The model for describing the width of double-gamma decay of the quadrupole state of spherical nuclei

A. P. Severyukhin[1,2] **and N. N. Arsenyev**[1]

[1]Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
[2]Dubna State University, 141982 Dubna, Moscow Region, Russia

E-mail: `sever@theor.jinr.ru`

Abstract. We study the competitive $\gamma\gamma$-decay of the 2_1^+ state of an even-even spherical nucleus. An influence of the phonon-phonon coupling on the double γ-decay width is analysed within the microscopic model based on the Skyrme energy density functional. It is shown that the $\gamma\gamma$-decay width is sensitive to the interaction of the one- and two-phonon configurations of the giant dipole resonance.

The $\gamma\gamma$-decay of a nuclear transition in competition with an allowed γ-decay has been discovered [1]. This is the observation of the $\gamma\gamma$-decay of the first excited $J^\pi = 11/2^-$ state of ^{137}Ba directly competing with an allowed γ-decay to the $J^\pi = 3/2^+$ ground state. The branching ratio of the competitive $\gamma\gamma$-decay of the $11/2^-$ isomer of the odd-even nucleus ^{137}Ba to the ground state relative to its single γ-decay was determined to be $(2.05 \pm 0.37) \times 10^{-6}$. This discovery has very recently been confirmed and the data were made more precise, in particular with respect to the contributing multipolarities [2].

The $\gamma\gamma$-decay reactions in the even-even nuclei are known, only, in three particular cases, ^{16}O [3, 4] and ^{40}Ca, ^{90}Zr [5, 6], where the first excited states of these even-even nuclei have spin and parity quantum numbers 0^+ and a single γ-decay is strictly forbidden by helicity conservation. The paper [7] reports on the more general situation, in which the $\gamma\gamma$-decay of the low-energy quadrupole state of the even-even nucleus occurs in a nuclear transition which could proceed by a single γ-decay in competition.

To describe the double γ-decay, a formalism relates the electromagnetic interaction up to second order in the electromagnetic operators and two-quantum processes in atomic nuclei, see e.g., [8]. Using $\hbar = c = 1$, the $\gamma\gamma$-decay width can be expressed as

$$\Gamma_{\gamma\gamma'} = \frac{64\pi}{42525}\left(E_{2_1^+}\right)^7 (\alpha_{E1E1})^2 (1+\delta)\,, \tag{1}$$

$$\alpha_{E1E1} = \sum_i \frac{\langle 0_{gs}^+||M(E1)||1_i^-\rangle\langle 1_i^-||M(E1)||2_1^+\rangle}{E_{1_i^-} - 0.5E_{2_1^+}}\,. \tag{2}$$

The $\gamma\gamma$-decay width is dominated by the $E1E1$ contribution, i.e., $\delta \ll 1$ [7]. In this connection, the electric dipole polarizability,

$$\alpha_D = \frac{8\pi}{9}\sum_i \frac{\langle 0_{gs}^+||M(E1)||1_i^-\rangle\langle 1_i^-||M(E1)||0_{gs}^+\rangle}{E_{1_i^-}}, \tag{3}$$

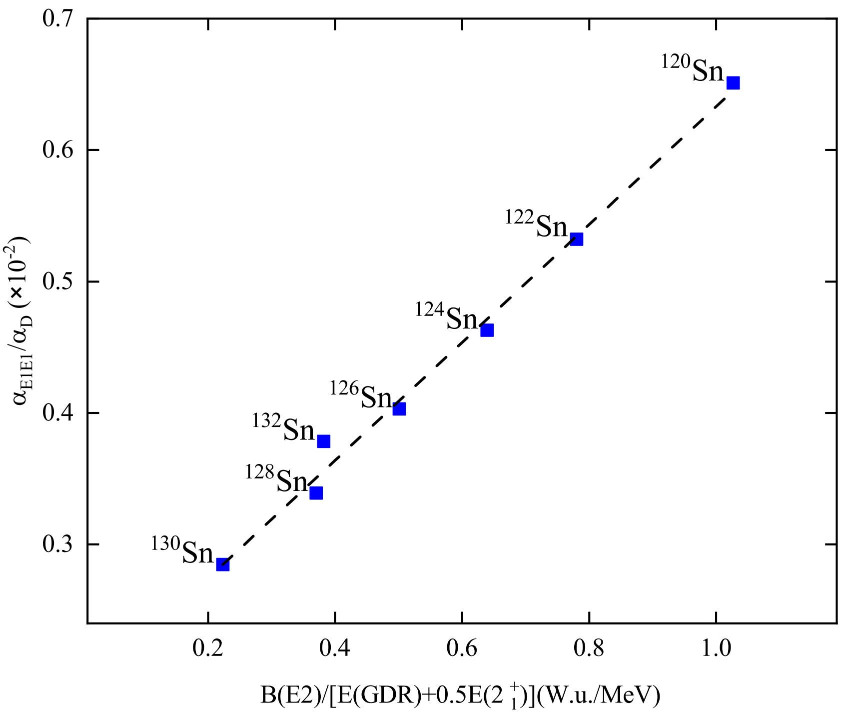

Figure 1. The ratio between polarizabilities α_{E1E1} and α_D as a function of $B(E2; 2_1^+ \to 0_{gs}^+)$ value with respect to the energy centroid of the GDR $[E(GDR)]$ and the 2_1^+ energy $[E(2_1^+)]$. The α_{E1E1}/α_D evolution of neutron-rich tin isotopes (squares) is calculated taking into account the phonon-phonon coupling based on the SLy5 EDF. The dashed line corresponds to the linear fit predicted Eq. (6).

is playing an important role, in particular, its value has strong implications in constraining the symmetry energy including its density dependence and the slope parameter of the nuclear equation of state [9]. As proposed in [7], the comparison between α_D and α_{E1E1} turns out particularly useful.

The solutions of the quasiparticle random phase approximation (QRPA) with the Skyrme energy density functional (EDF) [10] are treated as quasibosons. Among these solutions there are one-phonon states corresponding to collective states of the giant dipole resonance (GDR) and pure $1p-1h$ states. The configurations with various degrees of complexity can be built by combining different one-phonon configurations of fixed quantum number $J^\pi = 1^-$. Taking into account the basic ideas of the quasiparticle-phonon model (QPM) [11], the Hamiltonian is then diagonalized in a space spanned by states composed of one, two and three QRPA phonons [7, 12, 13]. The equations of the phonon-phonon coupling (PPC) have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the chosen Skyrme EDF without any further adjustments [14, 15]. As our test case we considered ^{48}Ca for which its dipole polarizability has recently been measured in the range from 10 to 25 MeV, $\alpha_D = 1.73 \pm 0.18$ fm^3 [16]. We use the SLy5 EDF adjusted to reproduce nuclear matter properties, as well as nuclear charge radii, binding energies of doubly magic nuclei [17]. The calculated value ($\alpha_D = 2.17$ fm^3) is in satisfactory agreement with the experimental data. It can be compared also to the generalized dipole polarizability, $\alpha_{E1E1}/\alpha_D = 0.0086$ in the energy region below 27 MeV [7]. The branching ratio of the competitive $\gamma\gamma$-decay relative to its single γ-decay is calculated as 3×10^{-8}.

As shown in [7], the $\gamma\gamma$-decay width is sensitive to the mixing of the collective one-phonon configurations and the two-phonon configurations composed of the 2_1^+ phonon. The full diagonalization can be reasonably well simulated by using the two-state mixing with the interaction V between the states,

$$|1_I^-\rangle = \alpha|GDR\rangle + \beta|GDR \otimes 2_1^+\rangle, \quad (4)$$
$$|1_{II}^-\rangle = -\beta|GDR\rangle + \alpha|GDR \otimes 2_1^+\rangle. \quad (5)$$

where the GDR state is built on the most collective one-phonon states [7]. This model can provide the simple expression

$$\frac{\alpha_{E1E1}}{\alpha_D} = \frac{9}{8\pi}\frac{V}{E(GDR) + 0.5E(2_1^+)}. \quad (6)$$

For the PPC calculation, the interaction V is closely related to the collectivity of the 2_1^+ phonon which can be characterized by the reduced transition probability ($B(E2; 2_1^+ \to 0_{gs}^+)$) in Weisskopf units (W.u.).

It is interesting to further investigate the evolution of the α_{E1E1}/α_D ratio in the nuclear chart. In this report we present our preliminary studies for the tin isotopic chain. As the parameter set in the particle-hole channel, we use again the SLy5 EDF. The pairing correlations are generated by the surface-peaked pairing force fixed in Ref. [18]. We restrict the PPC calculations by the simplest case of the coupling between the one- and two-phonon configurations. The results for the case of ^{48}Ca demonstrates theoretical validity of this approximation [7]. We have included in our model space different multipoles λ^π=1^-, 2^+, 3^-, and 4^+. This means that the two-phonon configurational space consists of the phonon compositions $[\lambda^\pi \otimes \lambda'^{\pi'}]_{QRPA}$, i.e., $[3^- \otimes 2^+]_{QRPA}$, $[3^- \otimes 4^+]_{QRPA}$, $[1^- \otimes 2^+]_{QRPA}$. The energies and transition probabilities of the first collective phonons represent important fingerprints for minimal two-phonon energy and the maximal matrix elements for coupling of the one- and two-phonon configurations. All one- and two-phonon configurations with energies up to 28 MeV are included. The inclusion of high-energy configurations plays a minor role in our calculations.

We consider the properties of the 2_1^+ states and the $E1$ strength distributions of ^{120,122,124,126,128,130,132}Sn. Generally there is a satisfactory agreement between theory and experiment [19]. By means of the PPC calculation we attempt to understand the complex structure of the $E1$ strength distribution, and gain insight into the $\gamma\gamma/\gamma$ branching ratio of the 2_1^+ state which is predicted from 1×10^{-9} (^{120}Sn) to 3×10^{-9} (^{130}Sn). The branching ratio of 9×10^{-9} in the case of ^{132}Sn corresponds to an evolution near closed shells. At the same time we find the gradual reduction of α_{E1E1} with increasing neutron number from 0.056 fm^3 (^{120}Sn) to 0.027 fm^3 (^{130}Sn). The opposite effect is seen for the α_D evolution. However, this does not counteract the similar evolution of the α_{E1E1}/α_D ratio, see Fig. 1. Figure 1 shows that the results of the PPC calculation confirms the findings (6), i.e., the linear correlation between the α_{E1E1}/α_D ratio and the $B(E2)$ value with respect to the energy centroid of the GDR and the 2_1^+ energy.

In summary, we discuss the special role of the mixing of the simple and complex configurations of the GDR for describing the double γ-decay width of the 2_1^+ state in the case of an even–even spherical nucleus. We use the Skyrme EDF SLy5 to create a single-particle spectrum and to analyze excited states of $^{120-132}$Sn. Our calculations take into account the coupling between one- and two-phonon terms in the wave functions. We conclude that the two-state scenario may provide a globally applicable analysis of the $\gamma\gamma$-decay width of the lowest quadrupole excitation. A further systematic study of the impact of the phonon-phonon coupling on the $\gamma\gamma$-decay width is clearly necessary and is in progress.

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References

[1] Walz C, Scheit H, Pietralla N, Aumann T, Lefol R and Ponomarev V Yu 2015 *Nature* **526** 406

[2] Söderström P-A, Capponi L, Açiksöz E, Otsuka T, Tsoneva N, Tsunoda Y, Balabanski D L, Pietralla N, Guardo G L and Lattuada D *et al.* 2020 *Nature Commun.* **11** 3242

[3] Watson B A, Bardin T T, Becker J A and Fisher T R 1975 *Phys. Rev. Lett.* **35** 1333

[4] Hayes A C, Friar J L and Strottman D 1990 *Phys. Rev. C* **41** 1727

[5] Schirmer J, D. Habs, R. Kroth, N. Kwong, D. Schwalm, M. Zirnbauer and Broude C 1984 *Phys. Rev. Lett.* **53** 1897

[6] Kramp J, Habs D, Kroth R, Music M, Schirmer J, Schwalm D and Broude C 1987 *Nucl. Phys. A* **474** 412

[7] Severyukhin A P, Arsenyev N N, and Pietralla N 2021 *Phys. Rev. C* **104** 024310

[8] Grechukhin D P 1963 *Nucl. Phys.* **47** 273

[9] Roca-Maza X and Paar N 2018 *Prog. Part. Nucl. Phys.* **101** 96

[10] Terasaki J, Engel J, Bender M, Dobaczewski J, Nazarewicz W and Stoitsov M 2005 *Phys. Rev. C* **71** 034310

[11] Soloviev V G 1992 *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Bristol and Philadelphia: Institute of Physics)

[12] Severyukhin A P, Voronov V V and Nguyen Van Giai 2004 *Eur. Phys. J. A* **22** 397

[13] Severyukhin A P, Arsenyev N N and Pietralla N 2012 *Phys. Rev. C* **86** 024311

[14] Nguyen Van Giai, Stoyanov Ch and Voronov V V 1998 *Phys. Rev. C* **57** 1204

[15] Severyukhin A P, Stoyanov Ch, Voronov V V and Nguyen Van Giai 2002 *Phys. Rev. C* **66** 034304

[16] Birkhan J *et al.* 2017 *Phys. Rev. Lett.* **118** 252501

[17] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 *Nucl. Phys. A* **635** 231

[18] Severyukhin A P, Voronov V V and Nguyen Van Giai 2012 *Prog. Theor. Phys.* **128** 489

[19] Severyukhin A P *et al.* in preparation