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ASTROPHYSICS AND COSMOLOGY

Spectrum of Primary Gravitational Waves in the Quantum Version of Conformal General Relativity

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The spectral power of primordial gravitational waves is calculated in the quantum version of conformal general relativity. The fundamental variables of quantum gravity in the used approach are special variables, which constitute the dynamic part of the spin connection, rather than components of the metric tensor. It has been shown that the proposed model in the Born approximation reproduces the standard spectral power of primordial gravitational waves generated in the canonical inflation process. This has made it possible to test the quantum version of the conformal theory of gravity in a specific phenomenological problem.

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1. INTRODUCTION

At present, the main model used to solve the problem of the cosmological horizon is the inflationary model based on the so-called semiclassical gravity theory, where gravity is still considered as a classical field and material fields as quantum. One of the main predictions of the inflationary model is the existence of primordial gravitational waves, which are expected to be detected in the foreseeable future. Their detection should allow the discrimination of different cosmological and gravitational models [1]. Thus, differences in the predictions of any modified theory of gravity for primordial gravitational waves from classical general relativity provide a remarkable opportunity to test both inflationary models and the theory of gravity in observational astrophysical experiments [2].

In this work, we study the generation of primordial gravitational waves in the model of quantum gravity obtained by quantizing the conformal version of general relativity in special variables and compare the power of their spectrum with the result obtained within classical general relativity. This work is also the first test of the phenomenological consequences of the quantum theory of gravity considered in our works [3, 4].

First, we specify the conformal transform of the metric $g_{\mu\nu}$ in the form

$$g_{\mu\nu}dx^{\mu} \otimes dx^{\nu} = e^{-2D}\,\tilde{g}_{\mu\nu}\,d\chi^{\mu} \otimes d\chi^{\nu}, \tag{1}$$

where $\tilde{g}_{\mu\nu}$ is the conformal metric and D is the dilaton.

The essence of the conformal modification of general relativity is as follows. The conformally noninvariant Einstein–Hilbert action

$$S_{\rm GR} = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{16\pi}(R - 2\Lambda) + L_{\rm matter}(g_{\mu\nu})\right] \tag{2}$$

is first modified by Weyl transforms to a form in which all conformal weights are explicitly separated [5, 6]

$$\begin{aligned} S_{\rm CGR} = \int d^4\chi\sqrt{-\tilde{g}}\Bigg[&\frac{\widetilde{M}_P^2}{16\pi}(\tilde{R} - 2\tilde{\Lambda}) \\ &+ \frac{3\widetilde{M}_P^2}{8\pi}(\tilde{g}^{\mu\nu}\nabla_{\mu}D\nabla_{\nu}D) + L_{\rm matter}(\tilde{g}_{\mu\nu})\Bigg]. \end{aligned} \tag{3}$$

Here, Λ is the cosmological constant, $\tilde{\Lambda} = e^{-2D}\Lambda$ is the conformal cosmological constant, M_P is the Planck mass, and $\widetilde{M}_P = M_P e^{-D}$ is the conformal Planck mass. The requirement of the conformal invariance is then imposed on Eq. (3). As a result, the action of the original general relativity is replaced by another conformally invariant action. It is assumed that the observed conformal symmetry breaking occurs spontaneously, which can be described in the nonlinear symmetry implementation approach [7]. Thus, conformal general relativity is incompletely equivalent to classical general relativity; it is a modified theory of gravity. Therefore, the action given by Eq. (3) will be called the action of conformal general relativity.

The key difference of conformal general relativity from classical general relativity is the assumption that we phenomenologically observe conformal values of all physical quantities rather than standard ones, as is assumed by default in classical general relativity. In particular, in conformal general relativity, it is assumed that if the observed quantities in a certain measurement are equal to the standard ones, this simply means that the conformal factor is equal to one.

In this case, the most pronounced differences between predictions of the conformal and classical general relativity should be expected in the case of their quantization.

We note in advance that four types of indices are used in this article. Four- and three-dimensional coordinate indices without parentheses are denoted by Greek and middle Latin letters i, j, ... letters and run through the values 0–3 and 1–3, respectively. Four- and three-dimensional spatiotemporal reference indices in parentheses are denoted by first Latin letters a, b, ... and middle Latin letters i, j, ... letters and take the values 0–3 and 1–3, respectively. The polarization indices of classical and conformal gravitons are denoted by (p) and (q).

The article is structured as follows. First, the foundation of the formalism used in this work is given. Next, a well-known method for obtaining the spectral power of primordial gravitational waves in the standard inflationary model based on general relativity is briefly described. Finally, following the analogy with the standard approach, the spectral power for the case of conformal general relativity is calculated and discussed.

2. PRELIMINARY REMARKS ON THE USED FORMALISM

In this paper, we use the formalism developed in our and other works on conformal general relativity. In this section, we will briefly present only those aspects of it that will be used to analyze primordial gravitational waves in conformal general relativity. A more detailed description can be found in the Appendix and in [3, 4, 8].

Tetrad fields and the spin connections are central notions in the formalism under consideration. The variables $\omega_{\alpha(a),}{}^{(b)}$ are components of the spin connection. In conformal general relativity, the spin connection is metric, implying that torsion and nonmetricity are absent [9]. In [4], we showed that the components of the spin connection in the tetrad representation are expressed by the formula

$$\begin{aligned}\omega_{(a),(b)(c)} &= \frac{1}{2}e^{\alpha}{}_{(a)}(e^{\beta}{}_{(c)}\partial_\alpha e_{\beta(b)} - e^{\beta}{}_{(b)}\partial_\alpha e_{\beta(c)}) \\ &+ \frac{1}{2}e^{\alpha}{}_{(b)}(e^{\beta}{}_{(a)}\partial_\alpha e_{\beta(c)} + e^{\beta}{}_{(c)}\partial_\alpha e_{\beta(a)}) \\ &- \frac{1}{2}e^{\alpha}{}_{(c)}(e^{\beta}{}_{(b)}\partial_\alpha e_{\beta(a)} + e^{\beta}{}_{(a)}\partial_\alpha e_{\beta(b)}) \\ &:= \omega^{L}_{(c)(b),(a)} + \omega^{R}_{(a)(c),(b)} - \omega^{R}_{(b)(a),(c)}.\end{aligned} \tag{4}$$

This formula is different in sign from those given in [8, 3] but this does not affect the results presented below.

The components $\omega^{L}_{(a)(c),(\alpha)}$ and $\omega^{R}_{(a)(c),(\alpha)}$ in the tetrad representation are given by the expressions

$$\omega^{L}_{(a)(c),\alpha}dx^{\alpha} = \frac{1}{2}\left(e^{\beta}{}_{(c)}de_{\beta(a)} - e^{\beta}{}_{(a)}de_{\beta(c)}\right), \tag{5}$$

$$\omega^{R}_{(a)(c),\alpha}dx^{\alpha} = \frac{1}{2}\left(e^{\beta}{}_{(c)}de_{\beta(a)} + e^{\beta}{}_{(a)}de_{\beta(c)}\right). \tag{6}$$

The substantiation and motivation for the introduction of $\omega^{R}_{(a)(b),(c)}$ and $\omega^{L}_{(a)(b),(c)}$ can be found in our previous works [8, 10, 3] and their relation to the Goldstone fields is given in [10, 11]. Below, the indices will be omitted for the sake of brevity, where this does not lead to the loss of meaning, and these components will be designated simply as ω^{R} and ω^{L}.

In [3, 4], it was shown that the differential of the metric tensor does not depend on the variables ω^{L}, and the following formula was obtained:

$$\begin{aligned}dg_{\mu\nu} &= dx^{\alpha}\partial_\alpha g_{\mu\nu} \\ &= (e_{\mu}{}^{(b)}e_{\nu}{}^{(a)} + e_{\mu}{}^{(a)}e_{\nu}{}^{(b)})\omega^{R}{}_{(b)(a),\alpha}dx^{\alpha}.\end{aligned} \tag{7}$$

In our approach, in contrast to teleparallelism and other tetrad approaches, it is postulated that the tetrad matrices $e_{\mu}{}^{(b)}$ are not objects of gravitational field dynamics, and the fundamental dynamics is entirely involved in the spin connection. In this case, the absence of ω^{L} in Eq. (7) for the differential means that the dynamics of the metric tensor can be contained in ω^{R}.

In this work, we consider only globally hyperbolic spacetimes. Then, it follows from Geroch's splitting theorem [12–14] that the spacetime under consideration $(\mathbb{M}, \mathbf{g})$ is represented as a direct product $\mathbb{M} = \mathbb{R}\times\mathbb{S}$, where $\mathbb{S}$ is the spacelike Cauchy 3-surface (hypersurface). Consequently, the Arnowitt–Deser–Misner formalism is valid for this spacetime. This formalism was described in detail in [15], but we need here only the lapse function $N(\chi^0,\chi^1,\chi^2,\chi^3)$. Without loss of generality, it can be represented in the form [16]

$$N(\chi^0,\chi^1,\chi^2,\chi^3) = N_0(\chi^0)\mathcal{N}(\chi^0,\chi^1,\chi^2,\chi^3). \tag{8}$$

Here, N_0 and $\mathcal{N}$ are the global and local parts of the lapse function, respectively. Then, taking into account Eq. (8), the volume form can be represented as

$$d^4x\sqrt{-g} = d\chi^0 N_0\, d^3\chi\sqrt{\gamma}\,\mathcal{N}, \tag{9}$$

where γ is the determinant of the spatial metric. This allows the separation of integration over the spatial coordinates in the action of conformal general relativity (3), which now takes the form

$$S_{\text{Gravitons}} = \int d\chi^0 N_0 \int d^3\chi\sqrt{\gamma}\,\mathcal{N}\,\frac{\widetilde{M}_P^2}{16\pi}\widetilde{R}. \qquad (10)$$

Thus, the temporal and spatial parts are separated in Eq. (10). We have also omitted the term Λ since its presence is not essential for the subsequent analysis.

3. PRIMARY GRAVITATIONAL WAVES IN THE CANONICAL INFLATIONARY MODEL

In this section, we will briefly outline the formalism necessary for the study of primordial gravitational waves. The study of primordial gravitational waves by means of transformations is usually reduced to a well-defined situation of scalar field waves. For this reason, we begin with a brief discussion of fluctuations of the scalar field $\phi(x)$.

The most important characteristic of radiation is its spectral power $P(\mathbf{k})$ defined by the formula

$$\langle\phi(\mathbf{k})\phi(\mathbf{k}')\rangle = P_\phi(k)\delta(\mathbf{k}+\mathbf{k}'). \qquad (11)$$

Below, the symbol $\mathbf{k}$ is used to designate the three-dimensional part of the 4-momentum when working with massless scalar fields, and its length is denoted in most cases as k. Note that the term spectral power is also used for another physical quantity $\mathcal{P}_\phi(\mathbf{k})$ that is related to $P_\phi(\mathbf{k})$ as $P_\phi(\mathbf{k}) = \frac{2\pi^2}{k^3}\mathcal{P}_\phi(\mathbf{k})$. Correspondingly, Eq. (11) can be represented in the form

$$\langle\phi(\mathbf{k})\phi(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_\phi(k)\delta(\mathbf{k}+\mathbf{k}'). \qquad (12)$$

The spectral power $\mathcal{P}_\phi(\mathbf{k})$ is related to the fluctuation of the scalar field by the formula

$$\langle\phi^2(x)\rangle = \int_0^\infty \frac{dk}{k}\mathcal{P}_\phi(k). \qquad (13)$$

Here, the so-called semiclassical averaging is present on the left-hand side, but quantum averaging over vacuum is defined in a similar way.

We calculate the spectral power for a massless Klein–Gordon field in Minkowski space. A free scalar field has the form

$$\phi(x) = \frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2k_0}}\left(e^{ik\cdot x}a^+(\mathbf{k}) + e^{-ik\cdot x}a(\mathbf{k})\right). \qquad (14)$$

Performing quantum averaging over the vacuum state and using the commutation relation $a(\mathbf{k})a^+(\mathbf{k}') - a^+(\mathbf{k}')a(\mathbf{k}) = \delta(\mathbf{k}-\mathbf{k}')$, we obtain the formula

$$\langle 0|\phi^2(x)|0\rangle = \frac{1}{(2\pi)^3}\int\frac{d^3k}{2k_0}\int_0^\infty\frac{dk}{k}\left(\frac{k}{2\pi}\right)^2 = \int_0^\infty\frac{dk}{k}\mathcal{P}_\phi(k). \qquad (15)$$

Thus, in the case of Minkowski space, the spectral power for the scalar field is given by the formula $\mathcal{P}_\phi(k) = (k/2\pi)^2$. A detailed discussion of these definitions, including their properties and motivations, can be found in [17–19]. Similar formulas can be obtained for tensor fields. The case of tensor fields can be reduced to the case of scalar fields by decomposition into the sum of polarizations using polarization operators [17, 18]. Now, we briefly discuss the specific of obtaining the spectral power of primordial gravitational waves in the canonical inflationary model based on general relativity.

The metric of a primordial gravitational wave in the canonical inflationary model in the simplest case is written as [20, 17]

$$ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]. \qquad (16)$$

By expanding the Einstein–Hilbert action into a series by metric perturbations and retaining only the terms quadratic in them, we obtain the action in the form

$$S^{(2)} = \frac{M_{Pl}^2}{8}\int d\eta\, d^3x\, a^2(\dot{h}_{ij}\dot{h}^{ij} - \partial_i h_{ij}\partial^i h^{ij}). \qquad (17)$$

The Fourier series of the perturbation of the metric with respect to the momentum has the form

$$h_{ij}(x) = \sum_{p=\pm 2}\int\frac{d^3k}{(2\pi)^{3/2}}h_{ij}^p(\eta,\mathbf{k})e^{i\mathbf{kx}}. \qquad (18)$$

Polarization operators have the following important properties, which are used below for calculations:

$$m_{ij}^{(p)}(\hat{\mathbf{k}})[m^{(q)ij}(\hat{\mathbf{k}})]^* = \delta^{(p)(q)}, \qquad (19)$$

$$[m_{ij}^{(p)}(\hat{\mathbf{k}})] = m_{ij}^{(-p)}(\hat{\mathbf{k}}) = m_{ij}^{(p)}(-\hat{\mathbf{k}}). \qquad (20)$$

Using the Fourier series given by Eq. (18), Eqs. (19) and (20), and the properties of the Dirac delta function, we obtain

$$\int d^3x\,\dot{h}_{ij}\dot{h}^{ij} = \sum_{p,q=\pm 2}\int\frac{d^3k}{(2\pi)^{3/2}}\frac{d^3k'}{(2\pi)^{3/2}}\frac{1}{2}\dot{h}^{(p)}(\eta,\mathbf{k})$$
$$\dot{h}^{(q)}(\eta,\mathbf{k}')m_{ij}^{(p)}(\hat{\mathbf{k}})m^{(q)\,ij}(\hat{\mathbf{k}}')\int d^3x e^{i(\mathbf{k}+\hat{\mathbf{k}}')\mathbf{x}}$$
$$= \sum_{p,q=\pm 2}\int\frac{d^3k}{(2\pi)^{3/2}}\frac{d^3k'}{(2\pi)^{3/2}}\frac{1}{2}\dot{h}^p(\eta,\mathbf{k})\dot{h}^q(\eta,\mathbf{k}')$$

$$\times m_{ij}^{(p)}(\hat{\mathbf{k}})m^{(q)\,ij}(\hat{\mathbf{k}}')(2\pi)^3\delta(\mathbf{k}+\hat{\mathbf{k}}')$$
$$= \frac{1}{2}\sum_{p=\pm 2}\int d^3k[\dot{h}^{(p)}(\eta,\mathbf{k})]^2.$$

Performing similar transformations for the second term, we arrive at the relation

$$\int d^3x\partial_i h_{jk}\partial^i h^{jk} = -\frac{1}{2}\sum_{p=\pm 2}\int d^3k k^2[h^p(\eta,\mathbf{k})]^2. \quad (21)$$

Combining the results, we obtain the expression

$$S^{(2)}_{\text{Gravitons}} = \sum_{p=\pm 2}\int dx^0\, d^3k\,\frac{M_P^2}{16}((\dot{h}^{(p)})^2 - h^{(p)}h^{(p)}). \quad (22)$$

Thus, the problem of calculating the spectral power of primordial gravitational waves can be reduced to calculating the spectral power of primordial perturbations of the scalar field ϕ.

4. PRIMARY GRAVITATIONAL WAVES IN CONFORMAL GENERAL RELATIVITY

In [4], we analyzed the quantum aspects of conformal general relativity on the example of a nonlinear gravitational wave described by the metric

$$\begin{aligned}\tilde{g} &= -dx^0\otimes dx^0 + dx^3\otimes dx^3\\ &+ e^{\sigma}dx^1\otimes dx^1 + e^{-\sigma}dx^2\otimes dx^2.\end{aligned} \quad (23)$$

This analysis showed that the general expression describing the propagation of a free plane gravitational wave in conformal general relativity has the form

$$\begin{aligned}\omega^R_{(a)(b),(c)} &= \int\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\\ &\times\, ik_{(c)}[\epsilon^R_{(a)(b)}(k)g_k^+e^{ikx} + \epsilon^R_{(a)(b)}(-k)g_k^-e^{-ikx}].\end{aligned} \quad (24)$$

Here, $\omega^R_{(a)(b),(c)}$ are considered as the fundamental variables of quantum gravity, i.e., as operators; $\epsilon^R_{(a)(b)}(k)$ and $\epsilon^R_{(a)(b)}(-k)$ are the standard polarization tensors; and g_k^+ and g_k^- are treated as the creation and annihilation operators of conformal gravitons, respectively, which are not direct analogues of standard gravitons and are not related to the last conformal transformations.

In this paper, we study primordial gravitational waves within the quantum version of conformal general relativity, starting with the results obtained in [3, 4]. An arbitrary gravitational wave, as well as the primordial gravitational wave metric in the canonical inflationary model, has two degrees of freedom.

To determine whether the spectral power of primordial gravitational waves generated in the inflation process, obtained within the conformal version of general relativity, coincides with that obtained in canonical general relativity, we assume that the standard metric is given by the same expression as in classical general relativity:

$$\begin{aligned}ds^2 &= a^2(x^0)\tilde{g}_{\mu\nu} = a^2(x^0)\\ &\times\left[-(dx^0)^2 + (\delta_{ij}+h_{ij})dx^idx^j\right],\end{aligned} \quad (25)$$

where x^0 is the conformal time.

First of all, we note that the action of conformal general relativity in this situation coincides with the action of classical general relativity. In fact, as noted in previous works, the average value of the dilaton should be related to the scale factor by the formula $\langle D\rangle = -\ln(a(x^0))$. Then, the square of the conformal Planck mass entering Eq. (3) is specified by the expression

$$\begin{aligned}\widetilde{M}_P^2 &= M_P^2e^{2\langle D\rangle} = M_P^2e^{-2\ln(a(x^0))}\\ &= M_P^2(e^{\ln(a(x^0))})^{-2} = M_P^2a^{-2}(x^0).\end{aligned} \quad (26)$$

As discussed in the preceding section, the gravitational part of the action in conformal general relativity is given by Eq. (10). The conformal scalar curvature is specified by the standard formula

$$\begin{aligned}\widetilde{R} &= \Omega^{-2}\\ &\times\left(R - 6g^{\mu\nu}\nabla_\mu\nabla_\nu\Omega + 6g^{\mu\nu}\nabla_\mu\ln\Omega\nabla_\nu\ln\Omega\right).\end{aligned} \quad (27)$$

Then, in view of Eqs. (26) and (27), the action of conformal general relativity (10) can be reduced to the action of classical general relativity [17]. The transition between the standard Einstein–Hilbert action and the action of conformal general relativity can be found in well-known monographs [17, 21]. In this case, after linearization, [17] we obtain the standard expression for the second-order terms of the gravitational action:

$$\begin{aligned}&S^{(2)}_{\text{Gravitons}}\int dx^0\,d^3x\,\frac{\widetilde{M}_P^2}{8}\left(\dot{h}_{ij}\dot{h}^{ij} - \partial_ih_{ij}\partial^ih^{ij}\right)\\ &= \sum_{p=\pm 2}\int dx^0\,d^3k\,\frac{\widetilde{M}_P^2}{16}\left((\dot{h}^{(p)})^2 - h^{(p)}h^{(p)}\right)\\ &= \sum_{p=\pm 2}\frac{M_P^2}{8}\int dx^0\,d^3k\;a^2(x^0)\frac{1}{2}\left((\dot{h}^{(p)})^2 + k^2(h^{(p)})^2\right).\end{aligned} \quad (28)$$

As shown in [4], the components of the metric tensor are generally related to the variables ω^R by means of differential equations

$$\frac{\partial\tilde{g}_{\mu\nu}}{\partial x^{(c)}} = \left(e_\mu{}^{(b)}e_\nu{}^{(a)} + e_\mu{}^{(a)}e_\nu{}^{(b)}\right)\omega^R_{(b)(a),(c)}. \quad (29)$$

Here, the expression $(e_\mu{}^{(b)}e_\nu{}^{(a)} + e_\mu{}^{(a)}e_\nu{}^{(b)})$ corresponds to the summation over polarizations. There are two important differences between our calculation and the standard one. First, Eq. (29) relates the variables $\omega^R_{(b)(a),(c)}$ to the conformal metric $\tilde{g}_{\mu\nu} = \delta_{\mu\nu} + \tilde{h}_{\mu\nu}$ rather than to the standard one $g_{\mu\nu}$. Second, as noted above, the basic variables $\omega^R_{(a)(b),(c)}$ in the quantization of gravity in our approach are considered to be the basic variables and, correspondingly, they are operators. The conformal metric $\tilde{g}_{\mu\nu}$ is constructed as a small perturbation of the Minkowski metric. In this case, Eqs. (29) take the form

$$\frac{\partial \tilde{h}_{\mu\nu}}{\partial x^{(c)}} = \left(e_\mu{}^{(b)}e_\nu{}^{(a)} + e_\mu{}^{(a)}e_\nu{}^{(b)}\right)\omega^R_{(b)(a),(c)}. \qquad (30)$$

We will consider weak gravitational waves. Since the metric ansatz given by Eq. (25) does not contain off-diagonal components, the components of the basis tetrads can be considered in the first approximation as identity matrices and Eqs. (30) can be easily integrated. In fact, the tetrad basis in this approximation coincides with the coordinate one. Then, integrating Eqs. (30), we get

$$\tilde{h}_{\mu\nu}(x) = (e_\mu{}^{(b)}e_\nu{}^{(a)} + e_\mu{}^{(a)}e_\nu{}^{(b)})\int\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}} \times [\epsilon^R_{(a)(b)}(k)g^+_k e^{ikx} + \epsilon^R_{(a)(b)}(-k)g^-_k e^{-ikx}]. \qquad (31)$$

Taking into account the presence of two polarizations, fluctuations of the tensor field can be reduced to fluctuations of the scalar field by substituting $\sum_{p=\pm 2}\frac{1}{\sqrt{2}}m^{(\pm 2)}_{ij}(x)h^{(p)}(x)$. In this case, Eq. (31) takes the form

$$\begin{aligned}\tilde{h}_{ij}(x) &= \sum_{p=\pm 2}\frac{1}{\sqrt{2}}m^{(\pm 2)}_{ij}(x)h^{(p)}(x)\\ &= (e_\mu{}^{(b)}e_\nu{}^{(a)} + e_\mu{}^{(a)}e_\nu{}^{(b)})\int\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\\ &\times\left[\epsilon^R_{(a)(b)}(k)g^+_k e^{ikx} + \epsilon^R_{(a)(b)}(-k)g^-_k e^{-ikx}\right]\\ &= \sum_{p=\pm 2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left[\epsilon^R_{\mu\nu}(k)g^+_k e^{ikx} + \epsilon^R_{\mu\nu}(-k)g^-_k e^{-ikx}\right].\end{aligned} \qquad (32)$$

In this section, we assume that the polarization operators in the coordinate representation are normalized by the condition $m^{(p)}_{ij}(x)m^{(q)|ij}(x) = 2\delta^{(p)(q)}$.

We now calculate the vacuum fluctuation using our representation for conformal gravitons. Since the observed metric in conformal general relativity is the conformal rather than the standard metric, these will be small perturbations over the Minkowski metric. First, we calculate the vacuum product of two gravitational fields at the points (x^0, x^1, x^2, x^3) and (y^0, y^1, y^2, y^3) at the same time, i.e., at $x^0 = y^0$. For the sake of brevity, (x^0, x^1, x^2, x^3) and (y^0, y^1, y^2, y^3) are below denoted as x and y, respectively, bearing in mind that $x^0 = y^0$. We obtain

$$\begin{aligned}\langle 0|\tilde{h}_{ij}(x)\tilde{h}^{ij}(y)|0\rangle &= \langle 0|\sum_{p,q=\pm 2}\frac{1}{2}m^{(p)}_{ij}(x)m^{(q)|ij}(y)\\ &\times h^{(p)}(x)h^{(q)}(y)|0\rangle = \langle 0|\sum_{p,q\pm 2}\frac{1}{2}\delta^{(p)(q)}h^{(p)}(x)h^{(q)}(y)|0\rangle\\ &= \langle 0|\sum_{p,q=\pm 2}\int\frac{d^3k\,d^3k'}{(2\pi)^3}\frac{1}{2\sqrt{\omega_k\omega_{k'}}}\Big[\epsilon^R_{\mu\nu}(k)g^+_k e^{ikx}\\ &+ \epsilon^R_{\mu\nu}(-k)g^-_k e^{-ikx}][\epsilon^R_{\mu\nu}(k')g^+_{k'}e^{ik'y} + \epsilon^R_{\mu\nu}(-k')g^-_{k'}e^{-ik'y}]|0\rangle\\ &= \langle 0|\sum_{p,q=\pm 2}\int\frac{d^3k\,d^3k'}{(2\pi)^3}\frac{1}{2\sqrt{\omega_k\omega_{k'}}}\\ &\times[\epsilon^{R(p)}_{\mu\nu}(-k)\epsilon^{R(q)|\mu\nu}(k')e^{-ikx+ik'y}]\,|\,0\rangle\\ &= \langle 0|\sum_{p,q=\pm 2}\int\frac{d^3k\,d^3k'}{(2\pi)^3}\frac{1}{2\sqrt{\omega_k\omega_{k'}}}\\ &\times[\delta^{(p)(q)}(g^+_{k'}g^-_k + \delta(\mathbf{k}-\mathbf{k}'))e^{-ikx+ik'y}]\,|\,0\rangle\\ &= \sum_{p=\pm 2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\Big[\delta^{(p)(p)}e^{-ik(x-y)}\Big].\end{aligned} \qquad (33)$$

Since the 4-vector of the gravitational wave is light-like, $-(k_0)^2 + (k_1)^2 + (k_2)^2 + (k_3)^3 = 0$ in Minkowski space. In this regard, for the sake of brevity, the length of its spatial part $|\mathbf{k}| = \sqrt{(k_1)^2 + (k_2)^2 + (k_3)^3}$ will be designated simply as k. Since the standard deviation at a single point $x = y$ is of interest, we get

$$\begin{aligned}\langle 0|\tilde{h}_{ij}(x)\tilde{h}^{ij}(x)|0\rangle &= \lim_{x\to y}\langle 0|\tilde{h}_{ij}(x)\tilde{h}^{ij}(y)|0\rangle\\ &= \lim_{x\to y}\sum_{p=\pm 2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}[\delta^{(p)(p)}e^{-ik(x-y)}]\\ &= 2\int\frac{d^3k}{(2\pi)^3}\frac{1}{2k} = 2\int\frac{dk}{(2\pi)^3}\frac{4\pi k^2}{2k}\\ &= 2\int_0^\infty d\ln|k|\left(\frac{k}{2\pi}\right)^2 = 2\int_0^\infty d\ln|k|\mathcal{P}_h(k),\end{aligned} \qquad (34)$$

where we used the commutation relation $g^-_k g^+_{k'} - g^+_{k'}g^-_k = \delta(\mathbf{k} - \mathbf{k}')$ for the creation and annihilation operators and the relation $\lim_{x\to y}e^{-ik(x-y)} = 1$. Thus, we have actually reproduced the classical result for vacuum fluctuations in Minkowski space. Thus, the spectral power of gravitational waves propagating against the background of Minkowski space is

$$\tilde{\mathcal{P}}_{h_{ik}}(k) = \frac{4}{M_P^2}\sum_{p=\pm 2}\tilde{\mathcal{P}}_h(k) = \frac{8}{M_P^2}\tilde{\mathcal{P}}_h(k) = \frac{8}{M_P^2}\left(\frac{k}{2\pi}\right)^2. \qquad (35)$$

Since k is the conformal momentum that is related to the standard one through the conformal factor according to the formula $q(x^0) = k/a(x^0) = H_k$, the expression for the conformal spectral power should be changed to take into account this factor. As a result, we obtain

$$\mathcal{P}_{h_{ik}}(k) = \sum_{p=\pm 2}\mathcal{P}_h(k) = \frac{8}{M_P^2}a^{-2}(x^0)\widetilde{\mathcal{P}}_h(k) = \frac{8}{M_P^2}a^{-2}(x^0)\left(\frac{k}{2\pi}\right)^2 = \frac{8}{M_P^2}\left(\frac{H_k}{2\pi}\right)^2. \quad (36)$$

Thus, the conformal spectral power differs from the standard one by the corresponding conformal weight $a^2(x^0)$, as one would expect. So, starting from the quantization of conformal general relativity in the variables ω^R, we can formally reproduce the spectral power of gravitational waves that arise in the canonical inflationary model, which is obtained from semiclassical reasons.

5. CONCLUSIONS

To summarize, an expression for the spectral power of primordial gravitational waves, which coincides with that obtained within the framework of classical general relativity, has been obtained by quantizing the conformal version of general relativity in the special variables ω^R. In our calculations, Eq. (24), which describes conformal gravitons, has been fundamentally used. Although the variables ω^R in our approach are assumed to be the fundamental variables of quantum gravity, the metric tensor still plays the main role in physical terms. This tensor in our approach is an operator, since it is related to ω^R through first-order differential equations (29), and ω^R itself is a quantum operator in construction.

The coincidence of an expression for the spectral power derived from Eq. (24) with the expression obtained from semiclassical reasons on the basis of classical general relativity means that our approach at least does not contradict the canonical model. In this context, it is noteworthy that our calculations have been performed in the leading (Born) approximation, which is in particular used to integrate Eqs. (31). Therefore, it would be very interesting to determine the spectral power of the gravitational radiation in this model including higher-order corrections. This requires a separate study, which will be reported elsewhere.

APPENDIX

TETRADS AND SPIN CONNECTION

Here, we present some formulas used in the tetrad formalism and the spin connection. The representation of a tetrad in a coordinate basis has the form

$$e_{(a)} = e^{\alpha}{}_{(a)}\partial_\alpha. \quad (A.1)$$

A cotetrad is represented in the basis of 1-forms (covectors) from the cotangent space as

$$e^{(a)} = e_\alpha{}^{(a)}dx^\alpha. \quad (A.2)$$

The following conditions are imposed on the cotetrads:

$$e^{\alpha}{}_{(a)}e_\alpha{}^{(b)} = \delta^{(b)}_{(a)}, \quad e^{\alpha}{}_{(a)}e_\beta{}^{(a)} = \delta^\alpha_\beta. \quad (A.3)$$

The quantities $e_{\nu(a)}$ are defined according to the formula

$$e_{\nu(a)} = \eta_{(a)(d)}e_\nu{}^{(d)}, \quad (A.4)$$

where $\eta_{(a)(d)}$ is the Minkowski metric. Since $\eta_{(a)(d)}$ are constants and do not depend on the point of the manifold, $\eta_{(a)(d)}$ can be freely introduced in any derivative and separated from it [9]:

$$\nabla_\alpha e_\beta{}^{(a)} = \partial_\alpha e_\beta{}^{(a)} - \Gamma_{\alpha\beta}{}^{\gamma}e_\gamma{}^{(a)} + e_\beta{}^{(b)}\omega_{\alpha(b),}{}^{(a)} = 0. \quad (A.5)$$

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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