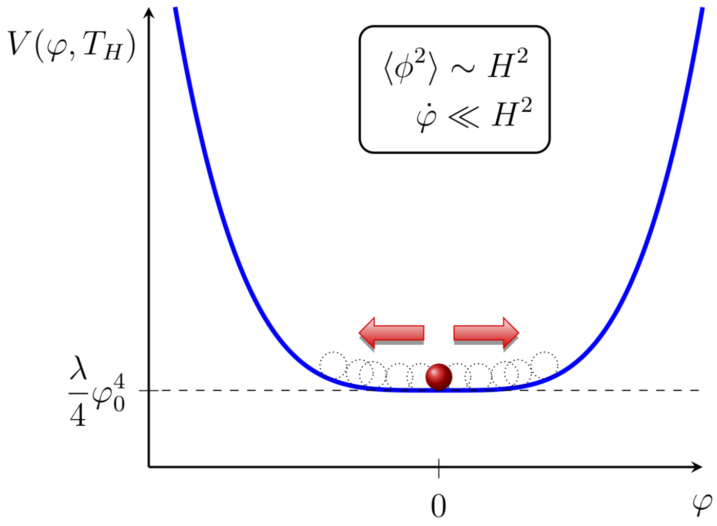


**Figure 2.3:** *Close to symmetry breaking the classical (mean) field $\varphi$ is almost stationary, but the quantum fluctuations $\langle\phi^2\rangle$ very large since the field is effectively light $|m^2| \lesssim H^2$.*

resentative value for the classical dynamics one may take the square root of the variance $\varphi^2 \equiv \langle\phi^2\rangle$. Here we settle for solving this expectation value by using the Langevin equation in the Hartree approximation [2] relegating a more complete discussion to section 2.3.1. From the Langevin equation Eq. (1.82) multiplying it by $\varphi$ and taking the stochastic average gives us the relevant equation [26, 27] for the second moment of the inflaton

$$\frac{d}{dN_e}\langle\phi^2\rangle = \frac{H^2}{4\pi^2} - \frac{2m^2}{3H^2}\langle\phi^2\rangle - \frac{2\lambda}{H^2}\langle\phi^2\rangle^2. \tag{2.28}$$

During this epoch there is little dynamics in $\langle\phi^2\rangle$. Initially, the field sits at the origin, the evolution of the variance is

$$\langle\phi^2\rangle = \frac{H^2}{4\pi^2}N_e\,, \tag{2.29}$$

[2]The Hartree approximation assumes that the PDF for the fluctuations is gaussian leading us to $\langle\phi^4\rangle = 3\langle\phi^2\rangle^2$ [26]. It is true for a massive field although for a massless self-interacting field using the stochastic formalism we find $\langle\phi^4\rangle/\langle\phi^2\rangle^2 = 2.2$.

until the gradient of the potential compares to the quantum fluctuations

$$\langle \phi^2 \rangle = \frac{H^2}{\pi\sqrt{8\lambda}}\,. \tag{2.30}$$

This occurs close to the symmetry breaking $m \sim 0$, but as the temperature drops the mass increases and the field will roll down to the minimum

$$\langle \phi^2 \rangle = \frac{-m^2}{3\lambda}\,. \tag{2.31}$$

The solution (2.30) is approached only at the saturated limit ($\frac{d}{dN_e}\langle\phi^2\rangle = 0$) when the system has been given enough time to equilibrate. The exact time this takes depends on the parameters of the potential. For a quartic theory the equilibration time scale in terms of $e$-folds is given by $1/\sqrt{\lambda}$ [27, 103], obtained by comparing Eqns. (2.29) and (2.30). If the time scale for the change in the Hubble rate in terms of $e$-folds or $\epsilon^{-1}$ (Eq. (2.10)) is much longer than the equilibration time our approximation of using the results at the saturated limit is valid. With (2.11) and (2.10) at $m \approx 0$ we can write the condition $\epsilon^{-1} \gg \lambda^{-1/2}$ as

$$\frac{15 g^4 N}{4\pi^2\sqrt{\lambda}} \gg 1\,, \tag{2.32}$$

which again is easy to satisfy for large $N$. Also, in our model the scale of symmetry breaking (2.27) can be tuned by choosing $\lambda$, $N$ and $g$ in our potential (2.5) with smaller scales corresponding to a slower dynamics as is evident from (2.10). Hence the condition in (2.32) can also be understood to imply the freedom to choose $H_{\rm SB}$ low enough such that the saturated expressions in (2.30) are a good approximation.

The inflaton is in a thermal bath of particles and their fluctuations will also affect the dynamics of the field [104]. To consider this effect we could add an extra stochastic term to consider the external forces than these fields will have on the inflaton. Another possibility is to decouple the equations of motion of the inflaton and remove the temperature dependence on the field, thus studying an inflaton field self-interacting (2.15). We use the later approach to prove than this effect is going to be negligible in our study.

In the Langevin equation, the gradient of the potential will affect the evolution of the field. The extra contribution from the thermal fields to the inflaton equation of motion when written explicitly as a function of the inflaton field is not trivial and results in

$$\frac{dV(\varphi)}{d\varphi} = \frac{d}{d\varphi}\left(\frac{\lambda}{4}\frac{\left(\varphi^2-\varphi_0^2\right)^2}{1-\frac{Ng^2\varphi^2}{24\pi^2 M_{\rm P}^2}}\right) = \left(\lambda\varphi^3+\left(Ng^2T_H^2-\lambda\varphi_0^2\right)\varphi\right)/\left(1-\frac{Ng^2\varphi^2}{24\pi^2 M_{\rm P}^2}\right)\,, \tag{2.33}$$

We can see from the denominator how if the field is not close to the critical value ($\varphi_{\rm crit}$), the extra effect from the stochastic nature of the thermal fields to the Langevin equation is negligible. Since we study the evolution of the field close to the symmetry breaking, this effect will not have an impact in our results. This extra term will mimick the contribution from the stochastic behaviour of the thermal fields, by adding an extra term to the Langevin equation of motion.

For completeness, for the parameters chosen later in Sec. 2.3, the value of the inflaton 60 $e$-folds before the end of inflation is

$$\frac{\varphi_{60}}{\varphi_{\rm crit}} = 0.004\,, \tag{2.34}$$

which reassure our assumption that this effect does not affect our calculation.

In the absence of the horizon entropy thermalising a large number of fields coupled to the scalar field, this kind of symmetry breaking would not lead to good inflationary initial conditions since the Kibble mechanism [105, 106] would lead to large fluctuations from horizon to horizon with large field gradients that would prevent inflation from starting in the first place. In this scenario, the thermal corrections to the potential prevent the kinetic energy dominated regions from running straight down to $|\varphi| = \varphi_0$. Eventually, therefore, as the thermal dissipation continues to gently facilitate the slow phase transition and the field's classical position and motion starts to dominate over its quantum fluctuations, a region should emerge somewhere

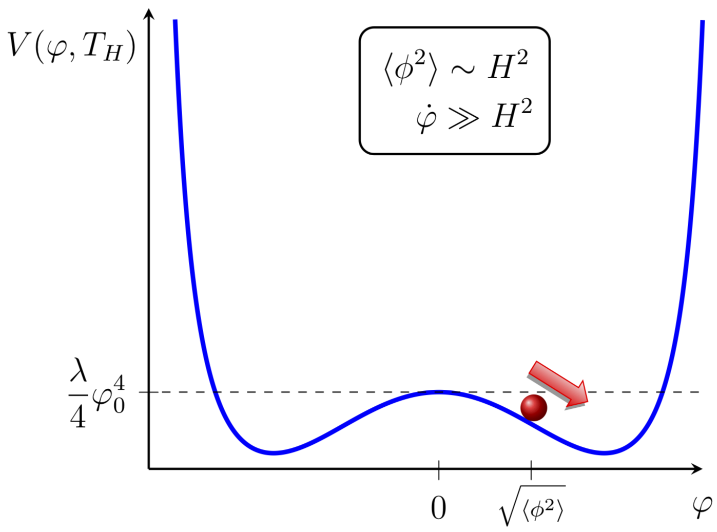


**Figure 2.4:** *The onset of classical rolling occurs when the steepness of the potential has increased enough. The initial value for the field is determined by the preceding random-walking epoch when* $|m^2| \lesssim H^2$.

where the field is coherent enough across several horizons such that the well-known difficulties obtaining initial inflationary conditions are overcome.

### 2.2.3 $m^2 \lesssim -H^2$: classical rolling

Strictly speaking one can only talk about a classical "rolling" once the minimum is further away from the origin than the stochastic vacuum expectation value ($\varphi^2 \equiv \langle\phi^2\rangle$) obtained from quantum fluctuations. As a first approximation one can say that the field starts rolling when in one $e$-fold the size of a single quantum jump ($H/2\pi$) is smaller that the distance traveled due to the classical rolling

$$|\dot{\varphi}|H^{-1} \gtrsim \frac{H}{2\pi}\,, \tag{2.35}$$

which is the opposite to the usual condition for eternal inflation [107] and equivalent to the condition $|V'| > H^3$, by using the equation of motion of the field during slow roll. More accurately we can use (2.28) for the dynamics of $\langle\phi^2\rangle$.

As shown in the previous section, at the time of symmetry breaking ($m = 0$), the variance of the field is approximately $2\frac{\lambda}{H^2}\langle\phi^2\rangle^2 = \frac{H^2}{4\pi^2}$. At times soon after symmetry breaking the mass term remains negligible $m^2\langle\phi^2\rangle \ll 3\lambda\langle\phi^2\rangle^2$ and the field will remain constant at the value acquired at symmetry breaking until the mass is relevant, i.e. $-m^2 - 3\lambda\langle\phi^2\rangle = 0$. At this point, the potential will be steep enough for classical slow roll to start and this classical motion will dominate over the quantum fluctuations (see Fig. (2.4)). Note that this condition agrees with the estimate (2.35) since $\frac{d}{dN_e}\langle\phi^2\rangle = \frac{H^2}{4\pi^2}$. Once the classical rolling starts the mass is given by

$$-m^2 \simeq 3\lambda\langle\phi^2\rangle = 3\lambda\frac{H^2}{\pi\sqrt{8\lambda}}\,, \tag{2.36}$$

and the value of the field and the speed are

$$\frac{d}{dN_e}\langle\phi^2\rangle = \left(\frac{H}{2\pi}\right)^2, \qquad \langle\phi^2\rangle = \frac{H^2}{\pi\sqrt{8\lambda}} = \frac{-m^2}{3\lambda}\,. \tag{2.37}$$

Once classical rolling has been triggered, our model gives rise to the usual slowly rolling inflation. We emphasise that the initial conditions for inflation are set dynamically by the large quantum fluctuations prior to classical rolling and hence are not free parameters. Similarly, the start of slow roll is triggered dynamically once the potential has acquired sufficient steepness and occurs for a wide range of values for $\varphi_0$, in particular also for $\varphi_0/H \gg 1$. Finally, we remind the reader that the neat attractor behaviour of the solutions prior to symmetry breaking (2.22) also exhibit independence from initial conditions. For these reasons, we can conclude that the model presented here successfully evades all the usual fine-tuning issues of inflationary models.

## 2.3 Inflationary predictions

To have a successful inflationary scenario, we need enough $e$-folds (at least 50-60) and we need to obtain the right perturbations $\Delta_s^2$ and spectral tilt $n_s$ 50-60 $e$-folds before the end of inflation. As shown in the previous section, more than 60 $e$-folds of expansion can be obtained very easily in this scenario - the scale of inflation $H$ can be set to be much smaller than the Planck mass, so $\epsilon \ll 1$ continues for many $e$-folds before $H$ differs significantly from the value at symmetry breaking $H_{\rm SB}$, meaning that we get enough inflation. Here we show how the perturbations are generated once the field is classically rolling to the minimum. The perturbations in the spatially flat gauge ($\Psi = 0$) are defined as [22] (Eq. (1.64))

$$\zeta = -\frac{H}{\dot{\bar{\rho}}}\delta\rho = -\frac{H\dot{\varphi}\,\delta\varphi}{\rho + p}\,, \tag{2.38}$$

where we use $\delta\rho = V'\delta\phi$ and also $3H\dot{\phi} = -V'$ but as opposed to what we did after Eq. (1.64), we can not say $\rho + p = \dot{\varphi}^2$. We note the pertubations in our model will lead to the usual expression encountered in models of warm inflation [108]. More explicitly, in our theory, the Hawking temperature is given by the vacuum solution of a conformal field inside the horizon, but, as well, for any conformal scalar field, the effective mass term of the Ricci scalar makes the field fluctuations around its mean value exponentially small, therefore we do not need to consider the temperature fluctuations in the calculation of the scalar perturbations.
The power spectrum takes the form

$$\Delta_s^2 = \langle\zeta\zeta\rangle = \left(\frac{H\dot{\varphi}}{\rho + p}\right)^2\left(\frac{H}{2\pi}\right)^2 = \left(\frac{H\dot{\varphi}}{-2\dot{H}M_{\rm P}^2}\left(\frac{H}{2\pi}\right)\right)^2\,, \tag{2.39}$$

where from now on we set $\varphi^2 \equiv \langle\phi^2\rangle$. In contrast to single field slow roll inflation $-2\dot{H}M_{\rm P}^2 = \frac{4}{3}N\frac{\pi^2}{30}T_H^4$, in the current setting the perturbations and the tilt are given by

$$\Delta_s^2 = \left(\frac{H\dot{\varphi}}{\frac{4}{3}N\frac{\pi^2}{30}T_H^4}\left(\frac{H}{2\pi}\right)\right)^2 = \left(180\pi\frac{\dot{\varphi}}{NH^2}\right)^2 = \left(60\pi\frac{m^2\varphi + \lambda\varphi^3}{NH^3}\right)^2 \tag{2.40}$$

$$n_s - 1 = \frac{d\ln\Delta_s^2}{d\ln k} = 6\epsilon - 2\eta\,, \tag{2.41}$$

where we used the slow roll condition $\dot{\varphi} = -V'/3H$, and dropped the subdominant kinetic piece $\dot{\varphi}^2$ in (2.8). Furthermore, we note that $\epsilon$ is negligible in the calculation of the tilt. The difference with the usual single field slow-roll inflation (Eq. (1.77)) comes from the contribution of the thermal sector to the dynamical Friedman equa-

tion (2.8), which both changes the vacuum energy decay and modifies the usual calculation for the spectrum because of $\dot{H} \propto H^4$. Because of this, the perturbations follow the inverse of the usual $\Delta_s^2 \propto H^4/\dot{\varphi}^2$ behaviour. This means that the usual expression for the tilt $n_s - 1 \propto \eta$ (1.77) is not true and we get $n_s - 1 \propto -\eta$ (since the scalar perturbations are the inverse of what we usually found up to an order of magnitude which is irrelevant for the logarithmic term in $n_s - 1$), where $\eta = M_{\rm P}^2 \frac{V''}{V}$ is the usual slow-roll parameter. So in order to obtain a red spectrum, the inflaton needs to be evolving in the regime where the potential is convex ($V'' > 0$), i.e. beyond the inflection point at $V'' = 0$, contrary to the usual small field case.
We also study if the perturbations are conserved in super horizon scales, since in many models of multifield inflation this can be an issue. The variation of the scalar pertubation is given by [22][3]

$$\dot{\zeta} = -H \frac{\delta p_{en}}{\rho + p} - \mathcal{O}\left(\frac{k^2}{a^2 H^2}\right) , \qquad \delta p_{en} = \delta p - \frac{\dot{p}}{\dot{\rho}} \delta \rho \,. \tag{2.42}$$

The contribution to the perturbations of the energy density and pressure are coming from the inflaton (since the temperature bath is formed of conformal fields), making $\delta p = -\delta\rho = -V'\delta\phi$. The scalar perturbations may not be conserved on super horizon scales if there is a significant non adiabatic contribution to the energy density, because the thermal addition to the Friedmann equations has a different equation of state to the inflaton. Therefore, we study more in detail the ratio

$$\frac{\dot{p}}{\dot{\rho}} = \frac{-\dot{\rho} - 2\ddot{H} M_{\rm P}^2}{\dot{\rho}} = -1 - \frac{2\ddot{H} M_{\rm P}^2}{6 H \dot{H} M_{\rm P}^2} = -1 - \frac{1}{3}\frac{d \ln \dot{H}}{d N_e} \,, \tag{2.43}$$

where we have used the second Friedmann equation (2.3) to substitute the temporal evolution of the pressure and the first for the energy density. If the second term in eq. (2.43) is much smaller than one, then $\delta p_{en} \approx 0$, making the perturbations conserved on super horizon scales. The kinetic inflaton term is proportional to $H^2$ close to symmetry breaking and in consequence is much smaller than the temperature term which is proportional to the number of spectator fields, $NH^4$ (this is also

[3]The exact equation can be found in [22], equation (A71)

corroborated in the simulations shown below). Then we can see that $\frac{\dot{p}}{\dot{\rho}} \approx -1$ since $\frac{d \ln \dot{H}}{dN_e} \propto 4\frac{d \ln H}{dN_e} = -4\epsilon \ll 1$, which is much smaller than one during inflation, and therefore the scalar perturbations are conserved on super horizon scales.

There are a variety of different combinations of parameters that can give rise to the correct inflationary perturbations, however in what follows we will present a situation where the 50-60 $e$-folds of inflation we are interested in starts very soon after the symmetry breaking occurs. In this situation, the value of $\eta$ is naturally small enough to give us the right tilt because soon after symmetry breaking $V'' = 0$ by definition. There is, therefore, no need to fine-tune our tree-level parameters to give us a flat potential near the origin effectively bypassing the issue that the natural tree-level values are argued to give rise to $\eta \simeq 1$ [109].

Our system has 4 free parameters ($N, \lambda, \varphi_0$ and $g$). However for the sake of clarity we choose here the final $e$-folds of inflation to occur soon after symmetry breaking and hence we can approximate the value of $\varphi_0$ during inflation by

$$\frac{\varphi_0}{M_{\mathrm{P}}} \approx \frac{(\varphi_0)_{\mathrm{SB}}}{M_{\mathrm{P}}} = \frac{4\pi\sqrt{3}}{\sqrt{N g}}\,, \tag{2.44}$$

which can be derived by making use of $m_{\mathrm{SB}} = 0$ in (2.12) and (2.27). With the above we effectively reduce the degrees of freedom from four to three. Also, if the observable part of the inflationary spectrum is going to take place soon after symmetry breaking then as soon as the classical rolling starts we want the field to be in the red tilt regime. For this to occur we need to ensure that $\frac{d}{dt}\langle\phi^2\rangle > \frac{d}{dt}\frac{-m^2}{\lambda}$ at the inflection point, otherwise the minimum $(-m^2/\lambda)$ would move faster away from the origin than the field. If this were to occur, the spectral tilt would be blue (after symmetry breaking). By making use of very similar steps that led to (2.44) and the Hartree approximation for $\langle\phi^2\rangle$ from (2.28) one may show this condition to lead to the order of magnitude constraint $\frac{1}{g^2} < \frac{15}{4\pi^2}$.

So from now on, we will set for simplicity $g = 1$. However, we also emphasise that a red tilt can be obtained a long time after symmetry breaking, so $g = 1$ is only a choice for the forthcoming calculation for the estimates of the perturbations and the tilt, and none of the above derivations relies on a specific value of g.

After this choice, we effectively have 2 degrees of freedom left $(N, \lambda)$ that will determine the perturbations and the spectral tilt soon after the classical roll to the minimum starts. It proves convenient to define a new parameter, $\alpha = \varphi/\sqrt{-m^2/3\lambda}$, where $\alpha = 1$ means that the field is at the inflection point and $\alpha = \sqrt{3}$ means that the field is at the minimum of the potential. We can then approximate the perturbations and the tilt from (2.41) and with the help of equations from section 2.2.3 by

$$\Delta_s^2 = \frac{2025}{\sqrt{2}\pi}\left(-\alpha + \frac{\alpha^3}{3}\right)^2 \frac{\sqrt{\lambda}}{N^2}\,; \qquad n_s - 1 = \frac{-1+\alpha^2}{\sqrt{2}\pi}\sqrt{\lambda}\,. \tag{2.45}$$

It is interesting to note that the order of magnitude of the spectral tilt does not depend on $N$ while the magnitude of the perturbations depends on both ($\lambda$ and $N$). A smaller value of $\lambda$ will make our spectrum become more scale invariant and will allow us to reduce the number of conformal fields that need to be present to give us the right spectrum. Hence we can shift the fine tuning between a small value of $\lambda$ and a large number of spectator fields.

The tensor to scalar ratio is given by the scale of inflation. Close to the symmetry breaking, we can make use of (2.45) and (2.11) to express it as a function of the parameters of our theory, as we have done for $\Delta_s^2$ and $n_s - 1$, depending only on $\lambda$, $g$ and $N$.

$$r = \frac{\Delta_t^2}{\Delta_s^2} = 8\frac{\sqrt{\lambda}}{g^4}\left(-\alpha + \alpha^3/3\right)^{-2}\,. \tag{2.46}$$

As we have seen, $g$ is setting how fast the minimum is moving away from the origin in relation with the value of the field (initially freeze at the Starobinsky value for approximately a $\varphi^4$ field). Therefore the value of $g$ will affect the slope of the potential after symmetry breaking and ultimately the perturbations. Smaller values of $g$ makes the minimum to move faster away from the origin than the field and, in consequence, the gradient of the potential increases, increasing the value of the scalar perturbations. The value of the tensor to scalar ratio is the smallest at the inflection point, which gives $r \approx 16\sqrt{\lambda}/g^4$. As the field rolls down to the minimum,

$r$ increases, diverging at the minimum since $V' = 0$.

Since in this model, inflation takes place between the inflection point and the new minimum emerging due to broken symmetry, the field excursion of $\varphi$ scales as $\varphi \sim \sqrt{-m^2/\lambda}$. Furthermore, since for our choice $g = 1$ this takes place soon after symmetry breaking where the mass parameter can be estimated as $-m^2 \sim \sqrt{\lambda} H^2$ (2.36), which with the Hubble rate from (2.27) and (2.44) indicates that for a large number of spectator fields the field excursions are sub-Planckian. For the parameters agreeing with observations, this turns out to be true. It indicates that despite begin a small field model of inflation, the initial conditions are not fine-tuned but arise naturally as the favoured attractor solutions, unlike what is usually encountered [110]. For equations (2.45), (2.46) and (2.44) agreeing with the observables in [23], we have three constraints for three observables, although the fact that inflation occurs soon after symmetry breaking imposes the value of $g$ to be close to unity, and also the value of the tilt is mostly dominated by $\lambda$, leaving the perturbations being determined by the number of fields. Of course there is some uncertainty in the value of $\alpha$ as well, which can only be resolved numerically. The approximations and estimates set out here agree with the numerical solution that we will turn to now.

### 2.3.1 Numerical solution

In this section, we look at the numerical solution corresponding to the parameter choices we have made above.

Before symmetry breaking the field lies at the origin $\varphi = 0$ until $\varphi_0$ drops sufficiently for the potential at the origin to become tachyonic. Classically, of course, the field would then not move anywhere because $dV/d\varphi = 0$ at the origin, and if we were to introduce a small perturbation away from $\varphi = 0$ as an initial condition, then classically our final solution would depend strongly on this perturbation. To get around this problem, we evolve the field using the Langevin equation which takes into account the stochastic quantum fluctuations the field receives

$$\frac{d\varphi}{dN_e} = -\frac{m^2\varphi + \lambda\varphi^3}{3H^2} + \frac{H}{2\pi}\xi\,, \tag{2.47}$$

where $\xi$ is Gaussian white noise with zero mean and unit variance.

For each combination of the remaining two free parameters $(N, \lambda)$ we perform $10^4$ numerical realisations of the Langevin equation and find the mean values of the magnitude of the perturbations $\Delta_s^2$ for each value of the tilt $n_s$ as the field evolves. These simulations are also used to verify that the dynamics of the field agrees with our analytical estimates from Sec. 2.2.

We know that to obtain a red spectrum ($n_s < 1$), the field must lie between the inflection point and the minimum.

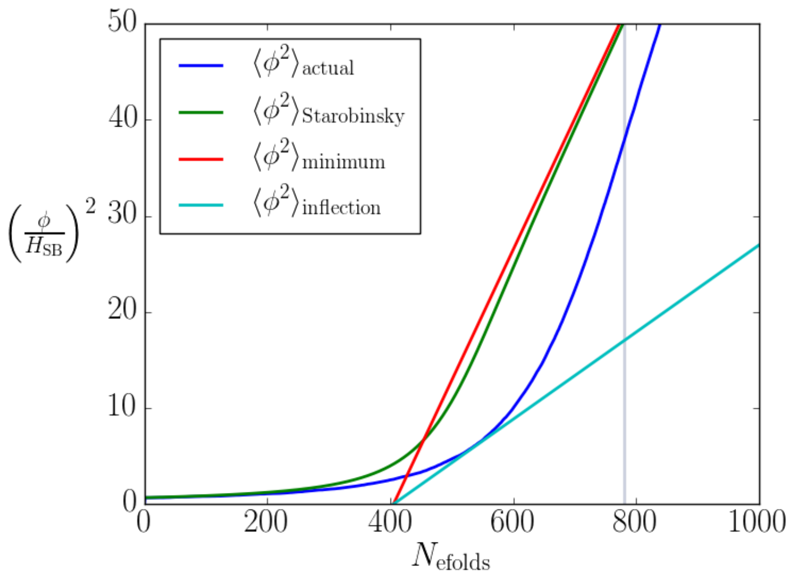


**Figure 2.5:** *Field dynamics with $\lambda = 10^{-3}, N = 10^{5.5}, \varphi_0 = 10^{-1.27}M_{\rm P}$ and $g = 0.7$. We plot the evolution of the field expectation vs. the Starobinsky estimate in units of $H_{SB}^2$ to show that they are different. We also show the time evolution of the minimum and the inflection point (which the field must remain beyond in order to obtain a red spectrum). $N_e = 780$ is the epoch when the spectrum of perturbations and the tilt match the observed values by Planck.*

Figure 2.5 shows how the variance of the field $\langle\phi^2\rangle$ evolves with respect to the number of $e$-folds of expansion for the parameters $\lambda = 10^{-3}, N = 10^{5.5},$ and $g = 0.7$ (making $\varphi_0 = 10^{-1.27}M_{\rm P}$). Note that the initial epoch where we set the horizontal axis equal to zero is chosen arbitrarily. We also plot in the same figure the variance of the field that one would expect using the Starobinsky-Yokoyama prescription for the variance in a perfect de Sitter space-time corresponding to a vacuum energy

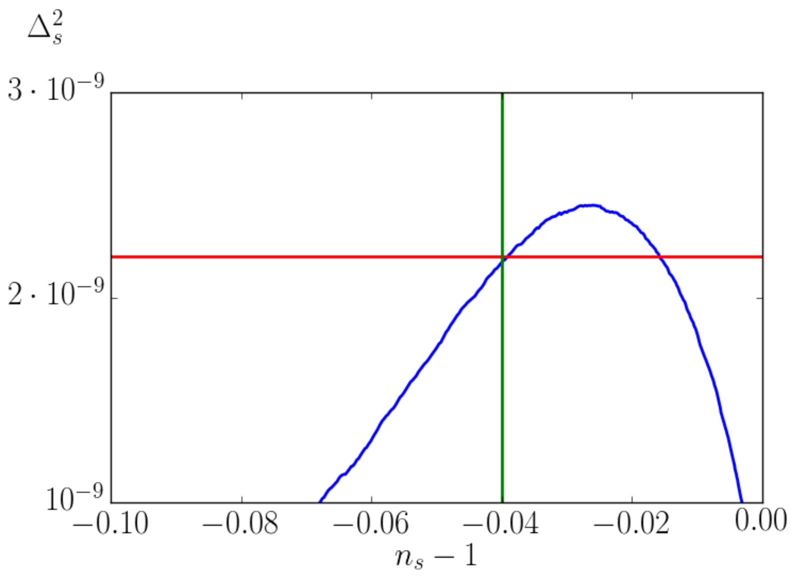


**Figure 2.6:** *Amplitude of the spectrum of perturbations versus the tilt with $\lambda = 10^{-3}, N = 10^{5.5}, \varphi_0 = 10^{-1.27} M_{\rm P}$ and $g = 0.7$ . The blue line is the simulation (average over $10^4$ realisations) and the green and red lines are the cosmologically observed value of the spectrum and amplitude of pertubations respectively.*

equal to our evolving vacuum energy [26], assuming instant equilibration for the probability distribution. The field expectation value lags behind this estimator, showing the importance of solving the Langevin equation. The field approximately remains constant at this value until the minimum has dropped enough to make the potential steep and for it to possess an inflection point. We also plot the position of the minimum as it moves out towards its zero temperature value and the position of the inflection point, showing that we remain on the good side to obtain a red spectrum. Note that in a period of time during inflation corresponding to 60 $e$-folds of expansion, the variation of $\varphi_0$ is negligible relative to the variation in $\varphi$ so we assume that it is constant for calculational simplicity.

Figure 2.6 shows the average evolution of the magnitude of perturbations for the same parameters as a function of the spectral tilt. We can see that these parameters can give rise to the correct combination of amplitude and spectrum for the perturbations to match what is observed in the Cosmic Microwave Background, i.e. $\Delta_s^2 = 2.2 \times 10^{-9}$ and $n_s = 0.96$ [111, 112].

The power spectrum of tensor fluctuations is given by the energy scale of inflation (1.75); therefore, for the parameters chosen, the tensor to scalar ratio that we get is $r = 0.07$. This value is still within the 95 % CL from the Planck data [42] although lower values can be obtained by choosing a different set of parameters, since the scale of symmetry breaking can be easily tuned, Eq. (2.27).

In order to end inflation, we assume the temperature corresponding to the expansion rate during inflation falls below the mass of the particles which form the thermal bath affecting the potential for the scalar field. We set this by hand to give us 60 $e$-folds after the epoch corresponding to the right values of $\Delta_s^2$ and $n_s$. Otherwise, the field will go to the minimum and continue inflating the universe as in the symmetric phase until $\varphi = \varphi_0$ when $H = 0$ and $T_H = 0$ resulting in an empty universe.

## 2.4 Conclusion

In this work, we have presented a possible new mechanism for inflating the early universe. We have argued that if one takes the Gibbons-Hawking temperature associated with the horizon of de Sitter space seriously, one is lead to a couple of conclusions that can affect the evolution of quantum fields in the early universe significantly. In particular, we have argued that if there are enough fields coupled to the scalar field which takes the role of the inflaton, their horizon induced temperature leads to thermal corrections to the potential which can affect its expectation value. Since these thermal fluctuations do not redshift as rapidly as normal radiation, this effect can last for a significant number of $e$-folds of expansion.

To summarise the mechanism - We consider a real scalar field with a $\mathbb{Z}_2$ symmetric potential with minima at $\pm\varphi_0$. If the field starts at the origin, the non-zero energy density leads to de Sitter expansion and a cosmological horizon with an entropy. The field is trapped at the origin due to its coupling to a large number of conformal spectator fields which have a non-zero temperature associated with this horizon entropy. The height of the potential at the origin decays slowly as a result of the back-reaction of the thermal radiation on the parameter $\varphi_0$ in the Lagrangian.

The expansion rate, therefore, decreases as does the temperature $T_H$ of the thermal radiation until the mass at the origin becomes tachyonic, at which point a phase transition occurs and the field rolls away from the origin towards its zero temperature minimum at $|\varphi| = \varphi_0$. This phase transition, however, occurs extremely slowly due to the same finite temperature corrections to the potential. During this period of rolling, we are able to obtain not only the correct number of $e$-folds but also the correct perturbations and spectral tilt as measured in the CMB. This setup has two attractive features:

- Normally, for a field to act as an inflaton, it must have a super Planckian expectation value and/or very finely tuned parameters. The mechanism we outline in this paper enables a potential which would otherwise not be flat enough to give rise to enough $e$-folds of inflation to do so due to the thermal effects of multiple spectator fields

- The initial conditions for inflation may arise naturally as attractor solutions due to a slowly occurring phase transition. This happens despite the fact that inflation occurs with sub-Planckian values for the inflaton. This is the result of the parameters in the Lagrangian changing due to the back-reaction of the thermal radiation.

It is clear that this scenario is not necessarily a panacea for all of the problems of inflation. We need to assume that parameters in the Lagrangian decay over time due to the back-reaction of Hawking radiation at the horizon. While there are many respected physicists who believe that such behaviour is probable and perhaps necessary, it is clear that further theoretical investigation and debate is required to put such speculation on a stronger footing. We also require quite a large number of fields which are coupled to the inflaton in order to obtain the thermal braking required to obtain enough $e$-folds of inflation, in the example we have put forward here about $3 \cdot 10^5$. The masses of the fields need to be chosen such that inflation ends 50-60 $e$-folds after the epoch where the good perturbations and spectral index are set. In return for this cost, we obtain a theory of inflation which doesn't require transplackian field excursions, does not require extremely small parameters in the Lagrangian (we use a value of $\lambda \sim 0.001$) and which may naturally explain the initial conditions for inflation. Finally, it seems quite challenging to understand how the difference between this scenario and normal inflation could be distinguished experimentally.

Chapter 3

# Gravitationally produced Top Quarks and the Stability of the Electroweak Vacuum During Inflation

## 3.1 Introduction

The measurement of the actual Higgs and the top quark masses at the LHC and other colliders [113–115] leads to an interesting effect when one calculates their Renormalisation Group running in that the quartic Higgs self-interaction coupling $\lambda$ becomes negative above around $10^{10}$ GeV [34, 35, 116, 117]. This high energy scale cannot be probed at current colliders but is much smaller than the Planck mass and is in a region where all the couplings remain perturbative, so there is no reason not to take this extrapolation seriously. Taking the central observed values for the Higgs mass ($m_h$), the top quark mass ($m_t$), and the strong coupling constant ($\alpha_s$) from [118], a calculation [34] of the running of $\lambda$ and $y_t$ is shown in Figure 3.1

The implication of this is clear: in the absence of physics beyond the Standard Model affecting the running of the coupling constants, our current electroweak vacuum favours a metastable solution over an absolute stable vacuum [119–124]. Fortunately when one calculates the lifetime for tunneling into the true vacuum

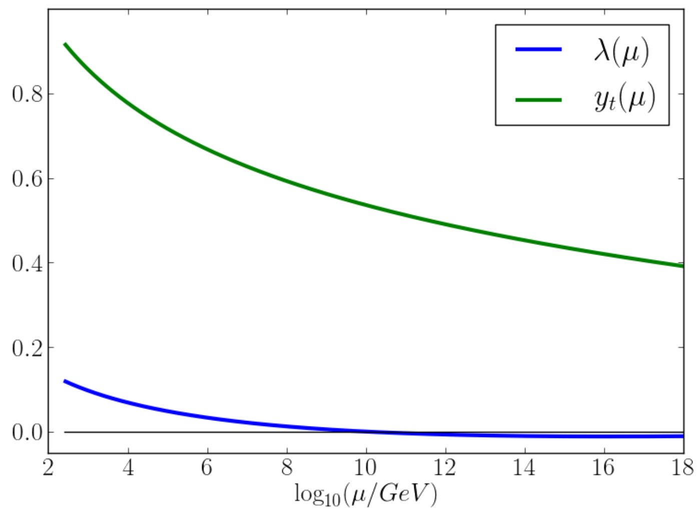


**Figure 3.1:** *Running of the Higgs self-interaction and the top yukawa coupling as a function of the energy scale $\mu$ up to 3-loops. The instability scale is at $10^{10}$ GeV, when $\lambda < 0$ and $y_t = 0.5$. To make this plot we have used $m_t = 173$ GeV, $\alpha_s = 0.1181, m_h = 125.18$ GeV.*

above $10^{10}$ GeV, one typically obtains numbers which are many orders of magnitude larger than the age of the universe [116], although it is still a subject of active research where new physics could modify the lifetime [125–130]. One might expect therefore that this unusual behaviour of the running at high scales is little more than a curiosity; however this situation changes when one considers the early universe.

Also, the basis of the Standard Model instability relies on the assumption of no new physics that will affect the running of the Higgs self interaction term from the approximately 100 GeV energy scale at which it is measured until $10^{10}$ GeV at which the coupling becomes negative. There are still many open questions in physics for which we still do not have an answer and may affect this analysis. For instance, the nature of dark matter could be a clue of physics beyond the Standard Model [131] if it does not have an astrophysical origin such as primordial black holes [132], another evidence is the discovery of the neutrino masses, for which the seesaw models suggest new physics above the TeV scale [133], maybe even larger than $10^{10}$ Gev [119]. Another clues may be the baryon asymmetry of the universe [134] and the huge difference in scales between the Higgs mass and the Planck mass which makes it difficult to study radiative corrections to the higgs mass [135], as well as possible higher dimensional operators which may affect its running [136].

For several decades the leading hypothesis for the earliest stages of the evolution of the universe has contained a period of cosmological inflation where the scale factor expanded exponentially, solving many cosmological problems and explaining the origins of astrophysical structure formation across many orders of magnitude in physical scale [17, 18, 43] (more details in Sec.1.2). While inflation has its own fine tuning problems (addressed and recasted in [40], Sec.2), there are not many compelling alternatives to inflation which have a simpler or even equally simple mathematical consistency [46].

Fluctuations in the Higgs field during inflation lead to stochastic growth in its expectation value which could push it to the region of instability at around $10^{10}$ GeV [26] [1]. The universe would then seemingly be overwhelmed by an anti-de Sitter (AdS) [2] region which would subsequently collapse, allowing no possibility of us being here today [35, 37, 38, 123, 139]. These AdS bubbles grow at the speed of light, but they are in a dS background which is expanding even faster and in consequence, they will never take over all dS space [35]. But even if they are not a problem during inflation, they can neither be present at the end of inflation in our past light cone, since after inflation ends, they will take over all our casual horizon [35, 140]. This is independent of the value of the Higgs field in the True vacuum, since the bubbles nucleate for any negative value of the potential. Because of these bubbles, there appear to be tight constraints upon the absolute scale of the expansion rate $H$ during inflation in order to evade the instability. This corresponds in a one-to-one fashion upon the magnitude of primordial gravitational waves which might be generated during inflation [22,141,142], which is parametrised by the tensor to scalar ratio $r$.

What we propose in this chapter is to take into account for the first time the gravitational particle production of fermions during inflation, in particular, the top

[1] It is usual to set the renormalisation scale $\mu$ to the expectation value of the Higgs $h$ when one considers effective potentials where the effects of loops are included as logarithmic corrections [137, 138].

[2] AdS is a solution to the Friedmann equations with a negative cosmological constant.

quark which has the strongest interaction with the Higgs field. The energy density of fermions produced during inflation is related to their mass [143], and since top quarks have a Yukawa coupling $y_t$ of order unity, their mass is given by the Higgs vacuum expectation value (vev) $m_t \sim y_t \cdot h$. The interaction term in the Lagrangian of the SM for the case of the Higgs and the top fermions is

$$\mathcal{L}_{\text{interaction}} = y_t \frac{h}{\sqrt{2}} \bar{\psi}\psi \, . \tag{3.1}$$

So as the Higgs field is pushed to higher values, the mass of the top quarks will increase, and the production of fermions will also increase, meaning that the contribution from the fermions $\bar{\psi}\psi$ to the Higgs potential will also rise. We aim to show that there are situations where this contribution to the potential can change the probability of ending in a catastrophic collapse during inflation.

The chapter is organised as follows, Section 3.2 reviews the instability of the electroweak vacuum during inflation. Section 3.3 describes the particle production of massive fermions in a de-Sitter background and their subsequent modification of the Higgs potential in the case of top quarks. In Section 3.4 we study the stability of the Higgs taking into consideration this effect before discussing the results in Section 3.5

## 3.2 The Instability of the Electroweak Vacuum during inflation

In this section, we will review the usual arguments which explain why a period of inflation is dangerous for the stability of the electroweak vacuum given the fact that the quartic coupling runs to negative values at high scales.

There is some discussion in the literature about the best choice of the scale $\mu$ and its relationship with the Higgs field expectation value $h$ when working with the Higgs in the early universe. It was recently proposed [33, 36–38] that when studying a quantum field in a curved space-time background, in order to cancel the logarithmic divergences that arise in the potential at one-loop order, the choice of the scale $\mu$ is different (3.2) from the choice that is usually assumed for the same situation in a flat space-time background where $\mu \approx h$ is chosen [35]. In this work, the results do not depend strongly on these two different choices of the scale, but for definitiveness, we choose to set the scale of the running as

$$\mu = \sqrt{h^2 + H^2}, \tag{3.2}$$

where $h$ is the Higgs vev and $H = \dot{a}/a$ is the Hubble parameter, although we will include an extension to our calculation to showcase the differences with the choice of scale $\mu = h$.

What is more widely agreed on is that during inflation, short wavelength fluctuations behave as classical noise acting on the dynamics of the Higgs field on super-Hubble scales and these fluctuations can be described using the Langevin equation [26, 27]

$$\frac{dh}{dN_e} = -\frac{V'(h)}{3H^2} + \frac{H}{2\pi}\xi\,. \tag{3.3}$$

Using this equation we can study how the expectation value of the Higgs field $\langle h^2 \rangle$ evolves with $N_e$ - the number of $e$-folds of inflation ($dN_e = d\ln a$, where $a$ is the scale factor). The evolution is due to a combination of two effects: the first is given by the classical equation of motion, where $V'(h)$ is the differentiation of the Higgs potential with respect to the Higgs vev, and the second is due to the stochastic noise, where $\xi$ is a Gaussian white noise with zero mean and unit variance. The Langevin equation is only valid for a light field $V'' \ll H^2$, since for a heavy field the fluctuations are suppressed (see footnote 1 in chapter 2). If the Higgs is initially at the origin ($h = 0$), the stochastic term dominates over the classical term and on average the Higgs vev after $N_e$ $e$-folds of inflation would be

$$\langle h^2 \rangle = \left(\frac{H}{2\pi}\right)^2 N_e, \tag{3.4}$$

until the classical term becomes as large as the stochastic term, which in the classical picture occurs after $N_e = 1/\sqrt{\lambda}$, and the Higgs would then acquire an equilibrium value given by

$$\langle h^2 \rangle = 0.13 \frac{H^2}{\sqrt{\lambda}}. \tag{3.5}$$

This is valid only if $\lambda > 0$ (and constant). In the case that $\lambda$ is not positive, then the Higgs vev motion would be unbounded. Note we are assuming here and throughout that the Higgs field starts at the origin 60 $e$-folds before the end of inflation. This assumption is somewhat important, but as long as $h$ starts somewhere below $H$, we expect very similar results. If $h$ starts with a very high value, then a different kind of analysis would have to be performed.

Therefore, even if we only assume 60 $e$-folds of de-Sitter expansion, on average, the value of the Higgs vev is going to be close to the energy scale of inflation ($h \approx H$) and the running of the Higgs self-interaction $\lambda(\mu) \approx \lambda(H)$, which is independent of the the choice $\mu = h$ or $\mu^2 = h^2 + H^2$. If the energy scale of inflation is high enough, then the Higgs field would move into the unstable region; in particular, for 60 $e$-folds, the scale of inflation should be about one order of magnitude smaller than the scale at the maximum of the potential ($\mu = h_{\max} \sim 10^{10}$GeV) [35, 123, 139].

$$\frac{H}{h_{\max}} < \frac{2}{3} \frac{\pi}{N_e} = 0.04 \tag{3.6}$$

From the non detection of CMB polarisation associated with primordial gravitational waves ($r < 0.12$) [144] we can set an upper bound on the energy scale of inflation $H < 10^{13}$ GeV and since the instability scale is around $\mu = 10^{10}$ GeV [35], we will focus on this energy interval $H = 10^9 - 10^{13}$ GeV.

There are many possible alternative solutions to this problem of combining inflation with the standard model. However, unlike what we are proposing here, they all invoke new physics, the most obvious and well studied are a simple coupling between the Higgs field and the inflaton [139, 145–147] and a non-minimal coupling between the Higgs field and the Ricci Curvature [33,36,148,149] or both at the same time [150]. See also [151] for the effect of the Gibbons-Hawking radiation during inflation on this problem or around an evaporating Black Hole in [152].

Having explained the problem and shown that for inflation with $H \gtrsim 10^9$GeV the electroweak vacuum can be unstable, we now move on to consider the gravitational production of fermions and how they might change this situation.

## 3.3 Massive fermion production

In this section we consider how fermions, in our case top quarks, can be produced gravitationally and what effect they will have upon the Higgs potential.
The gravitational creation of fermions is similar to the scalars as studied in Sec. 1.3.1, where due to the dynamical background, the defintion of vacuum is non unique. A crucial difference is that massless scalars in de-Sitter are infrared divergent [153] whereas fermions are conformally invariant.
We will start with the action for a spin-$\frac{1}{2}$ massive ($m$) Dirac fermion ($\psi$) in curved space-time [25, 143]

$$S = \int d^4x \sqrt{|g|}\bar{\psi}(i\gamma^a e^\mu_a \nabla_\mu - m)\psi\,, \tag{3.7}$$

where the spin-$\frac{1}{2}$ covariant derivative with vierbein dependent spin-connection, $\omega^{\alpha\beta}_\mu$, is defined as $\nabla_\mu\psi = \partial_\mu\psi + \frac{1}{8}\omega^{\alpha\beta}_\mu[\gamma_\alpha, \gamma_\beta]\psi$, $e^\mu_a$ is the vierbein and $\gamma_\alpha$ are the standard Minkowski space-time Dirac matrices. The vierbein is required to relate the metric $g^{\mu\nu}$ with the Minkowski metric, in order to obtain a curved space definition of the $\gamma$ matrices, $\bar{\gamma}^\mu(x) = e^\mu_a(x)\gamma^a$, which generalize the anticommutation $\{\bar{\gamma}^\mu, \bar{\gamma}^\nu\} = 2g^{\mu\nu}$
From the lagrangian (3.7) we get the equation of motion:

$$(i\gamma^a e^\mu_a \nabla_\mu - m)\psi = 0\,. \tag{3.8}$$

It is a generalization of the Dirac equation in curved space-time obtained by replacing the standard differentiation $\partial_\mu$ for the covariant $\nabla_\mu$ and $\gamma^a$ for $\bar{\gamma}^\mu$. Working in a flat

FRW metric $ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2)$, the vierbein is $e^\mu_a = a(\eta)\delta^\mu_a$, resulting in the curved space Dirac matrices $\bar{\gamma}^\mu = \gamma^\mu/a$.

The equation of motion (3.8) can be simplified by perfoming a Weyl trasnformation

$$g_{\mu\nu} = \Omega^2 \tilde{g_{\mu\nu}}\,, \qquad \psi = \Omega^{-3/2}\tilde{\psi}\,, \qquad e^\mu_a = \Omega^{-1}\tilde{e^\mu_a}\,, \tag{3.9}$$

where we choose $\Omega = a(\eta)$. The equation of motion in the rescaled frame looks like

$$(i\gamma^\mu\partial_\mu - a(\eta)m)\tilde{\psi} = 0\,. \tag{3.10}$$

In the Heisenberg picture, the field operator $\tilde{\psi}$ is expanded in a basis

$$\psi = \sum_i a_i U_i + b_i^\dagger V_i\,, \tag{3.11}$$

in which the canonical anticommutation relations are defined as

$$\{\psi_a(t,\mathbf{x}), \psi_b(t,\mathbf{x'})\} = 0\,, \tag{3.12}$$

$$\{\pi_a(t,\mathbf{x}), \pi_b(t,\mathbf{x'})\} = 0\,, \tag{3.13}$$

$$\{\psi_a(t,\mathbf{x}), \pi_b(t,\mathbf{x'})\} = i\delta_{a,b}\delta(\mathbf{x}-\mathbf{x'})\,, \tag{3.14}$$

where $\pi$ is the canonical momenta. These ensure the orthonormality of the mode functions and the statistics for fermions (*aka Pauli blocking*)

$$\{a_i, a_j^\dagger\} = \delta_{ij}\,, \qquad \{b_i, b_j^\dagger\} = \delta_{ij}\,. \tag{3.15}$$

The antiparticle state is related with the particle like: $V_i = -i\gamma^2 U_i^*$

The solution to the Dirac equation can be then separated in components like

$$U_{\vec{k},r}(\eta,\vec{x}) = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}} \begin{pmatrix} u_A(k,\eta)h_{\hat{k},r} \\ r u_B(k,\eta)h_{\hat{k},r} \end{pmatrix}\,, \tag{3.16}$$

in which $\hat{k}$ is the unit vector of $\vec{k}$, and $h_{\hat{k},r}$ the helicity 2-spinor which satisfies

$$\hat{k}\cdot\vec{\sigma}h_{\hat{k},r} = rh_{\hat{k},r}\,, \qquad r=\pm 1\,, \tag{3.17}$$

with $\vec{\sigma}$ being a vector with the Pauli matrices and $r$ is the parity of the state ($+1$ for right handed and $-1$ for left handed).
The normalization of the mode functions implies

$$h^{\dagger}_{\hat{k},r}h_{\hat{k},s} = \delta_{rs}\,, \tag{3.18}$$

$$|u_A(k,\eta)|^2 + |u_B(k,\eta)|^2 = 1\,, \tag{3.19}$$

then we end up with the equation of motion only depending on the conformal time

$$i\partial_\eta \begin{pmatrix} u_A(k,\eta) \\ u_B(k,\eta) \end{pmatrix} = \begin{pmatrix} a(\eta)m & k \\ k & -a(\eta)m \end{pmatrix} \begin{pmatrix} u_A(k,\eta) \\ u_B(k,\eta) \end{pmatrix}\,. \tag{3.20}$$

Since the choice of the orthonormal basis is not unique, we could define a different basis $\{\tilde{U}_i,\tilde{V}_i\}$, where $\psi=\sum_i a_i U_i + b_i^\dagger V_i = \sum_i \tilde{a}_i\tilde{U}_i + \tilde{b}_i^\dagger\tilde{V}_i$.

The vacuum state is defined by $a_i|vac\rangle = b_i|vac\rangle = 0$, so in the tilde basis, the number of particles measured over the initial (no-tilde) vacuum state is

$$\langle vac|\tilde{a}_i^\dagger\tilde{a}_i|vac\rangle = \sum_j |\beta_{ij}|^2\,, \tag{3.21}$$

where the relation between the two vacuum states is linear and parametrised by the Bogoliubov coefficients ($\alpha_{\vec{k}}$ ,$\beta_{\vec{k}}$): $\tilde{U}_{\vec{k}} = \alpha_{\vec{k}}U_{\vec{k}} + \beta_{\vec{k}}V_{-\vec{k}}$.

Defining the initial basis with the index 'in' and the tilde basis in which we measure the number of particles of the initial vacuum state as 'out', one obtains the following relation:

$$|\beta_k| = |u_A^{out}(k,\eta)u_B^{in}(k,\eta) - u_B^{out}(k,\eta)u_A^{in}(k,\eta)|\,. \tag{3.22}$$

The 'out' is set to be the instantaneous vacuum state (zeroth-order in adiabatic expansion), and it can be obtained by using the Wentzel-Kramers-Brillouin (WKB) approximation.

$$\begin{pmatrix} u_A(k,\eta) \\ u_B(k,\eta) \end{pmatrix}^{WKB} = \alpha_k \begin{pmatrix} \sqrt{\frac{w+am}{2w}} \\ \sqrt{\frac{w-am}{2w}} \end{pmatrix} e^{-i\int^\eta w(\eta)d\eta} + \beta_k \begin{pmatrix} \sqrt{\frac{w-am}{2w}} \\ -\sqrt{\frac{w+am}{2w}} \end{pmatrix} e^{i\int^\eta w(\eta)d\eta} \quad (3.23)$$

valid for

$$\left|\frac{\dot{w}}{w^2}\right|^2 \lesssim 1 \qquad \text{and} \qquad \left|\frac{\ddot{w}}{w^3}\right| \lesssim 1\,, \quad (3.24)$$

where $w^2 = k^2 + m^2a^2$ and due to the normalization of the modes (3.19), $|\alpha_k|^2 + |\beta_k|^2 = 1$. A vacuum state is defined as $\alpha = 1$ and $\beta = 0$.
This is a solution to (3.20) in a Minkowski space-time where the scale factor is constant and there is no particle production, which is why it is called the instantaneous vacuum, because as the scale factor changes with time, this vacuum would measure a different number of particles.

There are two general properties for the production of fermions that can be deduced independently of the details of the problem: first, fermions are conformally invariant, meaning that in the massless limit there is a conformal transformation from any Friedmann-Robertson-Walker (FRW) metric to Minkowski and therefore no particles are produced. This is explicitly seen in Eq. (3.20) where if $m = 0$ the dependence on the scale factor is lost. Second, particle creation is exponentially suppressed for the case of *heavy* fermions ($m \gg H$) and large momenta. The equation of motion (3.20) can be rewritten as

$$u''_{A,B} + \left(k^2 + m^2a^2\left(1 \pm i\frac{H}{m}\right)\right) u_{A,B} = 0\,, \quad (3.25)$$

now easily for the case of heavy fermions and large momenta the frequency looks like $w^2 = k^2 + m^2a^2$ and as we showed for the scalars (see footnote 1 in chapter 2), the production is exponentially suppressed. Both of these properties will be shown throughout the section.

The ‘in’ state is the Bunch-Davies vacuum state for a perfect de-Sitter background solution to (3.20), with $a(\eta) = -1/H\eta$ [154],

$$\begin{pmatrix} u_A(k,\eta) \\ u_B(k,\eta) \end{pmatrix}^{in} = \sqrt{\frac{\pi}{4}k\eta} \begin{pmatrix} e^{+\frac{\pi m}{2H}} H^{(1)}_{\frac{1}{2}-i\frac{m}{H}}(-k\eta) \\ e^{-\frac{\pi m}{2H}} H^{(1)}_{\frac{1}{2}+i\frac{m}{H}}(-k\eta) \end{pmatrix} . \tag{3.26}$$

In the limit $a \to 0$ ($\eta \to -\infty$) agrees with the WKB solution (3.23) at that time. Therefore at the beginning there are no particles since both states coincide with $\alpha = 1$ and $\beta = 0$.
In order to not create extra particles from the sudden measurement of particles in the instantaneous vacuum (3.23), we introduce a smooth exit from inflation into Minkowski space-time such that $H_\eta(\eta) = H(1 - \tanh((\eta - \eta_i)/\eta_0))/2$, where $\eta_i$ is the time at which inflation ends, $H$ is the value of the Hubble parameter during inflation and $\eta_0$ is the speed of the transition. Then (3.26) is the solution to (3.20) for $\eta \ll \eta_i$ and (3.23) is the solution at $\eta \gg \eta_i$ where we can unequivocally define the number of particles created during the de-Sitter period of expansion of the universe. The speed of the transition is set to $\eta_0 = 1/H$, the natural scale for inflation. In the low limit mass, $m/H \ll 1$, the calculation is unaffected by the speed of the transition. However, for masses $m/H \geq 1$ if the transition is faster, $\eta_0 \ll 1/H$, then more particles would be created because of the sudden change in the scale factor, and if $\eta_0 \gg 1/H$, the transition happens too slow and heavy fermions would be diluted leading to a smaller number of particles being produced. In Fig. (3.2) we show how for the product $\langle\bar{\psi}\psi\rangle$ (3.27), the light masses are unaffected by the speed of the transition whereas the heavy fermions can be artificially created by a fast change in the scale factor. For a more exhaustive study of the effect of the speed of the transition we refer the reader to the work done for the case of scalar fields in [155, 156].

The production of *heavy* fermions, $m \gtrsim H$, is exponentially suppressed by their mass, $|\beta_k|^2 = (1 + e^{2\pi m/H})^{-1}$, but for the case of *light* fermions, $|\beta_k|^2 = 1/2$ is

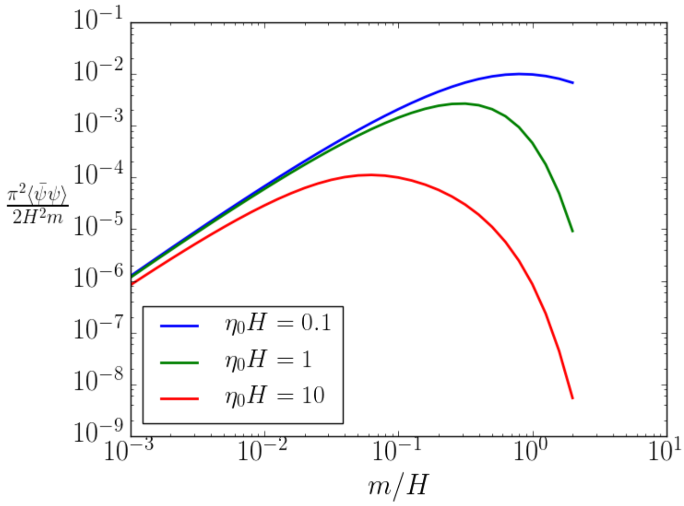


**Figure 3.2:** *Plot of the product $\frac{\langle\bar{\psi}\psi\rangle\pi^2}{2H^2m}$ as a function of the mass $m/H$. The speed of the transition ($\eta_0$) from de-Sitter to Minkowski only affects the production of heavy fermions. Production of light fermions can be seen to be unchanged regardless of the value of $\eta_0$.*

constant up to $k/a = m$ as shown in Figure 3.3.

The quantity we are interested in is the expectation value of an initial vacuum state for the product $\langle\bar{\psi}\psi\rangle$, and using (3.23) this takes the form

$$\langle\bar{\psi}\psi\rangle = \int \frac{d^3k_p}{2\pi^3}\frac{m}{w_p}|\beta_k|^2\,, \tag{3.27}$$

where the subscript $p$ stands for *physical* quantities, so $k_p = k/a, w_p = w/a$. Also the piece in the product coming from the initial vacuum and an oscillatory term has been discarded, as it has been done as well in [157, 158]; in the literature this is called normal ordering or renormalization of the product.
For light fermions we can obtain analytically the value of the product $\langle\bar{\psi}\psi\rangle$, since $|\beta_k|^2 = 1/2$ up to $k/a = m$

$$\langle\bar{\psi}\psi\rangle = \frac{2}{\pi^2}\int_0^m dk_p k_p^2 \frac{m}{k_p}\frac{1}{2} = \frac{m^3}{2\pi^2}\,. \tag{3.28}$$

This behaviour is observed in Fig. (3.2) which is independent of $\eta_0$. But for the instability of the EW vacuum, we are interested in the peak produced by this pro-

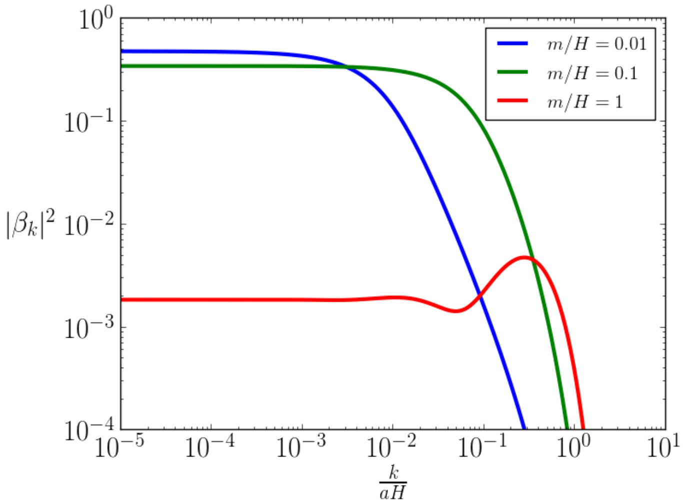


**Figure 3.3:** *Plot of $|\beta_k|^2$ as a function of $k/aH$ for different masses. If the fermions are light, the spectrum can be approximated as $1/2$ up to $k/a = m$; for the heavy fermions the spectrum is suppressed as $1/(1 + e^{2\pi m/H})$.*

duction, hence we perform a fit close to it (Fig. (3.4)).

In this way, we can obtain the expectation value for a massive fermion during inflation as a function of its mass as shown in Figure 3.4

$$\langle\bar{\psi}\psi\rangle = H^3 \frac{m}{H}\frac{2}{\pi^2}\frac{0.063\left(\frac{m}{H}\right)^{1.22}}{e^{4.92\frac{m}{H}} + 1} . \tag{3.29}$$

### 3.3.1 Addition to the Higgs potential

The full Lagrangian that determines the dynamics of the Higgs field is

$$\mathcal{L}_{\text{Higgs \& top}} = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{\lambda}{4}h^4 - 3y_t\frac{h}{\sqrt{2}}\bar{\psi}\psi + 3i\bar{\psi}\gamma^a e^\mu_a \nabla_\mu \psi . \tag{3.30}$$

The mass of the fermions is of course explicitly given by the Higgs expectation value. This coupling through the Yukawa coupling $y_t$ also leads to a term in the equation of motion for $h$ which is proportional to $\bar{\psi}\psi$; and therefore the fermions change its dynamics, and the factor 3 comes from the colour charge of the quarks in the Standard Model (as pointed out in [159]). The addition to the Higgs potential

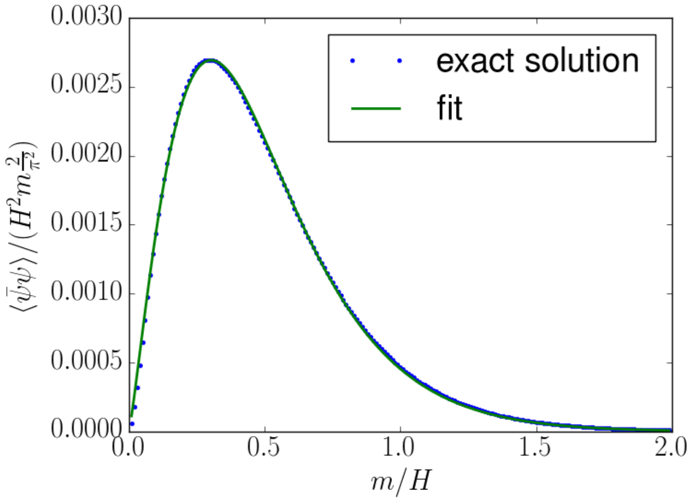


**Figure 3.4:** *Plot of* $\frac{\langle\bar{\psi}\psi\rangle\pi^2}{2H^2m}$ *as a function of* $m/H$*. If the fermions are light,* $\langle\bar{\psi}\psi\rangle \propto m^{2.2}$ *up to* $m/H = 0.49$*, above which it is exponentially suppressed.*

coming from the production of fermions is (using the result obtained in (3.29))

$$V(h) = V_{\mathrm{h}} + V_{\mathrm{f}} = \frac{\lambda}{4}h^4 + 3y_t\frac{h}{\sqrt{2}}\bar{\psi}\psi = \frac{\lambda}{4}h^4 + 3H^4\frac{0.013(y_t\frac{|h|}{\sqrt{2}H})^{3.22}}{e^{4.92(y_t\frac{|h|}{\sqrt{2}H})}+1}\,. \tag{3.31}$$

The condensate created from the production of fermions changes the Higgs potential, adding an extra term that peaks at $h_{\mathrm{peak}} = 0.96H/y_t$. At that value, the potential is

$$\frac{V(h_{\mathrm{peak}})}{H^4} = \frac{\lambda}{4}\frac{0.84}{y_t^4} + 0.00037\,. \tag{3.32}$$

So the contribution from the fermions to the Higgs potential can dominate if

$$y_t > 4.8 \cdot \lambda^{1/4}\,. \tag{3.33}$$

As can be seen in Fig. (3.5), the height of the barrier is increased and there is a visible shift in the scale of the instability; however, later we will see that this has a disappointingly small effect upon the overall probability of becoming unstable. Note that the effect of the fermion back reaction dominates the potential when the criterion (3.33) is fulfilled, i.e. very close to the point where $\lambda \sim 0$, at that scale $y_t \sim 0.5$, so $V_\mathrm{f}$ peaks at $h_\mathrm{peak}/H \sim 2$.

If we were to study another fermion with a different Yukawa coupling to the SM Higgs field, assuming that inflation occurs at a low energy scale where $\lambda \sim 0.1$, we would need a Yukawa coupling bigger than $y > 2.7$ in order for the fermions to dominate (after 60 $e$-folds of fermion condensate production). So the bigger the Yukawa coupling the bigger the effect, which is why we have been focusing on the top quarks throughout this paper. We note, however, that a fermion with a larger Yukawa coupling would destabilise the vacuum at a much lower value of the Higgs field.

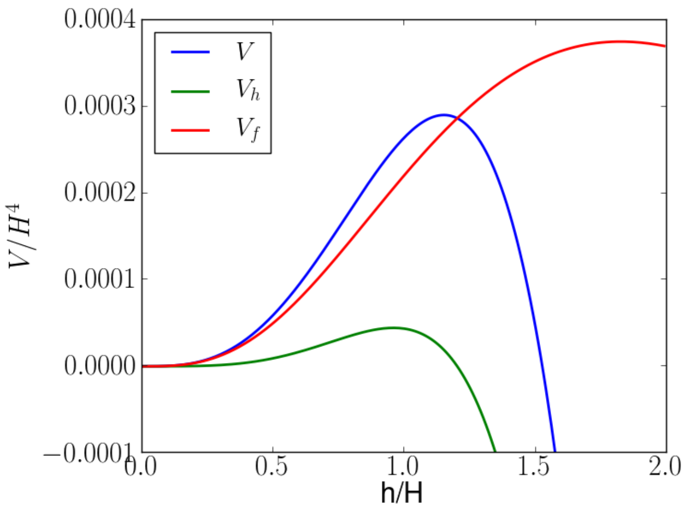


**Figure 3.5:** *Plot of $V/H^4$ as a function of $h/H$ for $H = 10^{10}\,GeV$. With the additional effect of the fermionic contribution to the potential $V_f$, the total potential $V = V_h + V_f$ has a barrier which is five times higher.*

The main difference in comparison with the calculation in Section 3.3 is that the fermion mass is not a constant but now depends on the Higgs vev. The relevant term in (3.20) is $ma(\eta) = y_t \frac{h}{\sqrt{2}} a(\eta)$ which clearly varies as $h$ changes. We need to establish if assuming that the mass is constant is a good approximation to trust our calculation. To do this we need to compare the variation with time of the Higgs field with that of the scale factor and ensure that $\frac{h'}{h} \ll \frac{a'}{a}$, where $' \equiv \frac{d}{d\eta}$. The variation with time of the Yukawa coupling is not considered since it would come from its running, but it should be close to zero since we are assuming close to perfect de-Sitter and $\mu \approx H$. If we assume that the renormalisation scale $\mu$ is given by $h$ and not $H$, then the only difference will be a larger value of $y_t$ during the first $e$-fold of inflation. However, during that time, the top quarks are almost massless ($h \ll H$) and their production negligible, so we do not expect the calculation to be sensitive to this choice.

The Higgs will jump stochastically due to quantum fluctuations, and in one $e$-fold the size of a single quantum jump is $H/2\pi$ [107]; therefore,

$$\frac{dh}{dN_e} = \frac{H}{2\pi}\,, \tag{3.34}$$

and from the definition of the Hubble parameter $a' = Ha^2$, the assumption of having a constant mass, in this case, is rewritten such that

$$\frac{h'}{h} \ll \frac{a'}{a} \Rightarrow \frac{H}{2\pi} \ll h\,. \tag{3.35}$$

From (3.4) this is true after the first $e$-fold. Before that, the Higgs vev is close to zero, making the fermion almost massless, and since the production of the fermions is proportional to their mass, it is safe to neglect the production from the time when (3.35) does not hold.

We have shown in this section how the gravitationally produced top quarks will contribute to the Higgs potential. In the next section, we will study how this might affect the stability of the electroweak vacuum during inflation.

## 3.4 Stability study

The Higgs field during inflation is moving stochastically. Even though the variance of the field is given by (3.4) or (3.5), the probability distribution function extends to infinity. Once the stationary solution is reached (3.5), from the Fokker Planck equation

$$P_{\text{static}} = \frac{2\sqrt{\pi\sqrt{2\lambda/3}}}{\Gamma\left(\frac{1}{4}\right) H} e^{-\frac{8\pi^2}{3H^4}\frac{\lambda h^4}{4}} \,. \tag{3.36}$$

Also, initially ($h = 0$) and close to the instability ($\lambda \approx 0$), a stationary solution is not reached, the variance is given by (3.4) and the mean is zero, then the Higgs acquires a Gaussian distribution

$$P(h, N_e) = \frac{1}{\sqrt{2\pi\langle h^2\rangle}} e^{-\frac{h^2}{2\langle h^2\rangle}} \,, \qquad \langle h^2\rangle = \left(\frac{H}{2\pi}\right)^2 N_e \,. \tag{3.37}$$

Therefore there is a possibility of going over the barrier and ending up in an anti-de Sitter region. Since this is not the case in our current Hubble horizon, which is composed of $e^{3N_e}$ causally independent regions, we need to impose the condition that the probability of going over the barrier is, at least, smaller than $e^{3N_e}$ because none of these regions can be in an anti-de Sitter space-time.
We do not study the evolution of the Higgs field after inflation and the possibility that even if the Higgs goes over the barrier, thermal effects can make it go back to the False vacuum. For more details concerning this, the reader should consult [35].

First, we solved numerically the Langevin equation (3.3) using the modified potential (3.31) and obtained the probability that after 60 $e$-folds of inflation, the Higgs field would have gone over the barrier, using both prescriptions to determine the scale, $\mu^2 = h^2 + H^2$ (Fig. (3.6)) and $\mu = h$ (Fig. (3.7)). To get reliable statistics we simulated $10^5$ realizations. The way we determined if the Higgs goes over the barrier is, after 60 $e$-folds, if $V'(h_{60}) < 0$, it has gone over the barrier and in the opposite case, it has not.

For the choice of scale $\mu^2 = h^2 + H^2$ and $H > 10^{10.2}$ GeV, the Higgs is always unstable (probability is always one). This is because the value of $\lambda$ is always negative and the Higgs, independently of its vev, ends up in an AdS vacuum. Once the

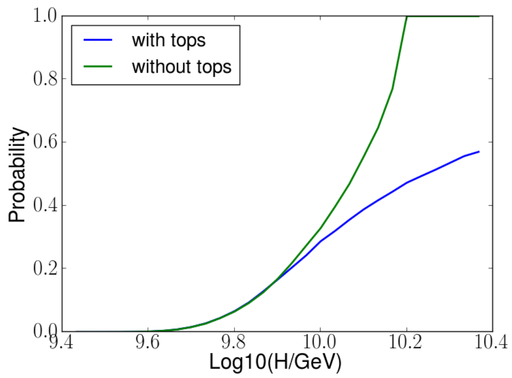


**Figure 3.6:** Probability of going over the barrier after 60 $e$-folds for a renomalisation scale $\mu^2 = h^2 + H^2$.

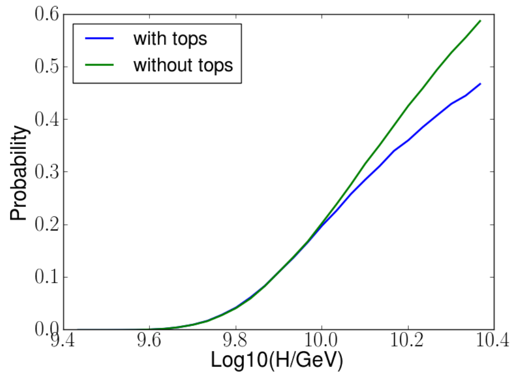


**Figure 3.7:** Probability of going over the barrier after 60 $e$-folds for a renomalisation scale $\mu = h$.

production of top quarks is taken into account, the probability is smaller than one since even though the value of $\lambda$ is negative, there is still a barrier generated by the top quarks preventing the Higgs from ending in an AdS region.

Comparing both plots, it can be seen that for the prescription $\mu^2 = h^2 + H^2$, one is more likely to end in an AdS region for values of the Hubble parameter close to $10^{10}$GeV than in the case where the scale is just given by the Higgs vev. It makes sense since in the former case there is a minimum value for the scale $\mu = H$, and therefore the value of $\lambda$ is smaller and closer to zero, independently of the Higgs vev. In this case, the Higgs feels *less* the potential and can acquire a larger vev during 60 $e$-folds of inflation.

Also, it can be seen that if the scale of inflation is reduced, there is almost no difference adding the top quarks to the potential - since the contribution from the fermions is determined by the scale of inflation, the smaller the scale, the smaller the effect. However, for the cases where it is crucial, the top quarks can reduce that probability by up to 50% in the prescription $\mu^2 = h^2 + H^2$ and 10% for $\mu = h$.

The stability condition is that the probability $P < e^{-3N_e}$. For 60 $e$-folds, it is not possible to study it numerically since the number of realisations that we would need is of the order $e^{3N_e}$. Instead we estimate the effect from the fermions analytically.

As can be inferred from Fig. (3.6) and (3.7), the effect from the top quarks for values of the scale of inflation $H < 10^{10}$GeV is very small, so we can treat this effect perturbatively.
Following the work in [33, 123] for the study of the Higgs instability without the top quarks, it was shown that for values of $H < h_{max}$ ($h_{max}$ is the Higgs vev at the maximum of the potential), within 60 $e$-folds of inflation, the Higgs acquires a constant variance, Eq. (3.5). Since the time it takes to reach an equilibrium distribution is given by $N_e = 1/\sqrt{\lambda}$ and we are studying the situation where $H < h_{max}$, $\lambda$ is never so small that the time it would take to reach the equilibrium distribution was longer than 60 $e$-folds. In this situation, the stochastic motion is compensated with the gradient of the Higgs potential and acquires an equilibrium distribution (3.36). Once an equilibrium has been reached

$$P(h > h_{max}) = 1 - \int_{-h_{max}}^{h_{max}} \frac{2\sqrt{\pi\sqrt{2\lambda/3}}}{\Gamma\left(\frac{1}{4}\right) H} e^{-\frac{8\pi^2}{3H^4}\frac{\lambda h^4}{4}} = \frac{\Gamma\left(\frac{1}{4}, \frac{8\pi^2}{3H^4} V_{\max}\right)}{\Gamma\left(\frac{1}{4}\right)}, \tag{3.38}$$

where $\Gamma(x, y)$ is the incomplete Gamma function [160], $V_{\max}$ is the value of the Higgs potential at its maximum and is larger than the scale of inflation, then $P(h > h_{max}) \approx e^{-\frac{8\pi^2}{3H^4}V_{\max}}$, so the stability condition is

$$\frac{8\pi^2}{3H^4} V_{\max} > 3N_e \,. \tag{3.39}$$

The difference from previous studies is that we now consider what is the effect of the top quarks, using (3.31) evaluated at a scale $\mu = h_{\max}$ for both prescriptions, since we are studying the cases where $H < h_{\max}$. The value of the Higgs self-interaction $\lambda$ close to $h_{\max}$ is estimated as in [35], $\lambda = 0.08/(4\pi)^2$. It is helpful to define $x = h_{\max}/H$, which for the case of the Higgs without the addition of the top quarks to the potential is $x_{\rm h} = 15.24$ for 60 $e$-folds of inflation. Giving us a bound on the scale of inflation to make the electroweak vacuum stable, $H < 8 \cdot 10^8$GeV.

The effect of the fermions coupled to the Higgs is parametrized like

$$\frac{x_{\mathrm{h+f}}}{x_{\mathrm{h}}} = 1 - \alpha \,, \tag{3.40}$$

where there is a minus sign since this effect makes the Higgs more stable, i.e. the bound on the maximum scale of inflation is slightly larger. In general, this can be generalised to any quark with a Yukawa coupling to the Higgs. The difference in the stability scale induced by this effect is shown in Fig. (3.8). The value of the top Yukawa coupling at the instability scale is 0.5 from Fig. (3.1), and therefore $\alpha_{\mathrm{top}} \sim 10^{-13}$.

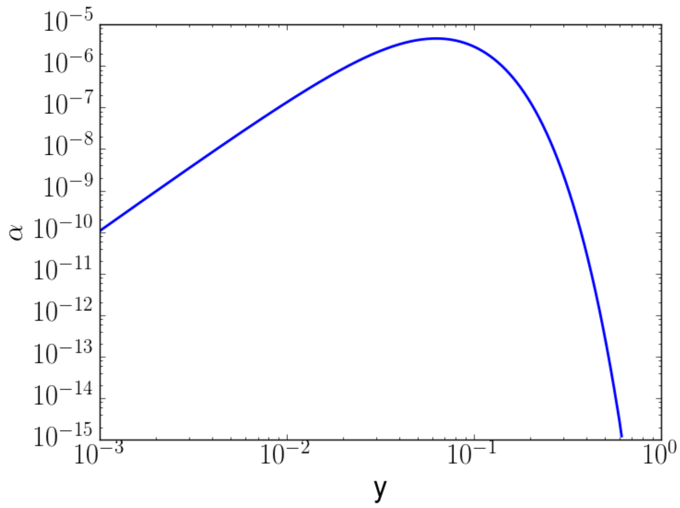


**Figure 3.8:** *The effect of the fermions on the stability of the EW vacuum, parametrized by $\alpha$ as a function of the Yukawa coupling $y(\mu = h_{max})$. It peaks at $y = 0.062$, close to the value for the bottom quarks. Larger Yukawa couplings have a smaller effect on the stability of the EW vacuum.*

The maximum difference on the stability scale comes from a Yukawa coupling of $y \sim 10^{-2}$, i.e. bottom quarks, and that would be $\alpha_{\mathrm{bottom}} \sim 10^{-6}$, still a small difference but orders of magnitude larger than the effect from the top quarks.
This makes sense since the value of the potential at the maximum is given by $V_{\mathrm{max}} = \frac{\lambda}{4} h_{\mathrm{max}} + V_f(h_{\mathrm{max}})$. The contribution from the fermions ($V_f$) peaks at $h_{\mathrm{peak}} = 0.96 H/y$, so the maximum contribution from them to $V_{\mathrm{max}}$ is when $h_{\mathrm{max}} = h_{\mathrm{peak}}$. Therefore for $x = 15.24$, it gives $y = 0.06$ as we see in Fig. (3.8). In

situations where $h_{\text{peak}} > h_{\text{max}}$ the larger the Yukawa coupling, the larger the effect on the stability of the EW vacuum.
Overall if there were to be a significant change to the study of the electroweak vacuum it would come from the top quarks since, proportionally, they modify the Higgs potential the most (see Fig. (3.6) and (3.7)). Although in a scenario where the scale of inflation is small enough such that there is not a problem with the stability of the Higgs, then the biggest effect would come from the bottom quarks despite being a tiny effect.

The Standard Model Higgs also gives mass to the ElectroWeak gauge bosons $(W^{\pm}, Z)$ and therefore their production during inflation may also affect the dynamics of the Higgs during inflation [159,161,162]. They are split into two transverse and one longitudinal degree of freedom. The former are produced as fermions or conformal scalar particles [161], meaning they are conformally invariant in the massless limit and their production grows proportional with the mass, but for masses larger than the scale of inflation, they are exponentially suppressed. Their contribution to the Higgs dynamics during Inflation is equivalent to the already studied fermions but with masses related to the Higgs as: $m_W = gh/2$ and $m_Z = \sqrt{g^2 + g'^2}h/2$, where $g$ and $g'$are the gauge couplings of $SU_L(2)$ and $U_Y(1)$, respectively. Therefore the study of the fermion production showed in here can be easily mapped to the transverse gauge bosons contribution from the Standard Model and also enhance the bounds on stability. The only difference in the contribution to the Higgs potential (besides their mass) is the number of degrees of freedom, which for $W^{\pm}$ bosons is four (two transverse modes per $W$) and two for $Z$ bosons as opposed to three from the colour charge of the quarks.
The longitudinal degrees of freedom behaves differently [161], for the low mass bosons their dispersion relation matches the one of a minimally coupled scalar, i.e. their variance grows inversely proportional to their mass, since the infrared modes can be tachyonically excited. Although for heavy bosons, their production is also exponentially suppressed. Since the maximum value for the Higgs is larger than the Hubble scale, the contribution to the stability from longitudinal degrees of freedom

is negligible in comparison with the contribution from fermions or transverse degrees of freedom [159].

## 3.5 Discussion

With only Standard model particle physics, the Higgs field $h$ seems to become unstable at renormalisation scale $\mu > 10^{10} GeV$ and from the non detection of primordial tensor perturbations, we know that during inflation $H < 10^{13} GeV$. If inflation occurs with a value of $H$ within this range, there is generically a problem with the stability of the Higgs field.

In this work, we have shown how without the addition of physics Beyond the Standard Model, the gravitational production of quarks during inflation changes the Higgs potential in such a way as to make it more stable.

Since the Higgs vev gives the quarks their mass, if it obtains a significant value during inflation, the fermions become relevant to the Higgs potential as shown in (3.31). This contribution can be large enough to prevent the Higgs from being pushed into the true vacuum during inflation in borderline cases (Figures (3.6) and (3.7) ).

It is also clear from the stability study (Sec. 3.4) that since we have not added anything new to the SM and there are no free parameters, there is no apparent possibility of improving these results. At the very least, it is possible to extend the stability of the Higgs a little bit (3.40).

Nevertheless, we find this an interesting and noteworthy effect. Possible future extensions of this work would be looking at the effect of fermions beyond the standard model to see if there is any way that they would change the situation. In summary, in the Standard Model, the Higgs field seems to be unstable during inflation, but slightly less unstable than before this effect is taken into account.

# Chapter 4

# Conclusion

The study of quantum fields in a curved background leads to several intriguing effects. In this thesis, we have focused mostly on the gravitational particle creation during inflation.In chapter 1, we motivated inflation as the theory to explain the current problems that face Big Bang Nucleosynthesis. Although other theories could also lead to the same observables, we argue that inflation stands out for its simplicity. Particle creation during inflation manifests itself generating the density perturbations in the CMB. Interestingly as well, at high energies, the Higgs self-interaction term runs to negative values, which is an issue during inflation. The Higgs field could be destabilized due to quantum fluctuations, making it difficult to explain how it is in the false vacuum nowadays.

In chapter 2, we address the issues of inflation. Despite their many successes, they all rely on the initial conditions of the inflaton field 60 $e$-folds before it ends. It not only relies on the initial value of the inflaton but also it requires a homogeneous initial configuration. Furthermore, there are several models where the inflaton acquires super-Planckian expectation values. To address these issues, we consider the Hawking temperature associated with the de-Sitter horizon. We study an inflaton field with Mexican-hat shape potential and N conformal fields in thermal equilibrium with the Hawking temperature. If the number of fields is large enough, we have shown how temperature corrections to the inflaton potential change its evolution. In our scenario, the field is initially trapped at the origin acting as a cosmological constant. The effect of the Hawking radiation makes it decays, extremely slow. At some point, the critical temperature is reached, and the symmetry is broken, at this point, the field is effectively light and acquires an expectation value due to quantum fluctuations. On average, the field is still at the origin until the temperature has dropped enough to make the gradient in the potential larger than the quantum fluctuations, and a quantum to classical transition occurs. Then the field starts to classically roll down the potential to the minimum. During this process, we have shown that the field produces the right perturbations observed by CMB measurements. 60 $e$-folds after the right perturbations are generated, inflation needs to stop and transfer all the energy in the universe to SM degrees of freedom.

In chapter 3, we have studied the SM during a high energy epoch of inflation. In particular, we know that if the scale of inflation is larger than $10^9$ GeV the Higgs field would have been excited to its true vacuum. We studied the effect of SM fermions on the stability of the EW vacuum. Generally, fermions during a de-Sitter epoch are excited; the production is larger for more massive fermions peaking at masses of the same order that the Hubble parameter. SM fermions mass is proportional to the Higgs vev. During inflation, the expectation value of the Higgs field grows to make the fermions massive. Therefore, they are produced and modify the evolution of the Higgs field. We found that the probability of the Higgs going over its barrier can be severely reduced thanks to the production of top quarks. However, in order to make our current observable universe stable, the probability of the instability happening needs to be extremely small. In these situations, the peak produced in the Higgs potential by the fermions is not high enough to make a sizeable contribution to the stability of the EW vacuum. For SM fermions, we found that bottom quarks increase the stability of the Higgs the most, although the effect is so small that the improvement is nothing else but a curiosity. Nevertheless, we notice that this effect could be interesting for fermions Beyond the SM.

In this thesis, we have investigated the physics of the early universe with a new model of inflation and the stability of the electroweak vacuum. We find our inflation model to have a few theoretical improvements to the current models, and we showcase the noteworthy contribution that the gravitational creation of fermions could have on the Higgs stability. We find all these effects fascinating and we hope that they may help towards a better understanding of our universe.

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