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# The Higgs Mechanism and Cosmological Constant Today

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**Abstract:** The Higgs mechanism, as responsible for the first inflation, powers the initial accelerated expansion and further preheating via the symmetry breaking from its false vacuum state corresponding to the Sitter vacuum of the GUT scale with $\Lambda = 8\pi G\rho_\Lambda$, whose decay provides necessary energetic support. Here we address the question of the possibility of symmetry restoration of the Higgs field at the presently observed vacuum scale which would make it responsible for the today value of the cosmological constant $\lambda = 8\pi G\rho_\lambda$. We find the existence of the possibility of symmetry restoration in the minisuperspace model of quantum cosmology and show that $\lambda$ today must have a non-zero value.

**Keywords:** Higgs mechanism; symmetry restoration; cosmological constant; de Sitter vacuum

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## 1. Introduction

The Higgs mechanism [1–3], as responsible for inflation, refs. [4–18] generically involves the de Sitter vacuum (false vacuum of the Higgs field) of the GUT scale $\Lambda = 8\pi G\rho_\Lambda$, which supports the accelerated expansion and powers the following stage—preheating—via the symmetry breaking of intrinsically involved scalar fields [19] (for a recent review, see [20,21]). At this stage the de Sitter vacuum decays into heavy Higgs and scalar particles which in turn quickly decay into the lighter species leading to the radiation-dominated stage [22–26] (for a review, see [27]).

Starting from symmetry breaking of the Higgs scalar field(s), the universe evolution goes consequently to its present state dominated by a dark energy with a negative pressure $p = w\rho$, $w < -1/3$ [28–30] and the best fit $w = -1$, $p = -\rho$ corresponding to the de Sitter vacuum with the cosmological constant $\lambda = 8\pi G\rho_\lambda$, reported by observations today [31–34] (for a review, see [35]).

In this paper, we consider a possibility of symmetry restoration of the Higgs field at the presently observed vacuum scale, which would make the Higgs field responsible for the today value of the cosmological constant. We apply the approach based on quantization of the cosmological constant [36] envisaged by the gauge noninvariance of quantum cosmology [37] which leads to a connection between a choice of the gauge and quantum spectrum for a certain physical quantity that can be specified in the framework of the minisuperspace model.

The key point is that the spontaneous symmetry breaking of the Higgs scalar fields implicitly and inevitably also involves the breaking of spacetime symmetry from the de Sitter group, when the symmetric false vacuum of the Higgs field—the maximally symmetric de Sitter vacuum

$$T_i^k = \rho_{vac}\delta_i^k = (8\pi G)^{-1}\Lambda\delta_i^k \quad (1)$$

evolves to its true vacuum state or to another vacuum state with the reduced symmetry as compared with (1) [38], releasing a vacuum energy driving inflation and further preheating (for a review, see [39]).

The analysis of the data of cosmological observations tells us that (i) in all presented inflation models with the initial de Sitter state, the de Sitter vacuum supports the accelerated expansion independently on an underlying particular model [40–42] (for a review, see [43,44]), and (ii) various models for dark energy (for a recent review, see [35]) lead to the equation of state $p = -\rho$ unambiguously identified as the de Sitter vacuum.

In fact, the cosmological observations testify for the spacetime symmetry breaking at the beginning of the universe evolution, at the GUT scale $\sim 10^{15}$ Gev, and suggest the spacetime symmetry restoration at achieving the stage dominated by the today value of the cosmological constant.

The standard model of particle physics provides the possibility of symmetry breaking from the false vacuum state at the GUT scale supporting the first inflation, and envisages the symmetry restoration of the Higgs field at the electroweak scale and its further breaking for supplying with masses vector bosons mediating the weak interactions (see [45] and references therein).

In this paper, we analyze the possibility of symmetry restoration for the Higgs field to its false vacuum state at the energy scale corresponding to the today value of the cosmological constant, conditioned by restoration of spacetime symmetry to the de Sitter vacuum state.

## 2. From Symmetry Breaking to Symmetry Restoration

In the literature there are a lot of papers devoted to the dynamics of vacuum density related to cosmological constant by (1), and motivated by the search for a possible solution to the cosmological constant problem [46]. Most of them apply the models for cosmic vacuum density evolving with the expansion rate [47–54], and the models with decaying or relaxing the cosmological constant [55–63].

In this paper, we apply the possibility of quantization of the cosmological constant granted by the gauge non-invariance of quantum cosmology which provides the existence of the connection between a gauge and a quantum spectrum of a certain physical quantity, that can be specified in the framework of the minisuperspace model. The appropriate gauge exists also for a cosmological constant $\Lambda$, in which it is quantized [36].

In quantum cosmology, the universe is described by a wave function $\Psi[h_{ik}(\vec{x}), \phi_m(\vec{x})]$ defined on the superspace of all 3-dimensional geometries $h_{ik}(\vec{x})$ and matter fields $\phi_m(\vec{x})$, and governed by the Wheeler–DeWitt equation [64,65]

$$\hat{H}\Psi = 0. \tag{2}$$

In the spherically symmetric homogeneous isotropic minisuperspace models typically applied for description of our universe, the four-dimensional line element is written in the form [66]

$$ds^2 = N^2(t)dt^2 - a^2(t)d\Omega_3^2 \tag{3}$$

where $N(t)$ is a lapse function, $a(t)$ is the scale factor, and $d\Omega_3^2$ is the metric on a unit 3-sphere. The units with $c = 1$ are adopted. In the synchronous gauge, $N = 1$, Equation (3) takes the standard FLRW form.

The action including the cosmological term

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R - 2\Lambda] \tag{4}$$

in the geometry (3) involves the Lagrangian [36,66]

$$\mathcal{L} = \frac{N}{2}\left[a\left(k - \frac{\dot{a}^2}{N^2}\right) - \Lambda a^3\right] \tag{5}$$

where $k$ is the curvature parameter, $k = -1, 0, +1$ for an open, closed, and flat model, respectively. In the canonical form of the Lagrangian $\mathcal{L} = p_a\dot{a} - \mathcal{H} = p_a\dot{a} - N(t)H$, the lapse function $N(t)$ plays the role of the Lagrange multiplyer, and the Lagrange equation $\partial\mathcal{L}/\partial N = 0$ gives the Hamiltonian constraint

$$H = -\frac{1}{2}\left[\frac{p_a^2}{a} + ka - \Lambda a^3\right] = 0 \quad (6)$$

which does not involve time dependence given by $N(t)$. The standard procedure of quantization $p_a \to -i\hbar d/da$ applied in the constraint (6), gives the Wheeler–DeWitt Equation (2) independent on the lapse function $N(t)$ [66].

An invariance of the theory under time reparametrization expressed by the Hamilton constraint, in fact, hides the gauge-noninvariance of quantum cosmology under field redefinition involving the lapse function $N(t)$ [37]. For example, in the synchronous gauge ($N = 1$) the rest-mass energy of a dust-filled universe is quantized in the closed FLRW model where the curvature generated potential ($k = +1$) represents the well with infinite walls [65], while adding cosmological term transforms an infinite well into a finite barrier which implies a possibility of the birth of a closed universe from nothing in the quantum tunnelling event [67]. In the conformal gauge ($N = a$) the quantized quantity taking the role of energy in the Wheeler–DeWitt equation corresponds to the contribution of the radiation, $p = \rho/3$, to the total energy density, which makes possible a quantum birth of a hot universe in the tunnelling event [68].

To find a gauge, appropriate for quantization of $\Lambda$, we express the line element in the form [36]

$$ds^2 = N^2(a)d\eta^2 - a^2(\eta)d\Omega_3^2; \;\; d\eta = dt/N(a) \quad (7)$$

which explicitly includes the field redefinitions involving the lapse function $N(a)$ in the configuration space, so that the dynamical system becomes clearly noninvariant under the gauge transformations. Equation (7) presents a generalization of the conformal gauge $N(a) = a$ ($dt = ad\eta$) [36].

In the canonical variables $q = \int \sqrt{a/N(a)}da$ and $p_q = \int \sqrt{N(a)/a}da$, the standard procedure of quantization, $p_q \to -id/dq$ gives the Wheeler–DeWitt equation in the form [36,66]

$$\left(-\frac{d^2}{dq^2} + V(q)\right)\Psi = 0; \; V(q) = \frac{1}{l_{Pl}^4}\left(N(a(q))ka(q) - \frac{8\pi G}{3}\rho(a(q))a^3(q)N(q)\right)\Psi = 0 \quad (8)$$

where $\rho$ is the energy density (we adopted the units with $c = 1$) which includes the contributions from non-interacting ingredients of the matter content,

$$\rho(a) = \sum_m B_m a^{-m}. \quad (9)$$

The parameter $m$ is related as $m = 3(1 + w)$ to the parameter $w$ specifying the equation of state $p = w\rho$.

The connection between the gauge function $N(a)$ and a certain quantized quantity $Q(a)$ is obtained by presenting $N(a)$ as $N(a) = l_{Pl}^{-m}a^{m-3}; \;\; q = a^{3-m/2}/(3 - m/2)$, which results in separation of a scale-factor-free term in the Wheeler–DeWitt Equation (8) [36]

$$-\frac{\hbar^2}{2m_{Pl}}\frac{d^2\Psi}{dq^2} + \frac{E_{Pl}}{2l_{Pl}^2}(U(a(q)) - Q_m)\Psi = 0; \;\; Q_m = \frac{8\pi G}{3}B_m. \quad (10)$$

Equation (10) describes a quantum system with a quantized contribution of a matter ingredient specified by the equation of state with $w = m/3 - 1$ in the potential, created by other components of the matter content. For example, the choice $m = 3$ and hence $w = 0$, which leads to $N = 1$ in Equation (7) and to $B_m = B_3$ in Equation (10), corresponds to the case of quantization of the dust-filled closed universe considered by DeWitt [64].

For the choice of the gauge $N(a)a^3 = 1$, correspondingto $m = 0$ and $q = a^3/3$, Equation (10) describes the quantum system with the quantized cosmological constant (vacuum energy density $\rho_{vac} = \rho_\lambda$) presented by $Q_0 = B_\lambda = \rho_\lambda/\rho_0$.

$$\frac{\hbar^2}{2m_{Pl}}\frac{d^2\Psi}{dq^2} + \frac{E_{Pl}}{2l_{Pl}^2}(B_\lambda - U(a(q)))a_0^{-2}\Psi = 0; \;\; a_0^2 = \frac{3}{8\pi G\rho_0}. \tag{11}$$

Normalizing the densities in (9) to $\rho_0 = \rho_{\mathrm{GUT}}$ and the scale factor $a$ to $a_0$, we present the potential in the dimensionless form

$$U(q(a)) = (k + B_s)(a_0/a)^2 - B_d(a_0/a)^3 - B_\gamma(a_0/a)^4; q = a^3/(3a_0^3). \tag{12}$$

It includes contribution of a dust-like, non-relativistic matter specified by $B_d$, of radiation specified by $B_\gamma$, and of an admixture of strings $B_s$ with the equation of state $p = -\rho/3$ and a negative deficit angle (see [68] and references therein). Contribution $B_s$ mimics the curvature term and provides the existence of a barrier (shown in Figure 1 below) needed for quantum tunnelling in the favored by observations case $k = 0$ [36,68].

The boundary conditions to the wave Equation (11) are $\Psi = 0$ at $a(q) = 0$ [64], and an outgoing wave function out of a barrier [69].

The wave function is given by the superposition of quantum states $\Psi = \sum c_j\psi_j$ satisfying

$$\frac{d^2\psi_j}{dq^2} - (U(q) - B_{\lambda\, j})\psi_j = 0. \tag{13}$$

The quasiclassical solution to Equation (13) outside of a barrier reads [36]

$$\psi_j = \frac{1}{\sqrt{(B_{\lambda\, j} - U(q))}} \exp\left[\pm i \int \sqrt{(B_{\lambda\, j} - U(q))}dq\right] \tag{14}$$

Evolution goes via transitions between subsequent quantum levels related to possible scales of symmetry breaking. The probability $|\psi_j|^2$ is maximal near the intersection of the potential curve $U(q)$ with $B_{\lambda\, j}$. At the beginning the probability $|\psi_j|^2$ is maximal for the initial vacuum scale $\Lambda$ shown in Figure 1 (Left), while making a measurement today we find a small value $\lambda$ with the bigger probability [36].

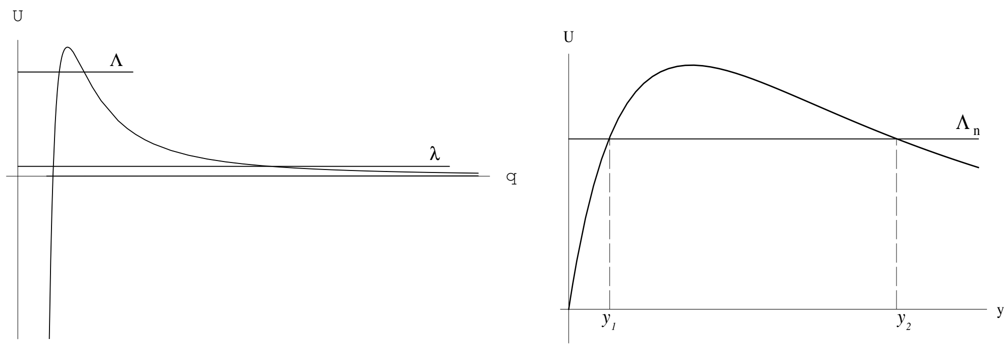


**Figure 1.** (**Left**): Typical behavior of a potential $U(q)$. Quantum levels are shown for the initial and the present vacuum states. (**Right**): Typical behavior of a potential $U(y)$.

Inthe region between $a = 0$ and $a = a_{(max\ U)} = \sqrt{(2B_\gamma/B_s)}a_0$, defined by $U' = 0$, we can apply quantization of $\Lambda$ by using the Bohr–Sommerfeld formula in the the WKB approximation [70], because for $a < a_{(max\ U)}$ the spectrum is discrete (for $a > a_{(max\ U)}$ the spectrum is continuous). As we are interested in the positive values of $\Lambda$, we can transform this region, by introducing the variable $y = a - a_{in}$ where $a_{in} = \sqrt{(B_\gamma/B_s)}a_0$ is defined by $U(a) = 0$, into the well between $y = 0$ and $y = y_{(max\ U)} = a_{(max\ U)} - a_{in}$, shown in Figure 1 (right), and formed by the axis $U(y = 0)$ and the potential curve $U(y)$ where

$$U(y) = \frac{B_s}{(y + a_{in})^2} - \frac{B_\gamma}{(y + a_{in})^4}. \tag{15}$$

In terms of the variable $y$ the Bohr–Sommerfeld formula reads

$$2\int_0^{q_f} \sqrt{2m_{Pl}(\Lambda_n - V)}dq = 2\int_0^{y_f} \sqrt{2m_{Pl}(\Lambda_n - V(y))}(y + a_{in})^2 dy = \pi\hbar\left(n + \frac{1}{2}\right) \tag{16}$$

where $\Lambda_n = (E_{Pl}/2l_{Pl}^2)B_{\lambda\ n}, \quad V(y) = (E_{Pl}/2l_{Pl}^2)U(y)$, and $y = y_f$ is determined from $(\Lambda_n - V(y) = 0)$.

To analyze a possibility of symmetry restoration at the present vacuum scale $B_\lambda$ predicted by observations, we look for a constraint on the quantum number $n$ in the $\Lambda$ spectrum.

According to the conventional scenario for the Higgs inflation, the first inflationary stage corresponding to the GUT scale symmetry breaking, $E_{\rm GUT} \simeq 10^{15}$ GeV, is followed by quick decay of vacuum energy resulting in the radiation dominated stage. We can expect that this situation takes place in the region of integration in front of the barrier; $\rho_\lambda$ achieves its present value at $a_\lambda \sim 10^{18}$ cm, and recombination occurs at $a_r \sim 10^{25}$ cm [71]).

Below we shall evaluate thevalues of $a_{in}$ and $y_f$ essential for the region of integration in (16), and show that $y_f \sim 10^3$ cm, which is much less than the recombination scale $a_r$, whereas $a_{in} > a_1 \sim 10^{-12}$ cm, where $a_1$ corresponds to the end of the first inflation and $a_{in}$ corresponds to the beginning of domination of radiation, so that the region of integration in (16) appears within the radiation-dominated region of cosmological evolution. This leads to $B_d \ll B_\gamma$ in the potential (12), and justifies the choice of noninteracting density components in (9). Interactions can be essential at the previous stage of appearance and further decay of heavy particles after the first inflation [22–24]; however this should not change the estimate given by the integral (16), the boundaries of which will be evaluated below from the observational data in the frame of the cosmological model of the Lemaitre class.

Theintegral in (16) can be written as

$$\int_0^{y_f} \sqrt{(B_\lambda - U)}(y + x_{in})^2 dy = \frac{\pi}{2}\frac{l_{Pl}^2}{a_0^2}\left(n + \frac{1}{2}\right); \;\; U = \frac{B_s}{(y + x_{in})^2} - \frac{B_\gamma}{(y + x_{in})^4} \tag{17}$$

where $y_f = x_f - x_{in}$, and the variables are normalized to $a_0$, so that $x_{in} = a_{in}/a_0$; $y = (a - a_{in})/a_0 = x - x_{in}$. The parameter $x_{in}$ is determined by $U = 0$ which gives $x_{in}^2 = B_\gamma/B_s$. The parameter $x_f$ is found as the real root of the equation for intersection of the potential with a quantum level for cosmological constant, $B_\lambda x_f^4 - B_s x_f^2 + B_\gamma = 0$, which gives

$$x_f^2 = \frac{B_s}{2B_\lambda}\left[1 - \sqrt{1 - \frac{4B_\gamma B_\lambda}{B_s^2}}\right] \approx \frac{B_\gamma}{B_s}\left[1 + \frac{B_\gamma B_\lambda}{B_s^2}\right] \tag{18}$$

with the constraint $(4B_\gamma B_\lambda) < B_s^2$ required for the real roots.

Theintegral (17) can be evaluated by approximation of the potential $U(x)$ as a series around $x = x_{in}$ and then transition to the variable $y$, which gives the constraint

$$\int_0^{y_f} \sqrt{(B_\lambda - U)}(y + x_{in})^2 dy < \frac{1}{3}\frac{B_\lambda^{3/2}}{B_s}x_{in}^5\left(1 + \mathcal{O}\left(\frac{B_\lambda}{B_s}x_{in}^2\right)\right). \tag{19}$$

This allows us to get the constraint on the quantum number $n$ in (17) as

$$n + \frac{1}{2} < \frac{2}{3\pi}\left(\frac{a_0}{l_{Pl}}\right)^2 \frac{B_\lambda^{3/2}}{B_s}x_{in}^5. \tag{20}$$

The parameter $B_\lambda = \rho_\lambda/\rho_0$ is estimated as $B_\lambda \simeq 2.77 \times 10^{-107}$ for the present value of $\rho_\lambda = 6.45 \times 10^{-30}$ gcm$^{-3}$ and $\rho_0 = 2.33 \times 10^{77}$ gcm$^{-3}$ for $E_{\rm GUT} = 10^{15}$ GeV. The parameter $B_s$ has been evaluated by the observational constraints including an upper limit on the CMB anisotropy in a rather narrow range; for a flat universe, $k = 0$, it is estimated as $B_s \simeq 3 \times 10^{-6}$ [68].

The estimate for the parameter $x_{in} = a_{in}/a_0$ can be obtained from the analysis of a regular solution describing a vacuum-dominated universe with several scales of vacuum energy [71]. At the classical level, transitions between quantum states of the operator $\Lambda$ corresponds to several stages in the universe evolution related to phase transitions with different values of vacuum density $\rho_{vac}$ [45].

The evolution of $\rho_{vac}$ can be consequently described by stress-energy tensors with partially reduced vacuum symmetry, such that the inflationary equation of state $p_\alpha = -\rho$ is satisfied in only one or two spatial directions [72]. This inevitably leads (by virtue of $\nabla_i T^i_k = 0$) to a dynamical vacuum energy. Stress-energy tensors of this class describe vacuum dark fluid [73]. In the spherically symmetric case they have the algebraic structure [72]

$$T^t_t = T^r_r; \;\; T^\theta_\theta = T^\phi_\phi \tag{21}$$

which implies the $r$-dependent anisotropic pressure (see [72] and references therein)

$$p_r = -\rho; \;\; p_\perp = -\rho - \frac{r}{2}\rho' \tag{22}$$

and requires description of a universe evolution by the Lemaitre class cosmological model

$$ds^2 = d\tau^2 - \dot{r}^2 dR^2 - r^2(R,\tau)d\Omega^2; \;\; \dot{r} = dr/d\tau \tag{23}$$

for an asymptotically flat universe. At approaching de Sitter vacuum stages, the Lemaitre metric (23) approaches the FLRW form with the relevant de Sitter scale factor

$$ds^2 = d\tau^2 - a_i^2(dR_i^2 + R_i^2 d\Omega^2); \;\; a_i = r_{0i}e^{c\tau/r_{0i}}. \tag{24}$$

The standard models of cosmology and particle physics predict a sequence of symmetry-breaking phase transitions in the course of the expansion and cooling history of the universe [45]. The first phase transition at the GUT scale $E_{\rm GUT} \simeq 10^{15}$ GeV, occurs at a temperature $T \simeq 10^{28}$ K, and involves the spontaneous symmetry breaking of the Higgs fields, leading to a vacuum decay resulted ultimately in a radiation-dominated stage (see [41] and references therein).

The second phase transition predicted by the standard model of particle physics at the electroweak scale, occurs at a temperature $T_{EW} \sim 10^{15}$ K. At $T > T_{EW}$, the symmetry is restored, and vector bosons mediating the weak interactions are massless; at $T < T_{EW}$ they acquire masses via the Higgs mechanism, but the photon remains massless [45].

The next phase transition predicted at the quantum chromodynamics (QCD) scale $E_{\rm QCD} \sim (100 \div 200)$ MeV drives a second inflationary stage at which a quasi-stable QCD vacuum state provides a relatively short inflation period (about 7 e-foldings) consequently diluting the net baryon to photon ratio to its presently observed value [74,75] (for a review, see [45]). The density $\rho_{\rm QCD}$ is smaller by a factor of $(E_{\rm QCD}/E_{\rm GUT})^4$ than the GUT density $\rho_{\rm GUT} = \rho_0$, i.e., of the order of the nuclear matter density.

The third, presently observed inflationary stage corresponds to the vacuum energy density $\rho_{vac} = \rho_\lambda$ which is about 107 orders of magnitude smaller than the GUT density.

The situation with three vacuum scales driving the first, second, and present inflation, can be modeled by a phenomenological density profile [71]

$$\rho = \rho_0 \left[1 - (1 - A_1)\exp(-r_1^k/r^k) - (A_1 - A_3)\exp(-r_3^k/r^k)\right], \tag{25}$$

which describes vacuum decay by the exponential functions as typical for a decay process. Here

$$A_1 = \rho_{\rm QCD}/\rho_0 \approx 10^{-64}, \qquad A_3 = \rho_\lambda/\rho_0 = B_\lambda \simeq 2.77 \times 10^{-107}; \qquad r_0 < r_1 \ll r_3. \quad (26)$$

The rate of decay is uniquely fixed as $k = 4$ by the strict constraints $k > 3$ required for analyticity, and $k \leq 4$ required by causality. The parameters $r_1$ and $r_3$ correspond to the end of the first and the second inflation, and have been constrained by the observational data [71].

At $r \sim r_1$ the second term in (25) becomes dominant, and for $r_1 < r < r_3$ behaves like $\rho \approx \rho_0(A_1 + r_1^4/r^4)$, which corresponds to $\rho \propto r^{-4}$, behavior typical for radiation, for $r < r_2 = r_1 A_1^{-1/4} = 10^{16} r_1$. For $r > r_2$ we have $\rho \approx A_1\rho_0 = \rho_{\rm QCD}$ corresponding to the second inflation when the metric approaches the FRW form (24) with $i = 2$.

The parameter $r_1$ is constrained by the requirement of the late-time isotropy, $A < 10^{-6}$ for agreement with the CMB observations, where the anisotropy parameter $A$ is defined as $A = \sum_1^3 (H_j^2/3H^2);\ H_j = \dot{a}_j/a;\ \ H = \sum_1^3 (H_j/3)$, and $a_j$ are scale factors [76,77]. In our case with the Lemaitre metric (23), $a_1 = \dot{r},\ a_2 = a_3 = r$. This gives $r_1 < 3 \times 10^{-12}$ cm and $r_2 = r_1 A_1^{-1/4} = 10^{16} r_1 < 3 \times 10^4$ cm [71]. By the restriction on $r_1$, the requirement of the late-time isotropy constraints the admissible interval for the e-folding $N_e = \ln(r_1/r_{01})$ where $r_{01} \simeq 0.83 \times 10^{-25}$ cm for the GUT scale vacuum $E \simeq 10^{15}$ GeV. The e-folding $N_e$ is constrained by $N_e < 31$. Adopting $N_e = 29$, we obtain the estimates $r_1 = 10^{-12}$ cm and $r_2 = 1.2 \times 10^4$ cm for beginning of the second inflation [71].

The value $a_{in}$ at which radiation starts to dominate at the level which provides $U(a_{in}) = 0$ in the period between the end of the first inflation and beginning of the second inflation, can be obtained from the detailed analysis of behavior of scale factors in [71] as $a_{in} \approx 0.6 \times 10^3$ cm.

The basic constraint (20) gives finally

$$n + \frac{1}{2} < 0.54 \quad (27)$$

which testifies that the vacuum density achieves the lowest level in its quantum spectrum. Therefore we can say about symmetry restoration, since in this case its zero level density has non-zero value for $n = 0$ in Equation (16), and a further decay is impossible in principle as forbidden by quantum mechanics. It follows also that the present value of the cosmological constant cannot be zero.

Two fundamental facts testify to the deep intrinsic relation between symmetry breaking/symmetry restoration for the Higgs field and for spacetime: (1) the false vacuum of the Higgs field corresponds (by its equation of state) to the maximally symmetric de Sitter vacuum and in this way is generically related to the spacetime symmetry. (2) Breaking of symmetry of the Higgs field from its false vacuum state intrinsically goes on with the spacetime symmetry breaking. It is natural to assume that restoration of spacetime symmetry to its present vacuum state is related to restoration of symmetry of the Higgs field dynamically responsible for the previous universe evolution to its false vacuum state.

## 3. Summary and Conclusions

Transition from the big value of the cosmological constant $\Lambda = 8\pi G\rho_{\rm GUT}$ responsible for the first inflation, to the present value $\lambda = 8\pi G\rho_\lambda$ given by observations, can be consequently described in the framework of the minisuperspase model of quantum cosmology, and in the framework of the Lemaitre class cosmology.

Applying the quantization of the cosmological constant related to a vacuum energy $\Lambda = 8\pi G\rho_{vac}$, and the values of involved parameters from observational data, we find, without any additional assumptions and special tuning, that the present value of the cosmological constant corresponds to the lowest level in its quantum spectrum, which testifies for restoration of spacetime symmetry. If not introduce an additional hypothetical matter source for the present value of $\rho_{vac}$, we can identify it with the false vacuum state of the Higgs field which powers evolution of the universe starting from the first inflation. This symmetry restoration of the Higgs field differs essentially from its symmetry restoration at the electroweak scale, because the presently observed value of $\rho_{vac}$ corresponds to an absolute lower limit $n = 0$ in its quantum spectrum (similar to zero-point energy $E_0 = \hbar\omega/2$ in the energy spectrum). No further vacuum decay or $\lambda$-relaxation is possible, which testifies for the final symmetry restoration for both spacetime and the Higgs field.

The responsibility of the Higgs field in its lowest false vacuum state for the today cosmological constant implies the existence of an additional small energy scale $E_\lambda$, related to symmetry restoration for the Higgs field generically related to restoration of spacetime symmetry. The energy scale $E_\lambda$ can be evaluated from $\rho_\lambda$ by relating it to the Planck scale $E_{pl} = 1.22 \times 10^{19}$ GeV by the basic relation, $\rho_\lambda = \langle T_0^0 \rangle = (E_\lambda/E_{Pl})^4 \rho_{Pl}$, where $\rho_{Pl} = 5.157 \times 10^{93}$ gcm$^{-3}$ and $\rho_\lambda = 6.45 \times 10^{-30}$ gcm$^{-3}$. This gives $E_\lambda = (\rho_\lambda/\rho_{Pl})^{1/4}$ $E_{Pl} = 22.9 \times 10^{-4} \simeq 23 \times 10^{-4}$ eV.

Let us summarize. Analysis of the possibility to relate the today value of the cosmological constant with the vacuum energy density of the Higgs field in its false vacuum state, in the frame of the minisuperspace model of quantum cosmology with applying the relevant cosmological parameters estimated in the frame of the Lemaitre class cosmological model, leads to the following conclusions:

(i) The Higgs field can be dynamically responsible for the universe evolution from the initial symmetry breaking to the final symmetry restoration.
(ii) The presently observed value of $\rho_\lambda$ corresponds to an absolute lower limit $n = 0$ in its quantum spectrum.
(iii) The value of $\rho_\lambda$ today must be non-zero in principle.
(iv) There exists the relevant energy scale, $E_\lambda \simeq 23 \times 10^{-4}$ eV, characteristic for symmetry restoration related to the observed value of the cosmological constant.

Currently we are working on a detailed analysis of the mechanism of symmetry restoration of the Higgs field on the scale $E_\lambda$ in the framework of quantum field theory.

Inquantum cosmology, there exists a certain gauge specified in the frame of the minisuperspace model, in which the cosmological constant $\Lambda = 8\pi G\rho_{vac}$ is quantized. This allows for the description of evolution of $\rho_{vac}$ for the false vacuum state of an involved Higgs field, from symmetry breaking at the initial GUT state, via intermediate state(s) related to phase transitions in the universe evolution, to symmetry restoration at the final state which corresponds to its lowest level in the $\lambda$-quantum spectrum.

In QFT cosmological models with time-dependent cosmological constant decaying or running from a false vacuum state were studied in detail in [78] without assuming a particular form for $\Lambda$-time dependence and with using the assumption that the transition from a false vacuum is the quantum decay process. During the initial stage, $T_0 < t < T_1$, where $T_1$ refers to the end of the inflationary exponential acceleration, $\Lambda = \Lambda_0$ calculated in the QFT frame. At times $T_1 < t < T_2$ it decreases as an oscillatory modulated exponential function to the value $\Lambda_{eff}$ given by [78]

$$\lambda_{eff} = \lambda_{bare} + \alpha_2 (H(t))^2 + \alpha_4 (H(t))^4 + \ldots \ll \Lambda_0 \tag{28}$$

where the Hubble parameter $H(t)$ defines the cosmological scale. Similar parametrization is considered in many papers (for a recent review, see [62]). At times $t > T_2$ cosmological constant $\Lambda$ evolves in time as $\Lambda_{eff}(t)$ and tends to $\Lambda_{bare}$ as $t \to \infty$ [62,78].

Recently it was shown that if a universe was born in the metastable false vacuum state, the big initial value of $\Lambda_0$ calculated in the QFT frame for the inflationary era, can be later quickly reduced to the very small values, provided that the transition from a false vacuum is the quantum decay process. This process was discussed for different scales [62].

In principle, this picture agrees with the cosmological evolutioin described in Section 2, in which the stages of an inflationary expansion due to values of $\rho_{vac}$ defined in the frame of quantum cosmology, are followed by a quick decay modelled in the frame of the Lemaitre cosmology.

Comprehensive analysis of evolution of cosmological constant in the frame of QFT for a scalar field non-minimally coupled to gravity at the FLRW background is presented in [52] for running vacuum models in which vacuum energy density $\rho_{vac}$ is presented by a constant term, and dynamical components $O(H^2)$ and $O(H^4)$ are considered as a series of the even powers of the Hubble parameter.

Calculations using a modified form of the adiabatic regularization, and involving dimensional regularization, result in the properly renormalized vacuum energy density which does not contain the unwanted contributions from quartic powers of particle masses $m^4$ at different scales [52]. In addition, the applied adiabatic regularization prescription allows for extraction of the precise form for $\rho_{vac}$ from the renormalized zero-point energy up to terms of the adiabatic order $O(H^4)$. The present value of $\rho_{vac}$ is dominated by the constant term; dynamical components are reduced to $\nu H^2$ where the running parameter $\nu$ satisfies $|\nu| \ll 1$ [52].

We expect that the above QFT regularization mechanisms can be helpful in the case of quantized vacuum energy density.

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## References

1. Englert, F.; Brout, R. Broken Symmetries and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.* **1964**, *13*, 321. [CrossRef]
2. Higgs, P.W. Broken symmetries and the masses of gauge bosons. *Phys. Rev. Lett.* **1964**, *13*, 508. [CrossRef]
3. Quigg, C. *Gauge Theories of the Strong, Weak and Electromagnetic Interactions*; Addison-Wesley Publishing Company: Redwood City, CA, USA, 1983.
4. Bezrukov, F.; Shaposhnikov, M. The Standard Model Higgs boson as the inflaton. *Phys. Lett. B* **2008**, *659*, 703–706. [CrossRef]
5. Bezrukov, F.; Gorbunov, D.; Shaposhnikov, M. On initial conditions for the Hot Big Bang. *J. Cosmol. Astropart. Phys.* **2009**, *2009*, 29. [CrossRef]
6. Bezrukov, F.; Magnin, A.; Shaposhnikov, M.; Sibiryakov, S. Higgs inflation: Consistency and generalisations. *J. High Energy Phys.* **2011**, 16. [CrossRef]
7. Bezrukov, F. The Higgs field as an inflaton. *Class. Quant. Grav.* **2013**, *30*, 214001. [CrossRef]
8. Bezrukov, F.; Shaposhnikov, M. Higgs inflation at the critical point. *Phys. Lett. B* **2014**, *734*, 249–254. [CrossRef]
9. Aguilar, J.E.M.; Zamarripa, J.; Monte, M.; Romero, C. Higgs inflation in complex geometrical scalar-tensor theory of gravity. *Phys. Dark Universe* **2020**, *28*, 100480. [CrossRef]
10. Benisty, D.; Guendelman, E.I.; Nissimov, E.; Pacheva, S. Quintessential Inflation with Dynamical Higgs Generation as an Affine Gravity. *Symmetry* **2020**, *12*, 734. [CrossRef]
11. Jain, M.; Hertzberg, M.P. Eternal Inflation and Reheating in the Presence of the Standard Model Higgs Field. *Phys. Rev. D* **2020**, *101*, 103506. [CrossRef]
12. Gialamas, I.D.; Karam, A.; Lykkas, A.; Pappas, T.D. Palatini-Higgs inflation with non-minimal derivative coupling. *Phys. Rev. D* **2020**, *102*, 063522. [CrossRef]
13. Bargach, A.; Bargach, F.; Mariam Bouhmadi-López, M.; Taoufik Ouali, T. Non-minimal Higgs inflation within holographic cosmology. *Phys. Rev. D* **2020**, *102*, 123540. [CrossRef]
14. Barrie, N.D.; Sugamoto, A.; Takeuchi, T.; Yamashita, K. Higgs Inflation, Vacuum Stability, and Leptogenesis. *J. High Energy Phys.* **2020**, *2020*, 72. [CrossRef]
15. Bojowald, M.; Brahma, S.; Crowe, S.; Ding, D.; McCracken, J. Quantum Higgs Inflation. *Phys. Lett. B* **2021**, *816*, 136193. [CrossRef]
16. Cheong, D.Y.; Lee, S.M.; Park, C.S. Progress in Higgs inflation. *J. Korean Phys. Soc.* **2021**, *78*, 897–906. [CrossRef]
17. Bostan, N. Palatini Higgs and Coleman-Weinberg inflation with non-minimal coupling. *JCAP* **2021**, *2021*, 42. [CrossRef]
18. Karydas, S.; Papantonopoulos, E.; Saridakis, E.N. Successful Higgs inflation from combined nonminimal and derivative couplings. *Phys. Rev. D* **2021**, *104*, 023530. [CrossRef]

19. Turner, M.S. Inflation after COBE. Fermilab-Conf.92/313-A. 1992. Available online: https://www.osti.gov/servlets/purl/10158604 (accessed on 24 May 2022).
20. Horn, B. The Higgs Field and Early Universe Cosmology: A (Brief) Review. *Physics* **2020**, *2*, 503–552. [CrossRef]
21. Hamada, Y.; Kawana, K.; Scherlis, A. On Preheating in Higgs Inflation. *J. Cosmol. Astropart. Phys.* **2021**, *2021*, 62. [CrossRef]
22. Dymnikova, I.; Krawczyk, M. First postinflationary particles equation of state. In *Birth of the Universe*; Occhionero, F., Ed.; Springer: Berlin/Heidelberg, Germany, 1994.
23. Kofman, L.; Linde, A.; Starobinsky, A. Reheating after inflation. *Phys. Rev. Lett.* **1994**, *73*, 3195. [CrossRef]
24. Dymnikova, I.; Krawczyk, M. Equation of state and temperature of massive nonrelativistic bosons arising in the universe at the first stage of reheating. *MPLA* **1995**, *10*, 3069–3076. [CrossRef]
25. Wu, Y.-P. Higgs as heavy-lifted physics during inflation. *J. High Energy Phys.* **2019**, *2019*, 125. [CrossRef]
26. Mantziris, A.; Markkanen, T.; Rajantie, A. Vacuum decay constraints on the Higgs curvature coupling from inflation. *J. Cosmol. Astropart. Phys.* **2021**, *3*, 77. [CrossRef]
27. Litsa, A.; Freese, K.; Sfakianakis, E.I.; Stengel, P.; Visinelli, L. Large Density Perturbations from Reheating to Standard Model particles due to the Dynamics of the Higgs Boson during Inflation. *Phys. Rev. D* **2021**, *104*, 123546. [CrossRef]
28. Riess, A.G.; Kirschner, R.P.; Schmidt, B.P.; Iha, S.; Challis, P.; Garnavich, P.M.; Esin, A.A.; Carpenter, C.; Grashins, R.; Schild, R.E.; et al. BV RI light curves for 22 type Ia supernovae. *Astron. J.* **1999**, *117*, 707–724. [CrossRef]
29. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; et al. Measurements of $\Omega$ and $\Lambda$ from 42 high-redshift supernovae. *Astrophys. J.* **1999**, *517*, 565–586. [CrossRef]
30. Bahcall, N.A.; Ostriker, J.P.; Perlmutter, S.; Steinhardt, P.J. The cosmic triangle: Revealing the state of the universe. *Science* **1999**, *284*, 1481–1488. [CrossRef]
31. Corasaniti, P.S.; Copeland, E.J. Constraining the quintessence equation of state with SnIa data and CMB peaks. *Phys. Rev. D* **2002**, *65*, 043004. [CrossRef]
32. Hannestad, S.; Mortsell, E. Probing the dark side: Constraints on the dark energy equation of state from CMB, large scale structure, and type Ia supernovae. *Phys. Rev. D* **2002**, *66*, 063508. [CrossRef]
33. Bassett, B.A.; Kunz, M.; Silk, J.; Ungarelli, C. A late-time transition in the cosmic dark energy? *Mon. Not. R. Astron. Soc.* **2002**, *336*, 1217–1222. [CrossRef]
34. Schmidt, B.P.; Barris, B.; Candia, P.; Challis, P.; Clocchiatti, A.; Cail, A.L.; Filippenko, A.V.; Garnavich, P.; Hogan, C.; Holland, S.T.; et al. Cosmological results from high-z supernovae. *Astrophys. J.* **2003**, *594*, 1–24.
35. Rivera, A.B.; Farieta, J.G. Exploring the Dark Universe: Constraint on dynamical dark energy models from CMB, BAO and Growth Rate Measurements. *Intern. J. Mod. Phys. D* **2019**, *28*, 1950118. [CrossRef]
36. Dymnikova, I.; Filchenkov, M. Gauge-noninvariance of quantum cosmology and vacuum dark energy. *Phys. Lett. B* **2006**, *635*, 181–185. [CrossRef]
37. Halliwell, J.J. Derivation of the Wheeler-DeWitt equation from a path integral for minisuperspace models. *Phys. Rev. D* **1988**, *38*, 2468. [CrossRef]
38. Dymnikova, I.; Khlopov, M. Decay of cosmological constant as Bose-condensate evaporation. *Mod. Phys. Lett. A* **2000**, *15*, 2305–2314. [CrossRef]
39. Dymnikova, I. The Higgs Mechanism and Spacetime Symmetry. *Universe* **2020**, *6*, 179. [CrossRef]
40. Dymnikova, I.G. Inflationary universe from the point of view of General Relativity. *Zh. Eksper. Theor. Phys.* **1986**, *90*, 1900–1907.
41. Olive, K. Inflation. *Phys. Rep.* **1990**, *190*, 309–403. [CrossRef]
42. Gibbons, G.W. Phantom Matter and the Cosmological Constant. *arXiv* **2003**, arXiv:hep-th/0302199v1.
43. Dymnikova, I. Dark energy and spacetime symmetry. *Universe* **2017**, *3*, 20. [CrossRef]
44. Dymnikova, I. Fundamental roles of the de Sitter vacuum. *Universe* **2020**, *6*, 101. [CrossRef]
45. Boyanovsky, D.; de Vega, H.J.; Schwarz, D.J. Phase Transitions in the Early and Present Universe. 2006 *Ann. Rev. Nucl. Part. Sci* **2006**, *56*, 441–500. [CrossRef]
46. Weinberg, S. The Cosmological Constant Problem. *Rev. Mod. Phys.* **1989**, *61*, 1–23. [CrossRef]
47. Overduin, J.M.; Cooperstock, F.I. Evolution of the scale factor with a variable cosmological term. *Phys. Rev. D* **1998**, *58*, 043506. [CrossRef]
48. Shapiro, I.L.; Solà, J.; Espana-Bonet, C.; Ruiz-Lapuente, P. Variable cosmological constant as a Planck scale effect. *Phys. Lett. B* **2003**, *574*, 149. [CrossRef]
49. Espana-Bonet, C.; Ruiz-Lapuente, P.; Shapiro, I.L.; Solà, J. Testing the running of the cosmological constant with type Ia supernovae at high $z$. *J. Cosmol. Astropart. Phys.* **2004**, *2004*, 6. [CrossRef]
50. Solà, J.; Gómez-Valent, A. The $\Lambda$CDM cosmology: From inflation to dark energy through running $\Lambda$. *Int. J. Mod. Phys. D* **2015**, *24*, 1541003. [CrossRef]
51. Solà Peracaula, J.; de Cruz Pérez, J.; Gómez-Valent, A. Possible signals of vacuum dynamics in the Universe. *Mon. Not. R. Astron. Soc.* **2018**, *478*, 4357. [CrossRef]
52. Moreno-Pulido, C.; Solà Peracaula, J. Running vacuum in quantum field theory in curved spacetime: Renormalizing ρ vac without $\sim m^4$ terms. *Eur. Phys. J. C* **2020**, *80*, 692. [CrossRef]
53. Moreno-Pulido, C.; Solà Peracaula, J. Renormalizing the vacuum energy in cosmological spacetime: Implications for the cosmological constant problem. *arXiv* **2022**, arXiv:2201.05827.

54. Solà Peracaula, J. The Cosmological Constant Problem and Running Vacuum in the Expanding Universe. *arXiv* **2022**, arXiv:2203.13757.
55. Kennedy, A.; Lazarides, G.; Shafi, Q. Decay of the false vacuum in the very early universe. *Phys. Lett. B* **1981**, *99*, 38. [CrossRef]
56. Sahni, V.; Starobinsky, A. The Case for a Positive Cosmological Λ-term. *Intern. J. Mod. Phys. D* **2000**, *9*, 373–443. [CrossRef]
57. Bamba, K.; Capozziello, S.; Nojiri, S.; Odintsov, S.D. Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests. *Astrophys. Space Sci.* **2012**, *342*, 155–228. [CrossRef]
58. Abdalla, E.; Graef, L.L.; Wang, B. A model for dark energy decay. *Phys. Lett. B* **2013**, *726*, 786–790. [CrossRef]
59. Szydlowski, M.; Stachowski, A.; Urbanowski, K. Cosmology with a decaying vacuum energy parametrization derived from quantum mechanics. *J. Phys. Conf. Ser.* **2015**, *626*, 012033. [CrossRef]
60. Dymnikova, I.; Dobosz, A.; Sołtysek, B. Lemaître Class Dark Energy Model for Relaxing Cosmological Constant. *Universe* **2017**, *3*, 39. [CrossRef]
61. Márián, I.G.; Jentschura, U.D.; Defenu, N.; Trombettoni, A.; Nándori, I. Vacuum energy and renormalization of the field-independent term. *J. Cosmol. Astropart. Phys.* **2022**, *3*, 62.
62. Urbanowski, K. Cosmological "constant" in a universe born in the metastable false vacuum state. *Eur. Phys. J. C* **2022**, *82*, 242. [CrossRef]
63. Aich, A. Phenomenological dark energy model with hybrid dynamic cosmological constant. *Class. Quant. Grav.* **2022**, *39*, 035010. [CrossRef]
64. DeWitt, B.S. Quantum Theory of Gravity. I. The Canonical Theory. *Phys. Rev.* **1967**, *160*, 1113. [CrossRef]
65. Wheeler, J.A. Superspace and the nature of quantum cosmology. In *Battelle Rencontres*; DeWitt, C.M., Wheeler, J.A., Eds.; W. A. Benjamin, Inc.: New York, NY, USA, 1968; pp. 242–308.
66. Vilenkin, A. Approaches to quantum cosmology. *Phys. Rev. D* **1994**, *50*, 2581. [CrossRef] [PubMed]
67. Vilenkin, A. Creation of universes from nothing. *Phys. Lett. B* **1982**, *117*, 25–28. [CrossRef]
68. Dymnikova, I.; Fil'chenkov, M. Quantum birth of the hot universe. *Phys. Lett. B* **2002**, *545*, 214–219. [CrossRef]
69. Vilenkin, A. Quantum creation of universes. *Phys. Rev. D* **1984**, *30*, 509. [CrossRef]
70. Landau, L.D.; Lifshitz, E.M. *Quantum Mechanics. Nonrelativistic Theory*; Pergamon Press: Oxford, UK, 1975.
71. Bronnikov, K.; Dymnikova, I.; Galaktionov, E. Multihorizon spherically symmetric spacetime with several scales of vacuum energy. *Class. Quant. Grav.* **2012**, *29*, 095025. [CrossRef]
72. Dymnikova, I. The algebraic structure of a cosmological term in spherically symmetric solutions. *Phys. Lett. B* **2000**, *472*, 33–38. [CrossRef]
73. Dymnikova, I.; Galaktionov, E. Vacuum dark fluid. *Phys. Lett. B* **2007**, *645*, 358–364. [CrossRef]
74. Borghini, N.; Cottingham, W.N.; Mau, R.V. Possible cosmological implications of the quark-hadron phase transition. *J. Phys. G* **2000**, *26*, 771–785. [CrossRef]
75. Boeckel, T.; Schaffner, J. A Little Inflation in the Early Universe at the QCD Phase Transition. *Phys. Rev. Lett.* **2010**, *105*, 041301. [CrossRef]
76. Harko, T.; Mak, M.K. Bianchi type I universes with dilaton and magnetic fields. *Int. J. Mod. Phys. D* **2002**, *11*, 1171. [CrossRef]
77. Bronnikov, K.A.; Chudayeva, E.N.; Shikin, G.N. Magneto-dilatonic Bianchi-I cosmology: Isotropization and singularity problems. *Class. Quant. Grav.* **2004**, *21*, 3389. [CrossRef]
78. Szydlowski, M.; Stachowski, A.; Urbanowski, K. The evolution of the FRW universe with decaying metastable dark energy—A dynamical system analysis. *J. Cosmol. Astropart. Phys.* **2020**, *4*, 29. [CrossRef]