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VACUUM ENERGY DENSITY IN $\mathcal{P}(\varphi)_2$ THEORIES. INDEPENDENCE OF (NONCLASSICAL) BOUNDARY CONDITIONS

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0. INTRODUCTION

Let $S'(R^2)$ be the space of tempered distributions over the two-dimensional space R^2 and let Σ be the Borel σ-algebra of $S'(R^2)$. By μ_0 let us denote a free field Gaussian measure. Any measure $\mu_{\mathcal{P}}$ on $\{S'(R^2), \Sigma\}$ is called tempered $\mathcal{P}(\phi)_2$ measure [1] iff:

i) $\mu_{\mathcal{P}}$ is locally absolutely continuous with respect to the measure μ_0.

ii) For any localized in the bounded region $\Lambda \subset R^2$ observable $F(\phi)$, the conditional expectation values with respect to the measure $\mu_{\mathcal{P}}$ and the σ-algebra $\Sigma(\Lambda^c)$ are given by the following formula:

$$E_{\mu_{\mathcal{P}}}\{F(\phi)|\Sigma(\Lambda^c)\} = E_{\mu_\Lambda}\{F(\phi)|\Sigma(\Lambda^c)\}, \tag{0.1}$$

where

$$\mu_\Lambda(d\phi) = (Z_\Lambda(\phi))^{-1}\exp(-\lambda\int_\Lambda \mathcal{P}(\phi)dx)\cdot\mu_0(d\phi),$$
$$Z_\Lambda = \int\mu_0(d\phi)\exp(-\lambda\int_\Lambda \mathcal{P}(\phi)(x)dx), \tag{0.2}$$

$\lambda \geq 0$ and $\mathcal{P}(\phi)$ is some bounded from below Wick ordered polynomial in the free field ϕ_0. A given $\mathcal{P}(\phi)_2$ measure $\mu_{\mathcal{P}}$ is called tempered, regular $\mathcal{P}(\phi)_2$ measure iff

$$\underset{c\in R_+}{\exists} : \underset{f\in H_{-1}(R^2)}{\forall} \int\phi^2(f)\,\mu_{\mathcal{P}}(d\phi) \leq c\,||f||^2_{-1} \tag{0.3}$$

and a completely regular $\mathcal{P}(\phi)_2$ measure iff

$$\underset{n=1,2,\dots}{\forall}\ \underset{c_n\in R_+}{\exists} : \underset{\substack{f_i\in H_{-1}(R^2)\\ i=1,\dots,n}}{\forall} \left|\int\prod_{i=1}^n\phi(f_i)\mu_{\mathcal{P}}(d\phi)\right| \leq c_n\prod_{i=1}^n||f_i||_{-1}. \tag{0.4}$$

Let us denote by $\mathcal{G}^t_r(\lambda)$ ($\mathcal{G}^t_{cr}(\lambda)$) the set of all regular (resp. completely regular) solutions of i) and ii).

It can be shown [2-4] that for $\mu_{\mathcal{P}}\in\mathcal{G}^t_r(\lambda)$ the following formulas for the conditional expectation values hold:

$E\mu_\Lambda\{F(\phi)\,|\,\Sigma(\Lambda^c)\}(\eta) =$

$$= (Z_\Lambda(\eta))^{-1}\cdot \int \mu_0^{\partial\Lambda}(d\phi)\,F(\phi+\Psi_\eta^{\partial\Lambda})\exp\Big(-\lambda\int_\Lambda \mathcal{P}(\phi+\Psi_\eta^{\partial\Lambda})(x)\,dx\Big), \qquad (0.5)$$

$$Z_\Lambda(\eta) = \int \mu_0^{\partial\Lambda}(d\phi)F(\phi+\Psi_\eta^{\partial\Lambda})\cdot \exp\Big(-\lambda\int_\Lambda dx\,\mathcal{P}(\phi+\Psi_\eta^{\partial\Lambda})(x)\Big). \qquad (0.6)$$

Here $\mu_0^{\partial\Lambda}(d\phi)$ means the free field measure with the Dirichlet boundary condition on $\partial\Lambda$, ϕ is the free field distributed according to $\mu_0^{\partial\Lambda}$. $\Psi_\eta^{\partial\Lambda}$ is the (unique) solution of the following stochastic Dirichlet problem

$$\begin{aligned} &(-\Delta + m_0^2)\,\Psi_\eta^{\partial\Lambda}(x)\,; && x\in \operatorname{Int}\Lambda \\ &\Psi_\eta^{\partial\Lambda}(x) = \eta(x)\,; && x\in\partial\Lambda\,. \end{aligned} \qquad (0.7)$$

The quantity (assumed to exist)

$$p_\infty^\eta = -\lim_{\Lambda\uparrow R^2}\frac{1}{|\Lambda|}\ln Z_\Lambda(\eta). \qquad (0.8)$$

(where the meaning of the symbol $\lim_{\Lambda\uparrow R^2}$ will be specified below) is called the infinite volume vacuum energy density conditioned by η. Our result is:

Theorema A. Let $\mu_{\mathcal{P}}\in\mathcal{G}_{cr}^{t}(\lambda)$. Take $\eta\in\operatorname{supp}\mu$ arbitrary, then p_∞^η exists and $p_\infty^\eta = p_\infty^{\eta=0}$.

This theorem says that in the case of completely regular solutions of the DLR equations i) and ii) (with fixed P) the infinite volume vacuum energy density does not depend on the boundary conditions.

Some results of the independence of boundary conditions of the vacuum energy density have been proven in /5/. However, from the point of view of Gibbsian approach this class of boundary conditions presumably is not of measure 1.

The class of completely regular solutions of the DLR-equations is rather large. It includes among others weakly coupled $\mathcal{P}(\phi)_2$ models /6/, ϕ^4-like models (due to the Newman Gaussian inequalities /7/).

1. PROOF OF THE THEOREM A

1.1. Shift Transformation

Let Λ be a bounded, with C^1-piecewise boundary $\partial\Lambda$, subset of R^2. Let $0<\epsilon<1$ and let $0\le\chi_\epsilon(x)$ be a C^∞-function on R^2 such that:

$$\chi_\epsilon(x) = \begin{cases} 1 & \text{for } x\in \operatorname{Int}_{-1}\Lambda \equiv \{x\in\operatorname{Int}\Lambda \,|\, \operatorname{dist}(x,\partial\Lambda)\ge 1\} \\ \le 1 & \text{for } x\in\operatorname{Int}^\epsilon_{-1}\Lambda \equiv \{x\in\operatorname{Int}\Lambda\,|\,1-\epsilon\le \operatorname{dist}(x,\Lambda)<1\} \\ 0 & \text{for } x\in\partial^\epsilon\Lambda \equiv \Lambda - (\operatorname{Int}_{-1}\Lambda\cup\operatorname{Int}^\epsilon_{-1}\Lambda). \end{cases} \qquad (1.1)$$

In formula (0.6) defining conditioned partition function let us perform the following shift transformation:

$$\phi\to\phi-(\chi_\epsilon\cdot\Psi_\eta^{\partial\Lambda}). \qquad (1.2)$$

Calculating the corresponding Radon-Nikodym derivative, we obtain:

$$\frac{Z_\Lambda(\eta)}{Z_\Lambda(\eta=0)} = \exp\Big(\frac{1}{2}\int\chi_\epsilon(x)\,\Psi_\eta^{\partial\Lambda}(x)\,J_\eta^\epsilon(x)\,dx\Big)\times \qquad (1.3)$$

$$\times\int\mu_\Lambda^{\partial\Lambda}(d\phi)\,\exp(-\phi(J_\eta^\epsilon))\cdot\exp(\Omega^\epsilon(\phi_0\Psi_\eta^{\partial\Lambda})),$$

where

$$J_\eta^\epsilon(x) = (-\Delta+m^2)(\chi_\epsilon\cdot\Psi_\eta^{\partial\Lambda})(x) =$$

$$= \begin{cases} 0 & \text{for } x\in\operatorname{Int}_{-1}(\Lambda) \\ (-\Delta\chi_\epsilon)\Psi_\eta^{\partial\Lambda}(x) + 2(\nabla\chi_\epsilon)(x)(\nabla\Psi_\eta^{\partial\Lambda})(x) & \text{for } x\in\operatorname{Int}^\epsilon_{-1}(\Lambda) \\ 0 & \text{for } x\in\partial^\epsilon\Lambda. \end{cases} \qquad (1.4)$$

Note that these manipulations have perfectly good mathematical sense as $\Psi_\eta^{\partial\Lambda}$ is C^∞-function inside Λ being solution (in the space $S'(R^2)$) of the elliptic homogenous equation.

The polynomial

$$Q^\epsilon(\phi,\Psi^{\partial\Lambda})(x) = \lambda[\mathcal{P}(\phi+(1-\chi_\epsilon)\Psi_\eta^{\partial\Lambda})(x) - \mathcal{P}(\phi)(x)] \qquad (1.5)$$

is supported in the set $\overline{\Lambda-\operatorname{Int}_{-1}\Lambda}$ and has degree $\deg Q^\epsilon = \deg\mathcal{P}-1$ in the variable ϕ.

By $\mu_\Lambda^{\partial\Lambda}$ we have denoted the finite volume $\mathcal{P}(\phi)_2$ measure with the full Dirichlet boundary condition on $\partial\Lambda$.

Applying Cauchy-Schwartz inequality we have

$$\frac{Z_\Lambda(\eta)}{Z_\Lambda(\eta=0)} \le \Pi^1_\Lambda(\eta)(\Pi^2_\Lambda(\eta))^{1/2}\,(\Pi^3_\Lambda(\eta))^{1/2}, \qquad (1.6)$$

where

$$\Pi^1_\Lambda(\eta) = \exp(\frac{1}{2} \int \chi_\epsilon(x)\, \Psi^{\partial\Lambda}_\eta(x)\;\; J^\epsilon_\eta(x)\, dx) .$$

$$\Pi^2_\Lambda(\eta) = \int \mu^{\partial\Lambda}_\Lambda(d\phi)\, \exp 2Q^\epsilon_\Lambda(\phi, \Psi^{\partial\Lambda}_\eta). \tag{1.7}$$

$$\Pi^3_\Lambda(\eta) = \int \mu^{\partial\Lambda}_\Lambda(d\phi) \exp - 2\phi(J^\epsilon_\eta).$$

1.2. Some Estimates on $\Psi^{\partial\Lambda}_\eta$

Let

$$K^{\partial\Lambda}(x,y) = (-\Delta + m^2)^{-1}(x,y) - (-\Delta^{\partial\Lambda} + m^2)^{-1}(x,y) . \tag{1.8}$$

Lemma 1.1. Let $\{\Lambda_n\}$ be any sequence of bounded subsets of R^2 with piecewise - C^1 boundaries $\{\partial\Lambda_n\}$ and such that $\Lambda_n \uparrow R^2$ monotonously and by inclusion.

Let $\mu_{\mathcal{P}} \in \mathcal{G}^t_{cr}(\lambda P)$ and a number $0 < \rho$ be given. Then there exists a subsequence $(n') \subset (n)$ and functions $C_1(\rho,\eta)$, $C_2(\epsilon,\eta)$ finite on the suppμ such that for every $s > 1$

$$\int_{(\partial_1\Lambda_{n'})} |\Psi^{\partial\Lambda_{n'}}_\eta(x)|^s\, dx \le C_1(\rho,\eta)\, |\partial\Lambda_{n'}|^{1+\rho} \tag{1.9}$$

and

$$\int_{(\partial^\epsilon_1\Lambda_{n'})} |\nabla\Psi^{\partial\Lambda_{n'}}_\eta|^2(x)\, dx \le C_2(\epsilon,\eta)\, |\partial\Lambda_{n'}|^{1+\rho} .$$

Here:

$$\partial_1\Lambda = \{x \in \Lambda \mid \; 0 \le \text{dist}(x,\partial\Lambda) \le 1\} \tag{1.10}$$

$$\partial^\epsilon_1\Lambda \;\; \{x \in \partial_1\Lambda \mid \;\; \epsilon \le \text{dist}(x,\partial\Lambda)\} .$$

Proof: This is a simple consequence of the assumed regularity of $\mu_{\mathcal{P}}$. Taking arbitrary $\rho > 0$ we have:

$$\int \mu_{\mathcal{P}}(d\eta) \frac{\int_{\partial_1\Lambda_n} (\Psi^{\partial\Lambda_n}_\eta(x))^s\, dx}{|\partial\Lambda_n|^{1+\rho}} \le$$

$$\le |\partial\Lambda|^{-1-\rho} \int_{\partial_1\Lambda_n} dx\, (\int \mu_{\mathcal{P}}(d\eta)(\Psi^{\partial\Lambda_n}_\eta(x))^s \le \text{const}(\eta)\, |\partial\Lambda_n|^{-\rho} .$$

The last estimate follows from the well-known fact that for every unit cube Δ we have $\|K^{\partial\Lambda}\|_{L^1(\Delta)} \le$ const (see Prop. 7.8.7 in [1])

uniformly in Δ and Λ and the assumed complete regularity of $\mu_{\mathcal{P}}$.

By an elementary calculation we have:

$$\Delta K^{\partial\Lambda}(x,x) + 2K^{\partial\Lambda}(x,x) =$$

$$= 2 \int_{\partial\Lambda} \int_{\partial\Lambda} \nabla_x P^{\partial\Lambda}(x, z_1) \nabla_x P^{\partial\Lambda}(x, z_2)\, dz_1 dz_2 ,$$

where $K^{\partial\Lambda}(x,x) = (-\Delta + m^2)^{-1}(x,x) - (\Delta^{\partial\Lambda} + m^2)^{-1}(x,x)$

for $x \notin \partial\Lambda$, and $P^{\partial\Lambda}(x,z)$ is the Poisson kernel for the problem (0.7). Moreover, $\Delta K^{\partial\Lambda}(x,x)$ has still exponential decay in dist $(x, \partial\Lambda)$ argument.

q.e.d.

Lemma 1.2. Let $\mu \in \mathcal{G}^t_{cr}(\lambda\mathcal{P})$. Then for any unit cube $\Delta \subset R^2$, any bounded $\Lambda \subset R^2$ with C^1-piecewise boundary, and any $j > 1$ there exists a constant $C(j,(\eta))$, finite $\mu_{\mathcal{P}}$ = a.e. and such that for all $\beta < (j+1)/2$ the following estimate holds:

$$\int_\Delta |\Psi^{\partial\Lambda}_\eta(x)|^j\, dx \le C(j,\eta)(\int_\Delta K^{\partial\Lambda}(x,x)\, dx)^\beta . \tag{1.11}$$

Proof: Let $c(R^2) = \cup_j \Delta_j$ be a partitioning of R^2 into unit cubes such that $\Delta \in c(R^2)$. Take $\delta > 0$ arbitrary and fixed. Assume $j = 2$ for simplicity (the general case can be proven in an analogous fashion). For $\mu_{\mathcal{P}} \in \mathcal{G}^t_{cr}(\lambda P)$ we have:

$$\mu_{\mathcal{P}}\{\eta \in S'(R^2) \mid \exists_j \int_{\Delta_j} (\Psi^{\partial\Lambda}_\eta)^2(x)\, dx \ge \frac{1}{\delta}(\int_{\Delta_j} K^{\partial\Lambda}(x,x)\, dx)^\beta\} \le$$

$$\le \sum_j \mu_{\mathcal{P}}\{\eta \in S'(R^2) \mid \int_{\Delta_j} (\Psi^{\partial\Lambda}_\eta)^2(x)\, dx \ge \frac{1}{\delta}(\int_{\Delta_j} K^{\partial\Lambda}(x,x)\, dx)^\beta\}.$$

(by Tchebyshev's inequality)

$$\le \delta \sum_j (\int_{\Delta_j} K^{\partial\Lambda}(x,x)\, dx)^{-\beta} \int \mu_{\mathcal{P}}(d\eta) \int_{\Delta_j} (\Psi^{\partial\Lambda}_\eta)^2(x)\, dx \le$$

$$\le \delta \cdot \text{const} \cdot (\sum_j \int_{\Delta_j} K^{\partial\Lambda}(x,x)\, dx)^{1-\beta} .$$

Whenever $\beta < 1$, the sum $\sum_j$ is finite due to the exponential decay of $K^{\partial\Lambda}$ and mild integrable singularities of $K^{\partial\Lambda}$ on $\partial\Lambda$. Since δ is arbitrary the proof follows.

q.e.d.

The dependence on Λ of the constants $C(j, \eta)$ in the above Lemma is very weak. For this we refer to our paper [8].

1.3. Existence and Shape-Independence of p^{η}_{∞}.

Let $\mu_{\mathcal{P}} \in \mathcal{G}^{t}_{cr}(\lambda P)$ and $\eta \in \mathrm{supp}\,\mu$ be arbitrary chosen. Then we have

$$Z_{\Lambda}(\eta) = \int_{S'(R^2)} \mu_0^{\partial\Lambda}(d\phi)\, \exp(-\lambda P_{\Lambda}(\phi))\, \exp\Omega_{\Lambda}(\phi, \Psi^{\partial\Lambda}_{\eta}), \tag{1.12}$$

where

$$Q_{\Lambda}(\phi, \Psi^{\partial\Lambda}_{\eta}) = -\lambda \int_{\Lambda} [\mathcal{P}(\phi + \Psi^{\partial\Lambda}_{\eta})(x) - \mathcal{P}(\phi)(x)]\, dx \tag{1.13}$$

is a polynomial in the field ϕ of degree $\deg\mathcal{P} - 1$ and with coefficients which are polynomials in the field $\Psi^{\partial\Lambda}_{\eta}$.
Let:

$$\mathrm{un}/\mu_{\Lambda}^{\partial\Lambda}(d\phi) = \mu_0^{\partial\Lambda}(d\phi)\, \exp(-\lambda \int_{\Lambda} :\mathcal{P}(\phi):_{\partial\Lambda}(x)\, dx), \tag{1.14}$$

where: $:_{\partial\Lambda}$ means the Wick ordering with respect to the covariance $(-\Delta^{\partial\Lambda} + m^2)^{-1}$.

Let $n = \deg\mathcal{P}$ and let us write

$$\Omega(\phi, \Psi^{\partial\Lambda}_{\eta})(x) = \sum_{j=1}^{n-1} Q^j(\Psi^{\partial\Lambda}_{\eta})(x) : \phi^j :_{\partial\Lambda}(x) - \lambda\mathcal{P}(\Psi^{\partial\Lambda}_{\eta})(x). \tag{1.15}$$

We apply variant of the finite volume: ϕ^j: estimates to $Z_{\Lambda}(\eta)$ [9]:

$$Z_{\Lambda}(\eta) \le \prod_{j=1}^{n-1} \exp|\lambda| (\|Q^j(\Psi^{\partial\Lambda}_{\eta})\|_{L^1(\Lambda)} + \|Q^j(\Psi^{\partial\Lambda}_{\eta})\|_{L^{n/(n-i)}(\Lambda)} \times \tag{1.16}$$
$$\times \exp - \lambda \int_{\Lambda} \mathcal{P}(\Psi^{\partial\Lambda}_{\eta})(x)\, dx.$$

Application of the Lemma 1.2 together with the Minkovski inequality gives that there exists a function $J(\eta)$ finite on $\mathrm{supp}\,\mu_{\mathcal{P}}$ such that

$$Z_{\Lambda}(\eta) \le \exp J(\eta)|\Lambda|. \tag{1.17}$$

This bound together with the standard subadditivity arguments (see, for example [6]) gives that for almost every η wr. to $\mu_{\mathcal{P}}$ the limit $p^{\eta}_{\infty} = \lim_{\Lambda\uparrow R^2} \frac{1}{|\Lambda|} \ln Z_{\Lambda}(\eta)$ exists, whenever $\Lambda \uparrow R^2$ in the sense of van Howe and such that $\partial\Lambda$ are C^1-piecewise curves. Moreover, the limit does not depend on the sequence $\Lambda \uparrow R^2$ chosen.

Remark: The existence of finite on $\mathrm{supp}\,\mu_{\mathcal{P}}$ function $J(\eta)$, such that (1.17) holds, can be also proven by the application of the Tchebyshev inequality.

1.4. $p^{\eta}_{\infty} = p^{\eta=0}_{\infty}$

Now we are ready to prove the second part of the Theorem A. From the Lemma 1.3. it follows that it is enought to control the limit $\lim_{\Lambda\uparrow R^2} p^{\eta}_{\Lambda}$ by passing to sebsequences. Let $\{\Lambda_n\}$ be any sequence of bounded with C^1-piecewise boundaries $\{\partial\Lambda_n\}$ subsets of R^2 such that $\Lambda_n \uparrow R^2$ monotonously and by inclusion and such that there exists a number $\rho > 0$ for which:

$$\lim_{n\to\infty} \frac{|\partial\Lambda_n|^{1+\rho}}{|\Lambda_n|} = 0.$$

For every n let χ^n_{ϵ} be a C^{∞}-function with the properties listed in (1.1). Additionally we assume

$$\sup_{n,x} \max\{|\partial_1 \chi^n_{\epsilon}|, |\partial_2 \chi^n_{\epsilon}|\} \le C_1 < \infty,$$

$$\sup_{x,n} |\Delta\chi^n_{\epsilon}| \le C_2 < \infty.$$

Using the: ϕ^j: -estimates together with the Lemma 1.1., we easily conclude from (1.9) that there exists for any given number $\rho > 0$ a function $C(\rho, \eta)$ finite on the support of $\mu_{\mathcal{P}}$ and such that for some subsequence $(n') \subset (n)$ we have the following estimate:

$$\frac{Z_{\Lambda_{n'}}(\eta)}{Z_{\Lambda_{n'}}(\eta=0)} \le \exp(C(\rho, \eta)|\partial\Lambda|^{1+\rho}).$$

From this estimate it follows that

$$\lim_{n'\to\infty} p^{\eta}_{\Lambda_{n'}} = \lim_{n'\to\infty} p^{\eta=0}_{\Lambda_{n'}}.$$

Taking into account Lemma 1.3. we conclude that

$$\lim_{n\to\infty} p^{\eta}_{\Lambda_n} = \lim_{n'\to\infty} p^{\eta}_{\Lambda_{n'}} = \lim_{n\to\infty} p^{\eta=0}_{\Lambda_n}.$$

This shows that for $\mu_{\mathcal{P}}$ almost every η the infinite volume vacuum energy density is equal to the Dirichlet pressure which is equal to other classically conditioned infinite volume pressures following from the results of [5].

Remark: A similar trick with the shift transformation has been used in a quite different context in paper [15].

2. CONCLUDING REMARKS

The above Theorem A has been formulated in the course of our studies on the DLR equations for the $\mathcal{P}(\phi)_2$ theories. We expect that in the region of convergence of the cluster expansion the obtained regular Gibbs measure is unique in the set $\mathcal{G}^t_{cr}(\lambda\mathcal{P})$. One of the possible ways of proof is to show that the cluster expansion converges uniformly in the boundary data. This is true for the classical boundary conditions [10]. As an application of the estimates proven in the subsection 1.3. we mention here the following result:

Theorem B. Let $\mu_{\mathcal{P}} \in \mathcal{G}^t_{cr}(\lambda P)$ be arbitrary and $\eta \in \operatorname{supp}\mu_{\mathcal{P}}$. Then there exists a number $\sigma = \sigma(\eta)$ such that for $|\lambda| < \sigma(\eta)$ the cluster expansion for the conditioned by η measure $\mu^{\eta}_{\Lambda}(d\phi)$ converges.

Note that this result is too weak to prove the uniqueness of the completely regular Gibbs states due to the η-dependence of the convergence radius of the corresponding high-temperature cluster expansions.

For the lattice models this problem has been resolved in [12] by a beautiful trick. Some uniqueness results have been proved for some other field theoretical models in [8, 12, 13, 14].

Estimates established in subsection 1.2. can be also used to prove independence of the boundary data of suitably perturbed energy densities.

The following generalized energy densities

$$p^{\eta}_{\Lambda}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f}) = -\frac{1}{|\Lambda|}\ln Z^{\eta}_{\Lambda}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f}), \tag{2.1}$$

$$Z^{\eta}_{\Lambda}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f}) = \int \mu^{\eta}_{\Lambda}(d\phi)\exp\sum_{i=1}^{n}\epsilon_i\phi(f_i)\ \exp\mu\phi(\chi_{\Lambda}), \tag{2.2}$$

where $\underset{\sim}{\epsilon} = (\epsilon_1, \dots, \epsilon_n)$, $\underset{\sim}{f} = (f_1, \dots, f_n)$ is a sequence of smooth functions localized in unit cubes, can be shown to be independent of typical boundary data: This remark opens a new possible way to attack the problem of uniqueness. From the remark above it follows:

$$\lim_{\Lambda\uparrow R^2} p^{(\eta)}_{\Lambda}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f}) = \lim_{\Lambda\uparrow R^2} p^{(0)}_{\Lambda}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f}) \equiv p_{\infty}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f}). \tag{2.3}$$

It follows then from the convexity at μ that whenever $p_{\infty}(\mu, 0)$ is differentiable at μ, then:

$$\frac{\partial}{\partial\mu}\lim_{\Lambda\uparrow R^2} p^{\eta}_{\Lambda}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f})\Big|_{\underset{\sim}{\epsilon}=0} = \frac{1}{|\Lambda|}\int_{\Lambda} dx < \phi(0) >_{\infty}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f})\Big|_{\underset{\sim}{\epsilon}=0} =$$

$$= \lim_{\Lambda\uparrow R^2}\frac{1}{|\Lambda|}\int dx < \phi(x) >^{\eta}_{\Lambda}(\mu, \underset{\sim}{0}, \underset{\sim}{0}). \tag{2.4}$$

Assuming moreover that $\lim_{\Lambda\uparrow R^2} < >^{\eta}_{\Lambda}$ has the translationally invariant first moment we conclude from (2.4) that

$$\lim_{\Lambda\uparrow R^2} < \phi(x) >^{\eta}_{\Lambda}(\mu, \underset{\sim}{0}, \underset{\sim}{0}) = < \phi(0) >_{\infty}(\mu, \underset{\sim}{0}, \underset{\sim}{0}) \tag{2.5}$$

does not depend on the typical boundary condition. In the region of convergence of the cluster expansion it is known that the right-hand side of (2.3) is analytic functions of the complex parameters $\underset{\sim}{\epsilon} = (\epsilon_1, \dots, \epsilon_n)$ at $\underset{\sim}{\epsilon} = (0, \dots, 0)$. Taking the derivatives of r.h.s. of (2.3) we obtain:

$$\frac{\partial^{n+1}}{\partial\epsilon_n \dots \partial\epsilon_1 \partial\mu} p_{\infty}(\mu, \underset{\sim}{\epsilon}, \underset{\sim}{f})\Big|_{\underset{\sim}{\epsilon}=(0,\dots,0)} = < \phi(0); \phi(f_1); \dots; \phi(f_n) >^{T}_{\infty}(\mu, \underset{\sim}{\epsilon} = 0). \tag{2.6}$$

Thus, the problem of uniqueness of the translationally invariant state has been reduced to the question about the possibility of interchanging the operation $\partial^n/\partial\epsilon_1 \dots \partial\epsilon_n\big|_{\underset{\sim}{\epsilon}=(0,\dots,0)}$ with the operation of taking the thermodynamic limit. This seems to be a much easier problem than the question about uniform convergence of the cluster expansion.

Even more perspective appears this possibility of proving the uniqueness for the case of $\mathcal{P}(\phi) = \lambda{:}\phi^4{:} + b{:}\phi^2{:}$ From the Lee-Yang theorem I it follows that the r.h.s. of (2.3) is then analytic at $\underset{\sim}{\epsilon} = (0, \dots, 0)$ whenever $\mu \neq 0$. Having in mind the Vitali theorem we see that the problem is reduced here to a volume uniform bound on $p^{\eta}_{\Lambda}(\mu, \underset{\sim}{\epsilon})$ in some complex neighbourhood of $\underset{\sim}{\epsilon} = (0, \dots, 0)$.

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