

LATTICE CORRECTION AND POLARIZATION ESTIMATION FOR THE FUTURE CIRCULAR COLLIDER $e+e^-$ *

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Abstract

Precise determination of the center-of-mass energy in the Future Circular Collider $e+e^-$ (FCC-ee) at Z and W energies can be achieved by employing resonant spin depolarization techniques, for which a sufficient level of transverse beam polarization is demanded under the presence of machine imperfections. In this study, the FCC-ee lattice has been modeled and simulated with a variety of realistic lattice imperfections, including misalignments, angular deviations, BPM errors, etc., along with refined orbit correction and tune matching procedures. The equilibrium polarization is calculated within the context of realistic machine models, aiming to understand the underlying reason for polarization loss and potentially improve polarization by lattice manipulation.

INTRODUCTION

The Future Circular Collider (FCC) project aims to significantly advance high-energy physics research by investigating the electroweak and Higgs sectors, top quark physics, and exploring physics beyond the Standard Model [1]. The FCC-ee, the electron-positron collider envisioned as the first phase of the FCC project, is designed to operate on the center-of-mass energies ranging from 88 GeV to 365 GeV [2].

The current precision targets for energy calibration at the Z and W energies stand at 4 keV and 100 keV respectively [3]. This precision goal is expected to be met by employing resonant depolarization techniques utilizing radio-frequency external electromagnetic fields. To ensure the efficacy of this method, it is imperative to maintain a minimum beam polarization of 5% to 10% in the presence of diverse machine imperfections [4].

Preliminary spin polarization calculations were conducted to assess the impact of lattice imperfections on polarization by introducing small errors to simulate the ideal residual orbit distortions after orbit correction [5,6]. For the purpose of evaluating tolerances on machine errors to ensure adequate polarization and proposing potential correction schemes to enhance polarization, this study explores the impacts of various machine errors with realistic magnitudes alongside a refined lattice correction procedure. In this study, orbit correction and optics tuning tasks are conducted using the MAD-X [7] code, meanwhile the equilibrium polarization levels at the Z energy (45.6 GeV) are estimated through analytical linearized spin-orbit motion using the Bmad software package [8].

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The Basics of Electron (Positron) Spin Dynamics

Particle spins undergo precession in electromagnetic fields following the T-BMT equation [9, 10]

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{\text{BMT}} \times \vec{S}, \quad (1)$$

where \vec{S} is the spin expectation value of single particle and $\vec{\Omega}_{\text{BMT}}$ is the spin precession vector. In a perfectly aligned flat ring without solenoids, the number of precessions per turn performed by spins on the closed orbit, referred to as the closed orbit spin tune ν_0 , is related to energy as

$$\nu_0 = a\gamma, \quad (2)$$

where a is the anomalous magnetic dipole moment and γ is the Lorentz factor, while ν_0 deviates from $a\gamma$ in the presence of lattice imperfections [11], resulting in a reduction in the precision of energy calibration attained through resonant depolarization.

The spin polarization of stored electron (positron) beams naturally increases due to synchrotron radiation emission, reaching a theoretical maximum level of 92.38% in uniform magnetic fields, which is known as the Sokolov-Ternov (ST) effect [12]. However, the spin diffusion caused by stochastic photon emissions, combined with non-uniform magnetic fields, induces radiative depolarization [11], exerting a counteractive influence against polarization buildup. The competition between these two factors ultimately yields an equilibrium polarization, which is sensitive to the beams orbits and optics. Hence, to ensure an adequate polarization for energy calibration, stringent standards are demanded for orbit correction and optics tuning.

ORBIT CORRECTION AND POLARIZATION ESTIMATION

In this study, adaptations have been implemented to the FCC-ee V22 baseline lattice configuration at the Z pole [13]. These adjustments involve adding one Beam Position Monitor (BPM) and one Orbit Corrector (OC) next to each quadrupole in the ring, as well as configuring sextupole knobs to proportionally control all sextupole strengths.

The designed orbit correction procedure is formulated to alleviate the effects of significant sextupole feed-down, which could otherwise distort the optics and subsequently alter the tune, potentially leading the machine towards resonance conditions. By interleaving orbit correction with gradual recovery of sextupole strength and tune matching, a stable closed orbit can be attained in the presence of substantial machine errors.

The selected critical machine errors under investigation include misalignments, dipole rotation around the s -axis, and BPM errors. They are incorporated into the lattice either individually or in combination. The values of these errors are randomly generated from truncated Gaussian distributions, with defined standard deviation σ and truncation at 2.5σ .

Initially, realistic random misalignments ($\sigma_{dx,dy,dz} = 30\ \mu\text{m} - 100\ \mu\text{m}$) are introduced to all non-IR elements, including dipoles, quadrupoles and sextupoles. Figure 1 shows an example of how horizontal and vertical orbits are minimized using the orbit correction procedure. In the presence of $50\ \mu\text{m}$ random misalignments in the arc, the orbits before are in the millimeter range (blue points), significantly reduced to the $10\ \mu\text{m}$ range after correction (orange points). The points refer to different seeds for the random errors applied.

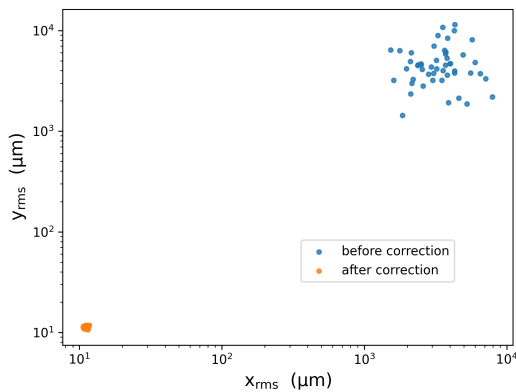


Figure 1: RMS horizontal and vertical orbits for 50 different machines (50 random seeds) before (blue points) and after correction (orange points) in the presence of $50\ \mu\text{m}$ random misalignments of arc elements.

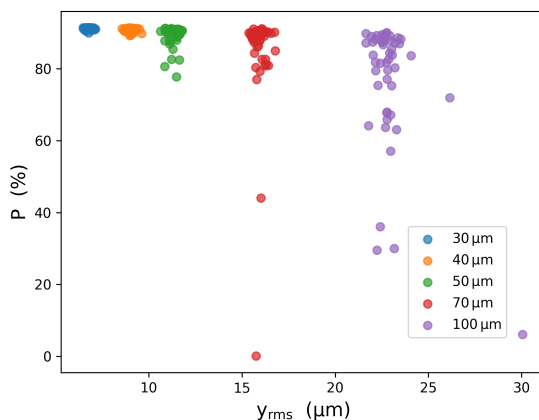


Figure 2: RMS vertical orbits after correction and the computed equilibrium polarizations of 50 seeds in the presence of arc misalignments ($\sigma_{dx,dy,dz} = 30, 40, 50, 70, 100\ \mu\text{m}$).

Varying the magnitude of arc misalignments, Fig. 2 depicts the RMS vertical orbits and equilibrium polarizations

of 50 random seeds after correction. As misalignments increase in magnitude, the orbits and polarizations of 50 seeds show a broader variation. Polarization exhibits a subtle correlation with vertical orbit distortion, although not in a linear manner. By examining cases with abnormally lower polarizations, it was found that factors such as large dispersion and chromaticity contribute significantly to the reduction in polarization.

In addition to misalignments in the arcs, more complex machine errors are introduced to explore their impact on the orbit distortion and polarization level. This includes misalignments in the Interaction Regions (IR) where elements demand more precise alignment due to their significant impact resulting from their very high strengths, dipole rotations around the s -axis ($\sigma_{d\psi}$) which introduce horizontal magnetic fields that can jeopardize polarization, and BPM errors. All the BPM errors that are taken into consideration are scaling errors (σ_{scale}) meaning the proportional deviation from actual reading, resolution (σ_{res}) which is BPM read error, 5% BPM nonavailability, and misalignments errors. The error settings are partially summarized in Table 1.

Table 1: Error Setting

Error Type	$\sigma_{dx,dy,dz}$ (μm)	$\sigma_{d\psi}$ (μrad)	σ_{scale} (%)	σ_{res} (μm)
Arc quad.+sext.	40/50	0	0	0
IR quad.+sext.	10/20	0	0	0
Arc dipoles	40/50	100	0	0
IR dipoles	10/20	0	0	0
BPMs	0/40	0	1	1

Table 2 displays the outcomes of the closed orbit search under different misalignment configurations (Case 1-6), while maintaining the other error settings consistent with those in Table 1. For each configuration, simulations are conducted with 200 different setups of the errors (random seeds), and the percentage of cases where a closed orbit cannot be found is indicated. As expected, larger errors in the IR elements have the most significant impact on closed orbit searching. The proportion of seeds where stable orbits cannot be found noticeably increases with larger errors applied in the IRs. Thus, a more stringent alignment procedure for IR should be developed in the real machine. While not all unsuccessful seeds may directly indicate real-machine challenges, the ratio of instances where a stable closed orbit cannot so far be found in simulation provides an approximate prediction of the misalignment limit, which will offer a target to the FCC-ee orbit correction and optics tuning. In simulation, the inability to find a closed orbit could potentially be addressed through further refinement of the correction process.

Figure. 3 depicts the polarization and the final RMS vertical orbit distributions of all survived seeds of Table 2. Despite potential survivorship bias, larger errors in the IR do not necessarily result in a significant decrease in equilib-

Table 2: Closed Orbit Searching Results

Case	Misalignments (μm)			Failed percentage (%)
	arc	IR	BPM	
1	40	10	0	17
2	40	20	0	39.5
3	50	10	0	21
4	50	20	0	44
5	40	10	40	18
6	40	20	40	38

rium polarization and a drastic increase in orbit distortion after correction, but compound the challenge of closed orbit searching. On the contrary, misalignments in the arc dominate the impact on the orbit. Thus, high standards for IR alignments and orbit correction are demanded to achieve sufficient polarization.

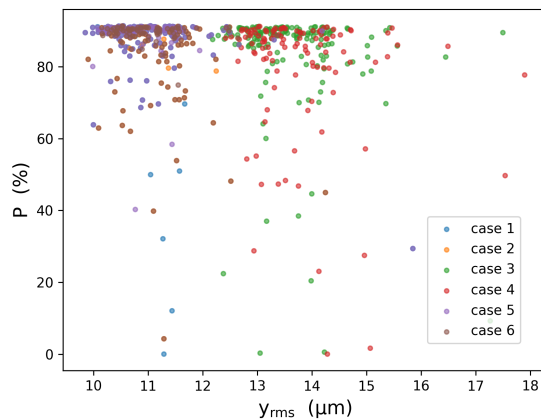


Figure 3: RMS vertical orbits after correction and equilibrium polarizations of 200 seeds under various error settings as in Table 1 and 2.

Spin Tune Shift

In a perfectly aligned flat machine, the beam energy correlates with the spin precession frequency as expressed in Eq. (2). However, this correlation is disturbed by factors such as energy-dependent momentum compaction and vertical orbit distortions, leading to systematic errors in the energy calibration by resonant depolarization [4]. Using the closed orbit spin tune ν_0 from simulations with linearized spin-orbit motion, Fig. 4 (above) illustrates the contribution of arc misalignments to the spin tune shift from ν_0 . As errors increase in magnitude, the variation in spin tune shift also expands. According to Fig. 4 (below), errors in IR drastically increase the spin tune shift variation, leading to a significant decrease in the precision of energy calibration. To achieve the current precision target, spin tune shift $|\nu_0 - a\gamma|$ should be lowered to the level of 10^{-4} , a goal that remains

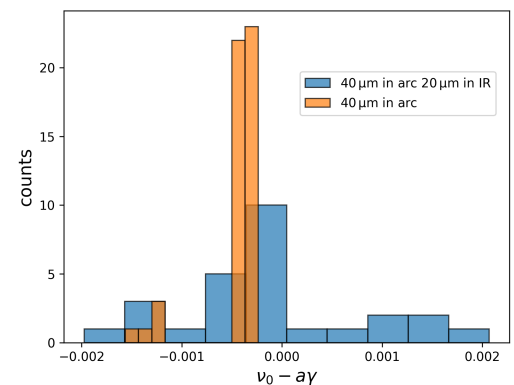
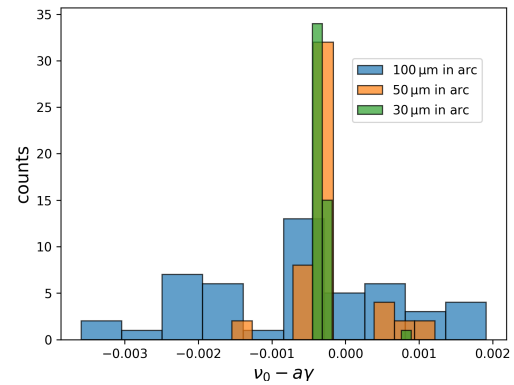


Figure 4: Comparison of spin tune shifts from $a\gamma$ in the presence of different magnitudes of arc elements misalignments (above), and with and without IR misalignments (below).

significantly beyond our current capabilities. Efforts will be dedicated to investigating the sources of systematic errors and devising strategies to minimize their impact.

CONCLUSIONS AND OUTLOOK

In this study, various types of machine errors have been modeled and applied to examine their impact on orbit distortion and polarization, building upon the refined correction and tuning procedure. In summary, high polarization is consistently observed when a stable closed orbit is attained. Notably, IR elements misalignments significantly impact stable closed orbit searching. Further enhancements in polarization levels could be realized through additional dispersion and chromaticity corrections. Future research will seek to establish the threshold for tolerable machine errors to guarantee a sufficient polarization level. More realistic machine errors, such as long-range alignment errors, will be modeled and simulated. Additionally, novel lattice correction techniques will be investigated to optimize machine performance. Lastly, efforts will be committed to exploring the origins of systematic errors and devising strategies to mitigate their effects in order to meet the energy calibration precision target.

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