Husimi distribution for Quark

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Introduction

The Wigner distribution, a quasi-probability distribution, was introduced by Wigner in 1932 while studying quantum corrections to statistical mechanics. Since then, the Wigner distribution has found applications in various branches of physics, including quantum chromodynamics (QCD) [1–5]. It is useful for studying the phase space probability distribution of quarks, but its non-positive definiteness makes it difficult to interpret as a true probability distribution and to use for defining quantum information entropy for quantum systems. The Husimi distribution [6], another phase space distribution, has an advantage over the Wigner distribution because it is always positive definite. This quality makes it suitable for both, interpretation as a true probability distribution and for defining quantum information entropy. In this article, we present the Husimi distribution alongside the Wigner distribution for quarks using the dressed quark model.

Wigner Distribution for Quark

The Wigner distribution for quark is defined as [1, 4]

$$\rho^{[\Gamma]}(x, b_\perp, k_\perp; S) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \times W^{[\Gamma]}(x, \Delta_\perp, k_\perp; S),$$

where $W^{[\Gamma]}(x, \Delta_\perp, k_\perp; S)$ is the quark-quark correlator, $b_\perp$ is the impact parameter, $k_\perp$ is the transverse momentum of the quark, $\Delta_\perp$ is the momentum transferred to the target in the transverse direction, S is the polarization of the target, and Γ is an element of the set $\{\gamma^+, \gamma^+\gamma^5, i\sigma^{+j}\gamma^5\}$. For an unpolarized quark inside an unpolarized target, $\Gamma = \gamma^+$, and the Wigner distribution ρ_{UU} is defined as

$$\rho_{UU}(x, b_\perp, k_\perp) = \frac{1}{2}\Big[\rho^{[\gamma^+]}(x, b_\perp, k_\perp; +e_z) + \rho^{[\gamma^+]}(x, b_\perp, k_\perp; -e_z)\Big].$$

For a quark dressed with a gluon at one loop, the Wigner distribution can be obtained using the overlap representation of the two-particle wavefunction as [1]

$$\rho_{UU}(b_\perp, k_\perp, x) = N \int d\Delta_x \int d\Delta_y \times \frac{\cos(\Delta_\perp \cdot b_\perp)}{D(q_\perp)D(q'_\perp)} \left(I_1 + \frac{4m^2(1-x)}{x^2}\right),$$

where m is the mass of the quark,

$$D(k_\perp) = \left(m^2 - \frac{m^2 + k_\perp^2}{x} - \frac{k_\perp^2}{1-x}\right),$$
$$I_1 = 4\left(k_\perp^2 - \frac{\Delta_\perp^2(1-x)^2}{4}\right)\left(\frac{1+x^2}{x^2(1-x)^3}\right),$$

and

$$q'_\perp = k_\perp - \frac{\Delta_\perp}{2}(1-x),$$
$$q_\perp = k_\perp + \frac{\Delta_\perp}{2}(1-x),$$

are the Jacobi momenta for quark in the symmetric frame.

Husimi Distribution for quark

The Gaussian smearing of the Wigner distribution results in the Husimi distribution. Therefore, the Husimi distribution for

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an unpolarized quark in an unpolarized target can be defined as [6].

$$xH_{UU}(x, b_\perp, k_\perp) = \frac{1}{\pi^2}\int d^2b'_\perp d^2k'_\perp \; e^{-(b_\perp - b'_\perp)^2/l^2 - l^2(k_\perp - k'_\perp)^2} \; x\rho_{UU}(x, b'_\perp, k'_\perp).$$

Here l is some arbitrary constant.

Result and Discussion

Figure 1 shows the 3-dimensional plot of the Wigner distribution for an unpolarized quark in an unpolarized target state in position space. The distribution is symmetric along the line $b_x = -b_y$ and is positive. Figure 2 presents the Husimi distribution for the same system, obtained by applying Gaussian smoothing to the corresponding Wigner distribution. The Husimi distribution is symmetric about the origin and is positive throughout the entire position space. For both plots, we have integrated out the fractional longitudinal momentum and set the transverse momentum of the quark to $k_\perp = 0.4\hat{j}$ GeV. Since

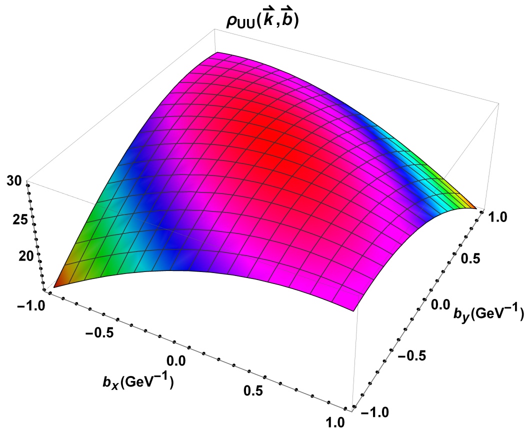

FIG. 1: Plot of Wigner distribution (ρ_{UU}) for quark in position space with mass, m=0.33 GeV.

both distributions are positive, either can be used to define the Wehrl entropy associated with the unpolarized quark in an unpolarized target. However, this may not hold true in momentum and mixed space.

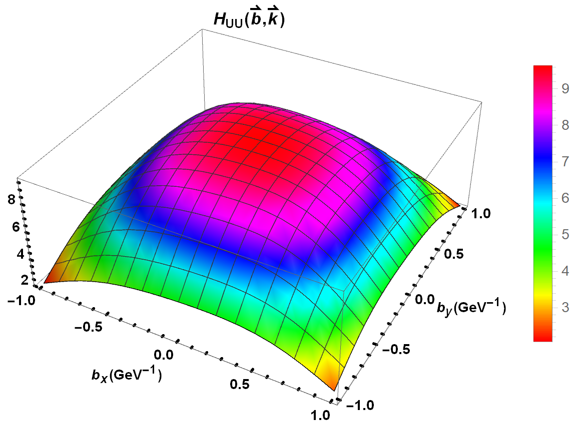

FIG. 2: Plot of Husimi distribution (H_{UU}) for quark in position space with mass, m=0.33 GeV, and $l = 0.5$.

Conclusion

We presented the Wigner distribution and Husimi distribution of quarks in the dressed quark model in position space. Both distributions are positive definite, allowing either to be used to define the Wehrl entropy, at least in position space. The Wigner and Husimi distributions for quark in momentum and mixed spaces needs to be further explored and may be presented in the symposium.

References

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