

# Instability Criterion and Uncertainty Relation

**T F Kamalov**

Moscow Institute of Physics and Technology, Dolgoprudny, Moscow Region, Russia

E-mail: [timkamalov@gmail.com](mailto:timkamalov@gmail.com)

**Abstract.** The main idea of I. Newton in Principia is a description of the laws of motion in Inertial Reference Frames by a second-order differential equation. The variation of the action functional  $S$  of stability trajectories equals to zero. The observational error is including the influence of the random fields' background to the particle. Are we must to use a high-order differential equation for the description in the random fields background? The high-order derivatives can be used as additional variables accounting for the influence of random field background. Trajectories due the influence of the random fields' background can be called instability random trajectories. They can be described by high order derivatives. Then the stability classical trajectories must be complemented by additional instability random trajectories. Quantum objects are described by the trajectory with neighborhoods. Quantum Probability can describe quantum objects in random fields'. The variation of the action functional  $S$  is defined by the Planck constant. For the common description of quantum theory and high-order theory, let us compare  $r$ -neighborhoods of quantum action functional with  $r$ -neighborhoods of the action functional of a high-order theory.

## 1. Introduction: Physics of Non-Inertial Reference Frames and Quantum Theory

Conversion of coordinates of a point particle between two Non-Inertial Reference Frames provided  $\tau$  is a time interval for averaging, shall be expressed as

$$Q = q(t) + \dot{q}(t)\tau + \Delta q(t),$$

$$\Delta q(t) = \sum_{k=2}^N (-1)^k \frac{1}{k!} \tau^k q^{(k)}(t).$$

And same holds for momentum

$$P = p(t) + \dot{p}(t)\tau + \Delta p(t),$$

$$\Delta p(t) = \sum_{k=2}^N (-1)^k \frac{1}{k!} \tau^k p^{(k)}(t),$$

$$Q = \langle q(t) \rangle = \frac{1}{2} [q(t + \tau) + q(t - \tau)].$$

Here,  $\Delta q(t)$  and  $\Delta p(t)$  are remainder terms of the Taylor expansion. The remainder terms  $\Delta q(t)$  and  $\Delta p(t)$  in Non-Inertial Reference Frame may be interpreted as uncertainties of coordinate

and momentum of a point particle in this reference frame. In quantum mechanics, uncertainties of coordinate and momentum of a micro particle obey to the rule

$$\Delta q(t)\Delta p(t) \geq \hbar/2.$$

In Non-Inertial Physics can be introduced an General Uncertainty Relation, as there always exist random small fields and forces influencing either the very system to be described or an observer, that is

$$\left[ \sum_{k=2}^N (-1)^k \frac{1}{k!} \tau^k q^{(k)}(t) \right] \left[ \sum_{k=2}^N (-1)^k \frac{1}{k!} \tau^k p^{(k)}(t) \right] \geq H/2$$

inertial one. In this case instability states of observing objects can be describe by Quantum Theory with high-order derivatives  $\psi(t) = |q, \dot{q}, \ddot{q}, \dots, q^{(n)}\rangle = |Q(t)\rangle$ . The transfer object from moment 1 to moment 2 can be describe by Lagrange function  $L(Q)$

$$\langle Q_1, t_1 | Q_2, t_2 \rangle = \int_{t_1}^{t_2} DQ \exp \left( \frac{i}{\hbar} L(Q) \right) dt.$$

And function  $A_{Q(R)}$  can be represent by Hamiltonian  $\hat{H}$

$$A_{Q(R)} = \left\langle q(t), \dot{q}(t), \ddot{q}(t), \dddot{q}(t), \dots, q^{(n)}(t) \left| \left[ i\hbar \frac{\partial}{\partial t} - \hat{H} \right] \right| q(t), \dot{q}(t), \ddot{q}(t), \dddot{q}(t), \dots, q^{(n)}(t) \right\rangle$$

## 2. Stability Principle

The stability principle may not only generalize, but also logically explain the basic laws of Nature. The stability principle enables using the stability of physical objects and their states for explanation and generalization of such fundamental laws of Nature as the least action principle, stability of atoms, stationarity of possible trajectories, etc. It may be employed as a generalized law explaining such a fundamental law of Nature as the least action principle. Therefore it can evidently be applied to all other laws following from the least action principle, such as Newton's laws, Euler-Lagrange equations, Schrödinger equation, laws of propagation of light and electromagnetic waves, etc. The Stability Condition in calculations of mechanical trajectories is put forward in publications of N.G. Chetayev [2]. According to him, "stability is probably an essentially general phenomenon that has to manifest itself in principal laws of Nature". In his opinion, stability is not a mere casualness, but rather, is a consequence of system being affected by persistent infinitesimal perturbations, which, no matter how small, affect the state of a mechanical system. This definition differs from Lyapunov stability definition. The state  $\Psi$  of a physical system will be considered stable if its deviations from a perturbed state  $F$  with random perturbations are minimal, i.e. for an arbitrarily small positive  $H$ , the inequality holds:

$$|\Phi - F| < H.$$

The condition of stability it is ordinary using in Mechanics and can be extending to other area of Physics.

In this case the condition of stability can be named Stability Principle. The stability principle is a generalization of basic fundamental physical laws, such as least action principle, Newton's laws, Euler-Lagrange equations, Schrödinger equation, et al.

**Stability principle:** *The state  $A$  of a physical system is considered stable if it returns to the initial state after finished the action of external factors.*

It is mean that the variance of the observable  $A$  with itself is equal to zero  $Var(A) = \sigma_A = 0$ .

We consider Non-Inertial Reference Frames due the influence of the background of random fields and waves because the variance of the action function  $S_r$  for instable trajectory with itself can be represented as  $Var(S_r) = \sigma_{S_r} = H$ , where for the classical stable trajectory the variance is  $\sigma_S = 0$  and for the instable trajectory  $\sigma_{S_r} = H$ .

Applying the stability principle for the action function  $S$ , we obtain General Uncertainty Relation [2,3]:

$$|S_r(q, \dot{q}, \ddot{q}, \ddot{\ddot{q}}, \dots, q^{(n)}, \dots) - S(q, \dot{q})| < H, \quad (1)$$

General Uncertainty Relation describes instability states due the influence of random small fields and waves. The supremum of the difference of the action function in Non-Inertial Reference Frames (with higher time derivatives of the generalized coordinate) from the classical mechanics action functions (without higher derivatives) is: In this case, higher derivatives are non-local additional variables and disclose the sense of the classical analog  $H$  of the Planck's constant. The constant  $H$  is the supremum of the influence of random fields onto the physical system and the observer. We shall analyze this case in terms of Non-Inertial Reference Frame. In this case,  $H$  defines as the supremum of the difference between a Non-Inertial Reference Frame and an inertial

$$\sup |S_r(q, \dot{q}, \ddot{q}, \ddot{\ddot{q}}, \dots, q^{(n)}, \dots) - S(q, \dot{q})| = H \quad (2)$$

where small random influences are described with higher time derivatives of generalized coordinates  $q$ . Instable Euler-Lagrange equations in Ostrogradsky formulation [4,5] accounting for random small influences in the form of higher derivatives will take on the form

$$\delta S = \delta \int L_r(q, \dot{q}, \ddot{q}, \ddot{\ddot{q}}, \dots, q^{(n)}, \dots) dt = \int \sum_{n=0}^N (-1)^n \frac{d^n}{dt^n} \frac{dL_r}{dr q^{(n)}} \delta r q^{(n)} dt = 0$$

or

$$\frac{\partial L_r}{\partial q} - \frac{d}{dt} \frac{\partial L_r}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L_r}{\partial \ddot{q}} - \frac{d^3}{dt^3} \frac{\partial L_r}{\partial \ddot{\ddot{q}}} + \dots + (-1)^n \frac{d^n}{dt^n} \frac{\partial L_r}{\partial q^{(n)}} + \dots = 0$$

This instability equation can be rewritten as

$$\begin{aligned} F - ma + \tau m \dot{a} - \frac{1}{2!} \tau^2 m a^{(2)} + \dots + (-1)^n \frac{1}{n!} \tau^n m a^{(n)} + \dots &= 0 \\ \dot{a}(t_0) = 0, \ddot{a}(t_0) = 0, \dots, a^{(n)}(t_0) = 0, \dots \end{aligned} \quad (3)$$

$\tau$  being the time interval, and  $a$  – acceleration,  $t_0$  is the moment of the time of stability trajectory. The second Newton's law is a second-derivative differential equation describing stability dynamics

$$F - ma = 0.$$

To investigate behavior of bodies in micro-world, we shall take as the constant  $H$  assessing the influence of random perturbations the Planck's constant  $h$ . Then the maximum value of random variables shall be limited with the Heisenberg uncertainty relation, and we derive from (2)

$$\sup |S(q, \dot{q}, \ddot{q}, \ddot{\ddot{q}}, \dots, q^{(n)}, \dots) - S_{In}(q, \dot{q})| = h/2.$$

From the Equivalence Principle and (3) it follows that in this case one can use the metric

$$ds^2 = \exp\left(-\frac{r_0}{r}\right) c^2 dt^2 - \exp\left(\frac{r_0}{r}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2,$$

which is the asymptotic of the Schwarzschild metric for the small  $r_0 < r$ .

### 3. Macro-experiments of Non-Newtonian Mechanics

Modern classical physics describes the dynamics of mechanical systems by differential equations of the second order. Many experiments with the introduction of experimental vibration force do not contradict the description of vibration effects by classical physics. They are described by second-order differential equations, as well as Newton's second law. But such descriptions do not give the correct direction of the resultant force, so they can be considered incomplete. A complete description of such experiments with the correct direction of the resultant force can be obtained by postulating a description of the dynamics of mechanical systems by differential equations with high-order derivatives of coordinates to time. This is true for complex motions and non-inertial reference systems with complexly changing inertia forces. If we neglect such effects, then we can restrict ourselves to second-order differential equations, since in mathematics, there is a method of lowering the order of a differential equation, for example, when a third-order differential equation can be replaced by two differential equations of order no higher than the second. But such a description will not be complete, because at the same time, the sign (direction) of the quantities which expressing the third derivative is lost.

The behavior of macroscopic mechanical systems in non-inertial reference frames can be described by higher-order differential equations. Here we consider the case when the contribution of higher derivatives is small compared to lower ones. Therefore, at this stage, we restrict ourselves to only the third derivatives of the coordinates with respect to time. There are many examples of the description of mechanical systems in non-inertial reference frames [3-6] due to the influence of the backgrounds of random fields and waves. Theoretical descriptions of such cases do not always fully describe the physical reality of the processes occurring in this process. Such cases include Kapica's pendulum, the movement of bulk materials upwards, against the action of gravity, Chalomey's pendulum, and others [7]. For describing vibrating mechanical systems, the principle of least action is traditionally used to obtain critical states of mechanical systems. All such cases are described by second-order differential equations. In this case, the direction of the resultant force remains uncertain. This is the main disadvantage of this method of description. Using the extended Newton's second law (3) we obtain the direction of the resultant force that coincides with the direction of the motion. In [7], the behavior of such systems is described by introducing experimental vibration forces. The introduction of vibration forces in these cases, in our opinion, is not justified and is introduced axiomatically.

Here we will use a third order differential equation. This allows, first, to get the correct direction of the resultant force. Secondly, it explains its occurrence and does not contradict the already known descriptions.

Comparing the two descriptions: the differential equations of the second order and the third order can be argued the consistency of these two descriptions. Indeed, in mathematics, there is a method of transition from higher-order differential equations to lower ones by changing variables. In our case, from a third-order differential equation, we can go to two equations of order not higher than the second.

For example, consider the description of the Kapica pendulum using the differential equation (3), limiting ourselves to the third order of the derivative of the coordinate with respect to time

$$F - ma + \tau m\dot{a} = 0 \quad (4)$$

where  $\tau = 1/\omega$  is the averaging time during the transition from the micro-world to the macro, the opposite of the average cyclic frequency. Using the substitution, we get

$$F + V = ma, \quad (5)$$

where the experimental vibration force  $V$  is equal to

$$V = mA\omega^2 \sin \omega t.$$

Thus, we have shown that equation (4) can be replaced by two equations (5) and (6). In this case, the description with high-order derivatives of mechanical systems is more complete than the description with second-order derivatives [8-10].

#### 4. Conclusion

Additional terms in the form of higher derivatives have non-local character, which enables their employment for description the non-locality of quantum mechanics. Random and instability variables in the form of high-order derivatives coordinates on the time can describe the effect of quantum correlations and non-locality of quantum states in Non-Inertial Reference Frames.

#### References

- [1] Ostrogradsky M V 1850 *Mem. Acad. St. Peterburg* **6** 4 p 385
- [2] Chetayev N G 1931 *Mathematics-1* Proc. Kazan State Univ. **91** 4 3
- [3] Kamalov T F 2013 *J. Phys.: Conf. Ser.* **442** 012051
- [4] Kamalov T F 2010 *AIP Conf. Proc.* **1316** 455 (*Preprint* 0708.1584)
- [5] Kamalov T F 2011 *Quantum Computers and Computing* **11** 52 (*Preprint* 1011.4256)
- [6] Kamalov T F 2018 *J. Phys.: Conf. Ser.* **1051** 012033
- [7] Blekhman I I 2000 *Vibrational Mechanics* Singapore (New Jersey, Hong Kong: World Scientific)
- [8] El-Nabulsi R A 2014 *Qual. Theory Dyn. Sys.* **13** 149
- [9] Woodard R P 2007 *Lecture Notes in Physics* 720 p 403 (*Preprint* astro-ph/0601672)
- [10] El-Nabulsi R A 2018 *Adv. Space Res.* **61** 12 2914