Hyperspherical three body calculation for exotic halo nuclei

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Introduction

In the recent years nuclear Physicists are giving signicant attention to the physics related to the weakly bound nuclear systems, particularly to those formed near the drip lines (proton and neutron dripline). Most striking fact in these region is the discovery of halo nuclei (mostly neutron halo). Tanihata et al first discovered ^{11}Li halo in 1985[1] after the advent of radioactive ion beam facilities. Study of the bound state properties, decay modes, life times including "halo structure" are still regarded as a very crazed topic still after three decades of its discovery. In the present work investigate the bound state properties along with some other interesting features of exotic halo nuclei- $^{14}Be,^{17}B,^{19}B,^{22}C$ etc in the framework of hyperspherical harmonics expansion method(HHEM). We assume a few-body (two- and three-body) model consisting of a structureless nuclear core plus one or two valence nucleon(s) forming a far extended low density tail by the quantum mechanical tunneling of the last one or two nucleon(s). Due to the presence of this extended low density tail in the nuclear matter density distributin, the average matter radii of halo nuclei are found to be significantly larger than those prdicted by the liquid drop model of nuclei. One of the interesting features of three-body halo nuclei is their Borromean property which resembels the ring of Borromeo formed by three interconnected rings, in which, if any one of the three rings is separated, the remaining two also separate automatically. Thus in a halo three-body system none of the binary sub-systems are bound.

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Halo nuclei are broadly categorised as neutron halo and proton halo which are above and below the line of stability respectively. Some of the experimentally observed neutron halo nuclei includes ^6He, ^8He, $^{11}Li,^{11}Be$, ^{14}Be, ^{17}B, ^{19}C, ^{22}C etc on proton halo side one may refer to ^8B, ^{26}P, ^{17}Ne, ^{27}S etc found in the literature[3]. In 1995, Giessen discovered proton halo in ^8B which was also confirmed by other group. Very recently Panda et al. [4], confirmed evidence of a proton halo in ^{23}Al.

Method

The label scheme and choice of Jacobi coordinates for a three-body system is schematically shown in Fig.2. We label the relatively heavier nuclear core as particle 'i' and the two valence neucleons as particles 'j' and 'k' respectively[5]. The three-body relative wave function is expanded in the complete set of hyperspherical harmonics(HH). Substitution of the wave function in the Schrodinger equation and use of orthonormality of HH lead to an infinite set of coupled differential equation(CDE).For the practical purpose the expansion basis is truncated to a finite set subject to the appropriate symmetry requirements. The truncated set CDE is solved numerically to get bound state energy and the wave function.

Results and Tables

For the NN pair we choose the standard GPT [7] potential and while for the core-nucleon interaction we choose SBB [8] and slightly modified SBB potential.

Math and Equations

Now for a given partition 'i'(in which the particle labelled 'i' is the spectator while the remaing two forms the interacting pair), the Jacobi co-ordinates are defined as:

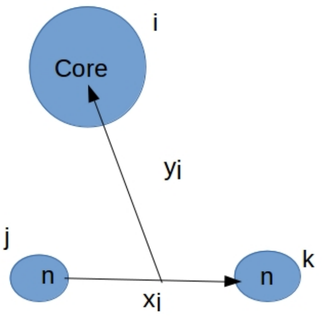

FIG. 1: Choice of Jacobi coordinates for the partion 'i'.

TABLE I: Trend of convergence in energy for increasing K_{max} values

Systems	K_{max}	N	E_{Cal}(MeV)
^{14}Be	4	6	-0.5251
	8	15	-1.0165
	12	28	-1.1821
	16	45	-1.2789
	20	66	-1.3402

$\vec{x_i} = a_{ij}(\vec{r_j} - \vec{r_k})$, $\vec{y_i} = a_{(jk)i}(\vec{r_i} - \frac{m_j\vec{r_j} + m_k\vec{r_k}}{m_j + m_k})$, $\vec{R} = \frac{1}{M}(m_i\vec{r_i} + m_j\vec{r_j} + m_k\vec{r_k})$. Where (i,j,k) are considered cyclically with the corresponding coefficients of jaxcobi coordinates as a_{jk}, $a_{(jk)i}$ are coefficient of Jacobi co-ordinates.$M = m_i + m_j + m_k$ is the total mass and R is the position of the centre of mass of the system. The hyperspherical coordinates are defined as: $x_i = \rho\cos\phi$, $y_i = \rho\sin\phi$. Therefore the Schrodinger equation in terms of hyperspherical coordinates becomes:$[-\frac{\hbar^2}{2\mu}\{\frac{1}{\rho^5}\frac{\partial}{\partial\rho}(\rho^5\frac{\partial}{\partial\rho}) - \frac{\vec{k}^2(\Omega_i)}{\rho^2}\} + V(\rho,\Omega_i) - E]\Psi(\rho,\Omega_i) = 0$ Where,$V(\rho,\Omega_i)$ is the total interaction potential and $\vec{k}^2(\Omega_i)$ is the square of hyper angular momentum operator.

TABLE II: Calculated energy of different exotic halo nuclei

Systems	E_{cal}(MeV)	$E_{expt.}$(MeV)	Others(MeV)
^{14}Be	-1.3402	-1.34± 0.11	-1.26 [9]
^{17}B	-1.3901	-1.39± 0.14[6]	-1.34[10]
^{19}B	-1.1401	-1.14± 0.44	-1.0[11]
^{22}C	-0.1108	-0.11± 0.06	-0.4[12]

Discussion

The hyper-spherical harmonics expansion(HHE) method adopted here is an essentially exact method where calculations can be carried out up to any desired precision by gradually increasing the expansion basis. Where the binding energies gradually attain a convergence with increasing K_{max} values. To accommodate the symmetry requirements among the valence nucleon and the core nucleons a repulsive term has been added in the core-nucleon potential. This approximate manages Pauli principle.

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