The Quark Gluon Plasma and relativistic heavy ion collisions in the LHC era

Francesco Becattini

Dipartimento di Fisica e Astronomia, Università di Firenze, Via G. Sansone 1, I-50019, Sesto Fiorentino (Firenze)

E-mail: `becattini@fi.infn.it`

Abstract.   In this talk we briefly review some of the achievements in the study of the QCD matter at high temperature produced in the nuclear collisions at very high energies. We focus on its bulk properties: hydrodynamical and thermodynamical.

1. Introduction

Quark Gluon Plasma (QGP) is a phase of strongly interacting matter (see figure 1) whose existence is predicted by Quantum-Chromo-Dynamics (QCD) to occur at sufficiently high temperatures and/or baryon densities. In the QGP phase, quarks and gluons are deconfined, i.e. they can move over distances much larger than a typical hadron size. This peculiar QCD prediction follows from a property of the theory known as asymptotic freedom: the interaction of the fundamental fields become weaker as the energy density increases. Therefore, for a strongly interacting system, there should be a critical temperature (at vanishing chemical potentials) separating the phase where quarks and gluons are deconfined from the hadron gas. It is by now established that, at least for $\mu_B = 0$, the transition from hadron gas to QGP is a continuous one, that is a crossover [1].

The experimental quest for Quark Gluon Plasma has been going on for more than 30 years by now. It was soon proposed that QGP could have been produced in high energy collisions of heavy nuclei. The main reason of colliding large nuclei is that, in order to obtain a proper deconfinement, the region of space where high energy density is achieved should be much larger than a typical hadron size. Therefore, for a fixed nucleon beam energy, it seems more likely to produce a deconfined QGP in collisions of heavy nuclei (several fm's of diameter) rather than, e.g., in proton-proton collisions, although, as we will see, there are striking similarities between the two systems.

A vaste experimental programme of heavy ion collisions at several centre-of-mass energies was then initiated; from few GeV (Au-Au collisions at BNL-AGS) to an intermediate range of 6-17 GeV (Pb-Pb collisions at CERN-SPS) up $O(100)$ GeV at BNL-RHIC, started in 2000 and finally, from 2009, collisions of Pb nuclei at the LHC at $\sqrt{s}_{NN}$ of few TeV. It is now widely believed that the threshold for QGP production has been overcome at RHIC and probably at SPS.

The main issue of the whole experimental programme is how to prove QGP formation, i.e. to find a signature thereof in the final observable particles produced in the collision events. Indeed,

most of the effort of theorists in this field has been spent into the search of the most powerful signatures or probes, besides the description of the collision itself. In this talk, I will just focus on soft hadronic probes and will not dwell on the so-called hard and electromagnetic probes, which would deserve a separate chapter.

Because of the complexity of the process of collision of two nuclei at high energy and the lack (or unfeasibility) of QCD first-principle calculations at the energy scale of interest, most phenomenology in this field has to resort to different approaches and models. One of the most successful models in relativistic heavy ion physics is the hydrodynamical model, which should be rather called "hydrodynamical description" because it is based on a general physical scheme rather than on peculiar assumptions for the specific process (like for instance the string model in pp collisions). The hydrodynamical description has indeed provided much insight and understanding of the collision process and bears out the QGP formation.
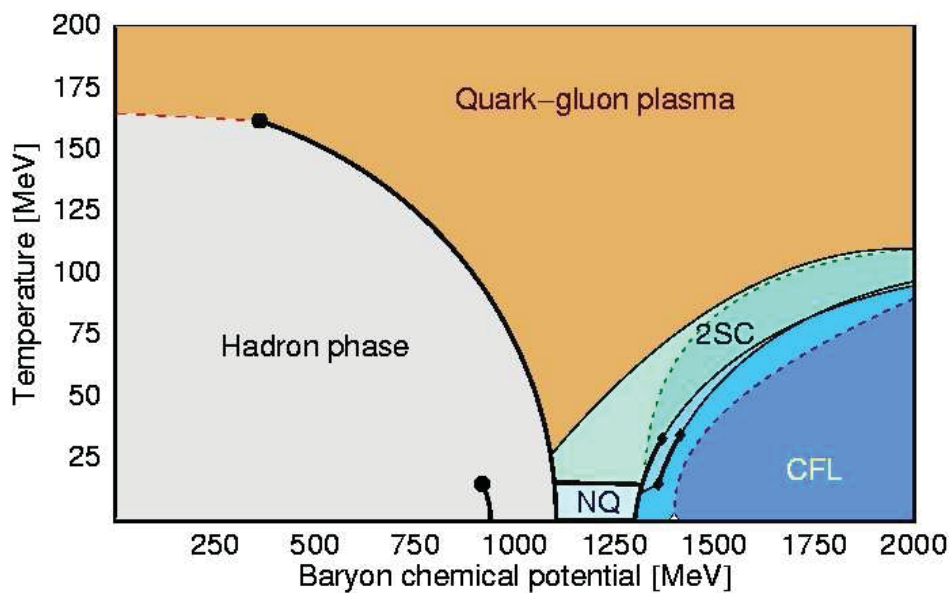

Figure 1. (Color online) QCD phase diagram in the $T - \mu_B$ plane.

2. The QGP as a nearly ideal fluid

For hydrodynamics to be a sensible description of the QGP evolution, two requirements must be met:

(i) the system should be close to local thermodynamical equilibrium for a sufficiently long time, that is local equilibrium must be achieved at some early stage, in the QGP phase;

(ii) the typical microscopic interaction scale (e.g. the mean free path in a kinetic framework) should be much smaller than the length over which temperature and energy density vary, hence much smaller of the size of the system.

The processes leading to the local thermalization in the plasma phase are non-perturbative and still subject of investigation. But if this assumption holds, much of the life of the plasma can be reproduced through the hydrodynamical continuity equations, until the phase transition into hadron gas takes place. The second requirement is generally fulfilled in relativistic heavy ion collisions as the fireball is estimated to have a radius of several fm's while the microscopic scale should be smaller than 1 fm (see later).

The hydrodynamical-based calculations try to reproduce measured particle spectra at low momenta and to determine the initial conditions (e.g. the initial energy density profile) adjusting the pertaining parameters, as well as the parameters (temperature and baryon chemical potential) at which the hydrodynamical regime ceases. The equation of state is an interpolation of lattice QCD results at high temperature and hadron-resonance gas at temperatures lower than critical one.

Mostly adopted initial conditions include a density profile calculated from the geometrical overlap of the two incoming nuclei and longitudinal flow profile according to Bjorken's scaling hypothesis with vanishing transverse flow. Most calculations indicate an early equilibration time of the order of 1 fm/c both at SPS and RHIC energies, with an initial temperature well above the critical value of ≈ 160 MeV.
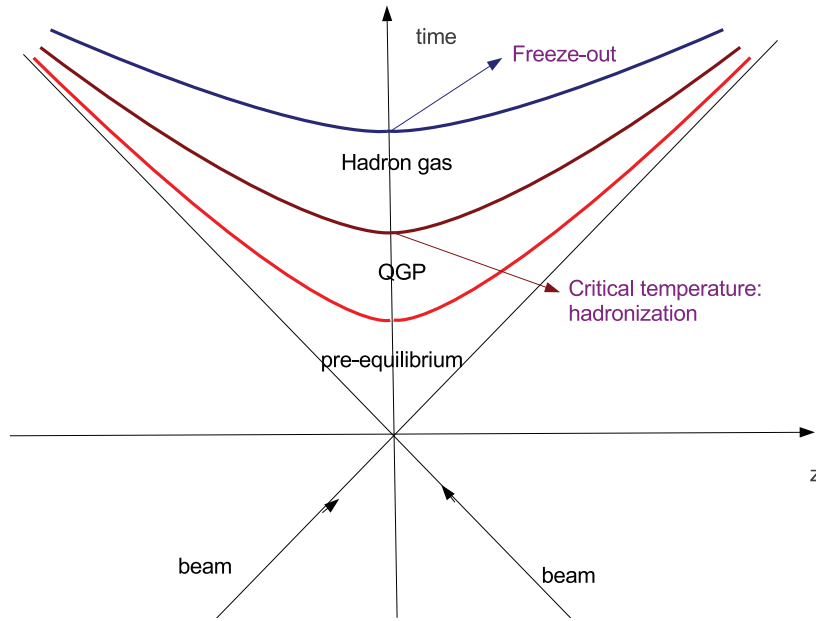

Figure 2. (Color online) Sketch of a relativistic nuclear collision process at high energy. Thermalization is achieved after some time, then plasma expands and hadronizes at the critical QCD temperature. Later, the hadron gas expands and freezes out; for the details of this process, see text.

Of special interest is the so-called *elliptic flow*, which shows as an anisotropy of differential azimuthal particle momentum spectrum in the reaction plane in peripheral collisions. Because of the non-zero impact parameter in peripheral collisions, the overlapping region of the two incoming nuclei has an initial almond shape, unlike in central collisions where it is spherical. If local thermal equilibrium sets in early, pressure gradient is significantly higher in the reaction plane than in the orthogonal direction and this drives an enhancement of the collective flow along the reaction plane, resulting in an enhancement of particle momentum. The magnitude of the elliptic flow is usually gauged with the second coefficient of the Fourier expansion of the $dN/dp_T d\varphi$ spectrum as a function of φ for fixed p_T, the so-called $v_2(p_T)$.

Necessary conditions for reaching high values of v_2 are an early equilibration time of and low values of viscosity, that is a nearly ideal fluid. The fact that QCD matter apparently behaves like an ideal fluid gave rise to the folklore that QGP is a strongly interacting fluid, in stark contrast

to the naive expectation of a gas of weakly interacting quarks and gluons. Indeed, lattice QCD calculations (for instance the stress-energy tensor trace anomaly) already indicate that the interaction energy is very significant around T_c, about $1/3$ of the total energy density. The estimated low ratio viscosity/entropy η/s from hydrodynamical fits reinforces this conclusion. This can be understood by simply reminding the familiar textbook formula of viscosity from kinetic theory:

$$\eta = \frac{1}{3}\rho v_T \lambda$$

where ρ is the mass density, v_T the particle mean thermal speed and λ its mean free path. We can rewrite the above formula with the mean thermal momentum p_T and particle density n:

$$\eta = \frac{1}{3}nm v_T \lambda = \frac{1}{3}p_T n \lambda$$

and make an educated guess of its extension to the relativistic domain by replacing the particle density n with the entropy density s (which measures the density of microscopic degrees of freedom). Therefore, one has:

$$\frac{\eta}{s} = \mathcal{O}(p_T \lambda)$$

that is the viscosity over entropy ratio is of the order of the ratio between the mean free path and the mean particle wavelength. For usual fluids, this is much larger than 1 [1]. If it is 1 or smaller than 1, as it seems to be the case for the QGP [2] where $\eta/s \approx 1/2\pi$ the mean free path is less than the thermal particle wavelenght, implying a breakdown of the kinetic and particulate description. Particularly, one has, at $T = 160$ MeV:

$$\lambda = \mathcal{O}\left(\frac{\eta}{s}\frac{1}{p_T}\right) \approx \frac{1}{2\pi T} \simeq 0.2 \text{ fm}$$

Indeed, the QGP near the critical temperature is a remarkable instance of a fluid which has no underlying kinetic substratum (in other words, no colliding particle or quasi-particle excitations), a features shared with the so-called unitary Fermi gas [3].

For this system, a kinetic description in terms of independent colliding particles is therefore questionable. Still, hydrodynamics and hydrodynamical description hold, including transport coefficients. Indeed, it is possible to formulate hydrodynamics using the mean value of the fundamental quantum stress-energy tensor operator:

$$T^{\mu\nu} = \text{tr}(\widehat{\rho}\widehat{T}^{\mu\nu})$$

and to show that all transport coefficients can be defined in terms of the two-point functions of this tensor at equilibrium through the Kubo formulae [4].

3. Freeze-out in an expanding hadron gas
The QGP expands and cools until it reaches the critical crossover temperature, where it hadronizes. The hadron gas keeps expanding for a while until particles do not interact anymore: the system is *frozen out*. In an expanding sytem of interacting particles freeze-out occurs when the mean scattering time τ_{scatt} exceeds the mean collision time τ_{exp}:

$$\tau_{\text{scatt}} = \frac{1}{n\sigma\langle v \rangle} > \tau_{\text{exp}} = \frac{1}{\partial \cdot u} \tag{1}$$

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[1] This situation should be contrasted with a familiar fluid, liquid water at $T = 0^o$C. The mean free path of water molecules can be estimated to be 0.3 nm, which is approximately equal to the square root of the cross section $\sqrt{\sigma}$, yet the mean thermal wavelength is of the order of 5 pm, mostly due to the large molecular mass.

u being the hydrodynamical velocity field and $\langle v \rangle$ is the mean velocity of particles. If the cross-section σ is the inelastic one, the freeze-out is called *chemical*, whereas if it includes elastic processes, the freeze-out is called *kinetic*. Chemical freeze-out of course precedes the kinetic as the inelastic cross section is smaller than the total.

We can obtain a gross approximation of the expansion time with the ratio $V/\dot{V}$ where $V(t)$ is the volume of the fireball at the time t. For a fireball which is spherical in shape with a radius R, this is $R/3\dot{R}$ and if the radius increases at approximately the mean particle velocity $\langle v \rangle$, we have the condition:

$$\frac{1}{n\sigma\langle v \rangle} = \frac{R}{3\langle v \rangle} \implies \frac{1}{n\sigma} = \frac{R}{3} \tag{2}$$

For a given number of particles N within the volume, this inequality yields the radius at which freeze-out occurs as a function of N and of the average cross-section:

$$R_{\text{fo}} = \sqrt{\frac{N\sigma}{4\pi}} \tag{3}$$

and the density at which freeze-out occurs, which decreases with N according to:

$$n_{\text{fo}} = \frac{N}{\frac{4\pi}{3}R_{\text{fo}}^3} = 3\sqrt{\frac{4\pi}{N}}\frac{1}{\sigma^{3/2}} \tag{4}$$

For instance, for $N = 1000$, typical of heavy ion collisions, and $\sigma = 30\,\text{mb} = 3\,\text{fm}^2$ one has $R_{\text{fo}} \simeq 15$ fm and $n_{\text{fo}} \simeq 0.06$ fm^{-3}, which are in the right ballpark taking into account the drastic approximations made. The estimates (3) and (4) are obviosuly crude, but it tells us that the freeze-out radius, for each particle, approximately scales with the square root of the number of scattering centers a particle can interact with and the related cross section.

An interesting question, which is relevant for a hadronic gas, is whether multi-body collisions are significant at some stage in the expansion process. One can write a simple generalization of the two-body collision rate formula:

$$\frac{dR}{d^4x} = n_1 n_2 \sigma v_{rel} = n_1 n_2 \sigma \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{\varepsilon_1 \varepsilon_2} \tag{5}$$

where p_i are the particle four-momenta and m_i their masses, n_i their densities and v_{rel} the relativistic relative velocity, to a three-body collision assuming that the particle 3 collides with a cluster (12) made of particles 1 and 2:

$$\left.\frac{dR}{d^4x}\right|_3 = n_3 n_{(12)} \sigma_{3(12)} v_{rel} P_{(12)} \tag{6}$$

where $n_{(12)}$ is the density of clusters, v_{rel} a suitable extension of the relativistic relative velocity function for the 2-body problem and $P_{(12)}$ the cluster formation probability. The latter can be assumed to be proportional to $n_1 n_2 \sigma_{12}^{3/2}$, i.e. proportional to the probability that 1 and 2 find themselves within a range of $\approx \sqrt{\sigma}$. Replacing $\sigma_{3(12)}$ and σ_{12} with a typical hadronic cross section σ one has:

$$\left.\frac{dR}{d^4x}\right|_3 \approx \frac{n_1 n_2 n_3}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \sigma^{5/2} f_{rel}(m_1, m_2, m_3, \sqrt{s}, ...) \tag{7}$$

where f is a function of relativistic invariants pertaining to a three-particle system. This approximated formula can be extended to N-body collisions:

$$\left.\frac{dR}{d^4x}\right|_N \approx \left(\prod_{i=1}^N \frac{n_i}{\varepsilon_i}\right) \sigma^{(3N-4)/2} f_{rel}(m_1, \ldots, m_N, \sqrt{s}, \ldots) \tag{8}$$

Binary collisions prevail when the left hand side of (5) largely exceeds the one in (7). Assuming that the ratio of the relativistic functions f_{rel} averages to $\mathcal{O}(1)$, this happens when:

$$n\sigma^{3/2} \ll 1 \implies \lambda = \frac{1}{\sigma n} \gg \sqrt{\sigma} \tag{9}$$

This is a physically very clear condition: the particle mean free path λ should be much larger than its effective interaction distance as determined by the square root of the cross section. This is by no means a trivial requirement; for an expanding hadron gas to have a stage where binary collisions prevail before freeze-out, this implies, according to equation (2):

$$R_{\text{fo}} = \sqrt{\frac{N\sigma}{4\pi}} = 3\lambda \gg 3\sqrt{\sigma} \tag{10}$$

that is the system should contain a *large* number of particles, $N \gg \mathcal{O}(100)$. If this condition is not fulfilled, the system decouples when ternary and, in general, N-nary collisions are still relevant. This is an important conclusion, to be emphasized: *in relativistic ion collisions, for the decoupling to occur in the binary collision regime, the number of particles should largely exceed $\mathcal{O}(100)$*. Thus, this may happen only for *heavy* ion collisions.

There is one more important condition to be addressed: for the freeze-out to be in the kinetic regime, meaning a system of quasi-free colliding particles, it is necessary that the mean free path of particles largely exceeds their mean (thermal) wavelength, a condition that we have seen is not fulfilled in the plasma phase:

$$\lambda = \frac{1}{\sigma n} \gg \frac{1}{\langle p \rangle} = \frac{1}{m\langle v \rangle} = \frac{1}{\sqrt{3Tm}} \tag{11}$$

where the last formula assumes the non-relativistic regime of most hadrons, which is a good approximation for a hadron gas in the actual observed range of temperatures. Hence, writing $n = N/(4\pi R^3/3)$ and using (3):

$$\frac{4\pi}{3}R^3 \gg N\sigma \frac{1}{\sqrt{3Tm}} \implies R \gg R_{\text{fo}}^{2/3} \left(\frac{3}{Tm} \right)^{1/6} \tag{12}$$

with the assumption that the number of particles N does not change till freeze-out. The last inequality dictates that we have a kinetic stage in hadron gas expansion, that is $R < R_{\text{fo}}$ provided that the radius at freeze-out is very much larger than the thermal wavelength of the hadrons. As a numerical example, for pions at $T = 160$ MeV the last factor equals 1.3 fm$^{1/3}$ and for $R_{\text{fo}} = 15$ fm, the above inequality demands $R \gg 8$ fm. We stress again that these numbers are crude approximations and only have an illustrative purpose, they should not be compared as such with actual measurements of the system size with e.g. pion interpherometry.

Another relevant question is whether multi-body collisions can occur in what we can properly call a kinetic regime. According to (9), multi-body collisions are relevant if the mean free path is of the order of the square root of the cross-section. Therefore, we are led to demand that:

$$\lambda \approx \sqrt{\sigma} \gg \sqrt{\frac{1}{3mT}} \tag{13}$$

Is this condition fulfilled? For pions at the typical temperature $T = 160$ MeV, the right hand side is about 1 fm, which is not much smaller than the square root of the typical hadronic cross-section. Under this circumstance, the hadron gas is in the very same condition of a strongly interacting fluid as described at the end of the previous section.

4. Hadronization and beyond: the statistical model

Among the models which have been proposed to account for the quantitative features of hadron production in relativistic heavy ion collisions, the statistical hadronization model can be considered as of today a reference. Besides its remarkable success in reproducing particle multiplicities with few parameters, this model is used in nearly all hydrodynamical calculations to turn the fluid cells into hadrons through the Cooper-Frye prescription, enforcing local thermodynamical equilibrium. Furthermore, it has been shown that this model is able to reproduce hadronic multiplicities in elementary collisions, from e^+e^- to pp and from low to high energy [5]. Therefore, it turns out that the statistical model grasps a peculiar universal feature of the hadronization process, hence of QCD in the non-perturbative regime, regardless of the kind of collision.

Indeed, the statistical model has proved to be successful in relativistic heavy ion collisions throughout the explored centre-of-mass energy range [5]. The interpretation of its success is, however, still subject of debate. Some proposed that thermalization is achieved at the level of hadrons through multiple collisions [6, 7]. Others [8], including the author, advocate that hadronization itself gives rise to an equilibrated system of hadrons (modulo the extra strangeness suppression in elementary collisions, not fully understood). In Hagedorn's words *hadrons are born in equilibrium* because it would be otherwise impossible to explain the statistical distributions observed in elementary collisions where multiplicities are such that freeze-out coincides with hadron, and, chiefly, the apparent full chemical equilibrium of strange particle abundances.

If hadronization itself gives rise to an equilibrated population of hadrons, can the later processes, if any, affect it? This is an important question for the understanding of the hadron production mechanism in both elementary and relativistic heavy ion collisions. We can sketch - in a very ideal fashion - the major steps in the process leading to the freeze-out in relativistic heavy ion collisions as points along a time-oriented axis as in figure 3. In fact, the freeze-out processes are continuous, species dependent, geometry dependent and overlapping. Nevertheless, the diagram of figure 3 is an useful tool for the purpose of illustration.
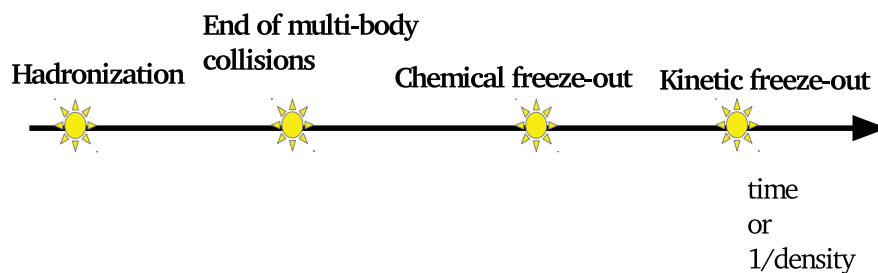

Figure 3. Sketch of the time sequence of processes in an expanding strongly interacting hadronic system: after hadrons are formed, a strongly interacting stage follows where particles may undergo multi-body collisions. Thereafter, mostly binary collisions survive till chemical freeze-out and, later, kinetic freeez-out.

After hadronization, the system begins to expand, but it can stay a little in the strong coupling

regime where it cannot be described as a kinetic system, according to previous discussion. At some point, this regime, where chemical equilibrium is presumably ketp, ceases and the era of binary collisions sets in, provided that the number of particles is $> \mathcal{O}(100)$. For some time, binary collisions can be inelastic (chemical freeze-out) thereafter only elastic till kinetic freeze-out occurs and the system finally decouples.

Is this picture borne out by the data? The existence of a stage where binary collisions prevail is confirmed by at least three major evidences:

(i) The suppression of short-lived resonances compared to long-lived particles in the statistical model fits [9, 10, 11], an effect which is not seen in elementary collisions [12]. It can be explained by the rescattering of the resonance decay products in the expanding medium, not possible for a small system.

(ii) The difference between the temperatures determined in multiplicity fits and those in the fits of transverse spectra [11]. This means that elastic interactions continue after chemical freeze-out. Also this effect is not seen in elementary collisions [12].

(iii) The dependence of kinetic freeze-out temperatures as a function of the impact parameter [11, 13]. Collisional decoupling depends on particle multiplicity, as we have seen, as well as geometry, and it occurs at a higher density if multiplicity is lower.

The existence of a kinetic stage where hadrons interact elastically is by now generally accepted. Do we have a similar evidence for the abundances? In other words, do hadrons chemically decouple after some stage of binary collisions? It should be stressed that, if such a stage exists, hadronic abundances should show deviations from an initial chemical equilibrium situation [14].

Until recently, there was no major clue of such a phenomenon as statistical model fits were in agreement with the data to a satisfactory degree of accuracy. However, new measurements of anti-proton yield at SPS energy [15] and at LHC [16] resulted in significantly lower values compared to the statistical model predictions [17, 18]. This result has been soon interpreted as an effect of post-hadronization inelastic rescattering, where anti-baryons are annihilated before chemical freeze-out increasing pion multiplicity [19, 20, 21]. We point out that such annihilation should occur in a stage where 2-body collisions prevail (see figure 3) and Monte-Carlo simulations such as UrQMD [22] only include binary collisions. Howeever, as has been mentioned, the separation of N-ary to binary collisions stages in fact cannot be sharp and indeed it was pointed out [23] that the back-reaction of N pions colliding to regenerate antibaryons could continue for a longer time.

As a matter of fact, the reanalysis of the data assuming equilibrium at hadronization and subsequent post-hadronization inelastic rescattering stage using UrQMD predictions as correction factors gives a better fit to the data and a hadronization (or latest chemical equilibrium point) in agreement with the extrapolated QCD lattice line (see figure 4) [24]. Even though UrQMD does not include $N > 2$-nary collisions, this is a strong indication that inelastic processes following hadronization do play a role and that chemical freeze-out does *not* coincide with hadronization. Still, taking these processes properly into account, it is possible to reconstruct the original hadronization conditions.

5. Conclusions

There is little doubt that QGP has been produced in relativistic heavy ion collisions at high energy. The experimental programmes from SPS to LHC energies have allowed a verification of several QCD predictions for the phases of the strongly interacting matter. There are several features of the phase diagram which are still to be verified, such as the existence of a critical point, which will be the physics case of a next generation of experiments taking place at JINR, CERN and GSI. From a theoretical viewpoint, QGP is, to my knowledge, the only instance
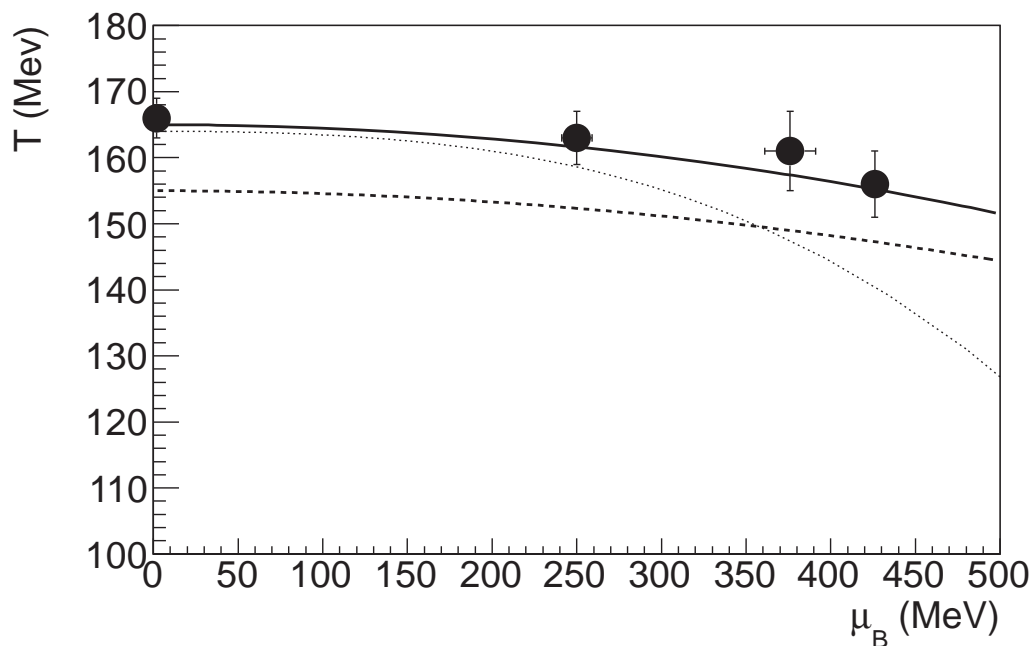
Figure 4. Reconstructed hadronization points in the (T, μ_B) plane compared with lattice QCD extrapolations (solid and dashed lines) and the chemical freeze-out line (dotted curve) (from Ref. [24]).

of fully relativistic matter (in the sense of locally equilibrated interacting system) created in a terrestrial laboratory, and this provided an unprecedented opportunity to develop, test and extend to the relativistic domain the theoretical tools utilized for the usual matter.

Acknowledgments

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