Nonlinear Phenomena in Complex Systems, vol. **17**, no. 4 (2014), pp. 451 - 454

Space–Time Non-Invariance of the Conformal Geometry and Its Possible Observable Manifestations

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(Received 05 September, 2014)

It is supposed that the geometry of the General Relativity flat limit can be described by semi-direct product of the Special Conformal Transformations and Lorentz groups, locally isomorphic to Poincare group. The possible observable manifestations of such a supposition are considered. It is shown that the detected Universe accelerated expansion can be treated as a purely kinematic effect of the proposed space–time geometry. The radar procedure of the distance determination in conformal space–time is described. It is shown that the space intervals conformal contraction gave rise to anomalous violet frequency shift during the monochromatic signal propagation over the closed path. Its relative value equals the Hubble constant multiplied by duration of propagation. The predicted phenomenon is the local manifestation of the cosmologic expansion and, in principle, is accessible to experimental detection.

PACS numbers: 03.30.+p; 11.25.Hf
Keywords: conformal geometry, Hubble law, radar distance, space–time deformation, violet frequency shift

We shall suppose that the geometry of General Relativity flat limit is defined by the group

$$\bar{\mathscr{P}} = SCT \rtimes L \tag{1}$$

where $\rtimes$ is designation of semi-direct product, L is the Lorentz group, SCT is the Special Conformal Transformations group

$$\left.\begin{aligned} x'^{\mu} &= \sigma^{-1}(x,b)\{x^{\mu} + b^{\mu}(x^2)\}, \\ g_{\mu\nu} &= \eta_{\mu\nu} = \mathrm{diag}\{1,-1,-1,-1\}, \end{aligned}\right\} \tag{2}$$

$$\sigma(x,b) = 1 + 2(xb) + (b^2)(x^2). \tag{3}$$

SCT (2) are nonlinear (space–time inhomogeneity) and singular (under $\sigma(x,b) = 0$) but conserving light cone equation (in the domain free of singularities). The group (1) is locally isomorphic the Poincare group. As it shown in [1–3] under choice

$$x^{\mu} = \{x^0 = ct, x, 0, 0\}, \quad b^{\mu} = \left\{0, -\frac{1}{2R_0}, 0, 0\right\} \tag{4}$$

where $R_0 = ct_0$ is a parameter with dimension of length, the light cone generatrix lines transformations under SCT are

$$\check{t}_{(\pm)} = \left(1 \pm \frac{\check{t}}{t_0}\right)^{-1} \check{t} \tag{5}$$

where $(\pm)$ corresponds to $x = \pm ct$.

Formula (5) gives the following dependence of the signal propagation duration $t(z)$ on the red shift:

$$t(z) = \check{t}_{(-)}(z) = t_0 \left\{1 - (1+z)^{-\frac{1}{2}}\right\}, \tag{6}$$

and, correspondingly, the expression

$$D(z) = ct(z) \tag{7}$$

for the distance covered by signal.

From (6), (7) and the well known formula

$$V(z) = c\frac{(1+z)^2 - 1}{(1+z)^2 + 1}, \tag{8}$$

for the relativistic longitudinal Doppler effect (V is the relative velocity) follows the analytic

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expression defining the Hubble law as a function on the red shift only

$$\frac{V(z)}{D(z)} = t_0^{-1} f(z) = t_0^{-1} \frac{(1+z)^{1/2}}{(1+z)^2+1} \cdot \frac{(1+z)^2-1}{(1+z)^{1/2}-1}. \tag{9}$$

Because of $\lim_{z\to 0} f(z) = 2$ it will be obvious that $t_0 = 2H_0^{-1}$ where H_0 is the Hubble constant. Formula (9) reproduce the contemporary cosmological expansion - relate experimental data (on interval $0.2 \leqslant z \leqslant 1.7$) in full accordance with observations (see Figs. 1, 2).

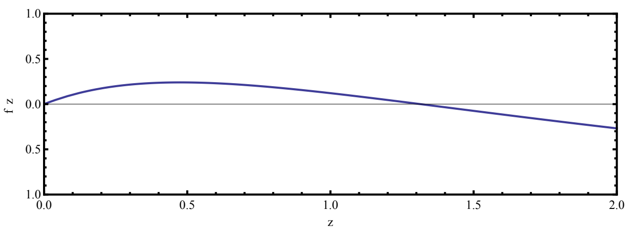

FIG. 1. The function $f(z)$. X-axis coincides with $f(z) = 1$, $z_{max} = 0.474$, $z_{intercept} = 1.315$.

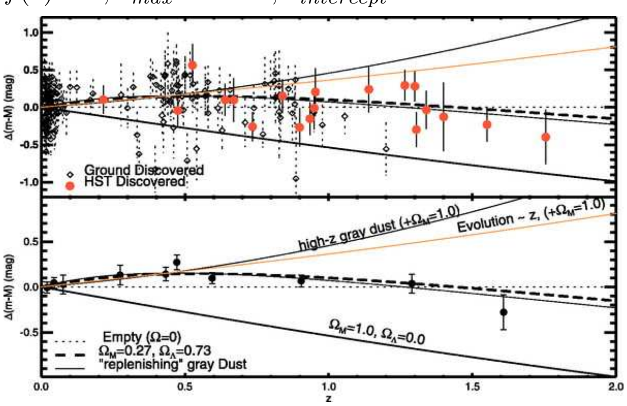

FIG. 2. Residual Hubble diagram (from [5]). X-axis coincides with dotted line $z_{max}^{exp} = 0.46 \pm 0.13$. (In colour).

The function $f(z)$ (Fig. 1) in fact coincides with the heavy dashed line (lower part of Fig. 2) describing the observable deflections from strictly linear Hubble law behaviour. By this means the explicit formula for Hubble law depending on the red shift only is reproducing the observed nonlinear deviations without any free parameters can be obtained as a purely kinematic outcome of light cone equation invariance under $\bar{\mathscr{P}} = SCT \rtimes L$ group transformations.

The possibility of an adequate physical interpretation of SCT supposes the restriction by singularity-free space–time domain and the existence of physically significant limit. In the case (4) SCT is the mapping $\{x, x^0\}$, singular on the light cone generating lines

$$x_{(\pm)} = -2R_0 \pm ct, \tag{10}$$

divining the $\{x, x^0\}$-plane into four singly-connected sectors. We shall consider one-to-one mapping:

open right sector of $\{x, x^0\}$-plane $\longleftrightarrow$, open left sector of $\{x', x'^0\}$-plane. Because of $R_0 = 2cH_0^{-1}$ the geometry of domain $\{x \geqslant 0, \quad |x^0| \geqslant 2R_0\}$ is practically coinciding with geometry of pseudoeuclidien semiplane $\{x \geqslant 0, \quad |t| < \infty\}$ as whole. In so doing (see [6, 7]):

$\{0,0\} \longleftrightarrow \{0,0\}'$, light cone $\longleftrightarrow$ light cone$'$. Every straight world line parallel to t-axis maps into hyperbola

$$(x' - \lambda(\alpha))^2 - c^2 t'^2 = \left(\frac{R_0}{\alpha}\right)^2, \; \lambda(\alpha) = \frac{R_0}{\alpha}(2\alpha - 1) \tag{11}$$

where

$$\alpha = 1 + \frac{x}{2R_0}, \quad (1 \leqslant \alpha < \infty). \tag{12}$$

Under conditions (small-neighbourhood approximation)

$$\frac{\Delta x}{R_0} \ll 1, \quad \frac{\Delta t}{t_0} \ll 1, \tag{13}$$

the SCT near $\{x, x^0\}$ origin coincide with Galilei–Newton transformations from inertial reference frame (RF) to the uniformly accelerating RF. Because of $t_0 \sim H_0^{-1}$ conditions (13) are really satisfied on the astrophysics scales. The mapping under consideration is presented on the Figs. 3, 4.

The clock synchronization and the radar distance definition under conformal geometry conditions are distinct from ones in the Special Relativity (SR) where (1) duration of signal propagation in forward $\left(\Delta t_{AB} = t_B - t_A^0\right)$

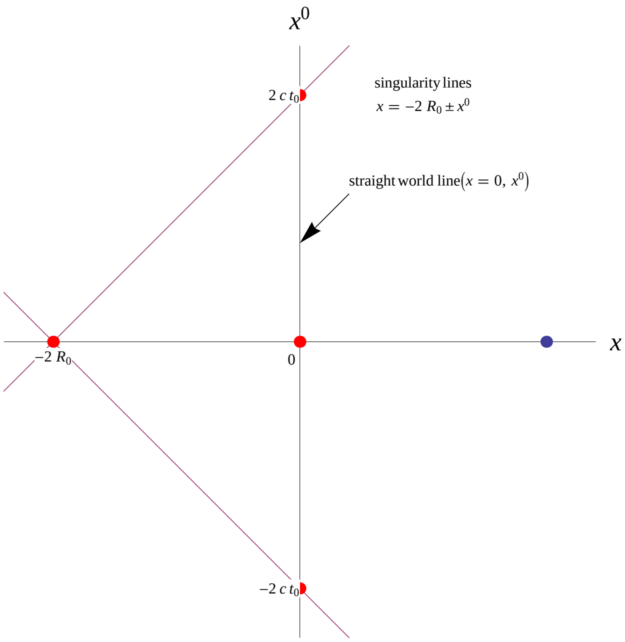

FIG. 3. The right sector of $\{x, x^0\}$. The line coinciding with t-axis is mapped to hyperbola $\alpha = 1$. (In colour).

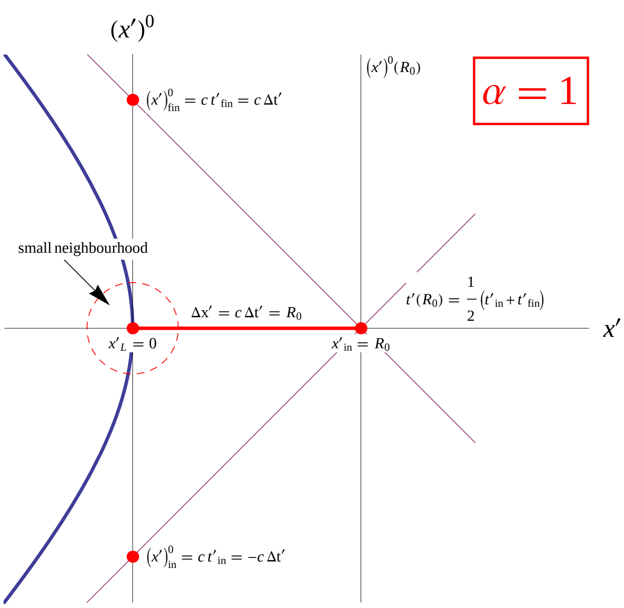

FIG. 4. The part of left sector $\{x', x'^0\}$. Small-neighbourhood of space–time domain of the co-ordinate origin $\{0, 0\}'$ is a model of contemporary part of Universe where real experiments and observations have been carried out. (In colour).

and backward ($\Delta t_{AB} = t_A - t_B$) directions are coinciding in every inertial RF, (2) the choice of t_A^0 is arbitrary, and (3) $t_B\left(t_A^0, t_A\right) = \frac{1}{2}\left(t_A^0 + t_A\right)$. These combined statements fails in conformal geometry. As it is evident from (5) the $\Delta t_{AB} = \Delta t_{BA}$ condition cannot be satisfied. Nevertheless the basic definition

$$\Delta t_r \overset{def}{=} \frac{1}{2}\left(\Delta t_{AB} + \Delta t_{BA}\right) = \frac{1}{2}\left(t_A - t_A^0\right) \qquad (14)$$

and, correspondingly, the radar distance $\mathfrak{D}_r = c\Delta t_r$ holds true (see [7]). Remember, this process can be realized practically in the small-neighbourhood approximation only.

The infinitesimal length ($d\overline{x}$) and duration ($d\overline{t}$) transformations under SCT in line with definitions $d\overline{x}' \overset{def}{=} dx'(x,t)|_{dt=0}$, $d\overline{t}' \overset{def}{=} dt'(x,t)|_{dx=0}$ defined by formulae

$$\left.\begin{aligned} d\overline{x}' &= \frac{dx}{\left(1 + \dfrac{x}{2R_0}\right)^2 + \left(\dfrac{ct}{2R_0}\right)^2}, \\ d\overline{t}' &= \frac{dt}{\left(1 + \dfrac{x}{2R_0}\right)^2 + \left(\dfrac{ct}{2R_0}\right)^2} \end{aligned}\right\} \qquad (15)$$

are parametrically dependent on t and x, correspondingly (see [2, 7]).

If the point of observation coincides with the co-ordinate origin we have after integration (15) the following expressions:

$$\Delta x' = \Delta x\left(1 + \frac{\Delta x}{2R_0}\right)^{-1}, \quad \Delta t' = 2t_0 \arctan\frac{\Delta t}{2t_0}, \qquad (16)$$

which define the conformal transformations of length and time intervals. Here $\Delta t = \Delta t_r = t_A/2$ defined by formula (14) and measured by the clock in coordinate origin.

The small-neighbourhood approximation gives

$$\Delta x' \cong \Delta x\left(1 - \frac{\Delta x}{2R_0}\right), \qquad \Delta t' = \Delta t. \qquad (17)$$

Using the notation $\Delta t = t = \frac{t_A}{2}$ we obtain the formula defining the conformal contraction of length:

$$\Delta x' = \Delta x - \frac{1}{2}\frac{c}{t_0}t^2 \qquad (\Delta x = ct). \tag{18}$$

Let us consider a signal propagation between two parallel perfectly reflecting mirrors divided by distance

$$\Delta x = x_B - x_A = ct \quad (\text{ in } \quad RF) \tag{19}$$

where we use the notation t for $\Delta t = \frac{t_A}{2}$.

The corresponding distance in RF' under condition $\frac{\Delta x}{R_0} \ll 1$ is

$$\Delta x' \cong \Delta x - \frac{(\Delta x)^2}{2R_0} = ct - \frac{c}{2t_0}t^2. \tag{20}$$

It is looking out as uniformally accelerating ($W_B = \frac{c}{t_0}$) motion of point B towards the point A with a time-dependent velocity

$$V_B(t) = W_B t = \frac{1}{2}cH_0 t. \tag{21}$$

The final result can be described as an effect of the monochromatic wave reflection from the mirror which is moving to the point of observation with velocity $V_B(t)$.

The well-known formula for the frequency change in the normal incidence case gives

$$\nu_{ref} = \nu_{inc}\frac{1+\frac{V}{c}}{1-\frac{V}{c}}\Bigg|_{V/c\ll 1} \cong \nu_{inc}\left(1+\frac{2V}{c}\right). \tag{22}$$

The relative violet frequency shift is

$$\frac{\Delta\nu}{\nu} = \frac{\nu_{ref}-\nu_{inc}}{\nu_{inc}} = \frac{2V}{c}. \tag{23}$$

If $V = V(t)$ is the instantaneous velocity (instantaneous reflection case) then it follows from (21), (23) that

$$\frac{\Delta\nu}{\nu} = 2W_B t = H_0 t. \tag{24}$$

In the case when $V = \overline{V(t)}$ is the average velocity the result will be $\frac{\Delta\nu}{\nu} = \frac{1}{2}H_0 t$. Such an anomalous violet frequency shift can be, in principle, observed experimentally. If the reflection is multiple, the individual time intervals (under the accepted approximation) will be summarized.

In summary it may be said that the observed accelerating Universe phenomenon can be described as the purely kinematic conformally-invariant approach without using the dark energy concept. This approach predicts: the form of the experimental Residual Hubble diagram by further increase of cosmological red shift, and the existence of anomalous violet frequency drift during the monochromatic signal propagation over the closed path as a local manifestation of the cosmological expansion.

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