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Triaxial rigid rotator approximation for odd-odd nuclei

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Abstract. By assuming an adiabatic separation of rotation and vibration degrees of freedom we suppose that the structure of the ground-state-band of the odd-odd nucleus may be determined by the triaxial-rigid-rotator motion. Secular equation of the rigid rotator approximation for considered nuclei has been obtained. Diagonalizing the hamiltonian in a symmetrized rotator basis we obtain a model description for the ground-state-band in heavy nuclei. At description of these states the K-mixing effect created by the model triaxial rotations is taken into account. As well as in derived expressions for matrix elements contributions of Coriolis interaction and interaction between unpaired nucleons were taken into account.

Key words: odd-odd nuclei, quadrupole deformation, collective states, effective triaxiality, components of moment of inertia, triaxial rotator.

I. INTRODUCTION

A nonadiabatic collective model [1], taking into account the relationship between rotational motion and longitudinal and transverse oscillations of the nuclear surface, makes it possible to explain a number of regularities observed in the excitation spectra of deformed nonaxial heavy nuclei [2]. The study of reactions with heavy ions on nuclei makes it possible to obtain information about the excited collective states of the energy levels of the ground, β- and γ-bands of these nuclei [3].

Consequently, the development of nonadiabatic collective models is required, taking into account the complex relationship of rotational and oscillatory motions, in which the collective variables are dynamic [3]. To explain many experimental data, it is enough to take into account the deviations of the nucleus shape from the spherical shape due to quadrupole-type deformations, i.e., it was enough to approximate the nucleus by a triaxial ellipsoid. Such a phenomenological nonadiabatic model of a quadrupole-type nucleus takes into account the connection of rotational motion with longitudinal and transverse vibrations of the surface of the nucleus [4]. Many calculations based on this model for various types of potential energies of surface vibrations provide a good description of rotational-vibrational excited levels of the even-even nuclei, including high-spin states.

The first single-particle excited level in even-even nuclei is approximately 2 MeV above the ground state [4, 5]. All levels up to 2 MeV can be considered as collective. In odd nuclei, the order of energy of collective and single-particle excited levels is approximately the same, although the adiabatic approximation is partially used to simplify calculations.

Collective-one-particle states of odd-odd nuclei have been little studied both theoretically and experimentally. It is usually assumed that in odd-odd nuclei the order of energy of collective and single-particle excited levels is the same and the adiabatic approximation of a rigid asymmetric rotator is not applicable. However, detailed studies of these states within this adiabatic approximation have not been carried out. In addition, high-spin states of odd-odd nuclei are very sensitive to the Coriolis force in the rigid rotator approximation, and currently look very tempting. Therefore, in this paper we make an attempt to describe how the adiabatic approximation of a rigid asymmetric rotator will behave in describing collective-single-particle states of odd-odd nuclei.

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II. TOTAL HAMILTONIAN

The Bohr Hamiltonian of an even-even nucleus with quadrupole and octupole deformations in the curvilinear coordinates has the following form:

$$\hat{H}_{\text{total}} = \hat{T}_\beta + \hat{T}_\gamma + \hat{T}_{rot} + V(\beta,\gamma). \tag{1}$$

where

$$\hat{T}_\beta = -\frac{\hbar^2}{2B}\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\left(\beta^4\frac{\partial}{\partial\beta}\right), \tag{2}$$

the operator of kinetic energies of β-vibration.

$$\hat{T}_\gamma = -\frac{\hbar^2}{2B_2}\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\left(\sin 3\gamma\frac{\partial}{\partial\gamma}\right), \tag{3}$$

the operator of kinetic energies of γ-vibration.

$$\hat{T}_{rot} = \frac{\hbar^2}{2B\beta^2}\sum_{\kappa=1}^{3}\frac{[\hat{I}_\kappa - \hat{j}_\kappa(p) - \hat{j}_\kappa(n)]^2}{\sin^2(\gamma - \frac{2\pi}{3}\kappa)}, \tag{4}$$

rotational energy operator of odd-odd nuclei, here I_κ – projection of the total angular momentum operator to the principal axis of even-even nucleus (κ = 1, 2, 3), $j_\kappa(p)$ and $j_\kappa(n)$ are the projections of the total angular momentum operators of the proton and neutron to the principal axis of even-even nucleus, respectively.

The general solution of the Schrödinger equation with the operator (1) depends on the chosen potential energy $V(\beta,\gamma)$. Usually this problem can be solved by choosing the phenomenological form of this potential energy. Often the general form of potential energy is expressed in following [6]:

$$V(\beta,\gamma) = V(\beta) + \frac{V(\gamma)}{\beta^2}, \tag{5}$$

where V(β) and V(γ) – potential energies of β- and γ-vibrations, separately.

A general solution to the Schrödinger equation with the operator (1) has not yet been found. Therefore, the study of collective excitations of the deformed even-even nuclei is carried out by introducing simplifying assumptions. One of the simplifying assumptions in the study of collective excitations of deformed odd-odd nuclei is the rigid rotator approximation, which we will consider in detail in the following sections.

III. GROUND STATE OF ODD-ODD NUCLEI

Total spin J of the ground state of odd-odd nuclei was found by Nordheim [7, 8] could be given by the rules:

$$J = |j_p - j_n|, \quad \texttt{if} \quad l_p + s_p + l_n + s_n \quad \texttt{is even}, \tag{6}$$

$$|j_p - j_n| \le J \le j_p + j_n, \quad \texttt{if} \quad l_p + s_p + l_n + s_n \quad \texttt{is odd}, \tag{7}$$

where s_p, s_n are the individual spins of the odd proton and odd neutron involved, and l_p, l_n their respective orbital quantum numbers.

In [9]

$$J = j_p + j_n, \quad \texttt{if} \quad j_p = l_p \pm s_p \quad \texttt{and} \quad j_n = l_n \pm s_n, \tag{8}$$

$$J = |j_p - j_n|, \quad \texttt{if} \quad j_p = l_p \pm s_p \quad \texttt{and} \quad j_n = l_n \mp s_n, \quad \texttt{"strong" rules} \tag{9}$$

The rule (7) is frequently given in the following less specific form:

$$|j_p - j_n| < J \le j_p + j_n, \quad \texttt{"weak" rules}, \tag{10}$$

IV. RIGID ROTATOR APPROXIMATION FOR ODD-ODD NUCLEI

In rigid rotator approximation the variables β and γ are replaced by their effective values [5]. The energy spectrum of the triaxial quadrupole-octupole rigid rotator is obtained by diagonalizing the $\hat{T}_{\text{rot}}$ in the basis of the symmetrized rotor functions $|Ij_pj_nKM\Omega_p\Omega_n\rangle$.

$$|Ij_pj_nKM\Omega_p\Omega_n\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\left[D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n} + (-1)^{I-j_p-j_n}D^I_{M,-K}\chi^{\tau_p}_{-\Omega_p}\chi^{\tau_n}_{-\Omega_n}\right], \tag{11}$$

where Ω_p and Ω_n are the projections of the total angular momentum operators of the proton and neutron to the axial axis of even-even nucleus, respectively. M and K are the projections of the angular momentum $\hat{I}$ on the third axes of the laboratory and intrinsic frames, respectively. $D^I_{MK}(\theta)$-Wigner function, θ-Euler angles, τ_p and τ_n the remaining quantum numbers of the odd proton and odd neutron, respectively.

In the present work we consider (under the assumption of the adiabatic approximation) the deformation parameters (or more precisely their average values) as constants whose values $\beta_{\mathfrak{eff}}$, $\gamma_{\mathfrak{eff}}$ effectively determine a rigid triaxial rotator. The Bohr hamiltonian in rigid rotator approximation for odd-odd nuclei:

$$\hat{H} = \hat{T}_{rot} + \hat{T}_{int}(p) + \hat{T}_{int}(n), \tag{12}$$

where

$$\hat{T}_{rot} = \frac{\hbar^2}{2B\beta^2_{\mathfrak{eff}}}\sum_{\kappa=1}^{3}\frac{[\hat{I}_\kappa - \hat{j}_\kappa(p) - \hat{j}_\kappa(n)]^2}{\sin^2(\gamma_{\mathfrak{eff}} - \frac{2\pi}{3}\kappa)}, \tag{13}$$

rotational energy operator of odd-odd nuclei, here I_κ – projection of the total angular momentum operator to the principal axis of even-even nucleus, $j_\kappa(p)$ and $j_\kappa(n)$ are the projections of the total angular momentum operators of the proton and neutron to the principal axis of even-even nucleus, respectively. The components of moment of inertia:

$$J_\kappa = 2B\beta^2_{\mathfrak{eff}}\sin^2(\gamma_{\mathfrak{eff}} - \frac{2\pi}{3}\kappa),$$

where J_κ (κ = 1, 2, 3) explicitly depend on the quadrupole mass and deformation parameters B, $\beta_{\mathfrak{eff}}$, $\gamma_{\mathfrak{eff}}$. Thus we obtain the quadrupole moment of inertia in a form depending on three arguments J_κ=$J_\kappa(B, \beta_{\mathfrak{eff}}, \gamma_{\mathfrak{eff}})$.

$$\hat{T}_{int}(p) = -T_p\beta_{\mathfrak{eff}}\left\{\cos\gamma_{\mathfrak{eff}}\left[3j_3^2(p) + j_p(j_p+1)\right] + \sqrt{3}\sin\gamma_{\mathfrak{eff}}\left[\hat{j}_1^2(p) - \hat{j}_2^2(p)\right]\right\}, \tag{14}$$

interaction energy operator of odd-proton.

$$\hat{T}_{int}(n) = -T_n\beta_{\mathfrak{eff}}\left\{\cos\gamma_{\mathfrak{eff}}\left[3j_3^2(n) + j_n(j_n+1)\right] + \sqrt{3}\sin\gamma_{\mathfrak{eff}}\left[\hat{j}_1^2(n) - \hat{j}_2^2(n)\right]\right\}, \tag{15}$$

interaction energy operator of odd-neutron.

Firstly we diagonalize the rotational operator (13). Enter the designation

$$a_\kappa^{-1}(\gamma_{\mathfrak{eff}}) = \sin^2(\gamma_{\mathfrak{eff}} - \frac{2\pi}{3}\kappa),$$

then we re-write the rotational operator (13) in the following form:

$$\hat{T}_{rot} = \frac{1}{4}\sum_{\kappa=1}^{3}[\hat{I}_\kappa - \hat{j}_\kappa(p) - \hat{j}_\kappa(n)]^2 a_\kappa(\gamma_{\mathfrak{eff}}) =$$

$$= \Gamma_1[\hat{I}^2 + \hat{j}^2(p) + \hat{j}^2(n) - \hat{I}_3^2 - \hat{j}_3^2(p) - \hat{j}_3^2(n)] + \Gamma_2[\hat{I}_1^2 - \hat{I}_2^2 + \hat{j}_1^2(p) - \hat{j}_2^2(p) + \hat{j}_1^2(n) - \hat{j}_2^2(n)] -$$

$$-2\Gamma_1[\hat{I}_1\hat{j}_1(p) + \hat{I}_1\hat{j}_1(n) - \hat{j}_1(p)\hat{j}_1(n) - \hat{I}_2\hat{j}_2(p) - \hat{I}_2\hat{j}_2(n) + \hat{j}_2(p)\hat{j}_2(n)] -$$

$$-2\Gamma_2[\hat{I}_1\hat{j}_1(p) + \hat{I}_1\hat{j}_1(n) - \hat{j}_1(p)\hat{j}_1(n) + \hat{I}_2\hat{j}_2(p) + \hat{I}_2\hat{j}_2(n) - \hat{j}_2(p)\hat{j}_2(n)] +$$

$$+\Gamma_3[\hat{I}_3 - \hat{j}_3(p) - \hat{j}_3(n)]^2. \tag{16}$$

where

$$\Gamma_1 = \frac{a_1(\gamma_{\mathfrak{eff}}) + a_2(\gamma_{\mathfrak{eff}})}{8}, \quad \Gamma_2 = \frac{a_1(\gamma_{\mathfrak{eff}}) - a_2(\gamma_{\mathfrak{eff}})}{8}, \quad \Gamma_3 = \frac{a_3(\gamma_{\mathfrak{eff}})}{4}.$$

The derivation of formula (16) is given in detail in Appendix **A**.

Now we will assume:

$$\hat{T}_{rot} = \hat{A}_1 + \hat{A}_2 + \hat{A}_3 + \hat{A}_4. \tag{17}$$

$$\hat{A}_1 = \Gamma_1[\hat{I}^2 + \hat{j}^2(p) + \hat{j}^2(n) - \hat{I}_3^2 - \hat{j}_3^2(p) - \hat{j}_3^2(n)], \tag{18}$$

$$\hat{A}_2 = \Gamma_2[\hat{I}_1^2 - \hat{I}_2^2 + \hat{j}_1^2(p) - \hat{j}_2^2(p) + \hat{j}_1^2(n) - \hat{j}_2^2(n)], \tag{19}$$

$$\hat{A}_3 = -2\Gamma_1[\hat{I}_1\hat{j}_1(p) + \hat{I}_1\hat{j}_1(n) - \hat{j}_1(p)\hat{j}_1(n) - \hat{I}_2\hat{j}_2(p) - \hat{I}_2\hat{j}_2(n) + \hat{j}_2(p)\hat{j}_2(n)] -$$

$$-2\Gamma_2[\hat{I}_1\hat{j}_1(p) + \hat{I}_1\hat{j}_1(n) - \hat{j}_1(p)\hat{j}_1(n) + \hat{I}_2\hat{j}_2(p) + \hat{I}_2\hat{j}_2(n) - \hat{j}_2(p)\hat{j}_2(n)], \tag{20}$$

$$\hat{A}_4 = \Gamma_3[\hat{I}_3 - \hat{j}_3(p) - \hat{j}_3(n)]^2. \tag{21}$$

First A_1 and forth terms A_4 of this expression are diagonal matrix elements. After diagonalization we write their as:

$$A_1 = \Gamma_1[I(I+1) + j(p)[j(p)+1] + j(n)[j(n)+1] - K^2 - \Omega_p^2 - \Omega_n^2], \tag{22}$$

$$A_4 = \Gamma_3[K - \Omega_p - \Omega_n]^2. \tag{23}$$

The terms A_1 and A_2 are non-diagonal matrix elements of the operator (17). Their diagonalized formulas are given in the appendix **B**.

Products $\hat{I}_1 \cdot \hat{j}_1(p)$, $\hat{I}_1 \cdot \hat{j}_1(n)$, $\hat{I}_2 \cdot \hat{j}_2(p)$, $\hat{I}_2 \cdot \hat{j}_2(n)$ in expression (20) describe Coriolis interaction. And products $\hat{j}_2(p) \cdot \hat{j}_2(n)$, $\hat{j}_2(p) \cdot \hat{j}_2(n)$ in expression (20) describe interaction between unpaired nucleons.

V. SECULAR EQUATION OF THE RIGID ROTATOR APPROXIMATION FOR ODD-ODD NUCLEI

We present wave function of the rigid rotator approximation for odd-odd nuclei in the following form:

$$W(\vartheta) = \sum_{K\Omega_p\Omega_n} A_{K\Omega_n\Omega_p}^{Ij_pj_n\tau_p\tau_n} |Ij_pj_nK\Omega_p\Omega_n\rangle \tag{24}$$

$A_{K\Omega_n\Omega_p}^{Ij_pj_n\tau_p\tau_n}$ – permanent coefficients, or mixing coefficients, they satisfy the following orthogonality relations:

$$\sum_{K\Omega_p\Omega_n} A_{K\Omega_n\Omega_p}^{Ij_pj_n\tau_p\tau_n} \sum_{K\Omega_p\Omega_n} A_{K\Omega_n\Omega_p}^{I'j_p'j_n'\tau_p'\tau_n'} = \delta_{II'}\delta_{\tau_p\tau_p'}\delta_{\tau_n\tau_n'} \tag{25}$$

Substitute (11) to Shrödinger equation with hamiltonian (12). Then we obtain system of algebraic equations

$$\sum_{K'\Omega_p'\Omega_n'} A_{K'\Omega_n'\Omega_p'}^{Ij_pj_n\tau_p\tau_n} d_{K\Omega_n\Omega_p}^{K'\Omega_n'\Omega_p'} - A_{K\Omega_n\Omega_p}^{Ij_pj_n\tau_p\tau_n}\varepsilon = 0 \tag{26}$$

where

$$d_{K\Omega_n\Omega_p}^{K'\Omega_n'\Omega_p'} = \langle Ij_pj_nK'\Omega_p'\Omega_n'|H|Ij_pj_nK\Omega_p\Omega_n\rangle. \tag{27}$$

The equation (26) has non-trivial solutions under the condition

$$det||d_{K\Omega_n\Omega_p}^{K'\Omega_n'\Omega_p'} - \varepsilon\delta_{K'K}\delta_{\Omega_p'\Omega_p}\delta_{\Omega_n'\Omega_n}|| = 0. \tag{28}$$

We obtained the secular equation of the rigid rotator approximation for odd-odd nuclei. The system of equations (28) will be solved numerically for obtained spectrum of energy levels and wave functions. Adjustable parameters of this model: energy factor $\hbar^2/(2B\beta_{\mathfrak{eff}}^2)$, $\xi_p = \hbar^2/(6BT_p\beta_{\mathfrak{eff}}^3)$, $\xi_n = \hbar^2/(6BT_n\beta_{\mathfrak{eff}}^3)$ and $\gamma_{\mathfrak{eff}}$.

VI. CONCLUSION

Collective-one-particle states of odd-odd nuclei have been little studied both theoretically and experimentally. In this work we present triaxial rigid rotator approximation for desctription ground-state-band of the odd-odd nuclei. Secular equation of the rigid rotator approximation for considered nuclei has been obtained. Detailed derivation of the formula of the operator rotational energy of odd-odd nuclei has been presented. The formulas for matrix elements for angular momentum operators rotational energy of odd-odd nucleus has been presented in detail. There K-mixing effect created by the model triaxial rotations for description of collective-one-particle states of the odd-odd nuclei is taken into account. As well as in derived expressions for matrix elements contributions of Coriolis interaction and interaction between unpaired nucleons were taken into account. An interesting point for further research is the study of an axially symmetric odd-odd nucleus, the triaxiality parameters are equal to zero, and K-mixing is not taken into account. Furthermore studies in this direction is the subject of further work, taking into account the “strong” and “weak” rules of coupling of unpaired nucleons with an even-even nucleus.

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APPENDIX

Appendix A

Detailed derivation of the formula of the operator rotational energy of odd-odd nuclei.

Rotational energy operator of odd-odd nuclei:

$$\hat{T}_{rot} = \frac{\hbar^2}{2B\beta^2_{\mathfrak{eff}}} \sum_{\kappa=1}^{3} \frac{[\hat{I}_\kappa - \hat{j}_\kappa(p) - \hat{j}_\kappa(n)]^2}{\sin^2(\gamma_{\mathfrak{eff}} - \frac{2\pi}{3}\kappa)}. \tag{A1}$$

Make notation:

$$a_\kappa^{-1}(\gamma_{\mathfrak{eff}}) = \sin^2(\gamma_{\mathfrak{eff}} - \frac{2\pi}{3}\kappa). \tag{A2}$$

Let us rewrite the rotational energy operator (A1) in the following form, and then begin to simplify it for convenient diagonalization:

$$T_{rot} = \frac{1}{4}\sum_{\kappa=1}^{3}[\hat{I}_\kappa - \hat{j}_\kappa(p) - \hat{j}_\kappa(n)]^2 a_\kappa(\gamma_{\mathfrak{eff}}) =$$

$$= \frac{a_1(\gamma_{\mathfrak{eff}})}{4}[\hat{I}_1 - \hat{j}_1(p) - \hat{j}_1(n)]^2 + \frac{a_2(\gamma_{\mathfrak{eff}})}{4}[\hat{I}_2 - \hat{j}_2(p) - \hat{j}_2(n)]^2 + \frac{a_3(\gamma_{\mathfrak{eff}})}{4}[\hat{I}_3 - \hat{j}_3(p) - \hat{j}_3(n)]^2 =$$

$$= \frac{a_1(\gamma_{\mathfrak{eff}})}{4}[\hat{I}_1^2 + \hat{j}_1^2(p) + \hat{j}_1^2(n) - 2\hat{I}_1\hat{j}_1(p) - 2\hat{I}_1\hat{j}_1(n) + 2\hat{j}_1(p)\hat{j}_1(n)] +$$

$$+\frac{a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)-2\hat{I}_2\hat{j}_2(p)-2\hat{I}_2\hat{j}_2(n)+2\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_3(\gamma_{\text{eff}})}{4}[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2=$$

$$=\frac{a_1(\gamma_{\text{eff}})}{4}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]-\frac{a_1(\gamma_{\text{eff}})}{2}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]+$$

$$+\frac{a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]-\frac{a_2(\gamma_{\text{eff}})}{2}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_3(\gamma_{\text{eff}})}{4}[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2=$$

$$=\frac{a_1(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]+\frac{a_1(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]++\frac{a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]-$$

$$-\frac{a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]+\frac{a_2(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]+\frac{a_2(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]+$$

$$+\frac{a_1(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]-\frac{a_1(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]-$$

$$-\frac{a_1(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-\frac{a_1(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]+$$

$$+\frac{a_2(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)-\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-\frac{a_2(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-$$

$$-\frac{a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]-\frac{a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_1(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]-\frac{a_1(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_3(\gamma_{\text{eff}})}{4}[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2=$$

$$=\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]+\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]+$$

$$+\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]+\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]-$$

$$-\frac{a_2(\gamma_{\text{eff}})-a_1(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-$$

$$-\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]-\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_3(\gamma_{\text{eff}})}{4}[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2=$$

$$=\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]+\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]+$$

$$+\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)]+\frac{a_2(\gamma_{\text{eff}})-a_1(\gamma_{\text{eff}})}{8}[\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]-$$

$$-\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-\frac{a_2(\gamma_{\text{eff}})-a_1(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]-$$

$$-\frac{a_2(\gamma_{\text{eff}})+a_1(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)]-\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{4}[\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_3(\gamma_{\text{eff}})}{4}[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2=$$

$$=\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)+\hat{I}_2^2+\hat{j}_2^2(p)+\hat{j}_2^2(n)]+$$

$$+\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{8}[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)-\hat{I}_2^2-\hat{j}_2^2(p)-\hat{j}_2^2(n)]-$$

$$-\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)-\hat{I}_2\hat{j}_2(p)-\hat{I}_2\hat{j}_2(n)+\hat{j}_2(p)\hat{j}_2(n)]-$$

$$-\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{4}[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)+\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\frac{a_3(\gamma_{\text{eff}})}{4}[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2.$$

Now, we make the following notations:

$$\Gamma_1=\frac{a_1(\gamma_{\text{eff}})+a_2(\gamma_{\text{eff}})}{8}, \tag{A3}$$

$$\Gamma_2=\frac{a_1(\gamma_{\text{eff}})-a_2(\gamma_{\text{eff}})}{8}, \tag{A4}$$

$$\Gamma_3=\frac{a_3(\gamma_{\text{eff}})}{4}. \tag{A5}$$

Then the rotational energy operator (A1) takes the following form:

$$\hat{T}_{rot}=\Gamma_1[\hat{I}_1^2+\hat{I}_2^2+\hat{I}_3^2+\hat{j}_1^2(p)+\hat{j}_2^2(p)+\hat{j}_3^2(p)+\hat{j}_1^2(n)+\hat{j}_2^2(n)+\hat{j}_3^2(n)-\hat{I}_3^2-\hat{j}_3^2(p)-\hat{j}_3^2(n)]+$$

$$+\Gamma_2[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)-\hat{I}_2^2-\hat{j}_2^2(p)-\hat{j}_2^2(n)]-$$

$$-2\Gamma_1[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)-\hat{I}_2\hat{j}_2(p)-\hat{I}_2\hat{j}_2(n)+\hat{j}_2(p)\hat{j}_2(n)]-$$

$$-2\Gamma_2[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)+\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\Gamma_3[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2=$$

$$=\Gamma_1[\hat{I}^2+\hat{j}^2(p)+\hat{j}^2(n)-\hat{I}_3^2-\hat{j}_3^2(p)-\hat{j}_3^2(n)]+$$

$$+\Gamma_2[\hat{I}_1^2+\hat{j}_1^2(p)+\hat{j}_1^2(n)-\hat{I}_2^2-\hat{j}_2^2(p)+\hat{j}_2^2(n)]-$$

$$-2\Gamma_1[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)-\hat{I}_2\hat{j}_2(p)-\hat{I}_2\hat{j}_2(n)+\hat{j}_2(p)\hat{j}_2(n)]-$$

$$-2\Gamma_2[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)+\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\Gamma_3[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2.$$

$$\hat{T}_{rot}=\Gamma_1[\hat{I}^2+\hat{j}^2(p)+\hat{j}^2(n)-K^2-\Omega_p^2-\Omega_n^2]+$$

$$+\Gamma_2[\hat{I}_1^2-\hat{I}_2^2+\hat{j}_1^2(p)-\hat{j}_2^2(p)+\hat{j}_1^2(n)-\hat{j}_2^2(n)]-$$

$$-2\Gamma_1[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)-\hat{I}_2\hat{j}_2(p)-\hat{I}_2\hat{j}_2(n)+\hat{j}_2(p)\hat{j}_2(n)]-$$

$$-2\Gamma_2[\hat{I}_1\hat{j}_1(p)+\hat{I}_1\hat{j}_1(n)-\hat{j}_1(p)\hat{j}_1(n)+\hat{I}_2\hat{j}_2(p)+\hat{I}_2\hat{j}_2(n)-\hat{j}_2(p)\hat{j}_2(n)]+$$

$$+\Gamma_3[\hat{I}_3-\hat{j}_3(p)-\hat{j}_3(n)]^2. \quad \text{(A6)}$$

A detailed derivation of the formula for non-diagonal matrix elements of the (A6) is given in Appendix B.

Appendix B

Detailed derivation of the formulas for matrix elements.

The operator of total angular momentum $\hat{I}$ depends on the Euler angles. $D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}$ are eigenfunctions of the angular momentum operators of even-even nucleus and unpaired nucleons.Then the following relationships for these angular momentum operators are valid:

$$\hat{I}^2 D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=\hbar^2 I(I+1)D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}, \quad \text{(B1)}$$

$$\hat{j_p}^2 D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=\hbar^2 j_p(j_p+1)D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}, \quad \text{(B2)}$$

$$\hat{j_n}^2 D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=\hbar^2 j_n(j_n+1)D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}, \quad \text{(B3)}$$

$$\hat{I}_3 D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=\hbar K D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}. \quad \text{(B4)}$$

$$\hat{j}_{p3} D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=\hbar\Omega_p D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}. \quad \text{(B5)}$$

$$\hat{j}_{n3} D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=\hbar\Omega_n D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}. \quad \text{(B6)}$$

For diagonalization $\hat{I}_1$ and $\hat{I}_2$ components total angular momentum, we use following relationships:

$$\hat{I}_1 D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}=-\frac{\hbar}{\sqrt{2}}\sqrt{(I+K)(I-K+1)}D^I_{M,K-1}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}, \quad \text{(B7)}$$

$$\hat{I}_2 D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n} = \frac{\hbar}{\sqrt{2}}\sqrt{(I-K)(I+K+1)}D^I_{M,K+1}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n}. \tag{B8}$$

The same as in (B7) and (B8), but for diagonalization $\hat{j}_{p1}$ and $\hat{j}_{p2}$:

$$\hat{j}_{p1} D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n} = -\frac{\hbar}{\sqrt{2}}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)}D^I_{M,K}\chi^{\tau_p}_{\Omega_p-1}\chi^{\tau_n}_{\Omega_n}, \tag{B9}$$

$$\hat{j}_{p2} D^I_{MK}\chi^{\tau_p}_{C}\chi^{\tau_n}_{\Omega_n} = \frac{\hbar}{\sqrt{2}}\sqrt{(j_p-j_p)(j_p+j_p+1)}D^I_{MK}\chi^{\tau_p}_{\Omega_p+1}\chi^{\tau_n}_{\Omega_n}. \tag{B10}$$

The same as in (B7) and (B8), but for diagonalization $\hat{j}_{n1}$ and $\hat{j}_{n2}$:

$$\hat{j}_{n1} D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n} = -\frac{\hbar}{\sqrt{2}}\sqrt{(j_n+K)(I-j_n+1)}D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n-1}, \tag{B11}$$

$$\hat{j}_{n2} D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n} = \frac{\hbar}{\sqrt{2}}\sqrt{(j_n-\Omega_n)(j_n+\Omega_n+1)}D^I_{MK}\chi^{\tau_p}_{\Omega_p}\chi^{\tau_n}_{\Omega_n+1}. \tag{B12}$$

Then we get the following relationships:

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_1\hat{j}_1(p)|I j_p j_n K-1 \Omega_p-1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(I+K)(I-K+1)}. \tag{B13}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_1\hat{j}_1(p)|I j_p j_n K+1 \Omega_p-1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(I-K)(I+K+1)}. \tag{B14}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_2\hat{j}_2(p)|I j_p j_n K-1 \Omega_p-1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(I+K)(I-K+1)}. \tag{B15}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_2\hat{j}_2(p)|I j_p j_n K+1 \Omega_p-1 \Omega_n M\rangle = -\frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(I-K)(I+K+1)}. \tag{B16}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_1\hat{j}_1(n)|I j_p j_n K-1 \Omega_n-1 \Omega_p M\rangle = \frac{1}{4}\sqrt{(j_n+\Omega_n)(j_n-\Omega_n+1)(I+K)(I-K+1)}. \tag{B17}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_1\hat{j}_1(n)|I j_p j_n K+1 \Omega_n-1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_n+\Omega_n)(j_n-\Omega_n+1)(I-K)(I+K+1)}. \tag{B18}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_2\hat{j}_2(n)|I j_p j_n K-1 \Omega_n-1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_n+\Omega_n)(j_n-\Omega_n+1)(I+K)(I-K+1)}. \tag{B19}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_2\hat{j}_2(n)|I j_p j_n K+1 \Omega_n-1 \Omega_n M\rangle = -\frac{1}{4}\sqrt{(j_n+\Omega_n)(j_n-\Omega_n+1)(I-K)(I+K+1)}. \tag{B20}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_1\hat{j}_1(p)|I j_p j_n K-1 \Omega_p+1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(I+K)(I-K+1)}. \tag{B21}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_1\hat{j}_1(p)|I j_p j_n K+1 \Omega_p+1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(I-K)(I+K+1)}. \tag{B22}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_2\hat{j}_2(p)|I j_p j_n K-1 \Omega_p+1 \Omega_n M\rangle = \frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(I+K)(I-K+1)}. \tag{B23}$$

$$\langle I j_p j_n K \Omega_p \Omega_n M|\hat{I}_2\hat{j}_2(p)|I j_p j_n K+1 \Omega_p+1 \Omega_n M\rangle = -\frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(I-K)(I+K+1)}. \tag{B24}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{I}_1\hat{j}_1(n)|Ij_pj_nK-1\Omega_n+1\Omega_pM\rangle = \frac{1}{4}\sqrt{(j_n-\Omega_n)(j_n+\Omega_n+1)(I+K)(I-K+1)}. \quad \text{(B25)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{I}_1\hat{j}_1(n)|Ij_pj_nK+1\Omega_n+1\Omega_nM\rangle = \frac{1}{4}\sqrt{(j_n-\Omega_n)(j_n+\Omega_n+1)(I-K)(I+K+1)}. \quad \text{(B26)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{I}_2\hat{j}_2(n)|Ij_pj_nK-1\Omega_n+1\Omega_nM\rangle = \frac{1}{4}\sqrt{(j_n-\Omega_n)(j_n+\Omega_n+1)(I+K)(I-K+1)}. \quad \text{(B27)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{I}_2\hat{j}_2(n)|Ij_pj_nK+1\Omega_n+1\Omega_nM\rangle = -\frac{1}{4}\sqrt{(j_n-\Omega_n)(j_n+\Omega_n+1)(I-K)(I+K+1)}. \quad \text{(B28)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_1(p)\hat{j}_1(n)|Ij_pj_nK\Omega_p-1\Omega_n+1M\rangle = \frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(j_n+\Omega_n)(j_n-\Omega_n+1)}. \quad \text{(B29)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|j_1(p)j_1(n)|Ij_pj_nK\Omega_p-1\Omega_n-1M\rangle = \frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(j_n-\Omega_n)(j_n+\Omega_n+1)}. \quad \text{(B30)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_2(p)\hat{j}_2(n)|Ij_pj_nK\Omega_p-1\Omega_n+1M\rangle = \frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(j_n+\Omega_n)(j_n-\Omega_n+1)}. \quad \text{(B31)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|j_2(p)j_2(n)|Ij_pj_nK\Omega_p-1\Omega_n-1M\rangle = -\frac{1}{4}\sqrt{(j_p+\Omega_p)(j_p-\Omega_p+1)(j_n-\Omega_n)(j_n+\Omega_n+1)}. \quad \text{(B32)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_1(p)\hat{j}_1(n)|Ij_pj_nK\Omega_p+1\Omega_n+1M\rangle = \frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(j_n+\Omega_n)(j_n-\Omega_n+1)}. \quad \text{(B33)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|j_1(p)j_1(n)|Ij_pj_nK\Omega_p+1\Omega_n-1M\rangle = -\frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(j_n-\Omega_n)(j_n+\Omega_n+1)}. \quad \text{(B34)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_2(p)\hat{j}_2(n)|Ij_pj_nK\Omega_p+1\Omega_n+1M\rangle = \frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(j_n+\Omega_n)(j_n-\Omega_n+1)}. \quad \text{(B35)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|j_2(p)j_2(n)|Ij_pj_nK\Omega_p+1\Omega_n-1M\rangle = -\frac{1}{4}\sqrt{(j_p-\Omega_p)(j_p+\Omega_p+1)(j_n-\Omega_n)(j_n+\Omega_n+1)}. \quad \text{(B36)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_1^2(p)-\hat{j}_2^2(p)|Ij_pj_nK\Omega_p-2\Omega_nM\rangle = \frac{1}{2}\sqrt{(j_p+\Omega_p)(j_p+\Omega_p-1)(j_p-\Omega_p+1)(j_p-\Omega_p+2)}. \quad \text{(B37)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_1^2(p)-\hat{j}_2^2(p)|Ij_pj_nK\Omega_p+2\Omega_nM\rangle = \frac{1}{2}\sqrt{(j_p-\Omega_p)(j_p-\Omega_p-1)(j_p+\Omega_p+1)(j_p+\Omega_p+2)}. \quad \text{(B38)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_1^2(n)-\hat{j}_2^2(n)|Ij_pj_nK\Omega_p\Omega_n-2M\rangle = \frac{1}{2}\sqrt{(j_n+\Omega_n)(j_n+\Omega_n-1)(j_n-\Omega_n+1)(j_n-\Omega_n+2)}. \quad \text{(B39)}$$

$$\langle Ij_pj_nK\Omega_p\Omega_nM|\hat{j}_1^2(n)-\hat{j}_2^2(n)|Ij_pj_nK\Omega_p\Omega_n+2M\rangle = \frac{1}{2}\sqrt{(j_n-\Omega_n)(j_n-\Omega_n-1)(j_n+\Omega_n+1)(j_n+\Omega_n+2)}. \quad \text{(B40)}$$

$$\langle K,\Omega|\hat{I}_1^2-\hat{I}_2^2|\Omega,K-2\rangle = \frac{1}{2}\sqrt{(I+K)(I+K-1)(I-K+1)(I-K+2)}. \quad \text{(B41)}$$

$$\langle K,\Omega|\hat{I}_1^2-\hat{I}_2^2|\Omega,K+2\rangle = \frac{1}{2}\sqrt{(I-K)(I-K-1)(I+K+1)(I+K+2)}. \quad \text{(B42)}$$