Yang-Mills supersymmetry and deformations of $AdS_5 \times S_5$ solution with the Yang-Baxter equation

A.Meirambay, K.K. Yerzhanov

L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan

E-mail: aidana2909@mail.ru,yerzhanovkk@gmail.com

Abstract. The Yang-Mills supersymmetry is considered using the Yang-Baxter equations as deformations of $AdS_5 \times S_5$ in which they are substituted as a twist of the conformal Drinfeld algebra. The operators here are represented as Killing vectors. The solution of this problem is represented as the Θ -matrix for which the deformation ϕ was found, which is a super gravitational solution.

1. Introduction

Integrability plays a key role for AdS (Anti de Sitter) / CFT (Comformal Field Theory) correspondence [1]. So integrability is important when studying superstrings on $AdS_5 \times S_5$ and its dual planar $N = 4$ supersymmetric Yang-Mills theory (SYM) [2, 3]. The AdS / CFT correspondence has an excellent integrated system between the superstring type IIB on the background of $AdS_5 \times S_5$ and the theory $N = 4$ SYM [4]. Here interesting is identification of the structure beyond the maximum symmetric parameter $AdS_5 \times S_5$. The earliest deformation preserving the integrability of $AdS_5 \times S_5$ [5, 1, 6] was created using non-commutative (NC) spaces in string theory [7, 8].

Yang-Baxter (YB) σ-model [9]-[12]deformations were generalized to the $AdS_5 \times S_5$ superstring. TsT transformations can be expressed as part of YB deformations of the σ-model, where YB deformations defined by r-matrices from classical Yang-Baxter equation (CYBE) [13].

We will investigate the $AdS_5 \times S_5$ deformations, which beginning is taken from quantum deformation of the $AdS_5 \times S_5$ [14]-[17] model. The article [18] said how Abelian and Jordan deformations twist the symmetries of the string $AdS_5 \times S_5$, which leads to the twisting of the Drinfeld Hopf algebra.

The open string data, which produced closed string data as a new metric g and B-field after inverting single matrix. Killing vector I receive from divergence of antisymmetric bivector Θ [18]:

$$\nabla_\mu \Theta^{\mu\nu} = I^\nu. \tag{1}$$

Building the open-closed string map of Seiberg & Witten [19]:

$$(g + B)_{\mu\nu} = (G^{\mu\nu} + \Theta^{\mu\nu})^{-1}, \tag{2}$$

where g, B are closed string and G, Θ are open string fields[20].

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Connection of the deformed solution $g_{\mu\nu}, B_{\mu\nu}, \phi$ and the original solution $G_{\mu\nu}, \Theta^{\mu\nu}, \Phi$ is[21]:

$$g_{\mu\nu} = (G^{-1} - \Theta \cdot G \cdot \Theta)^{-1}_{\mu\nu}, \tag{3}$$

$$B_{\mu\nu} = -(G^{-1} - \Theta)^{-1} \cdot \Theta \cdot (G^{-1} + \Theta)^{-1}, \tag{4}$$

$$\varphi = \Phi - \frac{1}{2} \ln \det(1 + G \cdot \Theta). \tag{5}$$

In the above $\cdot$ denotes matrix multiplication and G and Θ are to be viewed as two matrices. r-matrix solutions to the CYBE using an antisymmetric bivector Θ can be written [22]:

$$\Theta^{\mu\nu} = -2\eta r^{\mu\nu} (\mu, \nu = 0, ..., 3, z), \tag{6}$$

where z is the radial direction of AdS_5, η is the deformation parameter.

2. Deformation of Minkowski metric

We introduce null coordinates, $x^{\pm} = x^{\circ} \pm x^3$, so that the four-dimentional Minkowski metric is [13]:

$$ds^2 = -dx^+ dx^- + d(x^1)^2 + d(x^2)^2. \tag{7}$$

It is more convenient to write the metric in matrix form:

$$G = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

We can write non-zero components of $\Theta^{\mu\nu}$ [13]:

$$\Theta^{-+} = -4\eta x^+, \Theta^{-1} = -2\eta x^1, \Theta^{-2} = -2\eta x^2, \tag{9}$$

also in matrix form written,

$$\Theta = \begin{pmatrix} 0 & 4\eta x^+ & 0 & 0 \\ -4\eta x^+ & 0 & -2\eta x^1 & -2\eta x^2 \\ 0 & 2\eta x^1 & 0 & 0 \\ 0 & 2\eta x^2 & 0 & 0 \end{pmatrix}. \tag{10}$$

Following equations (3),(4) we arrive at a new solution of metric to generalized supergravity

$$ds^2 = \frac{4\eta^2 \left((x^1)^2 + (x^2)^2\right)}{16\eta^2 (x^+)^2 - 1} d(x^+)^2 + \frac{1}{16\eta^2 (x^+)^2 - 1} (dx^+ dx^- + dx^- dx^+) - \tag{11}$$
$$- \frac{8\eta^2 x^+ x^1}{16\eta^2 (x^+)^2 - 1} (dx^+ dx^1 + dx^1 dx^-) - \frac{8\eta^2 x^+ x^2}{16\eta^2 (x^+)^2 - 1} (dx^+ dx^2 + dx^2 dx^+).$$

Respectively,the components of the metric tensor can be written

$$g = \begin{pmatrix} \frac{4\eta^2((x^1)^2+(x^2)^2)}{16\eta^2(x^+)^2-1} & \frac{1}{16\eta^2(x^+)^2-1} & -\frac{8\eta^2 x^+ x^1}{16\eta^2(x^+)^2-1} & -\frac{8\eta^2 x^+ x^2}{16\eta^2(x^+)^2-1} \\ \frac{1}{16\eta^2(x^+)^2-1} & 0 & 0 & 0 \\ -\frac{8\eta^2 x^+ x^1}{16\eta^2(x^+)^2-1} & 0 & 1 & 0 \\ -\frac{8\eta^2 x^+ x^2}{16\eta^2(x^+)^2-1} & 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Thus, we easily find B-field

$$B = \begin{pmatrix} 0 & -\frac{4\eta x^+}{16\eta^2(x^+)^2-1} & \frac{2\eta x^1}{16\eta^2(x^+)^2-1} & \frac{2\eta x^2}{16\eta^2(x^+)^2-1} \\ \frac{4\eta x^+}{16\eta^2(x^+)^2-1} & 0 & 0 & 0 \\ -\frac{2\eta x^1}{16\eta^2(x^+)^2-1} & 0 & 0 & 0 \\ -\frac{2\eta x^2}{16\eta^2(x^+)^2-1} & 0 & 0 & 0 \end{pmatrix}, \tag{13}$$

$$B = \frac{4\eta x^+}{16\eta^2(x^+)^2-1}(dx^- \wedge dx^+) + \frac{2\eta x^1}{16\eta^2(x^+)^2-1}(dx^+ \wedge dx^1) + \frac{2\eta x^2}{16\eta^2(x^+)^2-1}(dx^+ \wedge dx^2). \tag{14}$$

3. Geodesic equation and its solutions for deformed metric

When finding cosmological solutions [23],[24] for the given metric, it is interesting to investigate the geodesic equation for the particle. The next goal of our work is to find geodesic equation of the obtained deformed metric that can be found using the equation (15)

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0, \tag{15}$$

where Γ^i_{kl}-Christoffel Symbols, they are calculated from the eq. (16)

$$\Gamma^i_{kl} = \frac{1}{2}g^{im}(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}). \tag{16}$$

Thus, nonzero components of Christoffel Symbols for metric tensor (12) are

$$\begin{aligned} \Gamma^{x^+}_{x^+x^+} &= \frac{32\eta^2 x^+}{-16\eta^2(x^+)^2+1}, \\ \Gamma^{x^-}_{x^+x^+} &= -\frac{32\eta^4 x^+((x^1)^2+(x^2)^2)}{-1+16\eta^2(x^+)^2}, \\ \Gamma^{x^-}_{x^1x^+} &= 4\eta^2 x^1, \\ \Gamma^{x^-}_{x^1x^1} &= -8\eta^2 x^+, \\ \Gamma^{x^-}_{x^2x^+} &= 4\eta^2 x^2, \\ \Gamma^{x^-}_{x^2x^2} &= -8\eta^2 x^+, \\ \Gamma^{x^1}_{x^+x^+} &= \frac{12\eta^2 x^1}{1-16\eta^2(x^+)^2}, \\ \Gamma^{x^2}_{x^+x^+} &= \frac{12\eta^2 x^2}{1-16\eta^2(x^+)^2}. \end{aligned} \tag{17}$$

At this stage, components of the geodesic eq. can be formulated in this form

$$\frac{d^2x^+}{ds^2} - \frac{32\eta^2 x^+}{16\eta^2(x^+)^2-1}\left(\frac{dx^+}{ds}\right)^2 = 0, \tag{18}$$

$$\frac{d^2x^-}{ds^2} - 8\eta^2\left(\frac{dx^+}{ds}\right)\left(\frac{4\eta^2 x^+((x^1)^2+(x^2)^2)\left(\frac{dx^+}{ds}\right)^2}{16\eta^2(x^+)^2-1} - x^1\frac{dx^+}{ds}\frac{dx^1}{ds} + \right. \tag{19}$$
$$\left. + x^+\left(\frac{dx^1}{ds}\right)^2 - x^2\frac{dx^+}{ds}\frac{dx^2}{ds} + x^+\left(\frac{dx^2}{ds}\right)^2\right) = 0,$$

$$\frac{d^2x^1}{ds^2} - \frac{12\eta^2 x^1}{16\eta^2(x^+)^2 - 1}\left(\frac{dx^+}{ds}\right)^2 = 0, \tag{20}$$

$$\frac{d^2x^2}{ds^2} - \frac{12\eta^2 x^2}{16\eta^2(x^+)^2 - 1}\left(\frac{dx^+}{ds}\right)^2 = 0. \tag{21}$$

From eq. (18) we obtain:

$$\frac{dx^+}{ds} = \left(16\eta^2(x^+)^2 - 1\right)\mathrm{e}^{C_1}. \tag{22}$$

Consequently, $x^+(s)$ and $s(x^+)$ take following forms

$$x^+(s) = -\frac{\tanh\left(4\eta(C_2 + s\mathrm{e}^{C_1})\right)}{4\eta}, \tag{23}$$

$$s(x^+) = \frac{-\operatorname{arctanh}\left(4\eta x^+\right) - 4\eta C_2}{4\eta \mathrm{e}^{C_1}}. \tag{24}$$

Then the eq. (20),(21) for the components x^1 and x^2 (they are the same) will be written as differential equation of second order, if we substitute values $\frac{dx^+}{ds}$, $x^+(s)$ and consider constants C_1, C_2 as zero:

$$\frac{d^2x^1(s)}{ds^2} + 12\eta^2\,\mathrm{sech}^2(4\eta s)x^1(s) = 0, \tag{25}$$

$$\frac{d^2x^2(s)}{ds^2} + 12\eta^2\,\mathrm{sech}^2(4\eta s)x^2(s) = 0. \tag{26}$$

Substituting the value of $s(x^+)$ from eq. (24), eq. (25),(26) have solutions

$$\begin{aligned} x^1(x^+) = \left(\cosh\left(2\arctan 4\eta x^+\right) + 1\right)^{3/4}\Bigg(C_3 \cdot F\left(\frac{3}{4}, \frac{3}{4}; \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\cosh(2\arctan 4\eta x^+)\right) + \\ + C_4\sqrt{\cosh(2\arctan 4\eta x^+) - 1} \cdot F\left(\frac{5}{4}, \frac{5}{4}; \frac{3}{2}; \frac{1}{2} - \frac{1}{2}\cosh(2\arctan 4\eta x^+)\right)\Bigg), \end{aligned} \tag{27}$$

$$\begin{aligned} x^2(x^+) = \left(\cosh\left(2\arctan 4\eta x^+\right) + 1\right)^{3/4}\Bigg(C_5 \cdot F\left(\frac{3}{4}, \frac{3}{4}; \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\cosh(2\arctan 4\eta x^+)\right) + \\ + C_6\sqrt{\cosh(2\arctan 4\eta x^+) - 1} \cdot F\left(\frac{5}{4}, \frac{5}{4}; \frac{3}{2}; \frac{1}{2} - \frac{1}{2}\cosh(2\arctan 4\eta x^+)\right)\Bigg), \end{aligned} \tag{28}$$

where $F(a, b; c; z)$-hypergeometric function.

4. Conclusion

Thus, in this paper was found a deformed metric for type IIB superstring on the $AdS_5 \times S_5$ correspondence with $N = 4$ super Yang-Mills theory by method Yang-Baxter deformations of the σ-model as generalized to the $AdS_5 \times S_5$ superstring. These solutions were used when finding the geodesic equation for this model, from which we obtained the solutions $x^1(x^+), x^2(x^+)$.

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