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Probability of Failure of "Doubled" Power Supplies in Series

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The reliability of the power supplies for the SSC correction system is of concern. With similar units in a series configuration, one failure puts the whole system down. If there are assumed to be N identical units and the failure rate for one unit is λ (units of inverse time), the simple probability of system survival after time t (treating the units as independent) is

$$P_0(t) = \exp(-N\lambda t) \tag{1}$$

Data [1] from the Tevatron, with similar supplies, shows that $\lambda \approx 8 \times 10^{-5} hr^{-1}$. For the SSC, with N = 2500, the mean time for a system failure would be

$$T_f \approx 1/N\lambda = 5 \text{ hours} \tag{2}$$

Even if repair time is short compared to 5 hours, this situation would be unacceptable.

One solution is to double up the power supplies, that is, put two units in parallel at each position, with associated circuitry that automatically switches from one to the other without interruption. The survival probability of a series system of these doubled units can be calculated as follows. Consider first one "unit" consisting of two supplies in parallel. The probability of survival of one supply at time t is $p_1(t) = \exp(-\lambda t)$ and the probability of failure is $[1-p_1(t)]$. The joint probability of failure of both supplies (again assuming independence) is

$$[1 - \exp(-\lambda t)]^2$$

The converse probability of survival of the doubled unit (the probability that not more than one supply of the two has failed) is

$$p(t) = 1 - (1-e^{-\lambda t})^2 \tag{3}$$

$$= 2e^{-\lambda t} - e^{-2\lambda t}$$

$$= e^{-\lambda t}\,(2-e^{-\lambda t})$$

The probability of survival of the N doubled units is

$$P(t) = e^{-N\lambda t}\,(2-e^{-\lambda t})^N \tag{4}$$

For a series hookup of N units, this is the survival probability of the system, not allowing for repair in finite time.

The attached graph shows P(t) versus $N\lambda t$ for N = 1000, 2500, 5000. The arrows indicate the values of $N\lambda t$ for N = 2500, t = 10 days, and λ = 6, 7, or $8 \times 10^{-5}\,hr^{-1}$. There is roughly at 50% chance of the failure of a doubled unit in a 10-day running period, if the Tevatron experience is relevant.

In the limit $\lambda t \ll 1$, Eq. (4) can be approximated by

$$P(t) \approx \exp\left(-\frac{x^2}{N} + \frac{x^3}{N^2} + \ldots\right) \tag{5}$$

where $x = N\lambda t$. Just the first term gives the dashed curve (for N = 2500) in the figure, clearly an excellent approximation in the range of interest. It is valid provided $N\lambda^3t^3 \ll 1$.

The first term in Eq. (5) can be obtained directly, if heuristically, by the following argument. At short times, the chance that one power supply has failed after time t is λt. The chance that one of a pair has failed is $2\lambda t$.

The <u>rate</u> of failure of the doubled unit is therefore $\lambda(2\lambda t)$. In other words,

$$\frac{dp}{dt} \approx -2\lambda^2 t\ p$$

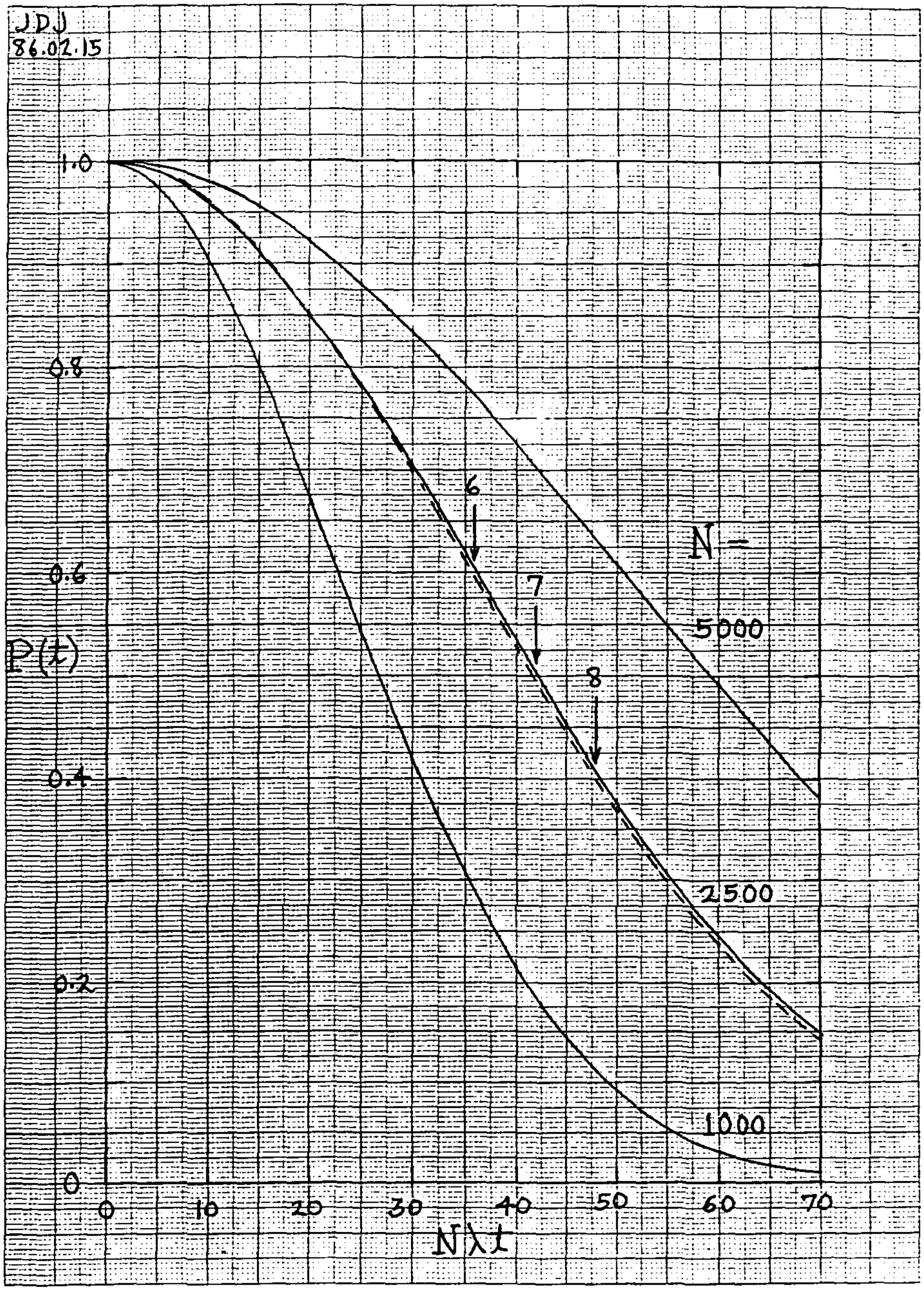
Integration then gives

$$P(t) = p^N \approx \exp(-N\lambda^2 t^2) \tag{6}$$

Note that the dependence on λ^2 puts a premium on improvement of the lifetime of the individual power supplies. A factor of two decrease in λ would change the survival probability from 50% to over 80% for a 10 day run.

References:

Private communication. Log book data from April through August, 1985 showed 44 failures among 220 correction dipole power supplies in 2600 hours of running.



JDJ
86.02.15
1.0
0.8
0.6
0.4
0.2
0
P(t)
0
10
20
30
40
50
60
70
Nλt
N =
5000
2500
1000
6
7
8