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NON-PERTURBATIVE NEW PHYSICS WITH FUNCTIONAL METHODS

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Non-perturbative new physics with functional methods

The Standard Model (SM) of particle physics stands as the most successful theory for describing the fundamental aspects of nature that experiments can access. Despite its numerous achievements, unresolved questions remain, necessitating the inclusion of physics beyond its current formulation. This thesis employs the functional renormalisation group, an advanced non-perturbative quantum field theoretical approach, to re-examine the SM and explore compelling, yet computationally demanding, extensions. The first part of this work studies the SM's phase structure, reinterprets its unresolved puzzles, and explores its embedding along asymptotically safe quantum gravity. Altogether, this provides new insights into the features of the model and establishes a framework for testing non-perturbative new physics self-consistently. The second part focuses on popular SM extensions that inherently require a non-perturbative treatment. We present a cartographical study of strongly interacting gauge-fermion theories, dissecting the interplay between confinement and chiral symmetry breaking, which is essential for many new physics models. Additionally, we investigate how this class of theories behaves in extra dimensions and how fermion masses can arise in Composite Higgs models. This work lays the foundation for studying new physics from a non-perturbative perspective, providing insights into novel dynamics and unexplored directions that could illuminate a more fundamental description of nature.

Nicht-perturbative neue Physik mit funktionalen Methoden

Das Standardmodell (SM) der Teilchenphysik ist die erfolgreichste bekannte Theorie zur Beschreibung der Natur auf ihrer fundamentalsten Ebene. Trotz seiner zahlreichen Erfolge bleiben fundamentale Fragen unbeantwortet, die eine Erweiterung der Physik über die aktuelle Formulierung des SM hinaus erforderlich machen. Diese Dissertation verwendet die funktionale Renormierungsgruppe, eine fortgeschrittene nicht-perturbative quantenfeldtheoretische Methode, um das SM neu zu analysieren und rechnerisch anspruchsvolle, aber physikalisch motivierte Erweiterungen zu untersuchen. Der erste Teil dieser Arbeit untersucht die Phasenstruktur des SM, interpretiert dessen ungelöste Rätsel neu und erforscht seine Einbettung in asymptotisch sichere Quantengravitation. Dies ermöglicht neue Einblicke in die Eigenschaften des Modells und etabliert eine Methodik zur selbstkonsistenten Untersuchung nichtperturbativer neuer Physik. Der zweite Teil konzentriert sich auf SM-Erweiterungen, die auf nicht-perturbativen Effekten beruhen. Wir präsentieren eine kartografische Untersuchung stark wechselwirkender Eich-Fermion-Theorien und analysieren das Zusammenspiel zwischen Confinement und chiraler Symmetriebrechung, das für viele Modelle neuer Physik essenziell ist. Darüber hinaus untersuchen wir wie sich diese Theorien in Extradimensionen verhalten und analysieren die Entstehung von Fermionmassen in Composite Higgs-Modellen. Diese Arbeit legt die Grundlage für die Untersuchung neuer Physik mit nicht-perturbativen Methoden und bietet Einblicke in neuartige Dynamiken sowie mögliche Erweiterungsrichtungen des SM.

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DISCLAIMER AND LIST OF PUBLICATIONS

The present thesis has been solely written by the author. However, it is based on work carried out in collaboration with others. The respective ideas and computations have been published or are currently under peer review, as documented in the following publicly available references:

- [1] **The Asymptotically Safe Standard Model:
From quantum gravity to dynamical chiral symmetry breaking**
Álvaro Pastor-Gutiérrez, Jan M. Pawłowski , and Manuel Reichert 
Published in SciPost Phys. 15, 105 (2023)
E-Print: hep-th | arXiv:2207.09817
- [2] **Flavor hierarchies from emergent fundamental partial compositeness**
Florian Goertz , Álvaro Pastor-Gutiérrez, and Jan M. Pawłowski 
Published in Phys. Rev. D 108 (2023) 9
E-Print: hep-ph | arXiv:2307.11148
- [3] **Ultraviolet completion of extra-dimensional Yang-Mills theory for Gauge-Higgs unification**
Álvaro Pastor-Gutiérrez and Masatoshi Yamada 
Published in SciPost Phys. 15 (2023)
E-Print: hep-th | arXiv:2205.13250
- [4] **Phase structure of extra-dimensional gauge theories with fermions**
Álvaro Pastor-Gutiérrez and Masatoshi Yamada 
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E-Print: hep-th | arXiv:2401.07908
- [5] **Gauge-Fermion Cartography:
From confinement and chiral symmetry breaking to conformality**
Florian Goertz , Álvaro Pastor-Gutiérrez, and Jan M. Pawłowski 
E-Print: hep-th | arXiv:2412.12254
- [6] **Unveiling new phases of the Standard Model Higgs potential**
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Additionally, this thesis contains material from unpublished work currently under preparation:

[7] Naturalness and predictivity in the Standard Model

Juan Pablo Garcés, Florian Goertz , Manfred Lindner , and Álvaro Pastor-Gutiérrez

The author of this thesis has played the role of first author in the development of all referenced works. Below, we explicitly outline the specific contributions to each publication, as agreed upon with all co-authors:

- ◊ In [1], I provided the motivation and derived the flow equations for the complete Standard Model. I implemented existing results from the co-authors' previous works for the deep infrared QCD and ultraviolet quantum gravity limits, carried out the analysis, and created the figures. All authors contributed to the manuscript and conclusions.
- ◊ In [2], I provided the motivation and conducted the analysis. The work was only possible with Florian Goertz's expertise in Composite Higgs model building and Jan M. Pawłowski's on the functional emergent composites approach. All authors contributed to the manuscript and conclusions.
- ◊ In [3], Masatoshi Yamada provided the original idea and motivation. Both authors derived the flows, while I performed the analysis and created the figures. All authors contributed to the manuscript and conclusions.
- ◊ In [4], I provided the motivation as an extension of [3]. Both authors contributed to the derivation of the flow equations, while I performed the analysis and created the figures. All authors contributed to the manuscript and conclusions.
- ◊ In [5], I provided the original idea and motivation as well as derived the flow equations and performed the analysis. Jan M. Pawłowski provided the expertise in the functional implementation of confinement and dynamical chiral symmetry breaking. All authors contributed to the manuscript and conclusions.
- ◊ In [6], I provided the original idea and conducted the analysis. All authors contributed to the manuscript and conclusions.
- ◊ In [7], I provided the conceptual idea and led the project. I conducted the analysis in collaboration with Juan Pablo Garcés. Discussions with all co-authors are acknowledged.

At the start of each Section, a disclaimer indicates the respective referenced works on which that part is based on. This thesis contains results from all cited works but formally is a accumulation of [1], [2] and [4] which are contextualised in Sections 3.4, 4.2 and 4.3 and explicitly printed in Appendices A to C.

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INTRODUCTION

The Standard Model (SM) of particle physics [8–13] stands as the most successful theory in describing nature at its most fundamental level so far known. It embeds all kinds of matter content and three of the four fundamental forces known to exist. With the discovery of the Higgs boson at the Large Hadron Collider (LHC) [14, 15] over a decade ago, all sectors of the model have been independently measured and no further milestones are expected in the near future.

And now what?

Theoretical predictions of the SM show an astonishing agreement with experimental results, demonstrating the theory’s success in describing nature across the accessible length scales. However, a range of observations indicate that the SM is incomplete. In the quest of formulating the fundamental theory of nature that encompasses all symmetries, forces, and matter, new physics beyond the SM (BSM) is required.

Experimental evidences

From the experimental side, three strong evidences for new physics are present. First the existence of dark matter (DM) has been established through various kinds of independent observations, from galaxy rotation curves to the cosmic microwave background power spectrum [16–21]. This suggests that some type of gravitationally interacting matter is not accounted for in the SM. Second, the observed universe shows a clear excess of matter over antimatter, implying that at some stage in the early universe, an asymmetry was triggered. However, the conditions necessary for the required asymmetry [22, 23] are not fully satisfied within the SM but could be addressed through the addition of new matter content or forces. Third, it has been experimentally confirmed [24] that neutrinos oscillate among their flavour eigenstates which implies the existence of a mass term. However, this is not accommodated within the known sym-

metries of the SM (in the absence of right-handed neutrinos) and calls for a generation mechanism.

Time for theory

On the other hand, the description of nature at its most fundamental level requires a mathematical formulation unifying quantum mechanics and special relativity. This is provided by the framework of quantum field theory (QFT), within which the SM is formulated. Under the assumption that both keystones hold at the most fundamental layers of nature, further evidence of new physics could be predicted from inconsistencies of the theory. A historical example of the astonishing predictive power achieved by imposing theoretical consistencies was proposed prior to the discovery of the Higgs boson, providing a *no-loose theorem* [25] for the presence of new physics at the LHC range of energies. Analysing the scattering of electroweak (EW) bosons, it was shown that the unitarity of the theory, or in simpler terms the conservation of probability, was at stake in the absence of new degrees of freedom. Thus, if the theoretical framework of QFT was to remain consistent, new physics (e.g. the Higgs boson at the time) had to be found below the TeV scale. In a similar fashion, studying the currently complete SM in a comprehensive manner could provide further hints toward the existence of new physics.

Landau poles and quantum gravity, not fundamental yet

Indeed, the SM cannot be formulated as a fundamental QFT valid at all energy scales. The theory exhibits a triviality problem given the fundamental couplings of the Abelian and scalar sectors turn singular at high energy scales [26–29]. This renders the four-dimensional SM not ultraviolet (UV) complete and hence unquantifiable as a fundamental theory of nature. For the measured values of the SM parameters, the theory predicts its own breakdown at energy scales of around $E \sim 10^{43}$ GeV. However, this issue is overshadowed by a more immediate problem emerging at much lower scales. At Planck-scale energies $E \sim 10^{19}$ GeV, quantum gravitational effects are expected to become significant and to alter the picture of the pure SM. For this to be achieved, gravity needs to be understood at a quantum level and contextualised along the other forces already included in the SM. It is worth noting that the assessment of quantum gravity could require of a qualitatively different picture which currently cannot be supported by experimental efforts. In spite of this, one can hope that quantum gravity together with some unknown new physics can upgrade the SM to a UV complete theory, absent of a triviality problem. These two caveats, the Landau pole and the absence of quantum gravity, prove that the SM should be viewed as a successful low-energy effective theory (LEFT) with highest validity limit around the Planck scale.

Plenty of space to fit anything

Furthermore, the range of scales between high-energy experiments $E \sim 10^3$ GeV and the Planck

scale appears as a great empty desert and offers plenty of space for unknown forces and symmetries which could resolve the current mysteries. Given the vast plethora of possibilities in this experimentally uncharted (and currently inaccessible) regime, guiding principles for model building are highly convenient. Ideally, theoretical inconsistencies of the model would provide a guiding principle, such as in the case of the no-lose theorem advertising the Higgs boson discovery at LHC [25].

Maybe metastable

A possible signature for new physics may in fact arise from the Higgs sector of the SM. This is central as it connects the mass generation mechanism for most fermions and EW gauge bosons. There is theoretical evidence for the existence of an additional vacuum in the Higgs potential at large field values, into which the present metastable minimum could decay [27, 30–32]. This implies that the current late universe where we are at today, could undergo a phase transition to a different vacuum where the particle masses are much heavier than the currently observed ones. Although this would have dramatic consequences, the existence of such a minimum and its properties are highly dependent on the measured values of the Higgs and top quark masses, which are challenging to determine with high precision. Also, this scenario relies on further theoretical assumptions, such as the absence of non-perturbative effects at high energies. Nonetheless, the fact that the present Universe has not yet decayed, places a lower bound on the lifetime of the current vacuum, challenging the plausibility of such a scenario. Despite these challenges, this potential issue remains a topic of intense research, as the lifetime could be dangerously shortened with the consideration of new physics or quantum gravity effects [33].

Puzzles and problems

Aside from the metastability issue, stronger constraints on the theoretical formulation of the SM are currently unknown. In the absence of such, extensions are being guided by unanswered *puzzles*. These long-standing puzzles have been mostly inspired by the principles of naturalness and minimality. While these do not rely on theoretical inconsistencies, they serve as guiding principles and *may* or *may not* be resolved when incorporated into a/the fundamental theory of nature.

Many and very different flavours

The SM is formulated as a QFT with nineteen free parameters, independently fixed by experiments. Nine of these correspond to the masses of fermions, quarks and leptons, which span over several orders of magnitude. Curiously, the top quark and the electron have the respective pole masses of 173 [34, 35] and $5 \cdot 10^{-4}$ GeV. The vast difference in magnitude between these naturally related parameters is known as the *flavour puzzle*, for which the SM offers no explanation. New physics extensions may introduce hierarchy generation mechanisms or unify the couplings under

a common origin, ultimately shedding light on their true nature.

Higgs mass and the electroweak scale

Another historical puzzle again concerns the Higgs boson. Its potential can be parametrised by two free parameters, which can be determined from the Higgs mass and the its self-interacting strength. Notably, the former encodes the sole parameter in the SM with a mass or energy dimension, namely the EW vacuum $v \approx 246$ GeV. Dimensionful quantities like this exhibit the theoretical peculiarity of being sensitive to other scales present in the theory. Consequently, a question arises regarding the magnitude of the EW scale when compared to the next expected scale, the Planck scale, which is many orders of magnitude larger. Such a large separation between scales renders the theory *unnatural*, motivating the existence of new physics at scales around the TeV regime. This (quadratic) sensitivity of the EW scale to higher unknown scales is known as the *hierarchy problem* and is constructed based on the principle of naturalness.

Strong CP problem

On a similar footing, the strong sector of the SM contains the theory of Quantum Chromodynamics (QCD) and allows a charge-parity (CP) violating term usually called the theta vacuum θ_{sCP} . Currently, experimental measurements of the neutron dipole moment constrain the magnitude of this parameter to $\theta_{\text{sCP}} < 10^{-10}$ [36–38]. Its small magnitude compared to the rest of the fundamental parameters in the SM hints towards the potential existence of a mechanism that provides a dynamical generation of such a small value. A common example proposal considers the existence of an *axion* particle [39].

Minimality and unification

Last, the electromagnetic, weak and strong forces could arise from a common origin allowing for a unified treatment at a more fundamental level [40]. In this case, the SM would be described in a more minimalistic manner given that the three fundamental parameters associated to the strength of the forces would be unified to a single one.

A modern view with old methods

All in all, the quest for a fundamental theory of nature and the resolution of the open experimental problems is a hard task with a large number of possible directions which currently are being intensively investigated. In the absence of inconsistencies of the SM, which could serve as guiding principles, theoretical coherence of different BSM extensions stands as a very powerful way to constrain possible model building candidates. In this thesis, we contribute to this task

by assessing the SM and extensions of it from a rather new perspective. Aiming for a better theoretical understanding of the model and to improve the development of SM extensions, we make use of *modern* and advanced QFT techniques whose power has not yet been exploited in this realm.

Quantum nature and renormalisation

Accounting for the quantum nature of a theory consists of resolving the quantum corrections in correlation functions. This crucial exercise has the name of renormalisation and it is necessary for understanding the fundamental properties of a theory. In fact, these quantum corrections connect physics at different energy scales and play an important role in the formulation of observable predictions in QFTs. The high energy character of the SM appears to be particularly simple given that the interaction strength of the fundamental forces is weak and hence the treatment of such quantum corrections can be done in a simplified manner. For example, perturbative expansions in weak coupling succeed in this task and provide very precise predictions in collider experiments.

Beyond perturbation theory

On the other hand, modern renormalisation methods based on the effective action formalism allow for a non-perturbative and more general treatment of quantum corrections. These are commonly used and developed in the areas of condensed matter and nuclear physics where perturbation theory is not successful in the description of relevant phenomena. This includes, for instance, the study of spontaneous symmetry breaking (SSB), or strong dynamics like the one present in colour confinement and dynamical chiral symmetry breaking (d χ SB). These powerful methods tend to offer a more complete picture and are particularly useful for the study of the phase structure of theories.

Functional renormalisation

A specially interesting approach is the one provided by the functional Renormalisation Group (fRG) [41, 42] whose application in several areas has confirmed its power, see [43] for a modern review. The main advantage of this method, aside from being non-perturbative, is that it allows to resolve the quantum corrections in a progressive manner, facilitating the task of renormalisation. This virtuous feature, consequence of its Wilsonian nature, distinguishes its applicability from other non-perturbative approaches such as discrete lattice simulations or Dyson-Schwinger equations (DSE). Moreover, it shows clear benefits when employed in large systems with multiple dynamical scales and thresholds such as in high energy physics. In short, the fRG is a quantitative and versatile method which allows to investigate dynamical phenomena in detail and also easily chart theory space.

Applying advanced methods to complex systems

These aspects make the approach very attractive for the assessment of the aforementioned high-energy questions and in this thesis we aim to make progress in this novel direction by exemplifying the methodology in various applications. As a straightforward consequence, perturbativity and other accompanying assumptions are lifted as an imposition on natural models and extensions. This opens two distinct avenues: first, the assessment of the SM in a novel way with a manifestly non-perturbative approach, and second, the emergence of a vast plethora of theory space to be explored as potential solutions to the present puzzles and problems.

The Standard Model class of theories

As the SM is a weakly interacting theory at high-energy scales, no dynamics or interesting features are to be expected. For this reason, together with the complexity of the model, Wilsonian or RG approaches have been thought not to be useful, or too cumbersome, for their application on high energy problems. Nonetheless, here we will show how its formulation within the fRG allows to investigate features unapparent to perturbation theory, such as SSB in the EW sector. This approach also allows for the systematic and comprehensive assessment of the multi-variable phase diagram of the SM, akin to studies in condensed matter systems, and offers a new perspective from which existing open questions can be reassessed, potentially motivating solutions or unveiling new ones. Furthermore, such a functional implementation provides a theoretical framework for implementing and testing non-perturbative BSM extensions. This non-trivial task is particularly relevant for self-consistently verifying the compatibility of these extensions with low-energy models.

Strong new physics

Many popular extensions of the SM consider non-perturbative physics to address the discussed problems and puzzles. Compelling directions include Composite Higgs [44–48] (CH) and Technicolour [49–52] (TC) models, which replace the Higgs mechanism and boson with phenomena analogous to the emergence of condensates and bound states in QCD. Similarly, strongly interacting dark sectors have been widely proposed to provide solutions to the DM problem. These approaches exploit the rich but challenging dynamics of strongly interacting systems and have been extensively studied despite the limited understanding of strong dynamics.

Charting the landscape

The fundamental building blocks of these ideas are gauge-fermion QFTs, within which also QCD is found, and whose dynamics are often assumed as this important physical theory has been the primary focus of non-perturbative studies. However, this class of theories spans a vast landscape with a rich variety of unexplored dynamics. This landscape is the subject of study

for non-perturbative approaches such as Monte Carlo lattice simulations and DSEs. The fRG approach stands out for this particular task due to its versatility and quantitative power. The methodology developed for confinement and $d\chi$ SB in the realm of QCD can be readily generalized to any theory within this class. This opens the door to a cartographical study of the landscape, unveiling a range of new phases and dynamics where the next extension of nature may be found.

Perturbatively non-renormalisable directions

Other popular BSM extensions have considered the existence of additional space-time dimensions [53–55]. In these extradimensional settings, the SM is a perturbatively non-renormalisable theory, meaning that its non-perturbative treatment is compulsory. To this end, confirming the theoretical coherence and phase structure of these theories is a key task to ensure the viability of these ideas. A similar behaviour is found when gravity is formulated as a relativistic QFT with a dynamical metric. This approach, known as *asymptotically safe quantum gravity* (ASQG) [56–58], is highly interesting, as it offers a QFT-based formalism which allows to test the compatibility of matter theories such as the SM (and beyond) at the most fundamental level of nature.

Non-perturbative new physics with functional methods

Assessing the standing problems and puzzles of nature using modern advanced tools provides a deeper understanding of the SM and helps to develop and constrain the possible extensions that could be realised in nature. In this thesis, we present a comprehensive analysis of the SM using the fRG and investigate diverse non-perturbative scenarios in detail. These include the unification of the SM with ASQG, the landscape of gauge-fermion QFTs, extra-dimensional gauge theories, and the flavour puzzle in CH models.

Outline

This thesis covers a broad range of topics where the power of functional methods has been exemplified in various contexts of high-energy physics.

- ◊ In [Chapter 2](#), we begin by introducing the main concepts of generating functionals, deriving the flow equation, and discussing its properties. In [Section 2.3](#), we provide a pedagogical introduction to field transformations and the generalised flow equation, with a focus on emergent composites and $d\chi$ SB due to their relevance in subsequent analyses.
- ◊ [Chapter 3](#) is devoted to the study of the SM using functional methods, with particular emphasis on features not apparent in perturbative computations.
 - ◊ In [Section 3.1](#) we review the features of the functional SM and contextualise with previous analyses with standard perturbative approaches. Here, we provide a condensed matter view on the class of SM theories and study its properties. The reader familiarised with the fRG may skip this Chapter and directly proceed to the results collected in the subsequent ones.
 - ◊ In [Section 3.2](#), we explore the benefits of this new framework in the context of BSM model building and contextualise in the line of naturalness and predictivity. This Section is based on [7].
 - ◊ In [Section 3.3](#), we uncover the Planck scale to unveil the UV phases of the Higgs potential and discuss their relevance when new physics is considered. This Section is based on [6].
 - ◊ [Section 3.4](#) summarises and contextualises one of the main publications of this thesis [1], where we confront the SM with ASQG to give birth to the asymptotically safe SM. The published version can be found in [Appendix A](#).
- ◊ In [Chapter 4](#), we explore direct applications of the functional approach to BSM scenarios requiring inherently non-perturbative treatments:
 - ◊ [Section 4.1](#) investigates the landscape of gauge-fermion QFTs using the first-principles power of functional methods. This Section is based on [5].
 - ◊ [Section 4.2](#) examines fermion mass generation in CH models, addressing the flavour puzzle. This Section summarises and contextualises [2] which published version can be found in [Appendix B](#).
 - ◊ [Section 4.3](#) shifts focus from the strongly correlated limit to study the phase structure of gauge theories in extra-dimensional settings. This Section summarises and contextualises [4] which published version can be found in [Appendix C](#).
- ◊ Finally, in [Chapter 5](#), we summarise the findings and discuss future directions.

FUNCTIONAL METHODS AND THE FLOW EQUATION

In this Chapter, we introduce the basic concepts of generating functionals and derive the flow equation, which plays a central role in the computations performed in this thesis. Particular attention is given to the properties of the non-perturbative functional Renormalisation Group, which are especially advantageous for the studies conducted in later Chapters. Additionally, we discuss the implementation of scale-dependent field transformations in the generalised flow equation, which is indispensable for describing dynamical phenomena such as chiral symmetry breaking, a mechanism widely present in beyond the Standard Model extensions.

The scientific method [59] requires a theoretical Ansatz, namely a hypothesis, from which physical observables can be derived and tested against experimental evidence. In fundamental physics, nature is formulated in terms of interacting quantum fields that respect certain symmetries, with theoretical predictions encoded in classical actions describing these fields. Expected observables are expressed as correlation functions of fields at specific space-time points.

To bridge the gap between theoretical ideas and actual measurements, the quantum and statistical character of nature plays a crucial role and needs to be accounted for. This process, known as *renormalisation theory*, unveils essential features of the theory, such as its dynamics and phases. In particular, quantum effects drive the variation of correlation functions and fundamental parameters, as encoded in the action, as a function of scale, energy, or length. Only by accounting for these effects can reliable predictions for observables be made. At this level, the initial theoretical Ansatz, represented by the classical action, is upgraded to the full effective action, which encodes all physical correlation functions.

The concept of the *renormalisation group* (RG) [60–64] establishes the connection between the *microscopic* regime (at large momenta or short distances), described by the classical action, and the *macroscopic* regime (at low momenta or large distances), captured by the full effective action, through scale transformations. The Wilsonian idea [63, 64] of RG introduces a scale-dependent effective action by progressively integrating out high-energy degrees of freedom above a certain UV scale. This procedure, which does not require detailed knowledge of the complete theory, offers significant generality over other methods.

This powerful idea has been implemented in the form of renormalisation equations or flows (see [65] and [66] for the Wegner and Polchinski flows, respectively). A particularly convenient construction involves introducing a momentum mode regulator term at the level of the classical action, enabling the smooth integration of momentum shells as originally proposed by Wilson. This approach led to the formulation of the flow equation [42] central to this work.

This Chapter provides a basic introduction to the effective action formalism and the properties of the functional Renormalisation Group (fRG). To this end, we derive the basic generating functionals and the flow equation, following the standard literature on generating functionals [67, 68] and general introductions to the fRG [69–71]. For more detailed discussions on the latter, we refer readers to specialised literature [72].

In the final Section, we place special emphasis on field redefinitions along the RG flow and their applicability in describing the emergence of composites. These powerful techniques, inherent to the fRG framework, will prove particularly useful in the various applications discussed throughout the remainder of this thesis.

2.1 Generating functionals and the effective action

We commence a series of functionals with the definition of the most general one, the *partition function*,

$$\mathcal{Z}[J] \equiv \mathcal{N} \int \mathcal{D}\varphi \exp\left(-S[\varphi] + \int_x J(x)\varphi(x)\right), \quad \text{where} \quad \int_x = \int d^d x. \quad (2.1)$$

Here, $\mathcal{N}$ stands for a normalisation constant of the path integral and $J(x)$ for the sources coupled to the fields $\varphi(x)$. $S[\varphi]$ is the classical action in Euclidean space-time which encodes the symmetries under which $\varphi(x)$ transform. As stated in the introduction, the classical action serves as the theoretical Ansatz imposing certain constraints of the model, such as for example symmetries. In statistical mechanics, the partition function encodes the thermodynamic properties of systems in equilibrium. From this generating functional, field correlation functions can be derived by performing functional derivatives with respect to the sources,

$$\langle \varphi(x_1) \cdots \varphi(x_n) \rangle = \frac{1}{\mathcal{Z}[J]} \left. \frac{\delta^n \mathcal{Z}[J]}{\delta J(x_1) \cdots \delta J(x_n)} \right|_{J=0} \quad (2.2a)$$

$$= \int_{\Lambda_{UV}} \mathcal{D}\varphi \varphi(x_1) \cdots \varphi(x_n) \exp(-S[\varphi]). \quad (2.2b)$$

The correlators appear as a weighted average of n -fields over all possible field configurations. The presence of the cutoff Λ_{UV} ensures the regularisation of the integral measure. Here it is evident how the normalisation constant $\mathcal{N}$ is of no importance given that it drops out from the definition of correlation functions in (2.2a). It is worth noting here that in Euclidean space, the correlation functions are weighted by the exponential of the action while in Minkowski signature with an oscillating phase $-S[\varphi] \rightarrow i S_M[\varphi]$.

The correlation functions generated in (2.2a) are of the most general case and encode all Green functions including those with disconnected parts. While this class of correlations may be relevant for certain purposes, in the present case the disconnected parts carry redundant information. Therefore it is convenient to derive a generating functional which only includes the connected parts of (2.2a). This is provided by the *Schwinger functional*,

$$\mathcal{W}[J] = \ln \mathcal{Z}[J], \quad (2.3)$$

Applying one functional derivative with respect to the source, leads to the definition of the *mean field value*

$$\langle \varphi(x_1) \rangle_c = \frac{\delta \mathcal{W}[J]}{\delta J(x_1)} = \frac{1}{\mathcal{Z}[J]} \frac{\delta \mathcal{Z}[J]}{\delta J(x_1)} \equiv \phi, \quad (2.4)$$

and two functional derivatives,

$$\langle \varphi(x_1) \varphi(x_2) \rangle_c = \frac{\delta^2 \mathcal{W}}{\delta J(x_1) \delta J(x_2)} \equiv G(x_1, x_2) \quad (2.5a)$$

$$= \langle \varphi(x_1) \varphi(x_2) \rangle - \langle \varphi(x_1) \rangle \langle \varphi(x_2) \rangle, \quad (2.5b)$$

to the definition of the quantum *propagator* $G(x_1, x_2)$. In (2.5b) the second term subtracts the disconnected part from the two-field correlator derived from the partition function.

Furthermore, the Schwinger functional still carries redundant information that may not be relevant for the definition of physical correlation functions. One can realise such by separating the connected correlation functions into two classes: one particle irreducible (1PI) and reducible. These are distinguished on whether the n -point function can be separated into two independent by the cutting of one internal propagator at the graph level.

The adequate generating functional for 1PI correlations is then the *quantum effective action*,

$$\Gamma[\phi] = \sup \left[\int_x J(x)\phi(x) - \mathcal{W}[J(x)] \right] = \int_x J_{\text{sup}}(x)\phi(x) - \mathcal{W}[J_{\text{sup}}(x)]. \quad (2.6)$$

which is the Legendre transform of the Schwinger functional (2.3) at the maximum or supremum of the source, namely,

$$\frac{\delta\Gamma[\phi]}{\delta J(x_1)} = \frac{\delta}{\delta J(x_1)} \left[\int_x J_{\text{sup}}(x)\phi(x) - \mathcal{W}[J_{\text{sup}}(x)] \right] = 0. \quad (2.7)$$

We may insert the definition of the Schwinger functional (2.3) into (2.6) reaching the convenient form,

$$\Gamma[\phi] = -\ln \left[\int_{\Lambda} \mathcal{D}\varphi \exp \left\{ -S[\phi + \varphi] + \int_x \frac{\delta\Gamma}{\delta\phi} \varphi \right\} \right], \quad (2.8)$$

which can actually be decomposed into two more intuitive contributions,

$$\Gamma[\phi] = S[\phi] - \ln \left[\int_{\Lambda} \mathcal{D}\varphi \exp \left\{ -S[\phi + \varphi] + S[\phi] + \int_x \frac{\delta\Gamma}{\delta\phi} \varphi \right\} \right]. \quad (2.9)$$

The first term corresponds to the initial theoretical Ansatz, the classical action and the second term accounts for the quantum nature of the theory, namely the quantum fluctuations. Performing a saddle-point expansion around vanishing fluctuation field $\varphi = 0$, we obtain

$$\Gamma[\phi] = S[\phi] + \frac{1}{2} \text{Tr} \left[\ln \left(\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\delta\phi(x_2)} \right) \right] + \dots, \quad (2.10)$$

recovering a perturbative “loop” expansion of the fluctuation terms. Here it is apparent how the effective action acts as the quantum analogue of the classical version evaluated at the *background* ϕ and including the quantum effects via fluctuations.

Now, performing a mean-field derivative of the quantum effective action and applying (2.4)

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x_1)} = - \int_{x'} \frac{\delta\mathcal{W}[J]}{\delta J(x')} \frac{\delta J(x')}{\delta\phi(x_1)} + \int_{x'} \frac{\delta J(x')}{\delta\phi(x_1)} \phi(x') + J(x_1) = J(x_1), \quad (2.11)$$

we reach the quantum version of the classical equations of motion (EoM) for the full effective action. These with (2.8) and the shift invariance of the path integral measure leads to the functional *Dyson-Schwinger equation* (DSE) [73–75],

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x_1)} = \left\langle \frac{\delta S[\phi + \varphi]}{\delta\varphi(x_1)} \right\rangle_J, \quad (2.12)$$

for arbitrary sources. This important master equation provides a closed form as well as access to full correlation functions starting from the classical action. The DSE shares a conceptual similarity with the flow equation and both approaches exhibit shared properties due to their functional nature. Throughout this thesis, we will revisit this expression, particularly in the context of strongly correlated systems, where its application proves indispensable.

At this stage, we have defined the full effective action in (2.8), which encodes all physically relevant information for our purpose. To explicitly recover the moments of $\Gamma[\phi]$ and the fully dressed correlation functions with all quantum corrections, we introduce the full n -point functions,

$$\Gamma^{(\phi_1 \cdots \phi_n)}[\phi](x_1, \dots, x_n) = \frac{\delta^n \Gamma[\phi]}{\delta \phi_1(x_1) \cdots \delta \phi_n(x_n)}, \quad (2.13)$$

where we generalise for the case with several fields. In some cases, we will simplify the superscript notation $((\phi_1 \cdots \phi_n) \rightarrow (n))$, indicating only the number of functional derivatives performed. The n -point functions can then be obtained in momentum space after performing a Fourier transformation.

While n -point functions are not necessarily measurable, they are key ingredients in calculating cross sections and other physical observables. For instance, they may include unphysical dependencies, such as gauge parameters, which cancel out when combined with other n -point functions to produce physical results.

In this Section, we reviewed how to derive physical observables starting from a theoretical Ansatz defined by symmetries, forces, and matter content. The resolution of quantum fluctuations in a non-perturbative manner and accessing the full effective action will be the focus of the next Section.

2.2 The functional renormalisation group

One of the main questions in QFT is resolving quantum fluctuations to uncover the dynamics of theories, predict particle spectra, and determine the scale dependence of fundamental parameters. This allows theoretical predictions to be confronted with experimental results.

Addressing this task within the effective action formalism leads us back to the DSE in (2.12), which encodes all quantum corrections simultaneously. By performing functional derivatives, the DSE generates equations for correlation functions of all orders. Moreover, the DSE for a given n -point function $\Gamma^{(n)}$ depends on higher-order correlation functions, necessitating truncations of the effective action. While this approach requires solving a coupled system of integral equations, it does not rely on expansions in the interaction strength, granting it a non-perturbative character. As we will see, this is essential for accessing strongly coupled regimes or describing phenomena such as phase transitions.

An alternative approach is the Wilsonian idea of renormalisation, which connects micro and macro physics via scale transformations [63, 64, 76], see [67] for a pedagogical introduction. Here, quantum fluctuations are progressively integrated out by reducing an IR cutoff of the path integral measure and integrating out progressively momentum shells.

A further refinement involves the concept of the *effective average action* Γ_k [41, 77, 78], tightly linked to the Wilsonian approach. Conceptually, it can be understood as the effective action for field averages over a volume of size k^{-d} , where k is a specific energy scale and d the spacetime dimension. This generalises the block-spin action [62, 63, 76] to continuous space, opening a range of applications. The spatial cutoff $l \propto k^{-1}$ in the discrete case corresponds to integrating out momentum modes $p^2 \gtrsim k^2$. It is important to note that while the block-spin action averages the physics of the respective scale, the average effective action accounts for all the integrated effects of momenta $k < \Lambda_{UV}$, adding a qualitative upgrade to the conceptual picture. Then, the effective average action interpolates between two important limits:

- ◊ For $k \rightarrow \infty$ or $k \rightarrow \Lambda_{UV}$ where Λ_{UV} is the UV cutoff of the theory, the classical action is recovered as no quantum fluctuations with momenta $p^2 > k^2$ are included: $\Gamma_{k \rightarrow \Lambda_{UV}}[\phi] = S[\phi]$.
- ◊ For $k \rightarrow 0$, all quantum fluctuations are accounted for, yielding the full effective action: $\Gamma_{k \rightarrow 0}[\phi] = \Gamma[\phi]$.

This idea of a sliding effective action can be realised by suppressing the modes of the path integral measure with momenta $p^2 \lesssim k^2$. This is implemented in a simple manner with a regulator functional in the fields which cancels such modes at the level of the classical action, namely

$$\int [\mathcal{D}\varphi]_{p>k} = \int \mathcal{D}\varphi \exp(-\Delta S_k[\varphi]) \quad (2.14)$$

with

$$\Delta S_k[\varphi] = \frac{1}{2} \int_p \varphi(-p) R_k(p) \varphi(p), \quad \text{and} \quad \int_p = \int \frac{d^d p}{(2\pi)^d}. \quad (2.15)$$

This IR regulating term quadratic in the fields has a clear resemblance to a mass term. In fact, this cutoff dependent mass provides a very natural and smooth way to decouple the light modes which advantages we will explore in the following Sections.

In order for the regulator function to correctly implement the suppression of IR modes, it needs to fulfil certain properties. First, it needs to satisfy its purpose and suppress the momentum modes below the cutoff scale k , namely,

$$\lim_{p^2 \rightarrow 0} R_k(p) > 0. \quad (2.16)$$

Needles to say, this condition automatically regularises any possible IR divergent modes. Second, for vanishing cutoff scales, the average action needs to be recovered and the regulator term vanish,

$$\lim_{k \rightarrow 0} R_k(p) = 0. \quad (2.17)$$

Last, in the UV limit,

$$\lim_{k \rightarrow \Lambda \rightarrow \infty} R_k(p) = \infty, \quad (2.18)$$

shows that the path integral measure is dominated by a stationary point and sets the agreement with the initial classical action $\Gamma_{\Lambda_{UV}} = S + \text{const.}$. The UV and IR limits, (2.18) and (2.17), correspond to the classical and full effective actions, respectively. In the IR limit at $k \rightarrow 0$, the regulator dependence vanishes entirely. However, while these asymptotic limits are uniquely defined, the trajectories of Γ_k in theory space depend on the choice of regulator. As a result, physical quantities and correlation functions are strictly obtained only after all quantum fluctuations have been fully integrated out.

It is worth noting that this framework introduces no explicit renormalisation scale, making the RG process purely dependent on momentum shell integration. Moreover, the cutoff scale k can be interpreted as the average momentum scale in loop diagrams. As we will demonstrate throughout this thesis, this feature allows for the extraction of non-trivial physical information even at finite cutoff scales.

While the asymptotic regulator shapes are fixed by the given conditions, the intermediate form at rescaled momenta $x = p^2/k^2 \sim 1$ can be set with relative freedom and used for optimisation of the computations [72]. In particular, different shapes implement various momentum shell integrations which choice can be suitable made depending for the task at hand. In the present thesis, we will mostly concentrate in the qualitative features of the flow equations hence computability and versatility plays an important role. Moreover, analytic regulators can be chosen in such a way that the momentum integration of the loops at vanishing external momenta display an

analytic form after the internal momenta have been integrated. These are known as Litim or *flat* regulators [79] and read

$$R_k^{(b)} = Z_{b,k} p^2 r^{(b)}(x) \quad \text{and} \quad R_k^{(f)} = Z_{f,k} \gamma_\mu p_\mu r^{(f)}(x) \quad (2.19)$$

for bosonic and fermionic fields respectively. $Z_{\varphi,k}$ is the wave-function renormalisation of the field φ and we have introduced the dimensionless shape functions

$$r^{(b)}(x) = (1/x - 1)\Theta(1 - x) \quad \text{and} \quad r^{(f)}(x) = (1/\sqrt{x} - 1)\Theta(1 - x) \quad (2.20)$$

with Θ being the Heaviside function.

With the mode-suppressed path integral measure, we can repeat the steps in Section 2.1 to derive the effective average action starting from the regularised classical action. The scale dependent partition function and Schwinger functional now read

$$\mathcal{Z}_k[J] = \exp(\mathcal{W}_k[J]) = \int \mathcal{D}\varphi \exp\left(-S[\varphi + \phi] - \Delta S_k[\varphi] + \int_x J\varphi\right). \quad (2.21)$$

Here, we can also translate the φ dependence of the regulator term into a derivative with respect to the external source

$$\mathcal{Z}_k[J] = \exp\left\{-\Delta S_k\left[\frac{\delta}{\delta J}\right]\right\} \mathcal{Z}[J]. \quad (2.22)$$

Moreover, with the base functionals defined and employing (2.6) we can define the interpolating average action,

$$\Gamma_k[\phi] = \sup \left[\int_x J \cdot \phi - \mathcal{W}_k[J] \right] - \Delta S_k[\phi]. \quad (2.23)$$

Here, the mean-field is k -independent

$$\phi(x) = \langle \varphi(x) \rangle_J = \frac{\delta \mathcal{W}_k[J]}{\delta J(x)} \quad (2.24)$$

while the sources absorb the k -dependency as the supremum $J_{sup} = J[\phi]$. The quantum EoM in the presence of a regulator term now reads

$$\frac{\delta \Gamma_k[\phi]}{\delta \phi(x)} = J(x) - \frac{\delta \Delta S_k[\phi]}{\delta \phi(x)}. \quad (2.25)$$

from where automatically follows,

$$\frac{\delta J(x_1)}{\delta \phi(x_2)} = \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi(x_1) \delta \phi(x_2)} + R_k(x_1, x_2) \quad \text{and} \quad \frac{\delta \phi(x_1)}{\delta J(x_2)} = \frac{\delta^2 \mathcal{W}_k[J]}{\delta J(x_2) \delta J(x_1)}. \quad (2.26)$$

This last equality introduces the *regularised connected propagator* $G_k(x_1, x_2)$, equivalent to (2.5a) in the presence of a regulator. With all these relations we can rewrite the identity as

$$\delta(x_1, x_2) = \frac{\delta J(x_1)}{\delta J(x_2)} = \int_{x_3} \frac{\delta J(x_1)}{\delta \phi(x_3)} \frac{\delta \phi(x_3)}{\delta J(x_2)} = \int_{x_3} \left(\frac{\delta^2 \Gamma_k[\phi]}{\delta \phi(x_1) \delta \phi(x_3)} + R_k(x_1, x_3) \right) G_k(x_2, x_3). \quad (2.27)$$

which in operator form reads

$$\mathbb{1} = \left(\Gamma_k^{(2)} + R_k \right) G_k, \quad \text{and} \quad G_k = \left[\Gamma_k^{(2)} + R_k \right]^{-1}. \quad (2.28)$$

In the last step, we have employed the definition of the n -point function for the effective average action equivalent to the definition in (2.13). Equation (2.28) relates the full regularised two-point function with the inverse of the regulated connected propagator.

With all important objects redefined in the presence of the regulator, let us discuss the scale dependence of the introduced functionals. To this end, we define the scale derivative

$$\partial_t = k \partial_k, \quad (2.29)$$

which introduces the RG-time,

$$t = \ln(k/\Lambda_{\text{UV}}), \quad (2.30)$$

with the corresponding limits $t(k = \Lambda_{\text{UV}}) = 0$ and $t(k = 0) = -\infty$. Applying this on the scale-dependent Schwinger functional one can show [71]

$$\begin{aligned} \partial_t \mathcal{W}_k[J] &= -\frac{1}{2} \int \mathcal{D}\varphi \partial_t (\Delta S_k[\varphi]) \exp \left\{ -S[\varphi] - \Delta S_k[\varphi] + \int J \cdot \varphi \right\}, \\ &= -\frac{1}{2} \int_p \{ G_k(p) \partial_t R_k(p) \} + \partial_t \Delta S_k[\phi], \end{aligned} \quad (2.31)$$

evaluated at fixed sources. Equivalently, we can derive the scale dependence of the 1PI analogue, namely the flow effective average action for fixed mean-field and evaluated at J_{sup} ,

$$\partial_t \Gamma_k[\phi] = \phi \cdot \partial_t J - \partial_t \mathcal{W}_k[J] \Big|_{\phi} - \partial_t \Delta S_k[\phi], \quad (2.32a)$$

$$= \phi \cdot \partial_t J - \frac{\delta \mathcal{W}_k[J]}{\delta J} \cdot \partial_t J - \partial_t \mathcal{W}_k[J] \Big|_J - \partial_t \Delta S_k[\phi]. \quad (2.32b)$$

In the last step, we have just propagated the scale derivative to evaluate at fixed sources. Using (2.24) and (2.31) we reach,

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int_p \{ G_k(p) \partial_t R_k(p) \} + \partial_t \Delta S_k[\phi] - \partial_t \Delta S_k[\phi], \quad (2.32c)$$

where the scale derivative of the regulator term exactly cancels. Recalling the regularised propagator definition in (2.28) we finally arrive at the compact operator form of the *flow* or *Wetterich* equation [42, 80],

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{STr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right], \quad (2.33a)$$

$$= \frac{1}{2} \text{ (loop diagram with a cross on the top) }. \quad (2.33b)$$

Moreover, in (2.33a) we use the super-trace over discrete and Lorentz groups and momenta integration $\text{STr} = \oint d^d p$. In the second line (2.33b), displays the diagrammatic form of the Wetterich equation, introducing the depiction of full propagators by thick lines and the regulator scale-derivative by a crossed circle.

2.2.1 Truncations and expansion schemes

Integrating the cutoff dependence in the flow equation (2.33a) is equivalent to accounting for quantum corrections and solving the theory. Once this has been done, we have at hand the full effective action from which correlation functions with all the physical information can be obtained simply by functional derivatives.

To reach the full effective action practically, one starts from the classical action, which contains a finite number of operators respecting the symmetries of the theory. As quantum corrections are accounted for and one infinitesimal step in cutoff is done ($k \rightarrow k - \Delta k$), an infinite number of operators are generated. At this point, we find ourselves with an effective average action, which requires accounting for the contribution of an infinite number of operators. However, this is practically challenging and usually unfeasible, forcing us to restrict the number of operators included in the computation to a finite one. This is known as *truncation* of the effective action when respects to the operators considered. While this is an important step common to most functional approaches (e.g., DSE and 2PI) and determines the precision of the computation, there are clear guidelines for constructing efficient, systematic and precise truncations. For example, the canonical dimension of operators is a good indicator of their irrelevance in the theory once quantum fluctuations are included. The more canonically irrelevant (higher dimension) an operator is, the larger the anomalous scaling of fields needs to be for it to enter in the dynamics or affect other operators. Therefore, neglecting the ones with very high-dimension does not lead to sizeable effects in the resolution of the full effective action.

Moreover, within the fRG techniques to include a large number of operators in a feasible manner exist. For example, the inclusion of explicit momentum dependencies in the flow of correlation functions carries, or rather generates, the contribution of initially truncated operators, see e.g. [81, 82] for examples in QCD. In other words, the right-hand side of the flow equation generates operators on the left-hand side not included in the initial Ansatz for the effective average action. An additional approach employs pseudo-spectral methods [83–86] which derive the flow equations for classes of operators, therefore facilitating the inclusion of arbitrary dimension contributions.

Given that a truncation is commonly necessary, choosing an expansion scheme for the effective action is an important step that determines the accuracy of the computation [72, 87, 88]. Different expansion schemes exist, and the choice can be conveniently made for the purpose of the

computation. A common one is the *derivative expansion*, which expands the action in momentum derivatives of the fields [89, 90]. This is usually employed for the analysis of condensed matter systems and the computation of critical exponents, see [91, 92].

Another expansion scheme consists in simply writing the effective average action as a finite summation of n -point functions, following the canonical criterion. In this context, the effective average action for a real scalar field with a non-trivial background $\bar{\phi}$ can be written as

$$\Gamma_k[\phi] = \sum_{n=0}^{N_{\max}^{\phi}} \int_{\mathbf{p}} \Gamma_k^{(\phi_1 \dots \phi_n)}[\bar{\phi}](\mathbf{p}) [\phi_n(p_n) - \bar{\phi}] \dots [\phi_1(p_1) - \bar{\phi}], \quad (2.34)$$

where we employ the n -point function definition in (2.13) and the multi-momentum variable $\mathbf{p} = (p_1, \dots, p_n)$. In this *vertex expansion* scheme, the flows of the n -point functions from $n = 0$ to $N_{\max}^{\phi}$ are derived, leading to a system of coupled differential equations. Hence, we access the effective action by computing its expansion coefficients $\Gamma_k^{(\phi_1 \dots \phi_n)}[\bar{\phi}](\mathbf{p})$ in terms of their fRG flows, which are one-loop exact relations. For the action (2.34), we have,

$$\begin{aligned} \partial_t \Gamma_k &= \frac{1}{2} \text{tr} [G_k \partial_t R_k], \\ \partial_t \Gamma_k^{(1)} &= -\frac{1}{2} \text{tr} \left[\Gamma_k^{(3)} [G_k \partial_t R_k G_k] \right], \\ \partial_t \Gamma_k^{(2)} &= -\frac{1}{2} \text{tr} \left[\left[\Gamma_k^{(4)} - 2 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] [G_k \partial_t R_k G_k] \right], \\ \partial_t \Gamma_k^{(3)} &= -\frac{1}{2} \text{tr} \left[\left[\Gamma_k^{(5)} - 6 \Gamma_k^{(4)} G_k \Gamma_k^{(3)} + 6 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] [G_k \partial_t R_k G_k] \right], \\ \partial_t \Gamma_k^{(4)} &= -\frac{1}{2} \text{tr} \left[\left[\Gamma_k^{(6)} - 8 \Gamma_k^{(5)} G_k - 6 \Gamma_k^{(4)} G_k \Gamma_k^{(4)} + 36 \Gamma_k^{(4)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right. \right. \\ &\quad \left. \left. + 24 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] [G_k \partial_t R_k G_k] \right], \\ &\vdots \end{aligned} \quad (2.35)$$

where the coupled structure of the flow of the n -point functions is apparent. Here it is clear how the flow of the two-point function, the inverse propagator or *Hessian*, is a key object entering the flow of all n -point functions. For its determination, the three- and four-point functions need to be resolved, which require the five- and six-point functions, and so on. In short, to the flow of the n th point function, only up to the $(n + 2)$ -point function directly feeds in and is suppressed by combinatorial factors. With this clear structure and the canonical dimension arguments previously mentioned, a truncation rather insensitive to neglecting higher-order operators can be constructed.

Along this thesis, we will employ this expansion scheme for different reasons. First, it easily allows for systematic improvement by including higher n -point functions, accounting for explicit momentum dependencies, or reading them off. Second, the derivation of the tower of flow equations is easily automatisable by performing functional field derivatives, and codes common

to other functional approaches exist [93, 94]. The vertex expansion scheme has been shown to provide a quantitative prescription even in strongly coupled systems and non-perturbatively renormalisable theories where the canonically irrelevant operators become relevant to the IR dynamics or determining the UV critical surface. Examples bridge over the fields of condensed matter [95, 96], QCD [81, 82, 97, 98] and quantum gravity [99–103].

The flows of correlation functions in this thesis are computed in the symmetric point configuration at vanishing external momentum, $\mathbf{p} = 0$. In this approximation, the angles between all external momenta of the n -point, $\mathbf{p}$, are symmetrically fixed and $p_i \cdot p_j = (n \delta_{ij} - 1)/(n - 1)p^2$, being δ_{ij} the Kronecker delta. In combination with analytic regulators such as the Litim regulator in (2.20), one obtains completely analytic expressions for the n -point function flows. We will see how, in this computationally *efficient* approximation, we obtain quantitative results in agreement with Monte Carlo lattice simulations even for strongly coupled theories like QCD.

2.2.2 Aspects of the flow equation

The flow equation (2.33a) shows numerous useful properties which are worth reviewing one by one and contextualising with other approaches:

- ◊ **Exact:** The derivation we have followed is free from any approximation or assumption, leading to an exact equation for the evolution of the effective average action. This is a powerful feature that provides a solid foundation for the approach. However, exact results are limited to certain theories and in practical computations of physical systems truncations need to be employed.
- ◊ **Non-perturbative:** The derivation is grounded in the effective action formalism and does not rely on a perturbative expansion of correlation functions in interaction terms. Moreover, the flow equations encode infinite orders in the coupling through the full n -point functions, which are captured in their resummed structure. This feature allows the framework to be applied to the study of power-counting non-renormalizable theories, such as quantum gravity, as well as to strongly coupled theories like QCD.
- ◊ **One-loop form and diagrammatic:** The flow equation is presented in a one-loop form, where all non-perturbative effects are encoded in the full n -point functions on which it depends. Moreover, as shown in (2.33a), the flow equation allows for a non-perturbative diagrammatic form, which offers an easy connection to the one-loop contributions. Nonetheless, higher order effects are encoded in the full correlation functions entering as vertices and propagators as well as its resummed form.
- ◊ **Differential form:** In contrast to other functional master equations, such as the DSE

(2.12), the flow equation takes the form of a differential equation. While the DSE includes quantum contributions via momentum integration in one step, the fRG includes them progressively as the k -dependence is integrated out along the flow. This feature considerably simplifies the computational (numerical) treatment and allows the study of intricate theories like the Standard Model, making it suitable for the purposes discussed in this thesis.

- ◇ **UV and IR finiteness:** The introduction of a regulator term provides an IR sliding cutoff, which progressively includes momentum shells. This straightforwardly ensures that all Hessians of the theory are IR finite. On the other hand, the scale derivative $\partial_t R_k(q^2)$ in the flow equation guarantees a UV regularization of all loop diagrams, effectively smearing loop-momenta such that $q^2 \approx k^2$. Therefore no additional UV regularisation is needed.
- ◇ **Wilsonian and mass-dependent scheme:** The fRG is a mass-dependent RG scheme in which threshold effects are automatically taken into account. The presence of an average and sliding momentum scale k allows for a natural interpolation across energy regimes in which different degrees of freedom dominate the dynamics. This facilitates the flow along effective field theories from a fundamental approach. As will be discussed in Chapter 3, this property is particularly useful in the study of the Standard Model, where we can flow through both the symmetric and broken phases of the Higgs sector and progressively integrate out the particle thresholds around the EW scale gaining access to threshold sensitive quantities such as the Weinberg angle, see Subsection 3.1.1.1.
- ◇ **Real-time formulation and complex couplings:** Although we do not exploit this feature in the present work, it is worth highlighting the power of the fRG to access real-time quantities of correlation functions [104] and study complex-valued effective actions [105, 106]. This is possible due to the formulation of the flow equation in the Minkowski space, which avoids sign problems commonly found in lattice formulations.

2.3 Field redefinitions and emergent composites

This Section in parts contains discussions from [5].

The fRG offers many interesting virtues, but one of its most powerful features is the ability to perform well-defined field redefinitions along the RG flow. In physics, phenomena like SSB lead to the emergence of effective degrees of freedom composed of more fundamental ones. For example, in condensed matter systems, superconductivity is described by the formation of bound states (Cooper pairs) of fermions (e.g., electrons). Similarly, in nuclear physics, the experimentally measured spectrum of mesons and baryons can be formulated in a more fundamental manner as multi-quark correlations. Field redefinitions help rearrange the higher-order correlations of the fundamental fields into forms that are more convenient for phenomenological descriptions.

Another application of scale-dependent field redefinitions is seen in simplifying the treatment of the effective action. These redefinitions can be used to reduce the set of operators that are relevant for defining the theory or solving its dynamics [72, 107–109].

To close this Chapter, we provide a pedagogical introduction to field redefinitions for fermionic systems and the methodology to address them in a well-defined manner, following the lines of [69, 110, 111]. We will focus on the first case of applicability due to its power to connect fundamental theories with their low-energy limits and its importance in gauge-fermion quantum field theories, which will be the subject of Chapter 4. The techniques introduced here will later be applied to understand and map the landscape of strongly coupled gauge-fermion QFTs, studying the effects of exotic baryon-like bound states. These are relevant in the BSM context for strongly coupled model building, such as in Composite Higgs models or strongly interacting dark matter.

2.3.1 Bosonisation: the Nambu–Jona-Lasinio and Yukawa models

The Nambu–Jona-Lasinio (NJL) model [112] provides a pedagogical example of *dynamical chiral symmetry breaking* (dχSB) in relativistic self-interacting fermionic theories. The action for the same type of model with N_f Dirac fermion fields is described by the Euclidean classical action

$$S_{4\psi}[\bar{\psi}, \psi] = \int_x \bar{\psi} \gamma_\mu \partial_\mu \psi - \lambda_{\text{SP}}^{(\sigma-\pi)} \left[(\bar{\psi} T_f^0 \psi)^2 - (\bar{\psi} \gamma_5 T_f^a \psi)^2 \right], \quad (2.36)$$

with

$$T_f^0 = \frac{\mathbb{1}}{\sqrt{2N_f}} \quad \text{normalised to} \quad \text{tr} [(T_f^0)^2] = 1/2, \quad (2.37)$$

and T_f^a being the generators of the $\text{SU}(N_f)$ flavour group. This action enjoys a global flavour symmetry which here acts as a chiral one under which the left-handed and right-handed fermions can be independently transformed. For simplicity in (2.36) we have left out of all the possible flavour and Lorentz combinations of four-fermion operators, and only considered part of the

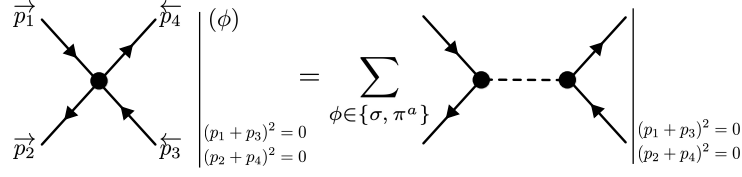


Figure 2.1: Reparametrisation of the four-Fermi resonant channel as the exchange of a scalar field in a particular momentum channel configuration. Figure adapted from [115].

complete scalar-pseudoscalar channel. The considered tensor structure is of particular interest given it accounts for the tensor structures related to the sigma (σ) and pion modes (π^a), dynamical in physical QCD. However, it is important to stress that for a qualitatively complete description of the dynamical appearance of chiral breaking the respective Fierz-complete basis with four independent tensor structures needs to be accounted, see Section 4.1.

On the other hand, we can write down a Yukawa-type of theory with the previously accounted tensor structures

$$S_{\psi\phi}[\psi, \bar{\psi}, \phi] = \int_x \bar{\psi} \gamma_\mu \partial_\mu \psi + \frac{m_\phi^2}{2} \phi^2 + h_\phi \bar{\psi} (T_f^0 \sigma + i \gamma_5 T_f^a \pi^a) \psi, \quad (2.38)$$

where $\phi = (\sigma, \pi^a)$ and we have intentionally not introduced a dispersion term for the scalar fields. So far, both actions, (2.36) and (2.38), describe different theories, one a fermionic self-interacting and the other that of a linear σ -model or quark-meson-like [113, 114]. The latter on the equations of motions (EoM), reads for the scalar fields

$$\sigma_{\text{EoM}} = -\frac{h_\phi}{m_\phi^2} (\bar{\psi} T_f^0 \psi) \quad \text{and,} \quad \pi_{\text{EoM}}^a = -\frac{h_\phi}{m_\phi^2} (\bar{\psi} i \gamma_5 T_f^a \psi), \quad (2.39)$$

which fed back on to (2.38) renders

$$S_{\psi\phi}[\psi, \bar{\psi}, \phi = \phi_{\text{EoM}}] = \int_p \bar{\psi} \gamma_\mu \partial_\mu \psi - \frac{h_\phi^2}{2m_\phi^2} \left[(\bar{\psi} T_f^0 \psi)^2 - (\bar{\psi} \gamma_5 T_f^a \psi)^2 \right], \quad (2.40)$$

We can now clearly see the similarity between both models (2.36) and (2.38) and their identical equivalence when

$$\lambda_{\text{SP}}^{(\sigma-\pi)} = \frac{h_\phi^2}{2m_\phi^2}, \quad (2.41)$$

is satisfied. In fact it is easy to see how both theories are also equivalent at the quantum level

$$\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi \exp \{-S_{\psi\phi}[\bar{\psi}, \psi, \phi = \phi_{\text{EoM}}]\} = \mathcal{N}_\phi \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp \{-S_{4\psi}[\bar{\psi}, \psi]\} \quad (2.42)$$

given that now the $\mathcal{D}\phi$ integral is Gaussian and contributes as a constant $\mathcal{N}_\phi$ that can be absorbed in the fermion measure.

This is an example of a Hubbard-Stratonovich transformation [116, 117] of a purely fermionic theory into its bosonised version, see Figure 2.1. Formally, the derivation departs from the partition function of (2.36) and proceeds by introducing sources for fermion bilinear operators in the desired channels, see [110] for a detailed derivation. There it is apparent how only the specific configuration of couplings (2.41) renders both theories the same.

On a different note, in (2.38) we have intentionally not included the dispersion relation of the scalar field. However, this appears when the four-Fermi interactions are considered to be non-local, $\lambda_{(\text{S-P})}^{(\sigma-\pi)}(p_1, p_2, p_3)$. Then, after the same bosonisation procedure, we reach a dispersion relation for the bosonised channels. We will consider such type of momentum-dependent transformation in Section 4.1 as we explore in detail the landscape of gauge-fermion QFTs.

But how can SSB, or more precisely d χ SB, be diagnosed in these theories? This phenomenon is well understood in the bosonic formulation through the appearance of a minimum in the scalar potential, where the theory no longer exhibits the symmetries of the classical action on the EoM [118, 119]. At the quantum level, this can be understood as quantum corrections reshaping the effective scalar potential [120]. If these corrections lead to the appearance of a non-trivial minimum, they result in a scalar condensate

$$\langle \sigma \rangle \propto \int_x \langle \bar{\psi}(x) \psi(x) \rangle, \quad \text{with} \quad \langle \sigma \rangle \neq 0, \quad (2.43)$$

where the theory no longer displays the symmetry present in the classical action.

In the broken phase, the scalar field is expanded around its solution to the EoM, namely the global minimum of the theory. This expansion in the Yukawa interaction in (2.38), produces a fermionic mass term,

$$m_\psi = \frac{h_\phi \langle \sigma \rangle}{\sqrt{2N_f}}. \quad (2.44)$$

This mass, known as the *constituent fermion mass*, breaks the chiral and global symmetries present in the original classical actions (2.38) and (2.40), upgrading the SSB mechanism to d χ SB. Starting from an action with chiral and global flavor symmetries, the mass term breaks these symmetries down to the diagonal subgroup,

$$\text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V. \quad (2.45)$$

In the classical symmetric theory, there are $2N_f^2$ modes: N_f^2 scalars and N_f^2 pseudoscalars. In the broken phase, only the diagonal subgroup remains, and the pseudoscalars become massless Nambu-Goldstone bosons (NGB). For simplicity, in the actions (2.38) and (2.40), we have written down only half of the scalar-pseudoscalar sector, which includes one radial mode (σ) and $N_f^2 - 1$ pseudoscalar NGBs (π^a). Furthermore, dynamical symmetry breaking implies the presence of a phase transition directly tied to the condensate in (2.43), supporting its role as the order parameter.

Thanks to the bosonized formulation of the NJL-type action, we can interpret SSB within the familiar framework of scalar field theory and recover it in the purely fermionic theory. Describing SSB requires accounting for quantum corrections. Here, we use mean-field theory results [69, 110] for a qualitative understanding. In this approximation, bosonic fluctuations (scalar loops) are neglected, assuming that the bosonic ground state corresponds to the classical bosonic potential. Corrections to the scalar field mass then arise from the fermionic loop,

$$\Gamma^{(\sigma\sigma)} = \text{const.} + \sigma^2 \left(m_\phi^2 - \frac{h_\phi^2 \Lambda_{\text{UV}}^2}{16\pi^2} \right) + \dots, \quad (2.46)$$

where the $\dots$ represent higher-order terms in σ . Here, Λ_{UV} denotes the UV cutoff of the theory, emphasizing the effective (rather than fundamental) nature of the formulation. When

$$m_\phi^2 < \frac{h_\phi^2 \Lambda_{\text{UV}}^2}{16\pi^2}, \quad \text{or equivalently,} \quad \lambda_{\text{SP}}^{(\sigma-\pi)} > \left(\frac{\Lambda_{\text{UV}}^2}{8\pi^2} \right)^{-1}, \quad (2.47)$$

the curvature mass of the σ -mode becomes negative. This induces a non-trivial shape in the scalar potential, resulting in the appearance of a condensate and, thus, SSB.

2.3.2 Dynamical chiral symmetry breaking in gauge-fermion theories

This fascinating phenomenon is indeed present in strongly interacting gauge-fermion theories such as physical QCD. The classical Euclidean action of a gauge-fermion QCD-like theory in the chiral limit reads

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} \gamma_\mu D_\mu \psi + \mathcal{L}_{\text{gauge-fixing}} + \mathcal{L}_{\text{ghost}} \quad (2.48)$$

where $F_{\mu\nu}^a$ is the field strength tensor containing the propagation and interaction of the non-Abelian gauge fields. D_μ is the covariant derivative acting on Dirac spinors charged on a unspecified representation of the gauge group. Remarkably, when quantum corrections are included, the theory shows a single marginally relevant coupling: the gauge coupling

$$\alpha_g = g^2/4\pi, \quad (2.49)$$

with a Gaussian fixed point at $g^* = 0$. This renders the theory perfectly well-behaved in the UV asymptotic limit [121, 122] and renormalisable. Consequently, the setting of the single gauge coupling defines the entire theory, including the scales of the dynamics and spectrum. A detailed derivation will be provided in Section 4.1.

When promoting the classical action to the full effective one in order to account for quantum effects, an infinite number of operators respecting the theory's symmetries are generated by the gauge coupling. Among them are four-Fermi operators analogous to those in the NJL action in (2.36). Importantly, these operators do not introduce new parameters but should be understood

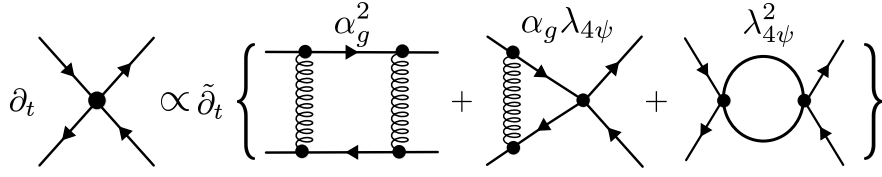


Figure 2.2: Sketch of the four-Fermi vertex flow, omitting the tadpole diagrams. Here, $\lambda_{4\psi}$ represents all possible four-Fermi interactions. The operator $\tilde{\partial}_t$ denotes the scale derivative defined in (2.29), acting only on the regulator dependences of the propagators.

as higher-order effects of the fermion-gauge coupling. As will become apparent later on, this key distinction highlights that while the NJL theory is non-renormalisable, gauge-fermion theories are. If we analyse the RG-flow of the four-Fermi interactions we can see how these are generated by the RG flow,

$$\partial_t \lambda_{4\psi} \propto c_1 \alpha_g^2 + c_2 \alpha_g \lambda_{4\psi} + c_3 \lambda_{4\psi}^2, \quad (2.50)$$

where $\lambda_{4\psi}$ includes all four-Fermi couplings, even those with tensor structures not yet discussed, and c_i are constants encoding volume, flavour, and colour factors from loop diagrams like the ones shown in Figure 2.2. In the deep perturbative regime near the Gaussian UV fixed point, the four-Fermi operators are generated through fermion-gauge box diagrams, with their strength proportional to α_g^2 . Once generated, these operators trigger additional contributions from mixed diagrams ($\alpha_g \lambda_{4\psi}$) and fish diagrams ($\lambda_{4\psi}^2$). We have omitted tadpole diagrams involving six-fermion vertices, as they are of order α_g^8 , strongly suppressed in the perturbative regime, and lack the resonant structure needed for relevance in the chiral symmetry-breaking regime.

In summary, the chiral dynamics is dominated by the diagrams shown in Figure 2.2 and we can discuss the emergence of $d\chi\text{SB}$ in a very concise way. To that end we concentrate on the β -function of the dimensionless scalar-pseudoscalar coupling for the sake of simplicity,

$$\bar{\lambda}_{\text{SP}} = \lambda_{\text{SP}} k^2, \quad \beta_{\bar{\lambda}_{\text{SP}}} \equiv \partial_t \bar{\lambda}_{\text{SP}} = (2 + 2\eta_\psi) \bar{\lambda}_{\text{SP}} + \text{diagrams}, \quad (2.51)$$

where η_ψ stands for the fermion field anomalous dimension and the diagrams are those generated by the flow of the fermion four-point function depicted in Figure 2.2 (divided by k^2). For the ensuing discussion we drop the other couplings for the sake of simplicity, hence $\bar{\lambda}_{4\psi} \rightarrow \bar{\lambda}_{\text{SP}}$. The full analysis is more complicated but similar, see e.g. [111].

All the diagrammatic contributions have negative prefactors and the resulting β -function is depicted in Figure 2.3. For $\alpha_g = 0$, the β -function collapses to that of a NJL type model. Then, it has a Gaussian IR attractive fixed point at $\bar{\lambda}_{\text{SP}} = 0$, and a UV attractive fixed point at $\bar{\lambda}_{\text{SP}}^*$. For initial values of the coupling at $k = \Lambda_{\text{UV}}$ below the UV fixed points, $\bar{\lambda}_{\text{SP}, \Lambda_{\text{UV}}} < \bar{\lambda}_{\text{SP}}^*$, the coupling is driven to zero. Note that this accommodates all finite values of the dimensionful four-Fermi coupling $\bar{\lambda}_{\text{SP}}$. Accordingly, no chiral symmetry breaking takes place. In turn, for initial values

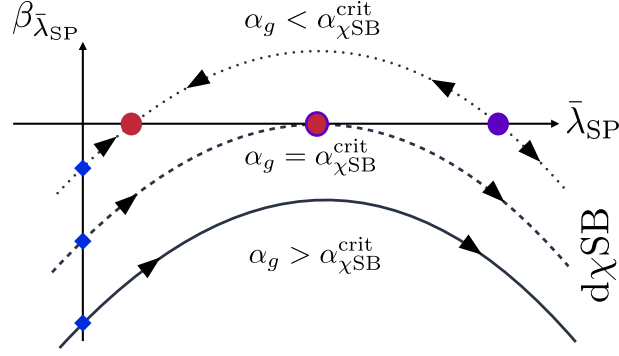


Figure 2.3: β -function of the dimensionless scalar-pseudoscalar four-Fermi coupling $\bar{\lambda}_{\text{SP}} = \lambda_{\text{SP}} k^2$. The arrows indicate the direction of the flow from UV to IR. The red dot marks the IR attractive fixed point and the purple dot, the UV repulsive one. At the critical gauge coupling ($\alpha_{\chi\text{SB}}^{\text{crit}}$), both fixed points merge. For larger gauge couplings, d χ SB occurs, see the discussion around (2.52).

above the UV fixed point, $\bar{\lambda}_{\text{SP}, \Lambda_{\text{UV}}} > \bar{\lambda}_{\text{SP}}^*$, the coupling is driven to infinity which signals the emergence of d χ SB. More details on this regime and full control with the emergent composite fRG approach will be provided in Subsection 4.1.2.

In stark contrast to NJL-type four-Fermi models, the four-Fermi coupling in gauge-fermion systems is generated by the gauge-fermion dynamics as we have discussed above. Consequently, the flows of all four-Fermi couplings start with $\bar{\lambda}_{\text{SP}} \approx 0$ at the blue squares on the y -axis in Figure 2.3. The fermion-gauge box diagrams in Figure 2.2 do not carry a dependence on the four-Fermi couplings and are proportional to α_g^2 . Thus, they simply shift down the NJL-type β -function. The mixed diagrams in Figure 2.2 exhibit a single one-gluon exchange and one four-Fermi coupling. They are proportional to $\alpha_g \bar{\lambda}_{\text{SP}}$ and lead to anomalous dimensions of $\bar{\lambda}_{\text{SP}}$ via $2\bar{\lambda}_{\text{SP}} \rightarrow (2 - \text{const.} \alpha_g) \bar{\lambda}_{\text{SP}}$ and only tilt the β -functions. These tilts are irrelevant for the current qualitative discussion even though they inform the value of the fixed points and also that of the critical coupling already mentioned in Subsection 2.3.1.

This structure leads us to the first important conclusion that d χ SB in a gauge-fermion system can only occur if the gauge coupling exceeds the critical coupling $\alpha_{\chi\text{SB}}^{\text{crit}}$ along the flow. This coupling is defined as the minimal coupling that leads to a negative or vanishing β -function,

$$\alpha_{\chi\text{SB}}^{\text{crit}} = \min \{ \alpha_g \mid \beta_{\bar{\lambda}_{\text{SP}}} \leq 0 \quad \forall \bar{\lambda}_{\text{SP}} \} . \quad (2.52)$$

Only for a completely negative β -function the coupling can crossover to the regime with chiral symmetry breaking. Accordingly,

$$\alpha_{g,k} > \alpha_{\chi\text{SB}}^{\text{crit}} , \quad (2.53)$$

for some k -range is a *necessary* condition for d χ SB. It is not *sufficient* as the analysis has been

done in the absence of any mass scales and confinement generates a gauge-field mass gap which leads to a decay of the box diagrams below the respective mass scale, effectively reducing the β -function to the NJL-type in the deep IR. This makes the general analysis very complicated in the regime where both scales are sufficiently close and the confinement and chiral dynamics are intertwined. Nevertheless, this will be the topic of study in Section 4.1.

To recap, as gauge dynamics strengthen toward the IR, four-Fermi operators grow accordingly, eventually reaching the critical coupling for $d\chi$ SB and then becoming singular (see the upcoming analysis in Figure 4.9). This is a straightforward consequence we already previewed in the mean-field example (2.47): given that the curvature of the effective potential in the bosonised picture vanishes, the respective four-Fermi coupling diverges.

The central question then becomes: how can we access the broken regime, where physical quantities such as the condensate value or the fermion constituent mass arise? This is crucial, as it amounts to solving the theory starting from a classically symmetric Ansatz. Furthermore, this is equivalent to deriving the quantities in the broken regime directly from the most fundamental Ansatz, thereby solving the theory from *first principles*.

To overcome the singular coupling in the four-Fermi RG language and access the broken phase within this formulation, one might consider applying a field transformation into the bosonic formulation at a specific step in the RG flow, as demonstrated in Subsection 2.3.1. For instance, near the scale where the critical coupling is reached, the transformation can be repeated to switch from the interacting fermionic theory to a σ -model-like description. Then, at the scale where bosonisation is performed, $\lambda_{\text{SP}}^{(\sigma-\pi)} = 0$ is set, and all relevant information is encoded in the bosonic h_ϕ and m_ϕ^2 . At this point, the introduced composite field provides a correct description.

However, as we continue the RG flow to the next step a complication appears: the gauge corrections in (2.50) regenerate the four-Fermi coupling, resulting in $\lambda_{\text{SP}}^{(\sigma-\pi)} \neq 0$. We are now left with both the bosonised action and the four-Fermi coupling. Flowing with both formulations simultaneously introduces an evident double-counting issue. Furthermore, the initially defined boson fails to serve its initial purpose as it does not account for the regeneration of the four-Fermi coupling. This issue arises not only in gauge-fermion QFTs but also in NJL-type theories, where the role of gauge interactions is replaced by the bosonised interaction channels themselves.

A viable solution to this problem is to perform the field reparametrisation dynamically at each scale of the RG flow. This approach ensures that gauge corrections are naturally incorporated into the bosonised formulation, enabling access to the broken phase starting from the symmetric one. In the following Section, we will demonstrate how this procedure is seamlessly integrated within the flowing fields formalism using the generalised flow equation.

2.3.3 The generalised flow equation

As previously mentioned, the solution to the singular four-Fermi coupling and the seamless switching between the two basis choices can be achieved by introducing scale-dependent fields and performing field redefinitions at each step in the RG flow [123]. This approach avoids potential double-counting issues in any approximation to the effective action. To illustrate this, let us revisit the pure fermionic action and extend the auxiliary field to a scale-dependent field transformation, $\phi \rightarrow \Phi[\phi]$.

In the presence of such a field, the flow equation at a fixed Φ is written as

$$\partial_t \Gamma_k[\Phi]|_\Phi = \partial_t \Gamma_k[\Phi] - \int \partial_t \Phi \frac{\delta \Gamma_k[\Phi]}{\delta \Phi}, \quad (2.54)$$

where the first term on the right-hand side corresponds to the full flow as given by (2.33a), and the second term characterises how the field transformation is adjusted at each scale. So far, we have not specified the functional $\partial_t \Phi$, which can be chosen to suit the purpose of the field redefinition.

In the context of the previously discussed case in Subsection 2.3.2, we want to choose $\partial_t \Phi$ such that all fermion self-interactions in a particular channel—for instance, the scalar-pseudoscalar channel—are eliminated in $\Gamma_k[\Phi]$, i.e., $\lambda_{\text{SP}} \equiv 0$. With this choice, $\partial_t \lambda_{\text{SP}}|_\Phi = 0$ for all scales due to the balance between the two contributions on the right-hand side of (2.54). As a result, the scale-dependent field Φ successfully accounts for the fermionic self-interactions at all scales.

Having explained the general idea, we omit the full derivation and refer the interested reader to the specialised literature [69, 72, 123]. We now present the flow equation in the presence of a field transformation, also known as the *generalised flow equation* [72],

$$\partial_t \Gamma_k[\Phi]|_\Phi + \int \partial_t \Phi \frac{\delta \Gamma_k[\Phi]}{\delta \Phi} = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2)}[\Phi] + R_k} \left(\partial_t + 2 \frac{\delta(\partial_t \Phi)}{\delta \Phi} \right) R_k \right]. \quad (2.55)$$

The terms on the left-hand side are those of (2.54), while on the right-hand side, we identify the standard form of the flow equation and an additional term accounting for the rotation of the composite field axis in the basis of fundamental fields.

The generalised flow equation has numerous applications in different contexts, ranging from computational simplifications to the investigation of dynamical phenomena. In this thesis, we employ this formalism with the practical goal of describing the emergence of degrees of freedom in a mathematically well-defined manner. Specifically, we will use it in two examples presented in Chapter 4: to chart the landscape of many-flavour gauge-fermion theories in Section 4.1 and to study the flavour puzzle in CH models in Section 4.2.

THE STANDARD MODEL WITH FUNCTIONAL METHODS

In this Chapter, we analyse the Standard Model using the functional Renormalisation Group. Particular attention is paid to the key features of the theory, such as spontaneous symmetry breaking and the phases of the Higgs potential, as well as to the advantages of implementing a Wilsonian approach. We study the quantum phase transition in this class of theories and discuss how the guiding principles of naturalness and predictivity are affected by the inclusion of new physics. Novel ultraviolet phases of the Higgs potential are introduced. In the final Section, we consider the coupling of the Standard Model to asymptotically safe quantum gravity, providing a pathway to UV completeness.

The SM describes to high precision and accuracy the interplay between all kinds of matter and almost all forces so far known to exist in nature. By matter we refer to fermionic fields such as quarks and leptons. On the other hand, the forces in nature are dictated by symmetry principles which set how different sectors communicate with each other. The SM can be conveniently separated into three different sectors based on the local gauge symmetries,

$$SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (3.1)$$

The first gauge group corresponds to the $SU(3)_C$ colour symmetry mediated by gluons. The only matter content charged under this symmetry are quarks and altogether constitute the strong sector of the SM. The remainder, $SU(2)_L \times U(1)_Y$, collects the left-handed chiral gauge symmetry and the hypercharge Abelian gauge group. These two symmetries build up the EW sector and are tightly linked via the remaining Higgs sector.

The Higgs boson plays a central role in the SM as it is responsible for EWSB and the generation of an important part of the mass in the SM. This occurs via the Higgs mechanism [11, 12] which exploits SSB and renders the SM a renormalisable theory [124, 125]. The Higgs condensate or *vacuum expectation value* (v) triggers a mass for the EW gauge bosons which “breaks” the EW group

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}. \quad (3.2)$$

Strictly speaking there is no breaking of the symmetry given that the theory remains gauge

invariant. However, the decoupling of massive modes leads to a low energy theory described by the ungapped gauge boson associated to $U(1)_{\text{EM}}$, namely the photon [112, 126, 127].

Moreover, while the Higgs is charged under the EW sector it is not under the colour group. This means that the strong force remains ungapped by the EW condensate. However, this sector senses the Higgs mechanism via the fermions. Moreover, EWSB is not only responsible for the masses of the EW bosons but also of the fermions. These originate via Yukawa interactions of the type in (2.38) between all fermions (excluding the neutrinos, maybe) and the Higgs scalar. Fermion masses explicitly break the chiral symmetry which, as will be discussed in Section 4.1, has an important impact in the spectrum of bound states in the strong sector.

The success of the Standard Model has been proven by the astonishing agreement between its theoretical predictions and experimental measurements in all of its sectors by independent experiments [128]. This serves as an example of the completeness and predictive power of the model which so far allowed to forecast the missing pieces and lead to the experimental observation of the Higgs boson.

Despite of this, the SM is a LEFT with a limited but unknown range of validity. As we have argued in the introduction of this thesis, there exists experimental evidence for the need of new physics and the model cannot be rendered fundamental due to two theoretical limitations: the Landau poles in the $U(1)_Y$ and scalar sectors and the absence of gravity.

The SM can be found as a particular theory within its class which englobes all possible IR configurations. This has nineteen free parameters which need to be independently fixed by measurements in order to set the physical theory within its class. These correspond to the canonically relevant or marginally relevant couplings which are:

- ◊ Three gauge couplings which need to be set to define the dynamical scales and masses of the different gauge sectors.
- ◊ Nine independent Yukawa couplings (twelve if neutrinos are Dirac fields and their masses are accounted within the SM) are needed in order to reproduce the current fermion masses via the Higgs mechanism.
- ◊ Two parameters describing the small field curvature of the Higgs potential, namely the Higgs' v and its mass.
- ◊ Four parameters, three angles and one complex CP violating phase, from the Cabibbo-Kobayashi-Maskawa [129, 130] (CKM) matrix (with massive Dirac neutrinos in the SM the analogous PMNS-matrix adds another four or five parameters).
- ◊ Last, one parameter responsible for the amount of strong CP violation, namely the QCD vacuum angle θ_{CP} .

In this Chapter we will study the SM at different energy scales employing the fRG approach.

The RG equations of the SM have been widely studied with perturbative methods, see e.g. [30], and the novelty of this analysis is the employment of non-perturbative ones with a mass-dependent character. This novel computational framework allows to diagnose SSB along the RG flow as well as smoothly interpolate between thresholds, connecting the physics of vastly different length scales. Moreover, the Wilsonian character provides a bird’s eye view on the phase diagram of this class of theories allowing to chart the different directions that can be taken when including deformations induced by new physics. The fRG allows to account for the delicate but relevant power-law scaling in the flow of dimensionful quantities. This contribution is key in describing the quantum phase transition within the theory class, as well as for the discussion of naturalness, which we will contextualize with the commonly employed perturbative approach.

Previously, sub-sectors of the SM have been studied employing functional methods, see e.g. [131–135, 135–138] for studies of the Higgs, [85, 139, 140] of the Yukawa and [141–146] of the gauge-Yukawa sectors. In this Chapter, we follow [1, 6, 7] where all sectors of the SM have been implemented self-consistently. As a note, in the discussed computations we have considered a diagonal CKM matrix and hence the associated free parameters have not been included in the flow of the rest of the system, this will be done in future work.

In the following Section 3.1 we commence with the derivation of the SM functional equations as well as discussing the integrated flows and the main features of this formulation. We continue analysing the phase structure of the SM treating it as a condensed matter system with different Higgs phases. In Section 3.2, we employ the present framework to find hints of new physics following the principles of naturalness and predictivity. In Section 3.3, we study the possible appearance of new phases in the Higgs potential when including new physics. Last, in Section 3.4, we confront the SM with the Planck scale, which includes the question of gravity and its compatibility with the known matter and forces. In this Section we study the self-consistent embedding of the SM along asymptotically safe quantum gravity (ASQG) rendering the *asymptotically safe SM* (ASSM).

3.1 The Standard Model flows

This Section is, in parts, based on [1, 6] and contains results to appear in [7].

In this Section we discuss the functional implementation of the SM paying particular attention on the features which are manifestly apparent in this formalism in comparison to others. While results have been obtained with the complete model implemented, here we focus on the derivation of the Higgs sector parameters and precisely on the Higgs mass flow. This parameter is of high relevance given that is the single dimensionful parameter at the classical level and carries information about the small-field curvature of the Higgs potential.

In the present analysis we consider a polynomial-type Higgs potential expanded up to the first marginal coupling,

$$u_k(\bar{\rho}) = V_{\text{eff},k}(\rho) / k^4 = \bar{\mu}_k^2 \bar{\rho} + \bar{\lambda}_k \bar{\rho}^2, \quad (3.3)$$

where

$$\bar{\rho} = Z_{\Phi,k} \frac{\text{tr } \Phi^\dagger \Phi}{k^2}, \quad \text{with} \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ H + i\mathcal{G}_3 \end{pmatrix}, \quad (3.4)$$

being H , $\mathcal{G}_{1,2,3}$ the Higgs and Goldstone real scalar fields, respectively, and $Z_{\Phi,k}$ the wave function renormalisation of the scalar doublet. The scale dependence of dressings and couplings will, from now on, be assumed in this Chapter and we will only invoke it in the subscript when referring to a particular scale.

The effective potential is parametrised with the dimensionless and renormalised Higgs curvature mass $\bar{\mu}^2 = \mu^2 k^{-2}$ and quartic self-coupling $\bar{\lambda}$. It is important to note that the effective potential includes an infinite number of operators and hence the form (3.3) neglects the canonically irrelevant ones. However, more accurate potentials can be considered. For example, in Section 3.4, we expand the UV fixed-point potential to high orders in its polynomial form in order to decrypt its shape when including quantum gravity corrections. Also, in Section 4.1, we will consider higher orders in the mesonic effective potential (similar to the present one) to account for non-perturbative contributions in the form of higher order fermionic operator and resolve the strong dynamics. In the literature non-trivial expansions [85, 134, 136, 139] have also been considered in similar contexts. For the study of the SM phase diagram as a low energy effective theory below a cutoff scale Λ_{UV} , higher dimensional operators are expected not to bring qualitative information given the weak coupling nature of the theory which renders these irrelevant in the IR. However, when addressing the SM as a LEFT below a cutoff scale Λ_{UV} , these parameters can become relevant, significantly complicating the analysis. For these reasons we have neglected them in the present analysis.

Furthermore, the scale dependent effective potential is expanded around its minimum value $\bar{\rho}_0$,

$$\overline{\text{Flow}}[V_{\text{eff}}] \times k^4 = \frac{1}{2} \text{ (dashed circle with cross) } + \frac{1}{2} \text{ (wavy circle with cross) } + \frac{1}{2} \text{ (gluon circle with cross) } - \text{ (dotted circle with cross) } - \frac{1}{2} \sum_{q,l} \text{ (fermion circle with cross) }$$

$H, \mathcal{G}^\pm, \mathcal{G}^0$ $W^\pm, Z^0, A^\gamma$ G^a $c_G^a, c_{W^\pm}, c_{Z^0}$ q, l

Figure 3.1: Diagrammatic flow of the SM Higgs potential defined in (3.9). From left to right, we show the contributions of scalar, EW gauge, gluon, ghost and fermionic loops. The lines denote full regularised propagators including all quantum fluctuations and the cut circles the insertion of the regulator scale derivative.

on which the minimum condition,

$$\partial_{\bar{\rho}} u(\bar{\rho})|_{\bar{\rho}_0} = 0 = \bar{\mu}^2 + 2\bar{\lambda}\bar{\rho}_0, \quad (3.5)$$

is satisfied. As will become apparent when we integrate the flow of the effective potential, the fRG allows to study the shape of $u(\bar{\rho})$ at finite values of the cutoff displaying its apparent shape at each k scale. This permits to trace the flow evolution between different phases of the potential and extract physical information. For cutoff values where $\bar{\mu}^2 > 0$ and $\lambda > 0$ the effective average potential displays a trivial shape and is expanded around $\rho_0 = 0$. In turn, for cutoff scaled where $\bar{\mu}^2 < 0$, the potential displays a non-trivial stable shape with minimum at

$$\bar{\rho}_0 = \bar{v}^2/2 = -\bar{\mu}^2/(2\bar{\lambda}) \geq 0 \quad (3.6)$$

where $\bar{v} = vk^{-1}$ is the dimensionless and renormalised Higgs vacuum expectation value. In this regime, the EW symmetry is broken and the potential is expanded around the non-trivial value. Here it is more convenient to parametrise the potential employing $\bar{\rho}_0$ and $\bar{\lambda}$ without loss of generality. Furthermore, we can define the Euclidean mass of the Higgs boson which is given by the small field curvature of the potential around the minimum

$$m_{\text{H}}^2 = k^2 \left[\partial_{\bar{\rho}} u(\bar{\rho}) + 2\bar{\rho} \partial_{\bar{\rho}}^2 u(\bar{\rho}) \right]_{\bar{\rho}=\bar{\rho}_0} = \begin{cases} \mu^2 & \bar{\rho}_0 = 0 \\ 2\bar{\lambda}v^2 & \bar{\rho}_0 > 0 \end{cases}. \quad (3.7)$$

This quantity is closely related to the pole mass in the physical $k \rightarrow 0$ limit.

The flow of the scalar sector parameters can be extracted by performing field derivatives of the flow of the effective potential,

$$\partial_t (\partial_{\bar{\rho}}^n u(\bar{\rho}))|_{\bar{\rho}_0} = \partial_{\bar{\rho}}^n \left[\partial_t|_{\bar{\rho}} u(\bar{\rho}) - \partial_t|_{\bar{\rho}} \bar{\rho} \cdot \partial_{\bar{\rho}} u(\bar{\rho}) \right]_{\bar{\rho}_0} + \partial_t \bar{\rho}_0 \cdot \partial_{\bar{\rho}}^{n+1} u(\bar{\rho})|_{\bar{\rho}_0}, \quad (3.8)$$

where we have rewritten the scale derivative for a fixed $\bar{\rho}$ and ∂_t is the RG-time derivative defined in (2.30). Now, employing the definition of the dimensionless diagrammatic flow of the potential depicted in Figure 3.1,

$$\overline{\text{Flow}}[V_{\text{eff}}] = \partial_t|_{\rho} V_{\text{eff}}(\rho)/k^4 = \partial_t|_{\rho} u(\bar{\rho}) + 4u(\bar{\rho}), \quad (3.9)$$

and

$$\partial_t|_{\rho} \bar{\rho} = -(2 + \eta_H) \bar{\rho}, \quad (3.10)$$

we reach the general expression,

$$\begin{aligned} \partial_t (\partial_{\bar{\rho}}^n u(\bar{\rho}))|_{\bar{\rho}_0} &= \partial_{\bar{\rho}}^n [\overline{\text{Flow}}[V_{\text{eff}}] - 4u(\bar{\rho})]|_{\bar{\rho}_0} \\ &+ (2 + \eta_H) \partial_{\bar{\rho}}^n [\bar{\rho} \partial_{\bar{\rho}} u(\bar{\rho})]|_{\bar{\rho}_0} + \partial_t \bar{\rho}_0 \cdot \partial_{\bar{\rho}}^{n+1} u(\bar{\rho})|_{\bar{\rho}_0}, \end{aligned} \quad (3.11)$$

where $\eta_H = -\partial_t Z_H / Z_H$ is the anomalous dimension of the Higgs field. For $n = 1$ and $\bar{\rho}_0 = 0$, we obtain the flow of the curvature mass in the symmetric phase,

$$\partial_t \bar{\mu}^2|_{\bar{\rho}_0=0} = \partial_t (\partial_{\bar{\rho}} u(\bar{\rho}))|_{\bar{\rho}_0=0} = (-2 + \eta_H) \bar{\mu}^2 + \partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}]|_{\bar{\rho}_0=0}, \quad (3.12)$$

with

$$\begin{aligned} \partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}]|_{\bar{\rho}_0=0} &= \frac{1}{8\pi^2} \left[-\frac{\bar{\lambda}}{(1 + \bar{\mu}^2)^2} \left(3 - \frac{\eta_{\mathcal{G}}^{\pm}}{6} - \frac{\eta_{\mathcal{G}}^0}{12} - \frac{\eta_H}{4} \right) + 3y_t^2 \left(1 - \frac{\eta_t}{5} \right) \right. \\ &\quad \left. - \frac{18g_1^2}{80} \left(1 - \frac{\eta_{Z^0}}{6} \right) - \frac{18g_2^2}{16} \left(1 - \frac{\eta_{Z^0}}{18} - \frac{\eta_{W^{\pm}}}{9} \right) \right], \end{aligned} \quad (3.13)$$

where we have dropped the non-top fermionic contributions and the η_i 's stand for the anomalous dimensions entering from the cut lines in Figure 3.1. In (3.12), we can distinguish two terms, the first is linear in $\bar{\mu}^2$ and contains the canonical dimension of the mass operator and the anomalous scaling of the field. The second term, is not linear in the curvature mass and leads to a power-law flow quadratic in the cutoff. This term is commonly referred to as the *quadratic divergence* and its relevance deserves a detailed discussion provided in the following Subsections.

Performing an additional $\bar{\rho}$ derivative in (3.11) we obtain the flow equation of the quartic self-coupling,

$$\partial_t \bar{\lambda}|_{\bar{\rho}_0=0} = \partial_t (\partial_{\bar{\rho}}^2 u(\bar{\rho}))|_{\bar{\rho}_0=0} = 2\eta_H \bar{\lambda} + \frac{1}{2} \partial_{\bar{\rho}}^2 \overline{\text{Flow}}[V_{\text{eff}}]|_{\bar{\rho}_0=0}. \quad (3.14)$$

Note that the flows are generally evaluated for arbitrary minima of the potential $\bar{\rho}_0 = \bar{v}^2/2$. While the expressions shown here are in the symmetric phase, in the broken one, additional terms do not vanish leading to further contributions.

The flows can also be derived directly from the scaling n -point functions of the SM effective average action, see [1] for the explicit forms. For example, the flow of the curvature mass can be derived from the Higgs two-point function flow evaluated at vanishing external momenta,

$$\partial_t \Gamma_k^{(HH)}(p^2 = 0) = \partial_t [Z_H (p^2 + \mu^2)]|_{p^2=0}, \quad (3.15)$$

and the quartic coupling from the four-point function

$$\partial_t \Gamma_k^{(HHHH)}(\mathbf{p}) = \partial_t [Z_H^2 \lambda] . \quad (3.16)$$

This derivation from the explicit n -point function flow permits to explicitly account for external momentum dependencies $\mathbf{p}$ in different channels and derive the form factors of the respective vertex dressing. In fact, these momentum dependencies (in particular in real-time) enter the derivation of scattering amplitudes and hence in the obtention of physical observables, see e.g. [147] for an example in this context. Moreover, as reviewed in Subsection 2.2.1, momentum dependencies are also relevant when seeking for a quantitative resolution of the flow equations as generate. Nonetheless, in this work we employ the more hands-on symmetric-point configuration with vanishing external momenta, as for the present purpose both computations yield the same qualitative result.

The flows of the remaining SM parameters, namely the

$$\text{gauge, } \quad \vec{g} = \{g_3, g_2, g_1\} \quad \text{and Yukawa, } \quad \vec{y} = \{y_t, y_b, y_c, y_s, y_u, y_d, y_\tau, y_\mu, y_e\}$$

couplings, have been computed in this manner from different n -point functions. Obtaining the same coupling from different n -point functions also gives access to the *avatars* of the same coupling. For example, we may obtain the gauge coupling from its avatar in the top quark-gluon, strange quark-gluon, three-gluon or ghost-gluon vertices (see (4.9) in Section 4.1 for explicit expressions). While these avatars agree in the asymptotic regime where strong dynamics and thresholds are absent, at low energy scales they are genuinely different. Such avatars sense differently the off-shell decoupling of diagrams and hence change the strength of a S-matrix sensitive to it. Having access to different avatars of the SM couplings allows us to semi-quantitatively obtain the cutoff profile of challenging quantities with phenomenological relevance such as the Weinberg angle, see discussion in Subsection 3.1.1.1.

The large system of flows that poses the SM has been derived in the vertex expansion truncation explained in Subsection 2.2.1. For it we employed software packages which generate the expression for n -point functions of the theory [148, 149] and the respective symbolic expressions for the diagrammatic form of the flow of n -point functions [150]. Computational details can be found in [1].

3.1.1 Sub-Planckian Standard Model

In Figure 3.2, we display the integrated flow trajectories for the SM which reproduce the experimental values measured below the TeV scale. In the top panel, we show the marginal strong (g_3), weak (g_2) and hypercharge ($g_1 = \sqrt{5/3} g_Y$) gauge couplings as well as the dominant top (y_t) and bottom (y_b) Yukawa couplings. In the lower panel, we show the integrated flows for the Higgs quartic coupling ($\bar{\lambda}$) and curvature mass ($\bar{\mu}^2$). The flows span from the Planck scale down

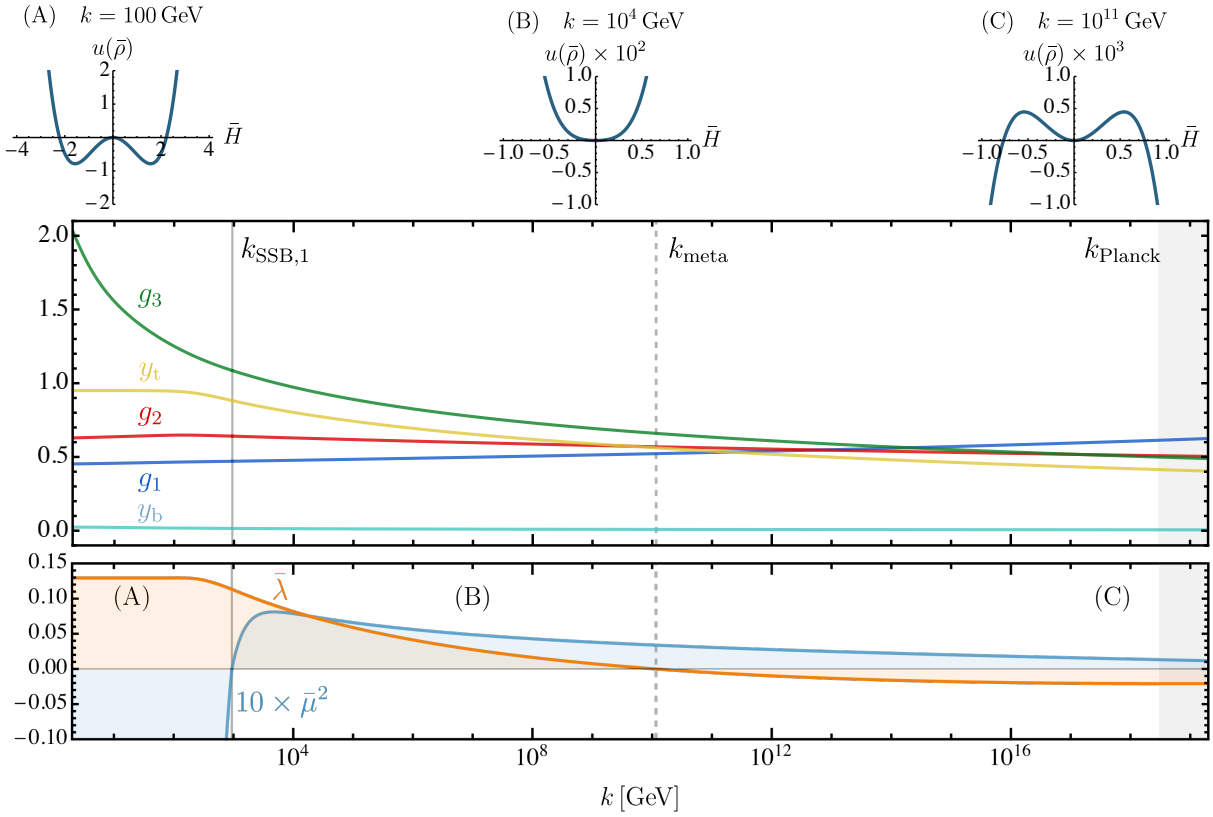


Figure 3.2: Central plot: trajectories of the SM gauge, top and bottom Yukawa couplings. These span from the Planck scale ($k_{\text{Planck}} \approx 10^{19}$ GeV) down to below the d χ SB scale ($k_{\chi\text{SB}} \approx 0.1$ GeV), over the EW ($k_{\text{SSB},1} \approx 10^3$ GeV) scale.

Bottom plot: trajectories of the dimensionless Higgs curvature mass (blue) and the quartic coupling (orange). These two parameters define, in the present truncation, the shape of the average potential and hence the effective phases of the Higgs field. Three different regions are marked by plain ($\bar{\mu}^2 = 0$) and dashed ($\bar{\lambda} = 0$) vertical lines.

Top panel: from left to right, we show the dimensionless effective average potential at $k = \{10^2, 10^4, 10^{11}\}$ GeV as a function of the dimensionless and renormalised Higgs field $\bar{H} = \sqrt{Z_H} H/k$.

to below the deep IR QCD scale ($k_{\chi\text{SB}} \approx 0.1$ GeV), over the EW ($k_{\text{SSB},1} \approx 10^3$ GeV) scale. For details on the truncation employed in the computation and the scale setting procedure followed here we refer to [1].

In Figure 3.2 we can identify three different regimes (see top plots) given by the shape of the effective average potential, namely the effective potential at finite k [77]. While the full effective potential is recovered as $k \rightarrow 0$, the effective average Higgs potential displays the *effective* shape at the respective scale and encodes the quantum fluctuations from Λ_{UV} to k . We extract its shape

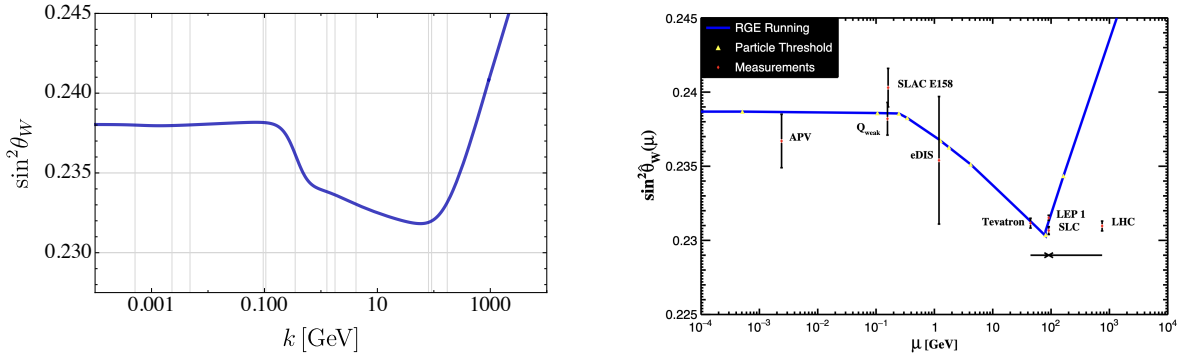


Figure 3.3: Left plot: cutoff profile of the Weinberg angle $\sin^2 \theta_W$ obtained with the fRG flows of g_1 and g_2 shown in Figure 3.2. The vertical lines mark the SM thresholds where different degrees of freedom (gauge and fermion) decouple.

Right plot: cutoff profile of the Weinberg angle in the $\overline{\text{MS}}$ scheme from [128, 151, 152], along with experimental measurements.

from the integrated values of $\bar{\mu}^2$ and $\bar{\lambda}$ at the respective scale. Starting at the Planck scale and flowing down towards the EW scale, the curvature mass is positive meaning that the potential appears stable around trivial values of the field. At $k_{\text{SSB},1} \approx 940$ GeV, the small-field curvature of the potential turns negative leading to the appearance of a non-trivial minimum of the potential. This corresponds to the Higgs' vacuum expectation value whose dimensionful form freezes at $v_{k \lesssim 200 \text{ GeV}} = 246$ GeV. This non-trivial vacuum generates the masses of the EW gauge bosons and fermions via the Higgs mechanism [11, 12, 126]. For the cutoff profile of all the SM Euclidean masses see [1]. Moreover, the potential evolves from a stable (see potential (B) in Figure 3.2) to a non-trivially stable (see (A)) shape. As will be discussed in Subsection 3.1.1.3, the cutoff mimics, in fairly good agreement, the finite temperature effects. Thus a physical interpretation of the present flows may be extracted at a qualitative level.

As the sliding cutoff scale decreases below $k < k_{\text{SSB},1}$, we find a continuous and progressive flattening of the couplings given by the smooth decoupling of degrees of freedom as we flow past the respective thresholds in the theory. For example, the top Yukawa coupling y_t freezes at $k \sim 165$ GeV which corresponds to the top quark Euclidean mass in the fRG scheme [1] and matches the physical pole mass [128].

3.1.1.1 Exploiting the mass-dependent nature: the Weinberg angle

The ability to smoothly flow across thresholds is intrinsic to mass-dependent RG schemes and proves convenient for deriving physical observables that are highly sensitive to off-shell decouplings. In the SM, an interesting quantity of this kind is the *weak mixing* or *Weinberg angle* [8], defined

as

$$\sin^2 \theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2}. \quad (3.17)$$

This angle quantifies the mixing between the chiral and Abelian gauge groups and is of particular interest as it is associated to the EW precision observables, offering significant constraining power to new physics scenarios, see e.g. [153–155]. The Weinberg angle is measured (or proposed) in a range of experiments across various energy scales, from the Z^0 pole mass at high-energy colliders [156–161] to the few MeV range and below [162–166].

In Figure 3.3, the cutoff dependence of the Weinberg angle computed via the fRG flows (left panel) and in the $\overline{\text{MS}}$ scheme [128, 151, 152, 167] (right panel) are shown. The mass thresholds are depicted as vertical grey lines on the left and yellow triangles on the right, with experimental values displayed in the right-hand plot. The fRG accounts for the smooth decoupling of the respective degrees of freedom along their thresholds through the $\propto (1 + \bar{m}_{i,k}^2)^{-1}$ factors in the flows. This allows to naturally and correctly reproduce the near-threshold effects as well as flowing over them. In contrast, the perturbative approach does not have a mass-dependent nature which is reflected in threshold functions with a limited validity range in scales. Moreover, here degrees of freedom are decoupled manually and the boundary RG expressions must be matched at the thresholds.

The precise momentum dependence of the Weinberg angle is crucial for bridging high-precision measurements at LHC energies with upcoming low-energy experiments, as reviewed in [154]. The increasing number of experimental proposals highlights the need for a quantitative theoretical computation. While the electroweak sector remains perturbative at low energy scales, several theoretical challenges arise which are effectively addressed in the fRG approach. First, it naturally and quantitatively incorporates thresholds, and second, it seamlessly accounts for the scale of strong dynamics. The left plot in Figure 3.3 demonstrates these contributions, including effects from strong-sector dynamics such as those arising from constituent fermion masses [81, 168]. Here, the up-quark avatars of the electroweak couplings in (3.17) were employed. Although the results are preliminary, they showcase the potential of functional methods in this novel application.

Moreover, the equivalent object can be derived accounting for explicit momentum dependencies within a real-time formalism, enabling direct comparisons to experiments. In conclusion, this functional framework incorporates the necessary features to quantitatively predict the low-energy Weinberg angle and allows to investigate the effects of BSM extensions, in particular the non-perturbative ones, on this parameter in a versatile manner.

3.1.1.2 Higgs metastability

Returning to the cutoff profile of the SM parameters in Figure 3.2, as scales grow $k \gg m_i$, the progressively ungapped fermionic corrections cause the curvature mass to turn positive at

$k_{\text{SSB},1}$. This leads to a trivially stable potential with a global minimum at $H = 0$. In this regime, $\bar{\rho}_0 = 0$, and the fermion and EW gauge boson Euclidean masses exactly vanish, leaving the Higgs curvature mass (3.7) as the only threshold in the flows. With $\bar{\mu}^2 \approx 0$ in this regime, the flow of the marginal couplings recovers its universal analytic form, as evidenced by the profile of the respective couplings, which aligns well with multi-loop perturbative computations [30, 169].

While $\bar{\mu}^2$ remains positive, the anti-screening fermionic corrections, dominated by the top Yukawa coupling, induce a zero-crossing in the scalar quartic coupling at $k_{\text{meta}} \approx 1.2 \cdot 10^{10} \text{ GeV}$. This well-known feature leads to an effective potential with a metastable shape (see potential (C) in Figure 3.2) up to cutoff values above the Planck scale. Moreover, to resolve the physical full effective Higgs potential, the full field dependence of the expansion coefficients in (3.3) is needed [170]. A common simple approximation involves RG-improving the field-dependent potential using the cutoff profile [30, 171], namely,

$$V_{\text{eff},k \rightarrow 0}(H) = \frac{\mu_{k \rightarrow 0}^2(H)}{2} H^2 + \frac{\bar{\lambda}_{k \rightarrow 0}(H)}{4} H^4 \approx \frac{\mu_{k \sim H}^2}{2} H^2 + \frac{\bar{\lambda}_{k \sim H}}{4} H^4. \quad (3.18)$$

The cutoff-improvement of the potential, or namely of the couplings, can be performed in different RG schemes. While the quartic coupling flow has a universal two-loop form the mass term is subject to the employed scheme already at one loop. This means that different schemes allow to decrypt different features along the RG flow and connect differently to physical quantities (e.g. pole mass). In the $\overline{\text{MS}}$ scheme with dimensional regularisation, the Higgs mass parameter runs logarithmically, yielding $\mu_{\overline{\text{MS}}}^2 \approx (-88 \text{ GeV})^2$ for all RG scales. This results in an RG-improved potential $V_{\text{eff}}^{\overline{\text{MS}}}(H) \approx \frac{\bar{\lambda}_{k \sim H}}{4} H^4$, neglecting the mass term for large field values. In contrast, the fRG flow of μ^2 changes significantly, even flipping its sign and giving rise to SSB (see Figure 3.2 and Subsection 3.1.1.3). The RG-improved potentials using both schemes are shown in Figure 3.4. The inclusion of the curvature mass flow slightly modifies the potential shape at field values where the curvature change takes place.

Independently of the mass treatment we observe that the zero-crossing in $\bar{\lambda}$ implies that the EW vacuum (solid blue line) is not the global minimum. Consequently, the present visible Universe residing in the EW minimum could tunnel to the global one, potentially leading to catastrophic consequences after a late-time phase transition. The key observable for this matter, is the lifetime of the EW minimum (solid blue line in Figure 3.4) over the Universe's evolution, which has been computed within the SM and with BSM extensions [27, 30–32, 171–179]. This quantity mostly depends on the position of the global minimum and the magnitude of the barrier which are given by the k -profile of $\bar{\lambda}$ at large energy scales. Furthermore, in the SM, the quartic coupling is particularly sensitive to the unsettled top quark pole mass, and secondarily, to the scale setting of the strong coupling [180]. Furthermore, the current experimental precision does not fully rule out the stable trajectory. In other words, the weaker the top Yukawa (lighter mass) becomes at high scales, the more the potential stabilises (see also discussion in Section 3.3 and Figure 3.21 for details).

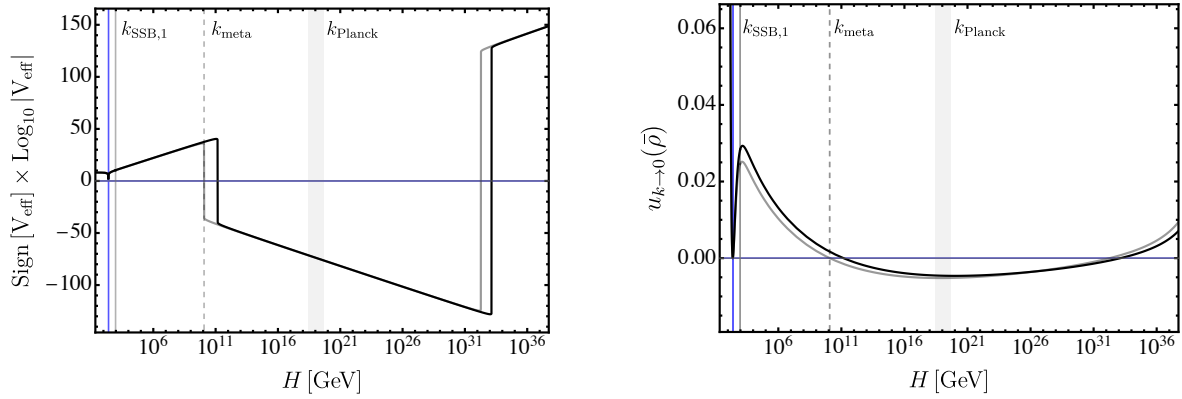


Figure 3.4: Left plot: RG-improved Higgs effective potential (see (3.18)) evaluated on the trajectories derived with the fRG (black line) and $\overline{\text{MS}}$ schemes (grey line). The solid blue line marks the position of the metastable EW vacuum and the vertical grey lines correspond to the SSB (plain), metastability (dashed) and Planck (shaded region) scales, respectively.

Right plot: the analogue Figure but for the dimensionless version of the RG-improved Higgs effective potential, $u_{k \rightarrow 0}(\bar{\rho}) = V_{\text{eff},k \rightarrow 0}(\bar{\rho})/k^4$.

Additionally, quantum gravity and BSM extensions are required to render an asymptotically stable potential ($\bar{\lambda}_{k \rightarrow \infty} \geq 0^+$) as the SM is embedded in a fundamental picture. Otherwise, the life-time of the EW vacuum would be shorter than the age of the Universe [33, 177]. This indeed has served as a guiding principle for BSM constructions in the search of new physics, see e.g. [169, 181].

However, fully diagnosing the metastability issue quantitatively is a challenging task and requires incorporating the field dependence of the potential along a robust ansatz. In perturbative computations and the current approximation, a polynomial potential (see (3.3)) is used in the lowest order which has a limited access to the global field dependence. In fact, non-perturbative studies have shown that stable global potentials reproducing the collider measurements of the Higgs sector [132, 135] can be constructed. This supports further quantitative determinations of the full field dependence with methodologies beyond the lowest order polynomial expansions targeting a conclusive answer.

3.1.1.3 Physical information in the effective average potential

We conclude this Section with a discussion on the physical information encoded in the effective average potential at finite k . In Figure 3.2, the effective average potential evolves across different phases as the curvature mass changes. The zero-crossing in the small-field curvature, $\bar{\mu}^2$, is only possible due to the power-law divergence in the curvature mass flow, see (3.12). Strictly speaking,

this evolution does not represent a physical phase transition since k is not a physical parameter. In the theory of critical phenomena, such transitions occur when a physical parameter such as the temperature of a system is varied and can be quantified via the dependence of an order parameter such as the Higgs vacuum expectation value. A notable feature of the fRG is its ability to encode finite-temperature information within a vacuum computation, such as this one.

As introduced in Section 2.2, the fRG regulator quadratic in the fields acts as a mass-like IR sliding cutoff, progressively incorporating quantum corrections. This regularisation procedure is analogous to the effect of temperature on thermal fluctuations. More precisely, while the RG steps progressively account for quantum fluctuations from UV to IR, thermal corrections have the analogue effect as temperature decreases. Furthermore, the arrangement of off-shell contributions along the RG flow mirrors their behaviour under varying physical temperature. In essence, as temperature is increased thermal corrections *undo* the quantum effects observed at the cutoff level.

In the thermal case, the integration of IR thermal modes is performed by the lowest non-zero Matsubara mode, $\omega_1 = 2\pi T$ for bosons and $\nu_1 = \pi T$ for fermions. In the SM Higgs potential, the leading quantum and thermal corrections stem from fermions, scalars, and EW bosons, with the top quark being the dominant contributor. However, since the phase transition is tied to the scalar potential, it is reasonable to approximate the dominant Matsubara mode as that of a scalar. Using this identification, the relevant mode maps to the zero-temperature cutoff as $k \approx 2\pi T$. This approximately yields a critical temperature for the EW phase transition of

$$T_{\text{SSB},1}^{\text{crit}} \approx \frac{k_{\text{SSB},1}}{2\pi} \approx 149.6 \text{ GeV}. \quad (3.19)$$

This rough estimate is surprisingly within 6% agreement with lattice computations of non-Abelian-Higgs theories which find $T^{\text{crit}} = 159.6 \pm 0.1 \pm 1.5 \text{ GeV}$ [182]. The identification here discussed is not new but well-known from various finite-temperature fRG computations, such as [183–185], which discusses the thermal fluctuations’ impact on the effective average action, and [134, 186, 187], which provide examples. However, while the fRG cutoff qualitatively resembles temperature, this analogy should not be interpreted quantitatively. Determining the precise order of the phase transition and the exact critical temperature requires explicit thermal contributions.

A similar case to EWSB is given by the quark-meson model in QCD, which successfully describes the dynamical emergence of a chiral condensate (analogous to the Higgs vacuum expectation value) via $d\chi_{\text{SB}}$. For more details, see Subsections 2.3.1, 2.3.2 and 4.1.2. This model provides accurate predictions below the confinement scale where gluon dynamics have decoupled due to the appearance of a mass gap. fRG studies in the chiral limit show that the vacuum cutoff transition $k_{\chi_{\text{SB}}} \approx 400 \text{ MeV}$ [98, 114, 188] leads to the appearance of a condensate at $T_{\chi_{\text{SB}}}^{\text{crit}} \approx k_{\chi_{\text{SB}}}/\pi \approx 140 \text{ MeV}$ [97, 98]. In this case, the identification aligns with the fermionic zero mode, supported by the inherent fermionic nature of the non-bosonised theory (see Subsection 2.3.1).

3.1.2 Phase diagram of the Standard Model class of theories

After having briefly introduced the technical and conceptual features of the SM fRG flows, here we extend to the whole class of theories where the SM can be found as a trajectory in theory space. The present computational framework allows to investigate the phase structure of gauge-Yukawa-gauge QFTs, a class in which the SM is found to good approximation. The first gauge stands for the EW sector, Yukawa for the top-Higgs part and the last gauge, for the QCD sector. All three pieces are necessary to describe the qualitative features of the theory and to properly reproduce the well-known zero-crossing of $\bar{\lambda}$ discussed in Subsection 3.1.1.2. In the present Section, we introduce some terminology and discuss structural properties of the flow such as fixed points and separatrices providing a comprehensive picture of the class.

3.1.2.1 Phase structure of the Higgs sector

To understand how the flow in theory space is arranged, we first need to introduce the basic structures that can appear. This concerns fixed-point solutions and related concepts, which are key in any RG analysis. Here, we follow standard literature [67, 68, 71, 189], and in the following, we will apply these concepts to the SM phase structure.

Fixed-point solutions in a theory are the roots of the full system of RG equations for all couplings in the theory. This encompasses all possible couplings associated with any operator allowed by the symmetries of the full effective action (including anomalous ones). Specifically, they are the roots of the full set of flow equations:

$$\partial_t g_i|_{g_i=g_i^*} = 0, \quad (3.20)$$

where g_i represents the dimensionless renormalised couplings of the theory, and g_i^* are their respective values at the fixed point. Precisely here, scale invariance is realised at a quantum level [170, 190].

While the existence of IR fixed-point solutions is necessary for IR finiteness and closeness of a theory, UV fixed points are required only for fundamental theories with a UV-complete description. For instance, in the SM, an IR fixed point is reached at $k \rightarrow 0$ as all massive degrees of freedom decouple below their respective thresholds (including gluons). The remaining sector is the IR finite electromagnetic interaction. Conversely, no UV fixed point is known to exist for the SM. Instead, Landau poles are present in the Abelian and scalar sectors.

Although not a fundamental theory, the SM retains predictive power as it can be formulated as a LEFT of a more fundamental framework. Its validity extends to scales below an unknown Λ_{UV} , beyond which the more fundamental description becomes essential. This approach is feasible in scenarios where degrees of freedom are integrated out at certain energy scales, ceasing to influence the evolution of parameters except as boundary conditions at the UV scale. The unknown UV

scale, Λ_{UV} , could correspond to the mass of a heavy scalar particle M_S coupled to the Higgs via a portal coupling λ_{HS} , or the condensation scale below which the Higgs emerges as a composite state of a more fundamental force. For energy scales $k > \Lambda_{UV}$, the properties of the heavy scalar or the underlying fundamental theory must be accounted for to properly describe physical observables sensitive to these scales. Furthermore, as discussed in the introduction, UV fixed points and their associated completeness might be realised through the inclusion of new physics and/or quantum gravity.

While the magnitude of a fixed point is not a universal quantity due to its dependence on the RG scheme, the RG flow in its neighbourhood is. This is encoded in the *stability matrix*,

$$\mathcal{B}_{ij} = \frac{\delta(\partial_t g_i)}{\delta g_j}, \quad (3.21)$$

which allows the derivation of the normalised *eigenperturbations* $\vec{e}_i$ with $\vec{e}_i^2 = 1$ such that

$$\mathcal{B}_{ij} \cdot \vec{e}_i = \Theta_i \vec{e}_i. \quad (3.22)$$

Here, Θ_i are the eigenvalues, or *critical exponents*. Around the fixed point, the theory can be described by linear perturbations,

$$\partial_t (g_i^* + \delta g_i) = \partial_t g_i^* + \mathcal{B}_{ij} \delta g_j + \dots \quad (3.23)$$

where the first term vanishes by definition. A perturbation in δg_i can thus be written as a combination of eigenperturbations, whose scaling is determined by Θ_i [67, 68, 71]. The magnitude and sign of the critical exponents define the number of operators needed to specify predictions. This is known as the dimension of the theory's *critical surface*. Moreover, depending on the critical exponent the couplings are

- (i) $\text{Re}[\Theta_i] < 0$: a relevant direction of the critical surface, sourcing UV-attractive or IR-repulsive trajectories. The respective coupling grows towards the IR.
- (ii) $\text{Re}[\Theta_i] > 0$: an irrelevant direction orthogonal to the critical surface, leading to UV-repulsive or IR-attractive trajectories. The respective coupling shrinks towards the IR.
- (iii) $\text{Re}[\Theta_i] \simeq 0$: a marginal direction of the critical surface. This case requires the inclusion of higher-dimensional operators to determine whether this operator defines a marginally relevant or irrelevant dimension of the critical surface. An example of this kind is found in pure non-Abelian gauge theories, where a single marginally relevant direction defines the critical surface and the scales of the theory.

A renormalisable theory, not necessarily perturbatively, has a finite number of relevant directions and hence a finite-dimensional critical surface [66, 76]. Importantly, since critical exponents vary from one fixed point to another, the relevance of parameters change and mix along the RG flow.

In the perturbative limit of the class of theories under investigation, most couplings (gauge and Yukawa) exhibit marginal scaling. However, the Higgs mass is the only canonically relevant

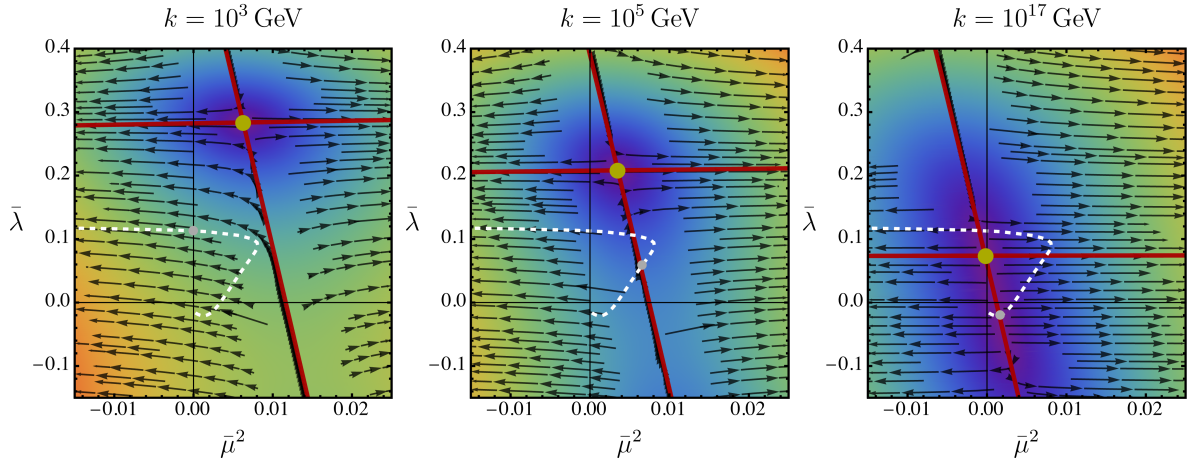


Figure 3.5: Phase diagram of the SM class of theories projected on the plane of scalar sector parameters $(\bar{\mu}^2 - \bar{\lambda})$. The black arrowed lines indicate the flow in theory space from UV to IR. The yellow dot and red lines correspond to the partial fixed-point solutions and the separatrix as defined in (3.24), (3.25) and (3.26), respectively. From left to right, we display slices of theory space at $k = 10^3$, 10^5 and 10^{17} GeV. The grey point indicates the position of the SM trajectory at the respective scales and the white dashed line, the full trajectory in the multi-dimensional volume projected on the plane of couplings.

parameter, with the sizeable $\text{Re}[\Theta_{\bar{\mu}^2}] \approx -2$ at the classical level. This negative critical exponent of large magnitude indicates that this operator significantly impacts the definition of the theory’s critical surface, or in other words, its definition in the UV predominantly determines IR.

The SM is a complicated multidimensional QFT with a priori nineteen relevant couplings. However, one may simplify the RG analysis by projecting onto the sub-dimensional theory space of the parameters of interest and compactifying, in a time-like manner, the remaining directions. This representation permits the analysis of complicated theory spaces in a simplified manner and helps identify important structures such as fixed points.

Following this idea, in Figure 3.5, we depict the SM phase diagram projected onto the $(\bar{\mu}^2 - \bar{\lambda})$ -plane of Higgs parameters at different RG-times or energy scales: $k = 10^3$, 10^5 , and 10^{17} GeV, from left to right. Here, the “compactified time direction” carries the evolution in the marginal gauge and Yukawa couplings dimensions as well as in the cutoff one. The black arrowed lines depict the flow in theory space from UV to IR. Given that we project on a plane of theory space, these vary along the RG-flow as the other dimensions (e.g. gauge and Yukawa couplings) of the theory also change. With a white dashed line, we mark the physical trajectory projected onto the plane of couplings.

This phase diagram allows us to identify different structures and phases. First, each of

the quadrants corresponds to different shapes of the effective average potential, which encode information about the shape of the Higgs potential at a finite cutoff. We have argued in the previous Subsection 3.1.1.3 how that actually resembles that of a physical temperature.

Moreover, there is a fixed-point-like solution, depicted by a yellow point, which satisfies,

$$\partial_t \bar{\lambda} (\bar{\mu}^{*2}, \bar{\lambda}^*, \vec{g}) = 0 \quad \text{and} \quad \partial_t \bar{\mu}^2 (\bar{\mu}^{*2}, \bar{\lambda}^*, \vec{g}) = 0, \quad (3.24)$$

with one attractive and one repulsive direction in this plane of couplings. Here $\vec{g}$ stands for the remaining set of marginal couplings, gauge and Yukawa. We stress that this is not a *full* fixed-point solution of the theory, given that it varies along the RG time and that $\vec{g} \neq \vec{g}^*$. Only two of the nineteen parameters of the theory satisfy the condition. Consequently, we denote this as a *partial fixed point of dimension 2*. The dimensionality corresponds to the number of directions in theory space whose flow vanishes. It follows, then, that the full fixed points of the theory are equivalent to an infinite-dimensional partial fixed point given that the full effective action is scale invariant. Furthermore, in Figure 3.5, there are further partial fixed-point solutions of dimension one, which appear as lines on the plane. These are depicted by red lines and satisfy

$$\partial_t \bar{\lambda} (\bar{\mu}^2, \bar{\lambda}, \vec{g}) \neq 0 \quad \text{and} \quad \partial_t \bar{\mu}^2 (\bar{\mu}^2, \bar{\lambda}, \vec{g}) = 0, \quad (3.25)$$

for the vertical one and

$$\partial_t \bar{\lambda} (\bar{\mu}^2, \bar{\lambda}, \vec{g}) = 0 \quad \text{and} \quad \partial_t \bar{\mu}^2 (\bar{\mu}^2, \bar{\lambda}, \vec{g}) \neq 0, \quad (3.26)$$

for the horizontal one. The enumerated structures manifest important features of the theory, which we will study one by one in the following Sections. Importantly, if the theory displays a full fixed point, it must be connected to any other partial fixed point in the subsystem of couplings. In other words, full fixed points are connected to the partial fixed points of different dimensions, e.g. two partial fixed points of dimension one cross at a partial fixed point of dimension two. In Figure 3.5, the respective partial fixed point has an IR attractive and repulsive directions in the plane of couplings shown. The former is provided by the large canonical scaling of the curvature mass and the latter by the Higgs quartic coupling. Moreover, for the moment we ignore the IR/UV repulsiveness in the other directions of theory space. However, we note that the relevance in theory space mixes between parameters as the stability matrix is commonly not diagonal.

Additionally, one can identify a boundary trajectory that separates qualitatively different IR scenarios: one with a non-trivially ($\bar{\rho}_{0, k \rightarrow 0} \neq 0$) and another with a trivially ($\bar{\rho}_{0, k \rightarrow 0} = 0$) stable shapes of the Higgs potential. This kind of trajectory is commonly referred as *separatrix*, and, only projected on this plane of couplings, it coincides with the $\bar{\mu}^2$ -partial fixed point (vertical red line). However, while such an agreement between separatrices and partial fixed points is trivial in the neighbourhood of the fixed point, it is not always ensured far from such solution. This will become apparent as further directions in theory space are considered.

More precisely, from the partial fixed-point condition for the curvature mass it follows that

$$\partial_t \bar{\mu}^2 = 0 \quad \forall k \quad \Rightarrow \quad \bar{\mu}^2 \approx \frac{\partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}] \big|_{\bar{\rho}_0=0}}{(2 - \eta_H)}, \quad (3.27)$$

which renders the cutoff evolution of the mass parameter as a function of the other couplings in the theory. While in (3.27) the relation is only approximate given that we have neglected the $\bar{\mu}^2$ dependence entering the diagrammatic flows (e.g. via the subdominant threshold functions), these can be accounted for leading to a resummed non-perturbative form. Furthermore, the cutoff profile of the curvature mass is now given, on this trajectory, as a function of the other couplings, $\bar{\mu}^2(\bar{\lambda}, \bar{g})$. Setting $\bar{g} = 0$ in (3.27), we recover the pure scalar theory in four dimensions where the fixed point in Figure 3.5 turns to the Gaussian one of the free theory, see [191, 192] for such analysis.

While this partial fixed point condition is rather straightforward and carries relevant information that will become apparent later on, the question of whether trajectories in theory space satisfy such a condition is more subtle. Although this can be found to be the case for simple theories, such as those in pure scalar and Yukawa systems, for more elaborated actions like in the SM, determining the fulfilment of such a condition becomes a challenging task.

The partial fixed point condition in (3.27) is also known as the *massless trajectory* given that on this, the scalar field never decouples, neither with a positive nor a negative curvature mass. Note that while in the pure scalar case the mass parameter $\bar{\mu}_{k \rightarrow 0}^2 \rightarrow \bar{\mu}^{*2} = 0$, in SM-like systems $\bar{\mu}_{k \rightarrow 0}^2 \rightarrow \bar{\mu}^{*2} \neq 0$. Nonetheless, in both cases a non-decoupling of the Higgs scalar takes place and no explicit scale is generated. We may refer to this precise solution as the trajectory *on* the critical surface [170, 192] as it minimises the deviation from the classically relevant scaling.

Partial fixed points, as the one in (3.27), are usually related with symmetries of the classical action but are even more general. In the present case with the curvature mass, it is clear how the massless condition renders scale symmetry [170, 193, 194]. Moreover, IR points are attracted in the UV towards this solution which renders the theory more predictive. This can be understood as a trade-off: a parameter is exchanged for a condition. This appealing direction will be the topic of discussion in Subsection 3.2.3.

Turning back to the SM phase diagram, in Figure 3.5 we observe that all trajectories emanate from the separatrix as they flow from the UV to the IR. This rapid repulsion from the separatrix is triggered by the large canonical dimension and its competition with the quadratic part of the flow, see (3.12). For the SM trajectory initiated at the Planck scale, the curvature mass is initially close to the separatrix and its flow exhibits a logarithmic growth towards the IR. At scales $k \sim 5 \cdot 10^3 \text{ GeV}$, the logarithmic flow is surpassed by the power-law contribution with opposite sign. This behaviour, depicted in Figure 3.6, leads to the curvature mass zero-crossing and hence the transition between the symmetric and broken phases of the effective average potential.

Moreover, it is important to note that the power-law part of the flow, $\partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}]$, is rather

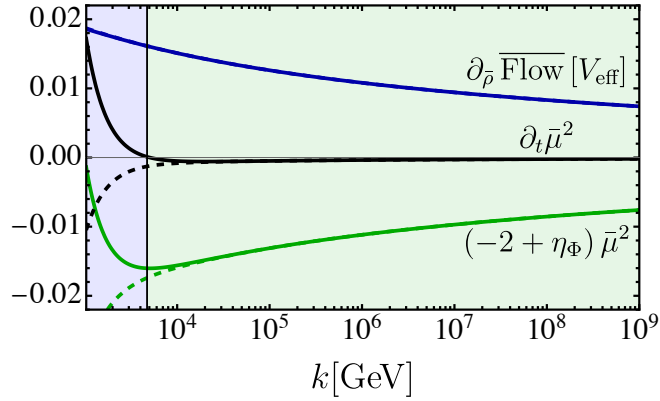


Figure 3.6: Total ($\partial_t \bar{\mu}^2$, plain black line) and partial (plain blue and green lines) contributions of the flow of the Higgs curvature mass for the SM trajectory. The power-law contribution is depicted in blue ($\partial_{\bar{\rho}} \overline{\text{Flow}} [V_{\text{eff}}] \big|_{\bar{\rho}_0=0}$), and the logarithmic $((-2 + \eta_H) \bar{\mu}^2)$ in green. The symmetric (shaded green) and broken (shaded blue) regimes are marked, where logarithmic and power scalings respectively dominate. The dashed lines show a neighbouring solution to the SM one which leads to no EWSB, as discussed in Figure 3.9.

independent of the Higgs curvature mass and instead determined by the Yukawa and gauge couplings of the theory, as shown in (3.13). Therefore, fixing a trajectory at Λ_{UV} to reproduce a given $k_{\text{SSB},1}$ translates into selecting a $\bar{\mu}_{\Lambda_{\text{UV}}}^2$ such that the logarithmic flow matches the power-law contribution at the desired UV scale. In other words, setting a trajectory in theory space is equivalent to tuning the initial value at a certain distance from the critical surface which vanishes the flow in the $\bar{\mu}^2$ direction (3.27) and renders a massless theory in the IR [170, 192, 193].

The position of the critical surface is inherently given by the power-law scaling, as seen in (3.27). This contribution is known to be scheme dependent [195], and we will elaborate on it later in Subsection 3.2.1. Furthermore, in the fRG approach, the threshold functions entering in (3.13) before integrating the loop momenta are particularly regulator dependent which is tantamount to a scheme choice [170, 191, 195]. Another example is the $\overline{\text{MS}}$ scheme with dimensional regularisation in which such contribution is apparently subtracted by the counterterms and stored on the UV cutoff. In conclusion, obtaining the power-law divergence and related quantities is particularly challenging but still of relevance. As previously discussed, the evolution along phases of the effective average potential encodes useful information which is only obtained in the change of sign of the curvature mass purely induced by the power-law scaling. Furthermore, the position of the special scale invariant solution in theory space is dependent on such contribution, see (3.27). Another example will be provided in Subsection 3.1.3, where this last feature is employed to derive the universal critical exponent of the quantum phase transition in this class of theories.

For completeness, in Figure 3.7 we show different slices of theory space projected onto the

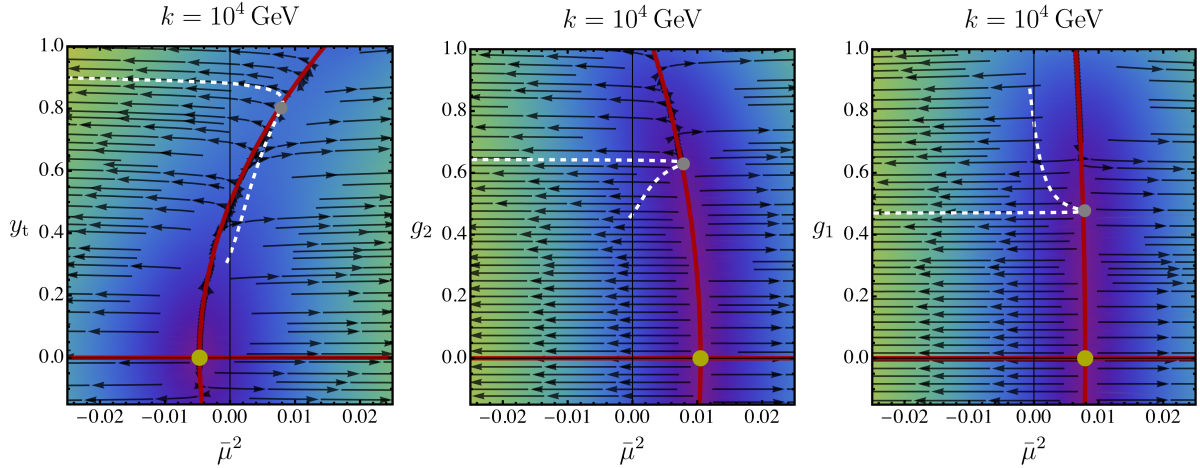


Figure 3.7: From left to right, we display the phase diagram of SM class of theories projected onto the plane of top Yukawa (y_t , left), weak (g_2 , middle) and hypercharge (g_1 , right) couplings versus the Higgs curvature mass at $k = 10^4$ GeV. The black arrowed lines indicate the flow in theory space from UV to IR. The yellow dot and red lines correspond to the partial fixed-point solutions of dimension one and two, respectively. The grey point indicates the position of the SM trajectory at the respective scales and the white dashed line, the full trajectory in the multi-dimensional volume projected on the respective planes of couplings.

plane of marginal gauge (g_1 and g_2) and top Yukawa (y_t) couplings versus the curvature mass. As in the previous analysis of the phase diagram, we identify further partial fixed point solutions of dimension one (red lines) and two (yellow points). By a white dashed line, we depict the physical trajectory of the SM. Notably, in these planes of couplings, the respective fixed points of dimension 2 only appear in the Gaussian limit of the marginal directions.

3.1.3 Quantum phase transition in the Standard Model class of theories

Now, we may continue analysing the phase diagram of the SM class of theories and wonder which variety of IR scenarios could be obtained by dialling neighbouring theories to the SM. To do so, we may first use the Higgs curvature mass at a Λ_{UV} scale relatively varied from the value reproducing the physical SM trajectory ($\mu_{k,\text{SM}}^2$),

$$\Delta \bar{\mu}_{\text{rel}}^2(\Lambda_{\text{UV}}) = \frac{\bar{\mu}_k^2 - \bar{\mu}_{k,\text{SM}}^2}{\bar{\mu}_{k,\text{SM}}^2} \Big|_{k=\Lambda_{\text{UV}}}, \quad (3.28)$$

as a control parameter for theory choice [146]. Strictly speaking, this exercise is equivalent to dialling over different theories and reflects the relation between scales in a certain QFT. However, as will be done in Subsection 3.2.1, the UV cutoff can be upgraded to carry a physical meaning,

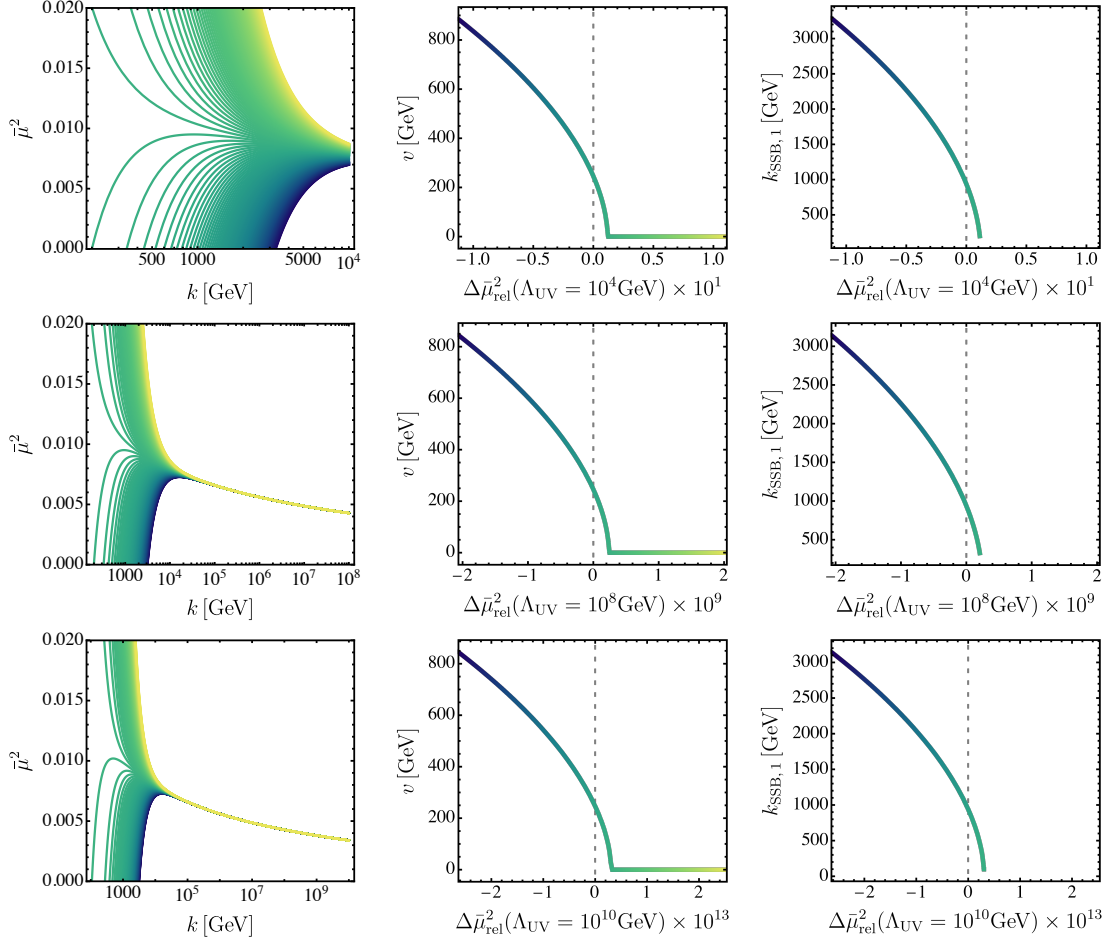


Figure 3.8: In the left-most panels, the Higgs curvature mass trajectories for various boundary conditions (yellow to blue lines), around the physical one. In the middle (rightmost) panel, we show the Higgs vacuum expectation value, v at $k \rightarrow 0$ (value of the $k_{SSB,1}$ scale), as a function of the relative variation of the boundary condition with respect to the physical trajectory ($\Delta\tilde{\mu}_{rel}^2(\Lambda_{UV})$). From top to bottom, each row shows to a different choice of $\Lambda_{UV} = \{10^4, 10^8, 10^{10}\}$ GeV, respectively.

for example that of a threshold where a heavy particle decouples or dynamics appear. In this case, the exercise of dialling boundary conditions on Λ_{UV} mimics the consideration of various realisations at the more fundamental embedding including new physics.

In the leftmost column of Figure 3.8, we depict the integrated flows of the Higgs curvature mass for different relative deviations from the SM trajectory and starting the flow at different Λ_{UV} scales (shown in different rows). In the middle and right columns, the Higgs' vacuum expectation value and the $k_{SSB,1}$ scale are shown as functions of the relative variation (3.28), respectively. This exercise clearly illustrates that two distinct IR regimes can be realised depending on the tuned UV values, one with a broken phase at $k \rightarrow 0$ and another that remains in the symmetric

regime. Noticeable from the range of relative variations, an exceptionally precise tuning of order $\mathcal{O}(k_{\text{SSB},1}^2/\Lambda_{\text{UV}}^2)$ needs to be performed to fix the IR parameters.

The boundary trajectory separating the two IR phases corresponds to the separatrix, which flows into the IR fixed point at $k \rightarrow 0$. We denote the boundary value of this specific trajectory at Λ_{UV} as the one initiated at $\bar{\mu}_{\Lambda_{\text{UV}},\text{SP}}^2$. In the middle and right panels of Figure 3.8, this corresponds to the leftmost point where $v = 0$. For $\bar{\mu}_{\Lambda_{\text{UV}}}^2 > \bar{\mu}_{\Lambda_{\text{UV}},\text{SP}}^2$, the Higgs potential remains in the symmetric phase with $v = 0$. In contrast, for $\bar{\mu}_{\Lambda_{\text{UV}}}^2 < \bar{\mu}_{\Lambda_{\text{UV}},\text{SP}}^2$, trajectories enter the broken phase. As $\bar{\mu}_{\Lambda_{\text{UV}}}^2$ decreases, $k_{\text{SSB},1}$ and v increase relative to the EW scale, effectively bringing Λ_{UV} and $k_{\text{SSB},1}$ closer together.

The existence of distinct phases in theory space at $k \rightarrow 0$ implies the presence of a *quantum phase transition*. For a review, see [196], and for examples in various contexts studied with the fRG, see e.g. [146, 197–200]. It is essential to differentiate between quantum and physical phase transitions; the latter occurs for a specific theory as a physical parameter, such as temperature, pressure, or chemical potential, is varied. Nonetheless, quantum phase transitions share analogous properties and can be described using the theory of critical phenomena [201]. In this framework, the Higgs expectation value serves as the order parameter, while the parameters or couplings of the effective action at a UV scale play the role of physical variables, akin to temperature. In Figure 3.8, we considered only one control parameter, namely the curvature mass at a high-energy scale, but others may be used as done in the following Sections.

The scaling of the vacuum expectation value near the critical separatrix trajectory can be universally described as [146, 201],

$$v \propto |\Delta\bar{\mu}_{\text{SP}}^2(\Lambda_{\text{UV}}) \times \Lambda_{\text{UV}}^2|^\beta, \quad (3.29)$$

where β is the critical exponent, and $\Delta\bar{\mu}_{\text{SP}}^2$ is the control parameter, defined as the distance to the separatrix trajectory,

$$\Delta\bar{\mu}_{\text{SP}}^2 = \bar{\mu}_{\Lambda_{\text{UV}}}^2 - \bar{\mu}_{\Lambda_{\text{UV}},\text{SP}}^2. \quad (3.30)$$

The critical exponent β is related to the properties of the scalar field at the critical scale [146], namely at $k_{\text{SSB},1}$ where $\bar{\mu}_{k_{\text{SSB},1}}^2 = 0$ and the correlation length is infinite. Then,

$$\beta = \frac{\nu}{2} \left(d - 2 + \eta_H|_{k_{\text{SSB},1}} \right), \quad (3.31)$$

where $\eta_H|_{k_{\text{SSB},1}}$ is the Higgs anomalous dimension evaluated at the critical point, and ν is the correlation length exponent, related to the RG scaling of the most relevant operator, namely that of the Higgs mass. Near a Gaussian fixed point, the corrections to the canonical scaling are given by the logarithmic contributions to the scalar field anomalous dimension,

$$\nu = -\Theta_{\bar{\mu}^2} \approx \left(2 - \eta_H|_{k_{\text{SSB},1}} \right)^{-1}, \quad (3.32)$$

following the convention introduced in (3.22). The quantum phase transition in the SM class of theories is second-order, as displayed in Figure 3.8 and studied in [146]. The nature of this transition is governed by the partial IR fixed point (yellow point). Depending on its position in theory space, the transition could take a different order (e.g., first-order). For the trajectory reproducing the SM in the IR, $\eta_H|_{k_{\text{SSB},1}} \approx 0.017$, which is close to the free theory, yielding a critical exponent of $\beta \approx 1/2$.

Moreover, the near-critical scaling of v close to the separatrix shows a power-law dependence of the order parameter on the control parameter. In order to properly describe such a scaling and the respective theories, accounting for the power-law scaling in the flow of dimensionful quantities is necessary. On the other hand, for large deviations of the order parameter (large condensates), the dependence is linear and those contributions do not play an important role and perturbative approaches suffice for the purpose.

Furthermore, we find there is a strong sensitivity of the order parameter to the control parameter of order $\mathcal{O}(k_{\text{SSB},1}^2/\Lambda_{\text{UV}}^2)$, as described by (3.29) and shown in Figure 3.8. This dependence requires to very precisely fix the parameters and the UV scale in order to obtain the desired theory in the IR. In the context of the BSM physics, this sensitivity between the EW scale and a new UV scale gives rise to the well-known *hierarchy problem*. Assuming the existence of a physical UV scale, *natural* choices [202, 203] of $\bar{\mu}_{\Lambda_{\text{UV}}}^2 \sim \mathcal{O}(1)$ would imply $k_{\text{SSB},1} \sim \mathcal{O}(\Lambda_{\text{UV}})$. However, collider experiments suggest no new scale below $\Lambda_{\text{UV}} \gtrsim 10 \text{ TeV}$. As higher energies are constrained by experiments, the naturalness argument becomes increasingly less predictive. Quantifying this issue is within the scope of the present framework and will be explored in Subsection 3.2.1.

Commonly, when the sensitivity of the EW scale physics to a UV scale is studied using perturbative methods, only the dependence on dimensionality is acknowledged. Moreover, in the present approach, we distinguish two sources of tuning: first, the standard one given by the scale at which new physics appears, Λ_{UV} ; second, the one provided by the existence of a quantum phase transition, namely the arrangement of parameters at Λ_{UV} . While only the former is usually acknowledged, the present formalism allows us to distinguish both. This will be made apparent as we investigate in depth the fine-tuning in the SM, in the following section Subsection 3.2.1.4.

It is important to emphasize that this motivation for new physics is not a theoretical inconsistency but rather an inherent feature of QFTs with dimensionful parameters. Therefore, following the classification in Chapter 1 we should refer to this question as a puzzle. However, to not complicate things we abdicate to the standard nomenclature.

We disclaim here that for the obtention of the results presented in Figure 3.8, the full entangled system of EW and strong gauge dynamics was not resolved for theories displaying $v \lesssim 10 \text{ GeV}$. In these scenarios, we did not resolve the full confining dynamics in the strong sector in the absence of a Higgs condensate but stopped the flow at cutoff scales where strong dynamics appeared. While resolving the full coupled dynamical system is within the scope of the framework, we leave

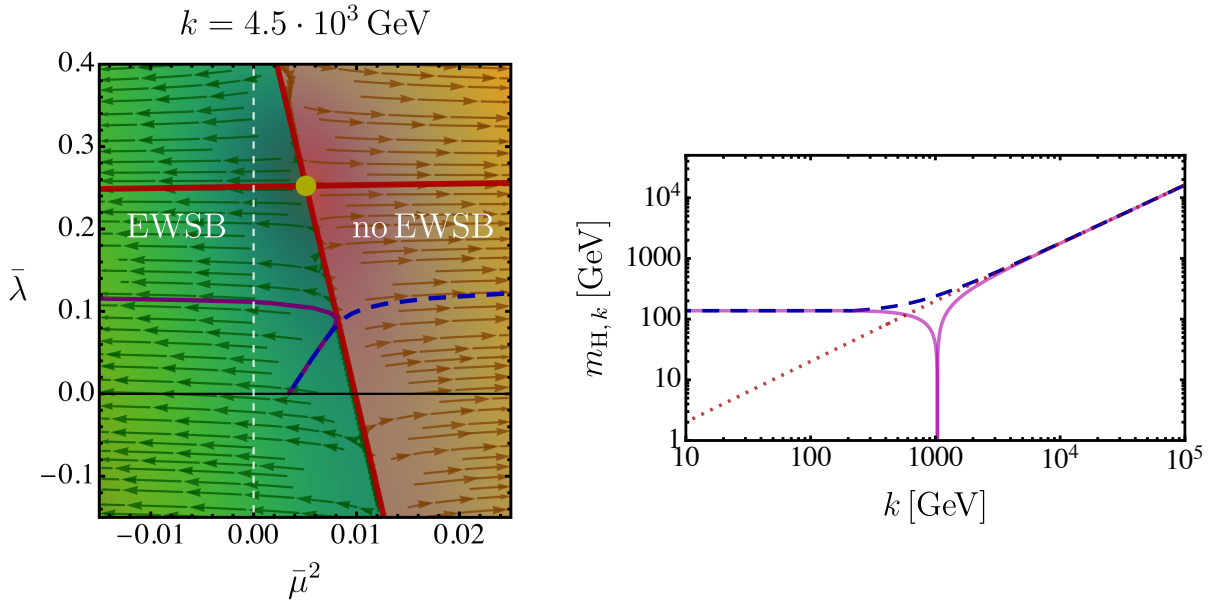


Figure 3.9: Left plot: phase diagram of SM class of theories projected onto the $\bar{\mu}^2$ – $\bar{\lambda}$ plane at a particular scale. We distinguish the SM trajectory (solid purple line) from a neighbouring one with no EWSB (dashed blue line). The red lines indicate the separatrices in (3.27), and the vertical dashed white line marks the zero crossing of the curvature mass, signalling the critical scale. The two shaded regions on either side of the separatrix highlight the two possible solutions.

Right plot: cutoff dependence of the Higgs Euclidean mass, as defined in (3.7), for the two trajectories depicted in the left plot (same colour coding) and for the massless condition (red dotted) given by (3.27).

this to a future work. We note that interesting findings could be uncovered via the non-trivial interplay of an elementary Higgs and the emergence of bound states. To conclude quantitatively the order of the phase transition, this point needs to be addressed. Recently, such a system has been studied in detail in [146], where evidence for a second-order quantum phase transition has been provided, supporting the results shown here.

3.1.4 Criticality of the Standard Model

Remarkably, the SM trajectory lies *close* to the phase boundary defined by the separatrix. By “close”, we mean that *natural* variations in a more fundamental formulation of the SM could lead to both SSB phases similar to the SM, as well as scenarios without EWSB. This proximity to criticality has interesting implications, which we discuss below. Furthermore, the notion of the naturalness will be further elaborated in Subsections 3.2.1 and 3.2.1.3.

In the left plot of Figure 3.9, we depict on the plane of Higgs couplings the SM trajectory

(purple plain line) alongside a neighbouring one with no SSB (blue dashed line). Both trajectories are initiated at $\Lambda_{\text{UV}} = 10^{10}$ GeV, and their Higgs Euclidean masses are nearly indistinguishable, as shown in the right plot. Here we additionally show the massless condition, partial fixed point or separatrix solution fulfilling (3.27) (dotted red line). In particular, the relative distance between the physical theory and the separatrix one is of $\Delta\bar{\mu}_{\text{rel}}^2(\Lambda_{\text{UV}}) \approx 0.02 (k_{\text{SSB},1}/\Lambda_{\text{UV}})^2$. The closeness of these solutions is remarkable even for low UV scales and definitely worsen for higher ones.

Here, we emphasize that drastically different IR theories, some with SSB in the Higgs sector and others without, are achievable within natural variations of the control parameter. Theories remaining in a symmetric phase would lead to a radically different IR scenarios, which merits discussion. Without a Higgs condensate, all fermions and EW bosons would remain massless. Consequently, the two non-Abelian gauge groups— $\text{SU}(3)_c$ and chiral $\text{SU}(2)_L$ —would independently tend towards confinement, one after the other. Due to the faster growth of the strong sector coupling, colour confinement would occur first. In the absence of explicit fermion masses from the Higgs mechanism, $d\chi\text{SB}$ would break the $\text{SU}(N_f)_R \times \text{SU}(N_f)_L$ global chiral flavour symmetry, resulting in pions as exactly massless NGBs. This dynamically generated condensate would indirectly produce the EW bosons and a fermion masses. In particular, the latter spectrum would be rather independent of flavour. Altogether, the scenarios without a Higgs v render radically different Universes to the present one.

The near criticality of the SM is an intriguing question which is not commonly investigated and explanations should be sought. Potential solutions, if needed, could be found in the context of the anthropic principle [204] while an alternative proposal will be given in Subsection 3.2.3.

3.2 Directions towards new physics

This Section contains results and analysis part of the work under preparation [7].

In the past Sections, we have discussed in depth the phase structure of the SM, employing a rather non-standard approach. We reviewed the features of this formulation, contextualized them with those present in perturbative theory, and discussed the most apparent ones in this context, such as quantum phase transitions. In this Section, we employ the constructed framework to investigate directions for SM extensions. We focus on the ones motivated by the principles of naturalness and predictivity which are particularly illuminating in the present Wilsonian framework and new insights can be gained.

The following Subsections are organized as follows. In Subsection 3.2.1, we introduce the naturalness principle and the hierarchy problem. We also compare the present fRG scheme, including power-law divergences, with commonly employed approaches. We proceed in Subsections 3.2.1.3 and 3.2.1.4 by introducing a measure of fine-tuning, benchmarking it on a well-known case, and investigating the fine-tuning in the SM in greater depth. In Subsection 3.2.2, we perform deformations of the SM phase diagram to assess existing proposed directions to solve the naturalness puzzle and investigate the existence of potential new ones. Lastly, in Subsection 3.2.3, we review and discuss directions for increasing the predictivity of the theory, namely by reducing the number of relevant parameters to be fixed.

3.2.1 Naturalness and the hierarchy problem

In Subsections 3.1.2.1 and 3.1.3, we observed how all trajectories in theory space with a significant separation of scales are rapidly attracted towards the separatrix solution in the plane of the Higgs mass and quartic coupling. This clustering of theories in the UV is an inherent feature linked to the presence of canonically relevant operators at the fundamental level of the theory. As a result, we found a strong sensitivity of the Higgs vacuum expectation value v to the UV scale and the configuration of UV parameters,

$$v^2 \propto |\Delta\bar{\mu}_{\text{SM}}^2(\Lambda_{\text{UV}}) \Lambda_{\text{UV}}^2| , \quad (3.33)$$

where $\Delta\bar{\mu}_{\text{SM}}^2$ is the relative variation of the dimensionless curvature mass of the Higgs potential at the UV scale Λ_{UV} , with respect to the value required to reproduce the SM. In the present Wilsonian RG, the dialling of different boundary conditions at the UV scale is equivalent to dialling over more fundamental theories where the heavier fields are already integrated out. Consequently from (3.33) it is apparent how the higher the scale of new physics, the greater the precision and fine-tuning required to recover the observed IR from the more fundamental configuration. This example defines the *hierarchy problem* as the substantial challenge for a UV theory to consistently yield the SM as a LEFT. This problem is rooted in the principle of naturalness, which is violated

by the need of significant fine-tuning of parameters [202, 203, 205, 206]. This concept has guided the development of models that predict new physics near the TeV scale, where the degree of fine-tuning remains moderate.

Before delving into this topic, it is crucial to emphasize that the present Wilsonian formulation makes apparent an additional source of fine-tuning. For a fixed Λ_{UV} , even if the scale is low enough to minimize fine-tuning, varying the control parameter can lead to vastly different IR scenarios. Specifically, as discussed in Subsection 3.1.4, theories without SSB are in proximity to the SM, rendering the theory almost critical. Although this *unnatural* feature is typically overlooked given the unknown position of the critical surface in standard perturbative approaches, here we will examine it thoroughly along the lines of fine-tuning, the hierarchy problem and predictivity.

3.2.1.1 Power-law scaling in different schemes

In the present functional RG approach, fine-tuning is explicit at the level of the flow equation of the Higgs curvature mass. However, in high-energy physics, particularly BSM studies, the standard tool for computing RG evolution is the $\overline{\text{MS}}$ scheme with dimensional regularisation [13, 207] where the manifestation of fine-tuning is slightly more obscure. While marginal couplings exhibit universality up to two loops in mass-independent schemes, the flow of dimensionful quantities differs at the one-loop level [208]. Marginal couplings in the perturbative limit align with the physical momentum profile inferred from scattering processes. However, mass parameters in different schemes connect differently to physical observables, namely the pole mass. For example, the Euclidean mass (3.7), evaluated at $k \rightarrow 0$, is closely linked to the Higgs pole mass in the $\overline{\text{fRG}}$ scheme while for large scales, see Figure 3.9, it is similar to a Debye mass. The scheme dependence of dimensionful parameters does not prevent extracting information from them at scales far from the IR; it simply reflects that these parameters encode different information. However, one needs to be aware of the properties, implicit assumptions and approximations in the RG being employed. In particular, the scheme dependence sources from the power-law divergences whose treatment is subtle.

To clarify this whole discussion, let's recall the flow of $\bar{\mu}^2$ in (3.12),

$$\partial_t \bar{\mu}^2 = \left(-2 + \frac{3y_t^2}{8\pi^2} \right) \bar{\mu}^2 + \left(\frac{3y_t^2}{8\pi^2} - \frac{3\bar{\lambda}}{32\pi^2(1 + \bar{\mu}^2)^2} \right), \quad (3.34)$$

where here we have considered the symmetric regime and omitted subleading Yukawa and gauge corrections, as well as higher-order effects. Equation (3.34) has two contributions: one linear in $\bar{\mu}^2$ and another non-linear. The first term captures the canonical scaling and the Higgs field's anomalous dimension (top polarization diagram), leading only to a logarithmic dependence on the scale. The second contribution in (3.34) is nonlinear in $\bar{\mu}^2$ and sets a reference point for the quadratic sensitivity of the IR to the deep UV configuration of the theory. While the flow

equation for the dimensionless curvature mass does not explicitly include a UV scale, this term introduces a quadratic sensitivity to the UV-fixed parameter $\bar{\mu}_{\Lambda_{\text{UV}}}^2$. To make this point more explicit, let's integrate (3.12), assuming a fixed anomalous dimension and power-law contribution,

$$\bar{\mu}_k^2 = \left(\frac{k}{\Lambda_{\text{UV}}}\right)^{-2+\eta_H} \left\{ \bar{\mu}_{\Lambda_{\text{UV}}}^2 - \frac{\partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}]|_{\bar{\rho}_0=0}}{(2-\eta_H)} \left[1 - \left(\frac{k}{\Lambda_{\text{UV}}}\right)^{2-\eta_H} \right] \right\}. \quad (3.35)$$

In the absence of the $\partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}]|_{\bar{\rho}_0=0}$ term, the dependence between UV and IR values is purely logarithmic. The power-law contribution, however, is as important, and impacts the determination of quantities and the diagnosis of fine-tuning which will be discussed further below. In fact, here it can be seen how choosing a particular trajectory in theory space (that which renders $\bar{\mu}_k^2$) is equivalent to fixing the distance to the critical surface at Λ_{UV} . This is just explicit in the second term in the brackets which we previously encountered in (3.27).

It is worth reminding that the momentum integral associated with the power-law contribution exhibits a well-known sizeable regulator dependence [144, 191, 195], which relates to the scheme (coordinate) choice of this parameter. In the context of the functional RG, a different regulator choice corresponds to a different scheme. As discussed in Section 2.2 not any kind of regulator is allowed but only those that admit certain conditions. Hence, the power-law term cannot be engineered to take any particular form or vanish. Furthermore, this contribution is particularly relevant for various features of the computation. Without it, no zero-crossing of the curvature mass can be described and we lose the nice mapping between vacuum and finite temperature computations explained in Subsection 3.1.1.3. In fact, such mapping is so efficient given that power-law contributions play an important role at finite temperature and in particular the determination of a critical temperature. Furthermore, the properties of the quantum phase transitions are not so apparent in the absence of the power-law flow, given the critical exponent is extracted at the critical scale where the Higgs curvature mass vanishes. In conclusion, the power-law divergences cannot be justifiably dropped.

Nonetheless, Wilsonian RGs are commonly not used in the realm of high energy physics as have been traditionally thought to be too intricate to be effectively applied to large systems with multiple scales and phenomenology. The $\overline{\text{MS}}$ scheme along with dimensional regularisation is a simpler approach as well as well-suited for perturbative computations. Given its popularity in the high energy area, it is important to contextualise the present discussion with this scheme.

Moreover, this approach incorporates a series of implicit simplifications tied to the regularisation procedure. Specifically, when employing mass-independent regularisations such as dimensional regularisation, the power-law contribution in (3.34) is cancelled by the counter-terms and stored at the Λ_{UV} boundary scale; see [209, 210] for a comprehensive derivation and discussion. In essence, mass-independent regularisation schemes such as dimensional regularisation offer a convenient shortcut that combines both steps of the physical Wilsonian EFT calculation into a single process: the integration of quantum corrections and the tuning on the critical surface. As a result, the

RG evolution of the mass is determined solely by the logarithmic running of the scalar field’s anomalous dimension, making the $\overline{\text{MS}}$ mass effectively a scale-independent parameter. For an illustrative example, see [30].

Given that the power-law term is “obscurely” stored in the perturbative RG equations at the boundary of the renormalisation, one may question whether fine-tuning is actually present. Firstly, if only a logarithmic dependence were present, as in the case of dimensional regularisation, the hierarchy problem would not arise due to the absence of quadratic sensitivity. However, fine-tuning can be explicitly manifested by introducing a threshold in the theory, thereby uncovering the power-law contributions encoded at Λ_{UV} .

Commonly, this is achieved by introducing a new heavy particle with mass M_S . In this scenario, the theory splits into two regimes: one below and one above M_S . Connecting the RG equations in both regimes requires performing a matching at the threshold. At this point, the threshold now plays the role previously held by Λ_{UV} in the low-energy theory, thereby encoding the associated power-law divergence. Furthermore, this is carried over in the matching to the RG equations in the high-energy regime. This explicitly manifests the apparently hidden quadratic divergence and, consequently, the fine-tuning [210, 211].

In other words, the matching between low- and high-energy regimes, performed at $k = M_S$, results in a picture where logarithmic running is observed both above and below M_S , while the quadratic fine-tuning emerges as a large, non-differentiable jump at the threshold, see [211]. This demonstrates how mass-independent regularisation schemes necessarily require an *Ansatz* about the more fundamental structure of the theory above the initially agnostic scale Λ_{UV} in order to manifest this feature, which is already explicitly present in the Wilsonian framework. Nevertheless, this does not imply that the theory in the regime $M_S - \Lambda_{\text{UV}}$ is lacking of quadratic sensitivity to the arrangement of parameters at the most fundamental scale. As previously mentioned, fine-tuning is merely “hidden on the other side”, remaining to be addressed in the case of a more fundamental layer being invoked.

We remark that the perturbative approach often incorporates further implicit assumptions. An important example is the deep Euclidean region or asymptotic symmetry [212], which assumes all physical mass scales or external momenta are small with respect to the loop momenta of the fluctuations, hence implicitly justifying neglecting them, see [141, 143–145] for a discussion of its effect. In contrast, the fRG framework does not assume its relevance in the UV but accounts for its dependence and decoupling explicitly.

After this technical discussion, we conclude with a reflection on whether there is or is not a hierarchy problem in the absence of new physics. Consider, for example, a fictional scenario where the SM is a UV complete theory, either asymptotically free or safe. If desired, a quantum field theoretical formulation of gravity could be incorporated into the story without further complication. In any case, if an elementary scalar field is present at the fundamental level, it is

expected to have a critical exponent of magnitude $\Theta_i \sim 2$, which would be notably relevant. In this situation, quadratic fine-tuning in the neighbourhood of the fixed point is still necessary to choose the theory with the desired IR. The fine-tuning remains given there is a highly relevant direction in the critical surface, but now its interpretation does not align with the defined hierarchy problem. In other words, the fine-tuning is still required to select the theory with the desired IR but its physical interpretation lacks given the absence of a physically interpretable UV scale.

3.2.1.2 Historical solutions to the hierarchy problem

After this detailed technical discussion of the hierarchy problem, we conclude this section with a brief review on common approaches aimed at resolving the fine-tuning puzzle.

- (i) **Composite Higgs and Technicolour models:** These are theories in which the Higgs is not a fundamental scalar field but rather a bound state of new constituents confined by a new force. Here, the absence of the fine-tuning problem becomes clear: the fundamental action of the SM, augmented with an additional strongly interacting force, does not contain any dimensionful parameter. Instead, such parameters are dynamically generated by strong interactions à la QCD. Examples include Composite Higgs (CH) [44–48] and Technicolour (TC) [49–52] models (see [213] for a review), where the Higgs emerges as a bilinear of new fermionic fields.
- (ii) **Supersymmetry:** Another solution involves new physics that systematically cancels the power-law divergent terms. This is elegantly achieved through a symmetry between bosonic and fermionic fields, namely Supersymmetry (SUSY) [214–216]. In such models, SUSY is broken below a certain scale, typically expected around the TeV scale for naturalness reasons. At higher scales, the exact symmetry linking fermionic and bosonic degrees of freedom cancels the power-law corrections to the Higgs mass, leaving only the tuning required up to the SUSY breaking scale [217].
- (iii) **Extra dimensions:** A third family of solutions introduces additional space-time dimensions, which can be realised in various forms, such as compactified flat [53, 218] or warped extra dimensions [54]. In these scenarios, the hierarchy problem is addressed in different ways: either by effectively lowering the Planck scale to the TeV regime [54, 55] or (and) by interpreting the Higgs as a remnant of the fifth-dimensional components of electroweak bosons [219].

Despite extensive efforts to develop models motivated by the hierarchy problem, experiments have probed energy scales where experimental evidence of these more natural solutions was expected. On the one hand, the absence of superpartner detections suggests a higher SUSY breaking scale, making this approach less attractive. On the strong dynamics side, no experimental evidence for additional bound-state resonances has been observed, leading to the conclusion

that the strong sector must be more exotic than initially expected from QCD dynamics or the Higgs should be realised as a *better* pNGB of a slightly broken global symmetry [220, 221]. Both scenarios necessitate raising the associated scales where new physics degrees of freedom appear, introducing an additional layer of complexity and tuning. If these solutions remain viable, the hierarchy between the Higgs mass and the BSM physics scales must now also be explained—a challenge known as the *little hierarchy problem*. In the analysis that follows, the approach is entirely agnostic regarding the fundamental realisation of the Higgs or the scale of new physics. Consequently, the present framework can be applied to both scenarios, big and little hierarchies.

As a final remark, points (i) and (iii) inherently exhibit non-perturbative features, either due to strong dynamics or their formulation in higher-dimensional embeddings. In Chapter 4, we will demonstrate the utility of functional methods in this domain by charting the landscape of gauge-fermion QFTs, where new CH and TC models may reside, and by investigating the properties of non-perturbative renormalizable five-dimensional theories.

3.2.1.3 Quantifying naturalness

To provide a comprehensive and systematic analysis of the fine-tuning in theory space, it is essential to quantify the sensitivity of IR physics to the UV arrangement of the theory. Various approaches to quantify fine-tuning exist [206, 222–224]; however, in this work, we employ the Barbieri-Giudice (BG) measure [203, 225]:

$$\Delta_{\text{BG}} = \max_{i,j} \left| \frac{\partial \log \mathcal{O}_i}{\partial \log g_{j, \Lambda_{\text{UV}}}} \right|, \quad (3.36)$$

where $\mathcal{O}_i$ represents the set of independent observables and $g_{j, \Lambda_{\text{UV}}}$ corresponds to the values of the fundamental parameters at the UV scale. This is a normalised measure, making it an adequate quantity for comparisons between different theories.

The BG measure quantifies the sensitivity of IR observables to the variation of the theory parameters at a UV scale. However, given that we are varying with respect to RG variant parameters, this measure is intrinsically scheme dependent. For parameters associated to marginal operators such as gauge couplings, the RG evolution is universal up to two-loops in mass independent regularisation schemes and hence a direct mapping between the RG flow and strength of physical correlation functions can be made. However, we have discussed how the RG equations for mass parameters is scheme dependent at one loop and hence not universal. One can reformulate (3.36) in a scheme-independent fashion. Instead of using RG parameters at the UV scale one can look at the relative variation between UV and IR sensitive observables. In other words, we may trade $g_{j, \Lambda_{\text{UV}}}$ for $\mathcal{O}_j$ where this latter is an observable sensitive to the UV physics. A good example is that of the pole mass of a very heavy scalar field approximately of Λ_{UV} . Now, one can think of varying quantities of the more fundamental theory (e.g. the potential of the heavy scalar, its portal coupling to the visible sector, etc.) and quantify the variation of the

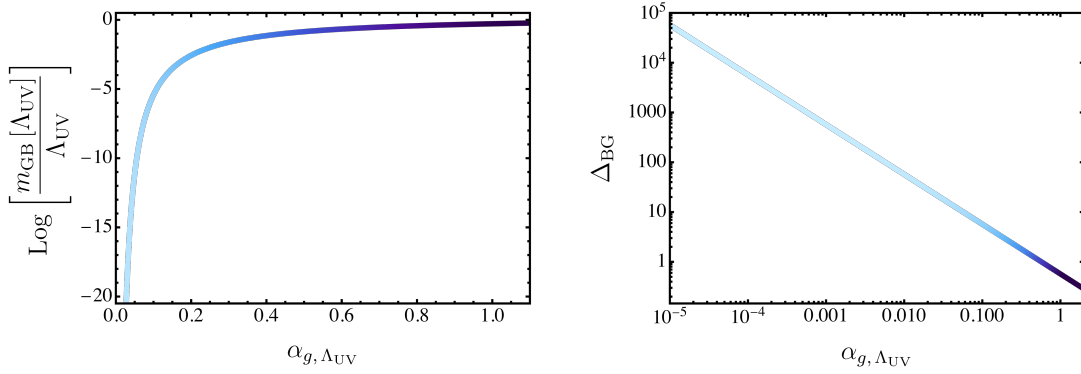


Figure 3.10: Left plot: dependence of the lightest gauge boson bound state mass as a function of a gauge coupling value at an arbitrary UV scale Λ_{UV} .

Right plot: BG tuning measure defined in (3.36) as a function of the gauge coupling at the UV scale.

IR observables. This exercise in fact resurrects the power-law contribution at the heavy scalar threshold in mass independent regularisation schemes like dimensional regularisation. Hence we see how although in some cases not apparent, the fine-tuning appears when studying physics.

Although we have defined a scheme independent quantity this does not imply that we will find a precise agreement between its derivation in different schemes. As with the computation of other quantities, such as critical exponents, different methods capture relevant information in distinct ways, leading to varying levels of accuracy in the results. This is for example the case if we compare fRG and $\overline{\text{MS}}$ results. As discussed the perturbative approach manifests the fine-tuning only when assuming a UV embedding and introducing a threshold. On the other hand, the Wilsonian cutoff approach accounts for the interplay between logarithmic and quadratic corrections along the flow.

Given the definition (3.36), let's discuss its applicability on a well understood fundamental theory such as pure Yang-Mills. This is beneficial as in the present Section we will not only study the dependence on the UV scale but also on the arrangement of parameters. Pure Yang-Mills is a theory uniquely defined by a single marginally relevant operator, the gauge coupling, which has a UV Gaussian fixed point. While this kind of theory is not known to exist in nature in its pure state, one can still define a physical observable such as for example, the pole mass of the lowest lying bound state of gauge bosons (e.g. in QCD it is the scalar glueball). This can roughly be approximated to the scale at which the gauge dynamics become strong $m_{\text{GB}} \sim \Lambda_{\text{YM}}$. We may now ask, how much does this observable change if we downgrade this fundamental theory to a LEFT below a cutoff Λ_{UV} at which the gauge coupling is unknown. The dialling of $\alpha_g = g^2/4\pi$ at the UV is depicted in the left panel of Figure 3.10. For large gauge couplings values, the $m_{\text{GB}} \sim \Lambda_{\text{UV}}$ given that the theory is commenced closely to the critical coupling for strong dynamics. On the other hand, as we decrease the value in the UV the appearance of the confining scale is delayed

in an exponential manner. By dialling different gauge couplings we can obtain any infinitesimally small spectrum of bound states. However, obtaining very small masses (with respect to Λ_{UV}) requires increasing fine-tuning of the gauge coupling to small values. This appears more clear as we compute the BG measure parameter shown on the right panel of Figure 3.10.

To contextualise this discussion with the numerical investigation in the upcoming Subsection, let's put some numbers for the case of the QCD gauge coupling considering different UV cutoff scales. For $\Lambda_{UV} = k_{\text{Planck}}$, $\alpha_{g, \Lambda_{UV}} = 0.013$, which leads to $\Delta_{BG} \approx 40$. For a lower choice, for example from the EW scale $\Lambda_{UV} = k_{\text{SSB},1}$, $\alpha_{g, \Lambda_{UV}} = 0.08$ which corresponds to $\Delta_{BG} \approx 10$. In conclusion, the BG parameter can be understood as to which precision the parameters in the UV must be known in order to render the IR within the desired order of magnitude. In the present case of a marginal coupling, these are mild fine-tunings showing the *technically natural* character of this coupling [202].

The Yang-Mills case is interesting given that it is a classically scale invariant theory, namely no explicit dimensionful quantity is present at the level of the classical action. This appears in the fact that we can freely reconsider a UV limit to the effective theory and the sensitivity will be properly rescaled. In the case on non-scale invariant theories, as the SM, an inherent scale is present by the Higgs mass term.

3.2.1.4 Fine-tuning in the Standard Model

Having introduced a measure for naturalness (3.36) and exemplified its applicability, we proceed with the case of interest. We now recall the analysis done in Subsection 3.1.3 where we studied the dependency of v and the SSB scale ($k_{\text{SSB},1}$) on the curvature mass of the Higgs potential which played the role of a control parameter for theory choice, see Figure 3.8. While these two quantities are strictly speaking not observables, v enters directly in the determination of the EW, Higgs and fermion pole masses, and $k_{\text{SSB},1}$ we have argued to be related to the critical temperature of the EW phase transition.

For simplicity, let us first consider v as the most susceptible IR observable and $\bar{\mu}_{\Lambda_{UV}}^2$ as the most sensitive parameter at the UV scale. In Figure 3.11 we show the BG measure as a function of the relative variation with respect to the trajectory rendering the SM value, (3.28), for different choices of $\Lambda_{UV} = \{10^4, 10^8, 10^{10}\}$ GeV. In fact, these results have been obtained from the presented in Figure 3.8. The BG parameter for theories far from the critical solution is of order of the ratio between scales,

$$(\Delta\Lambda)^2 \equiv (\Lambda_{UV}/k_{\text{SSB},1})^2. \quad (3.37)$$

For example, for a UV scale $\Lambda_{UV} = 10^{10}$ GeV (left-most panel in Figure 3.11), $(\Delta\Lambda)^2 \approx 10^{14}$ as the curvature mass is relatively varied $\Delta\bar{\mu}_{\text{rel}}^2(\Lambda_{UV}) \approx (\Delta\Lambda)^{-2}$. This explicitly manifests the quadratic sensitivity of IR physics to the alignment of UV parameters. If Λ_{UV} is considered in

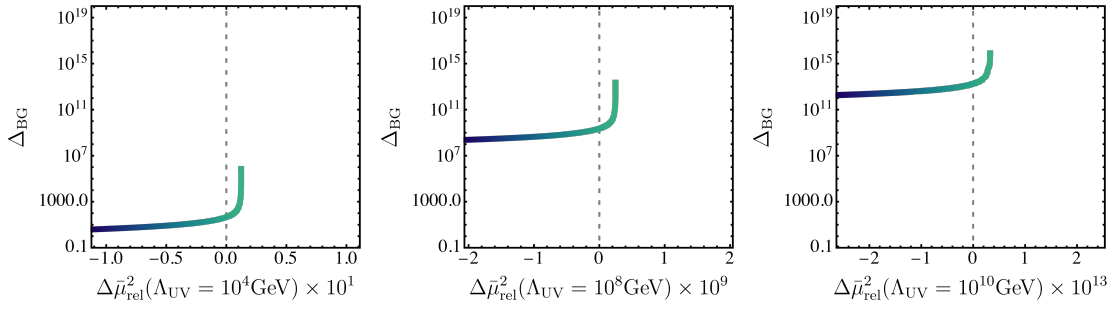


Figure 3.11: The BG tuning measure as a function of the relative variation of the UV boundary conditions with respect to the physical trajectory reproducing the SM (vertical dashed grey line), $\Delta\bar{\mu}_{\text{rel}}^2(\Lambda_{\text{UV}})$. From left to right, three choices of the UV scale, $\Lambda_{\text{UV}} = \{10^4, 10^8, 10^{10}\}$ GeV, are shown in correspondence with the trajectories discussed in Figure 3.8.

the 10 TeV regime, see left-most panel, the fine-tuning required is compatible with that of a theory described by marginal operators, namely that of the previously discussed Yang-Mills case in Figure 3.10. A natural separation of scales of up to $(\Delta\Lambda) \approx 10^2$ is still allowed in the scalar field theory and also manifests in other physical systems [203]. For this reason, the naturalness principle has provided motivation for new physics in the 1 – 100 TeV regime, potentially reachable by collider-based efforts.

Figure 3.11 already makes apparent the two sources of tuning previously previewed. On one hand, there is the well-known and dominant dependence on the choice Λ_{UV} scale, and on the other, the present Wilsonian approach also allows to easily identify which particular arrangement of UV parameters requires most fine-tuning. This last point is inherent to the existence of a quantum phase transition in the class of theories. In particular, the critical solution given by the separatrix is the boundary to theories where no broken IR is found. In terms of naturalness, it is hard to quantify the large difference between qualitatively different IRs, one with EWSB and another without. Previously in Subsection 3.1.4 we have motivated this observation in the same line as naturalness. A fair question to ask is whether a more fundamental realisation of the SM could ensure a broken IR by distancing the separatrix solution.

However, while the separatrix trajectory resembles the largest fine-tuning, this theory is particularly interesting as shows enhanced predictivity. In other words, the Higgs mass parameter can be rendered as a prediction of the rest of the parameters of the theory. In the following Subsection 3.2.3 we will discuss this point further and how it could actually relax the criticality of the SM.

The BG measure, as defined in (3.36), singles out the most sensitive direction in theory space as the one defining the amount of fine-tuning. For Figure 3.11 we have only considered the variation of $\bar{\mu}^2$ at the UV scale and the change of v in the IR. However, we need to check the other

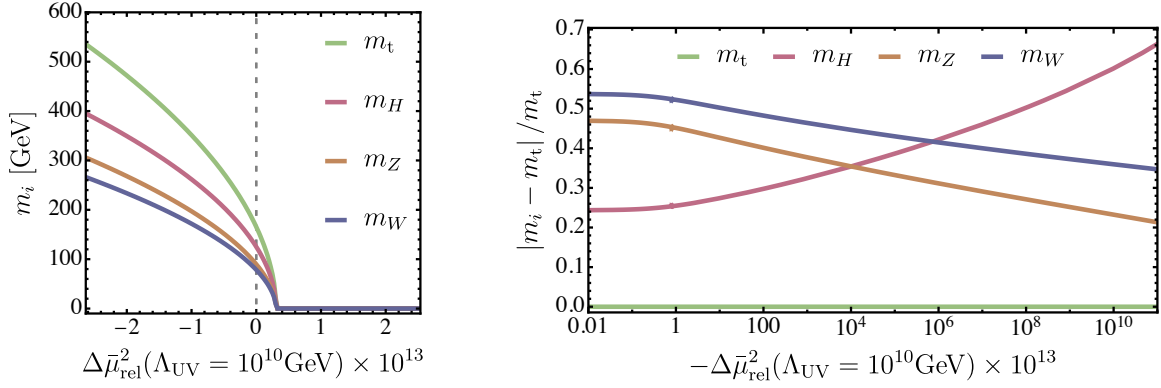


Figure 3.12: Left plot: Euclidean masses of the top quark, Higgs and electroweak bosons as a function of $\Delta \bar{\mu}_{\text{rel}}^2(\Lambda_{\text{UV}})$.

Right plot: relative difference between the particle masses with respect to the top quark mass for different $\Delta \bar{\mu}_{\text{rel}}^2(\Lambda_{\text{UV}})$. Here, we consider relative variations up to $\Delta \bar{\mu}_{\text{rel}}^2(\Lambda_{\text{UV}}) \leq 10^{-3}\%$ which suffice to show how the spectrum of the SM particles qualitatively changes.

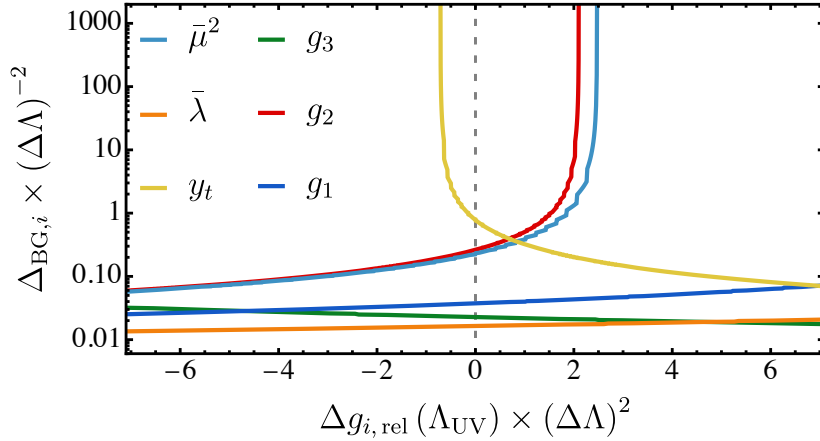


Figure 3.13: Dimensionless BG measure as a function of the relative variation of the different SM parameters at an arbitrary UV scale $\Lambda_{\text{UV}} = \Delta \Lambda \cdot 1 \text{ TeV}$.

directions of theory space in the UV and also, other observables in the IR. As an example, in Figure 3.12 we show the spectrum of masses of the theory as a function of the relative variations of the curvature mass at $\Lambda_{\text{UV}} = 10^{10} \text{ GeV}$. While we observe how the spectrum of masses in the theory changes qualitatively when varying the curvature mass less than 0.0001%, for trajectories close to the SM one, the most sensitive IR parameter remains to be v . Therefore, from now on we will take this quantity as the providing the most sensitive IR observable.

Furthermore, in Figure 3.13 we show the dimensionless version of the BG measure, where we divided the amount of fine-tuning by $(\Delta \Lambda)^{-2}$, as a function of the relative variation in different

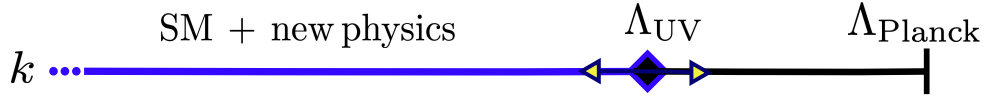


Figure 3.14: Sketch of the approach to new physics followed in this Section. We consider that some form of new physics coexists with the SM, acting as a viable LEFT below an unknown Λ_{UV} scale.

directions in theory space (i.e. couplings). Nonetheless, this Figure was obtained with trajectories initiated at $\Lambda_{UV} = 10^8 \text{ GeV}$ and a mild difference is found when considering others. The IR observables are, in fact, most sensitive to the variation of the top Yukawa coupling, followed by g_2 and $\bar{\mu}^2$. This interesting observation is well understood as the canonical relevance of the curvature mass term mixes with the other directions in theory space and the Yukawa and gauge couplings are the dominant contributions in (3.12). This relevant mixing can be seen in the off-diagonal entries of the stability matrix.

3.2.2 Phase diagram deformations and new physics

Having set the stage, we are now ready to expand our analysis in the direction of new physics model building. We may ask ourselves whether, with the current powerful framework that identifies different sources of fine-tuning, new directions to alleviate it can be realised by considering new physics. A similar question has been studied in [222, 226] and here, we expand on it.

So far, we have followed a hard cutoff approach that mimics the presence of new physics beyond a certain UV scale. At this point, we remain completely agnostic regarding the nature of the new physics, as the Wilsonian approach does not require specific assumptions to manifest the fine-tuning. However, it is conceivable that below the considered Λ_{UV} scale, the SM may not be what it seems, and some form of new physics could couple to the Higgs sector in a way that renders the theory (more) natural. In this scenario, such new physics could remain weakly coupled and communicate only with the Higgs sector, explaining why it has not been observed yet.

Then, the possibility of new physics effectively active at energies below the considered Λ_{UV} remains open. To study this, we adopt the approach depicted in Figure 3.14, where new physics coupled to the Higgs is assumed to provide a more accurate description of the SM below a certain UV scale. This UV scale is preserved as a “thermometer” of fine-tuning and can be understood as the validity limit of the present LEFT.

The main aim with this strategy is to find non-trivial configurations of parameters at the UV scale which could specially render the theory more natural. Furthermore, this path is also particularly relevant in the context of the little hierarchy problem, where a more fundamental embedding is already ensured above Λ_{UV} . Through this analysis, we can interpret which modifications—such as changes in matter content—should be implemented in the Higgs-like

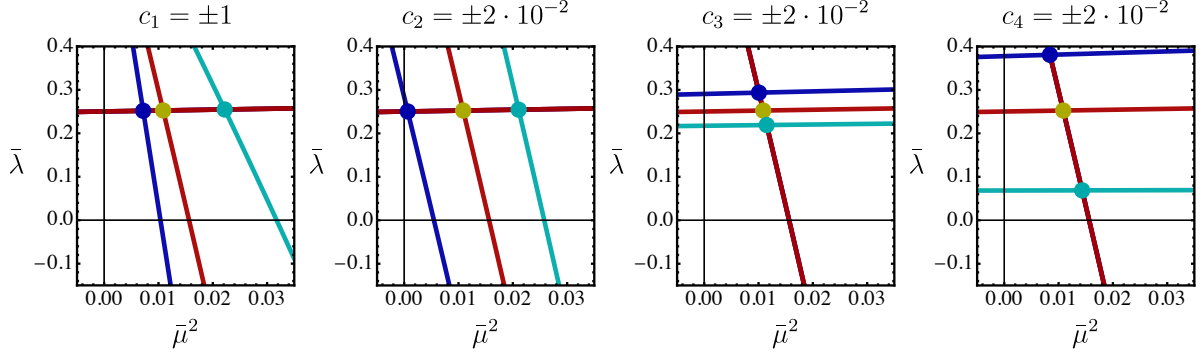


Figure 3.15: Sketch of the deformations of the SM's phase diagram. From left to right, we vary $c_1 = \pm 1$, $c_2 = \pm 2 \cdot 10^{-2}$, $c_3 = \pm 2 \cdot 10^{-2}$, and $c_4 = \pm 2 \cdot 10^{-2}$ respectively. The points mark the partial fixed points in the plane of Higgs couplings, while the vertical and horizontal lines represent the separatrices. Dark blue points and lines depict the negative sign deformations, whereas cyan points and lines indicate the positive sign deformations. The yellow point and the red lines correspond to the SM case at a benchmark scale.

LEFT to balance the increase of the UV scale, as required by pressing experimental constraints.

In this first analysis we consider the effect of some agnostic new physics only coupled to the Higgs sector. Its effects can be mimicked on the phase diagram by deforming the RG flow of the SM Higgs potential, leading to

$$\partial_t \bar{\mu}^2 = (\partial_t \bar{\mu}^2)_{\text{SM}} + c_1 \cdot \bar{\mu}^2 + c_2 \quad \text{and} \quad \partial_t \bar{\lambda} = (\partial_t \bar{\lambda})_{\text{SM}} + c_3 \cdot \bar{\lambda} + c_4, \quad (3.38)$$

as the new flows. Here, $(\partial_t \bar{\mu}^2)_{\text{SM}}$ and $(\partial_t \bar{\lambda})_{\text{SM}}$ stand for the RG flows of the SM class of theories, and c_i parametrize the deformations. This simplified approach facilitates the realisation of the special solutions, and on the other hand, it helps parametrize the effect of new physics in a model-independent way. For example, the set of deformations c_i can be understood as the effect of some new fields only coupled to the Higgs sector, where

$$\begin{aligned} c_1 &= f_1(-g_N^2, y_N^2), & c_2 &= f_2(-\lambda_{HS}, -g_N^2, y_N^2), \\ c_3 &= f_3(-g_N^2, y_N^2), & c_4 &= f_4(\lambda_{HS}^2, g_N^4, -y_N^4), \end{aligned} \quad (3.39)$$

are functions of the g_N , y_N , and λ_{HS} couplings to dark gauge bosons, fermions, or scalars only communicating to the Higgs, respectively. Different values of c_i affect different parts of the flows of the scalar sector. For example, c_1 alters the contributions to the anomalous dimensions of the Higgs field, c_2 the power-law part of the flow, and c_3 and c_4 the flow of $\bar{\lambda}$. Evidently, when considering some specific new physics, the relation between several deformations $\{c_i\}$ will be uniquely given. However, at the present stage of the analysis, we keep them free. Additionally, we simplify even further the setup by neglecting the flow of the couplings underlying c_i . It is evident

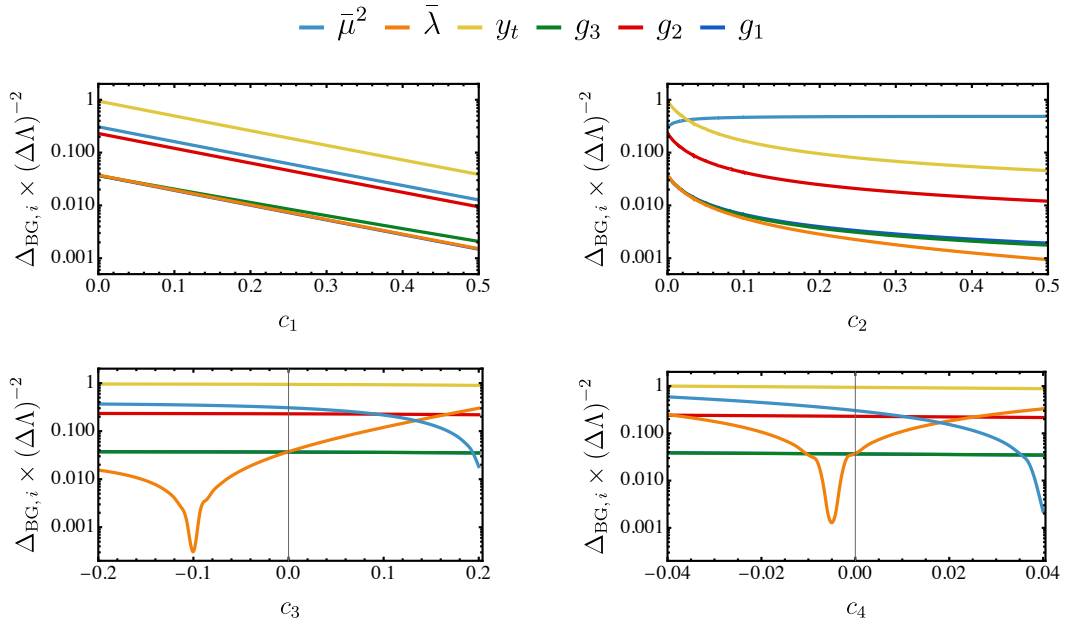


Figure 3.16: Dimensionless version of the fine-tuning parameter, $\Delta_{\text{BG},i} \times (\Delta\Lambda)^{-2}$, as a function of the deformations introduced in (3.38) which mimic the inclusion of new physics, see (3.39). The coloured lines show the sensitivity along variations in different directions of theory space, namely in the various couplings.

that the present approach aims for a qualitative understanding of the possible deformations of theory space, and for a detailed study, a more quantitative implementation is needed.

The effect of the deformations in (3.39) on the location of the “partial” fixed points in the plane of Higgs parameters is summarized in Figure 3.15.

Furthermore, for the following analysis, we simplify the complete SM to the setup including the flow of the g_1 , g_2 , g_3 , and y_t , in addition to the scalar sector parameters. We identify the physical theory as that which reproduces the SM values at $k_{\text{SSB},1}$, namely $\bar{\lambda}_{k \rightarrow k_{\text{SSB},1}} \approx 0.13$. The rest of the SM gauge couplings and Yukawas do not sense significant modifications; hence, we do not check their value in the IR. Lastly, for simplicity we do not flow into the broken phase and hence use the $k_{\text{SSB},1}$ scale as a placeholder for v . As shown in Figure 3.11, this is well justified.

In Figure 3.16, we display the value of the BG measure for the trajectory displaying the IR compatible with the SM and considering different directions of theory space with the presence of new physics parametrised by the different c_i ’s. The final Δ_{BG} is determined by the maximum among all variations in theory space, which corresponds to the highest of the curves. Moreover, although the results were all obtained for $\Lambda_{\text{UV}} = 10^6$ GeV, we show the dimensionless version of the BG measure where we divided by $(\Delta\Lambda)^2$ defined in (3.37). Varying the UV cutoff has a minor impact in the c_i -dependence but the overall picture remains the same.

In the first top-left panel, we observe that the largest sensitivity arises when varying the top Yukawa, as previewed in Figure 3.13. Nonetheless, there is a constant decrement of the fine-tuning in all directions of theory space as c_1 is increased. This parameter enhances the Higgs anomalous dimension, decreasing the dimensionality of the Higgs mass operator. In other words, such deformations render the curvature mass operator more canonically irrelevant at Λ_{UV} and hence, reduces the fine-tuning.

In the top-right panel, we vary c_2 , which enters as a correction to the power-law contribution in the flow of the curvature mass. Such a deformation could be generated from bosonic tadpole diagrams, which due to their momentum independence do not enter on the anomalous dimension. For large values of c_2 , we observe how the fine-tuning in the direction of the marginal couplings decreases, while it mildly increases and eventually freezes in the direction of the curvature mass. In combination, we see only a very mild improvement in fine-tuning. Negative variations of c_2 are also allowed and these displace the fixed point on the $\bar{\mu}^2 - \bar{\lambda}$ -plane towards the regime with a non-trivially stable shape. However, trajectories with these deformations in the present setup display also a non-trivially stable shape at the UV scale Λ_{UV} . This is the same case scenario as the presented in [6] and studied in Section 3.3. Given that this requires a more delicate treatment we do not discuss them here.

In the bottom panels, we vary c_3 (left panel) and c_4 (right). Positive deformations of these parameters lower the position of the horizontal partial fixed point, or separatrix, as shown in Figure 3.15. In fact, this triggers the stabilisation of the quartic coupling towards the UV and ultimately lowers the Landau pole scale in the scalar sector. Nonetheless, these deformations are interesting given they improve or even cure the metastability of the Higgs sector discussed in Subsection 3.1.1.2. However, these deformations only reduce the fine-tuning in the direction of the Higgs parameters in theory space, while the sensitivity in the other marginal coupling directions remains rather constant. This minimisation occurs at two instances. First, at the maximum values of the deformations shown, $c_3 \simeq 0.2$ and $c_4 \simeq 0.04$, which correspond to the scenarios in which the trajectories displaying the SM have a stable shape and $\bar{\mu}_{\Lambda_{UV}}^2 = 0$ entering the same class as that discussed in [6] and studied in Section 3.3. The second case occurs for $c_3 \simeq -0.1$ and $c_4 \simeq -0.005$, and corresponds to the case where the quartic coupling vanishes at the UV scale, illuminating it as *technically natural* [202]. As the dominant sensitivity is still sourced in the Yukawa direction which is rather insensitive to these deformations, the total BG measure remains unchanged and no fine-tuning is relaxed. Nonetheless, we emphasize that in more realistic model embeddings, where specific combinations of c_i are required, the dependence on deformations could differ, potentially revealing directions that reduce the overall fine-tuning.

In conclusion, we observe that it is not easy to find more natural configurations in theory space other than by reducing the canonical dimension of the Higgs mass operator. The dominant sensitivity in the marginal coupling directions arises as a direct consequence of the mixing between

the relevant direction and the marginal couplings. The most promising pathway appears to be the simplest one: introducing new physics that effectively lowers the dimension of the Higgs mass operator. In fact, this has been explored in the context of *self-organisation* [227] and quantum gravity [228]. Moreover, this direction incorporates considerations of the Higgs as a composite of other more fundamental fields. In this case, Higgs field operators can be formulated in a more fundamental way with higher-order fermionic interactions. While these appear canonically relevant below a certain IR scale, they are actually canonically irrelevant at a more fundamental perturbative one. Additionally, other interesting implementations in this direction could be achieved by coupling the elementary Higgs to a strong sector or by considering a large multiplicity of new fields. All these directions require of a non-perturbative treatment of the new physics scenario which will be explored in an independent work.

The present investigation allows us to identify specific configurations that merit further discussion. One is that which exactly cancels the power-law part of the flow, $\partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}]|_{\bar{\rho}=0} = 0$ in (3.12), at the UV scale. In Figure 3.16, this occurs for $c_2 \simeq -10^{-2}$, $c_3 \simeq 0.2$, and $c_4 \simeq 4 \cdot 10^{-2}$. This particular solution is known as the *Veltman condition* [229], which gained relevance with SUSY extensions, as it marks the respective symmetry-breaking scale. Other interpretations for the energy scale at which the Veltman condition is satisfied have been proposed; see [230–232] for examples related to the appearance of quantum gravity and [233, 234] for applications in BSM extensions. In Section 3.3, we will discuss how this condition, in fact, reflects a change in the curvature of the effective potential and leads to the emergence of UV phases of the Higgs potential [6].

Another particular solution is the one that places the theory on the partial fixed point, which precisely vanishes the *total* flow of $\bar{\mu}^2$ at all scales. As discussed in Subsection 3.1.3, this is known as the massless or critical condition [170, 192, 193], where the Euclidean Higgs mass satisfies $m_{H,k \rightarrow 0}^2 \simeq 0$. This solution effectively realises scale symmetry, as no mass scale is generated at $k \rightarrow 0$ —neither via SSB nor through an explicit Higgs mass term. In the present simplified analysis, this solution is satisfied only at the Λ_{UV} scale for parameter values that approximately also satisfy the Veltman condition. At lower scales, the theory flows away from the partial fixed point along a marginally relevant direction in theory space. Consequently, to achieve a fully scale-invariant solution, the flow of the curvature mass must vanish at all scales. This will be further discussed in Subsection 3.2.3.

Another interesting question to explore in future works is whether the class of SM theories exhibits any regime in its multidimensional phase diagram where the fine-tuning induced by the canonical dimension of $\bar{\mu}^2$ is reduced without invoking new physics. This question was recently investigated in [146], where the interplay between the chiral symmetry breaking mechanisms (d χ SB and the Higgs mechanism) was studied within a unified framework. It was demonstrated that increasing the QCD scale leads to a reduction in the sensitivity to UV scales and hence to

more natural theories. This is achieved as when drawing both chiral symmetry breaking scales together, the corrections of the strong sector on the Higgs mass reduce its canonical relevance. This example highlights that intriguing possibilities can emerge within the theory space, even purely within the SM.

To conclude, we recall that we have been assessing the deformations of the SM guided by the principle of naturalness and the sensitivity of the EW scale to UV scales, primarily motivated by the hierarchy problem. Nonetheless, one important question remains unaddressed: why is the SM realised so close to the critical condition that separates theories exhibiting EWSB from those that do not? Extensions of the SM could shift the physical point farther from the critical solution, thereby also alleviating this issue. While this particular puzzle has not been the focus of our analysis (nor that of others), the same methodology could be applied.

3.2.3 Predictivity as a guiding principle

We close this Section with a discussion on a different but popular direction towards new physics: predictivity. Here, we briefly review relevant studies and contextualize the benefits of the present approach in this realm. Specifically, we discuss how a scale-invariant SM could be realised and how it fits into the broader context of fine-tuning and criticality. While lacking the complete analysis here, we focus on the conceptual framework and refer to [7] for the concluded study.

We may seek theories that are described in a more predictive manner, requiring fewer free parameters and thus achieving a more minimal formulation. A clear example of such a scenario is provided by symmetries. For instance, let us consider a theory where, in principle, all possible parameter choices in theory space are allowed, but only specific ones satisfy a given symmetry. When this theory is upgraded to include quantum corrections, the symmetric configurations manifest as distinct structures on the phase diagram. For example, if quantum corrections respect the symmetry (i.e., it is not anomalous), these configurations appear as the partial fixed-point solutions previously discussed. Hence, as long as the symmetry is preserved at the quantum level, symmetric theories remain on the respective partial fixed points at all scales. Furthermore, symmetries reduce the dimensionality of theory space as the freedom of choosing a parameter is constrained by a condition imposed on a different one.

It is important to emphasize that not all partial fixed-point solutions are associated with *classical* symmetries of the theory (e.g., discrete symmetries or scale invariance). However, partial fixed points are even more powerful than symmetries, as they not only incorporate them but also provide additional solutions with enhanced predictivity [170].

Pendleton and Ross [235] were the first to exploit such special solutions to obtain physical predictions. They used fixed-point-like solutions in the flow of gauge and Yukawa couplings to try predict the mass of the top quark [235–239]. The flow of the ratio between the strong and

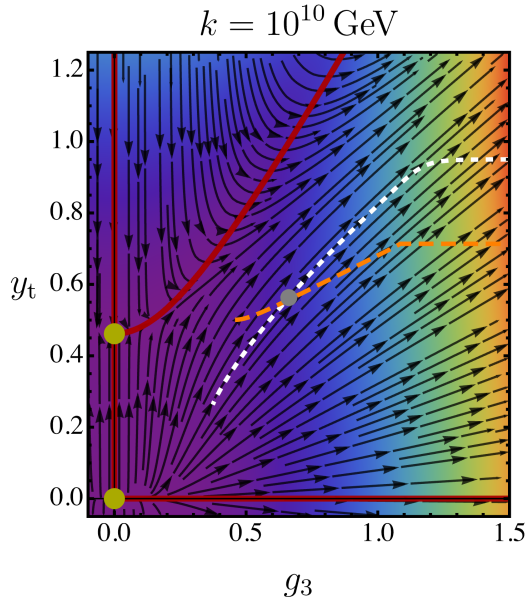


Figure 3.17: Phase diagram of the SM class of theories projected onto the plane of top Yukawa and strong gauge couplings. The white dashed line depicts the SM trajectory, and the orange dashed line, the predicted from the Pendleton-Ross fixed point, (3.41). The red lines and yellow dots depict partial fixed points as in Figure 3.5.

top Yukawa couplings, $r = y_t/g_3$,

$$\partial_t r = r \frac{\partial_t y_t - \partial_t g_3}{g_3} \approx \frac{g_3^2 r (9r^2 - 2)}{32\pi^2}, \quad (3.40)$$

has a fixed-point solution,

$$\partial_t r = 0 \rightarrow r^* = \sqrt{2}/3, \quad (3.41)$$

in addition to the Gaussian one. At this configuration of couplings in theory space, a more predictive theory can be realised since the fixed point acts as a *quantum scale symmetry* relation between the strength of the gauge and Yukawa couplings. In other words, the Yukawa coupling can be predicted by measuring the strong coupling and imposing the fixed-point condition in (3.41). We depict this in Figure 3.17 where we have also included for the contributions from other SM sectors not shown in (3.40) which make the line deviate from a constant relation. We see that the physical SM trajectory and the one predicted by (3.41) are qualitatively similar but the latter renders a much lighter top mass.

This approach was shortly after generalized by Zimmermann and dubbed *reduction of couplings* [240, 241]. It was used to provide predictions for the Higgs and top masses prior to their discovery [242–244], and since then, this idea has been applied as a guiding principle towards new physics across a broad range of settings, see [245] for a review.

With these examples, it is clear that the previously introduced partial fixed points are not the only kind of solutions present in theory space as others may be hidden in relations between couplings. Here we will not delve into the search of such solutions but focus on the realisation of a particular one. Moreover, we note that for the investigation of this type of solutions in the plane of marginal couplings, perturbative beta functions can suffice for the purpose. On the other hand, the mass-dependent approaches such as the fRG are beneficial for the analysis in the presence of thresholds (as degrees of freedom may decouple) and of dimensionful quantities.

Particularly, accounting for the power-law part of the flow allows for the identification of the trajectory that renders scale invariance, namely the massless condition in (3.27). We have learned from the analysis in Subsection 3.1.3 and Figure 3.5 how this trajectory is relevant as it delimits two different IR phases and that the vanilla SM trajectory does not fulfil such a condition. In the case of seeking its fulfilment, new physics is needed to realise the known IR within a scale-invariant embedding. Furthermore, the existence of a trajectory that satisfies such a condition for all scales is not always ensured [191] and is only possibly achieved for certain configurations of the remaining couplings in the theory, as $\bar{\mu}^2(\bar{\lambda}, \bar{g})$ is now a prediction.

While power-law contributions are well-known to not trigger scale-invariance breaking [246] as the trace anomaly is only generated by logarithmic corrections, they carry the information about the position of the critical surface and namely that condition which flows into the massless theory [192]. This scale-invariant solution can be checked to fulfil the respective quantum Ward-Takahashi identities (or rather their modified version in the presence of a regulator), namely the Callan-Symanzik equation [247, 248], along the RG flow. In the fRG, this reduces to the vanishing of the flow equation as shown in (3.27). In fact, the trajectory on the partial fixed-point is the only one not displaying any power-law scaling but only logarithmic along the RG flow.

In order to render the SM scale invariant, the theory needs to be placed on the massless condition (critical surface) for all scales, and EWSB must be generated by the SSB of scale symmetry. This can be achieved in several ways. One option involves the generation of a non-trivial minimum of the Higgs potential via the effect of higher-order scalar operators. In this case, the curvature mass is positive on the partial fixed point along the RG flow, and the scale-symmetry breaking vacuum is generated in the configuration of marginal and irrelevant couplings, for example by $\bar{\lambda} < 0$ and $\bar{\lambda}_6 > 0$ (being the dimensionless H^6 operator dressing). This path follows the line of the Coleman-Weinberg mechanism [120] and has been investigated in different contexts, see e.g., [85, 191, 249].

Scale-invariant realisations of the SM have been studied in a broad range of settings [246, 250–254] using perturbation theory. Given that this solution is based on respecting scale symmetry, it is convenient to employ a regularisation procedure that minimizes its breaking. Perturbation theory along with dimensional regularisation fulfils this purpose and grants easy access by tuning onto the critical surface by subtracting the power-law contributions. The classically-scale-invariant

condition is then satisfied by imposing $\bar{\mu}_{\Lambda_{UV}}^2 = 0$, which, in the absence of any power-law contributions, does not generate any non-vanishing curvature mass along the RG flow. In this scenario, EWSB and the Higgs' v are commonly generated following the Gildener-Weinberg mechanism [255] (in the lines of Coleman-Weinberg), which requires the inclusion of an additional scalar field. The interplay between all classically scale-invariant scalars generates, via the quartic and portal couplings, a non-trivial shape in both the Higgs and new scalar field directions. This approach appears especially useful, both technically and computationally while it does not require any intel on the position of the massless condition in theory space.

Furthermore, a qualitatively different direction to trigger SSB of scale invariance is possible. The partial fixed-point solution related to scale invariance can naturally trigger a non-trivial shape of the Higgs potential, this occurs if such a condition induces a change in the sign of the curvature mass. For this to happen, it is necessary that the IR partial fixed point of dimension 2 in the plane of Higgs parameters is found in the quadrant of theory space with a non-trivially stable effective average potential (top left in Figure 3.5) at scales approximately those of $k_{\text{SSB},1}$. More concretely, the partial fixed-point solution should display SSB at approximately the measured values known in the SM, namely at $\bar{\lambda}_{k_{\text{SSB},1}} \approx 0.13$, following present experimental observations.

The different approaches to realise SSB in scale-invariant theories appear to be more or less constraining. On one hand, the standard approach following the Coleman-Weinberg path is agnostic to the massless condition and enjoys relative freedom in model construction. On the other hand, if we want to realise SSB without invoking higher-dimensional operators and purely by arranging the massless condition to induce a change in the sign of the curvature mass, the scale-invariant condition is highly restrictive with strong predictive power.

Concerning the scheme choice, once one is chosen, the critical surface position is unambiguously determined. Nonetheless, as previously stated, its position has a scheme dependence manifested by the regulator sensitivity [195] of the power-law contribution, which acts as a coordinate choice. While we have shown how mass-independent schemes absorb such information into the counterterms, making it less accessible, the discussion in previous sections evidences how physical information can be extracted from them, such as the critical temperature of the EW phase transition. Therefore, in this sense, the scheme choice can be exploited for the purpose of the analysis. Here, we are interested in keeping track of the power-law contributions and their effects on the massless condition such that we can identify a particular realization of scale invariance. For this, we use the fRG scheme with the Litim cutoff which we have confirmed to reproduce physical features not really apparent in other schemes.

Recalling the analysis in Subsection 3.2.2 and Figure 3.15, the massless condition could be imposed at the physical point with deformations of the kind $c_2|_{k_{\text{SSB},1}} \simeq -0.02$ (see the grey point in Figure 3.18). These can technically be achieved with additional scalar fields coupled to the Higgs through a portal interaction, as shown in (3.39) and Figure 3.15. Here, we do not construct

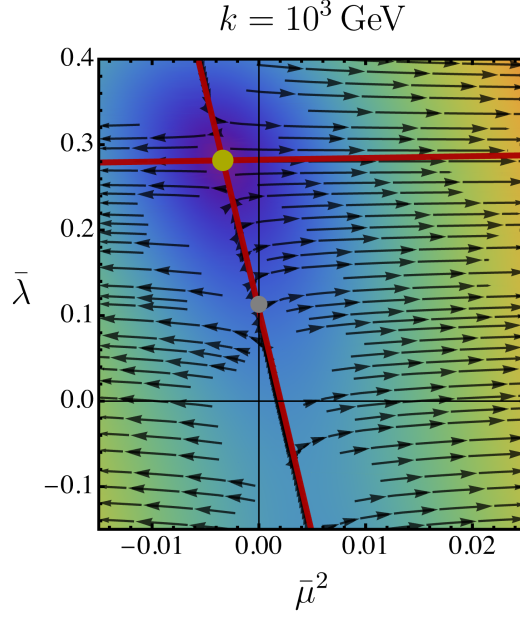


Figure 3.18: Phase diagram of the SM Higgs couplings evaluated at $k = k_{\text{SSB},1} \simeq 1 \text{ TeV}$ and considering a deformation of $c_2|_{k \approx k_{\text{SSB},1}} \simeq -0.02$. At this scale, the extended SM position (grey dot) undergoes SSB of scale invariance.

a particular model but focus on its qualitative realisation. In Figure 3.18, we depict the result after varying the phase diagram with the indicated deformation. We stress that to render the SM scale-invariant, a deformation is required to position the theory on the partial fixed point for all scales and additionally, the new physics extension must satisfy that SSB occurs at the physically measured IR, leading to the condition:

$$\left. \frac{\partial_{\bar{\rho}} \overline{\text{Flow}}[V_{\text{eff}}] |_{\bar{\rho}_0=0} + c_2}{2 - \eta_H - c_1} \right|_{k \approx k_{\text{SSB},1}} = 0, \quad (3.42)$$

derived with (3.27). Provided that the denominator does not become singular, this constraint primarily acts on the power-law contribution, analogous to the Veltman condition [229] discussed earlier. Given that we have not specified the origin of the c_i 's, the fulfilment of (3.42) directly follows from the measured values of the Higgs' vacuum expectation value and mass, and sets a condition on the model-building extension.

We note in passing that, given the structure of the phase diagram in this setup, *precondensation* phenomena in the Higgs potential [187, 199] could be expected. The Higgs potential could enter the broken phase, and bosonic fluctuations could induce a restoration of its shape.

It is interesting to reflect on how scale invariance connects to the previous fine-tuning and hierarchy problem discussion. The scale-invariant extended SM could be thought of as a highly specific theory among many others that do not respect this symmetry and hence be considered

finely tuned. However, imposing the symmetry reduces the dimensionality of theory space, and only this subset should be considered when making a tuning analysis. Consequently, we should only vary over theory space that respects the partial fixed-point condition in the $\bar{\mu}^2$ direction for all scales. This subset of theories is not subject to the effect of power-law contributions, and hence the tuning comes only from the marginal directions of theory space. In this sense, the hierarchy problem is solved.

We close on a different note by pointing out that scale-invariant extensions of the SM may additionally relax the criticality question. The control parameter in this new class of theories will be provided purely by marginal couplings, such as a portal coupling between the Higgs and a new scalar singlet [255]. As with the fine-tuning discussion, this brings a much milder dependence of the order parameter on the couplings at a more fundamental scale. While we still expect a quantum phase transition, this will probably be of a qualitatively different nature. To address this interesting question, it is necessary to consider a specific model, which will be done in an independent work.

3.3 Ultraviolet phases of the Standard Model Higgs potential

This Section is based on [6].

So far, we have discussed the properties of the SM as a LEFT below the Planck scale, focusing on features manifestly apparent in the fRG implementation. In particular, we have reviewed phenomenological aspects such as the metastability of the Higgs potential, the appearance of SSB along the RG flow, and the presence of a quantum phase transition in this class of theories. In this Section, we extend the analysis beyond the Planck scale into an uncharted regime, exploring the properties of the Higgs potential within the present approach.

To investigate further properties of the SM as a QFT, we analyse the interplay of different sectors and discuss the coupling profiles above k_{Planck} . In this exploration we neglect the effects of quantum gravity or equivalently assume a weak gravity scenario, where quantum gravitational effects do not qualitatively alter the matter sector. Furthermore, in the following Subsections, we demonstrate that when well-motivated new physics extensions are considered, the effects discussed here manifest below the Planck scale.

Uncovering the flows above the Planck scale, we find that $\bar{\mu}^2$ turns negative at $k_{\text{inst}} \approx 2 \cdot 10^{26}$ GeV, see Figure 3.19. This signals that the curvature of the Higgs potential at small field values turns negative, which, in combination with a negative quartic coupling, renders an unstable shape ((D) in Figure 3.19).

One may speculate that as this phase is reached, the Higgs field evolves towards infinite (or very large) field values, causing the decoupling of all degrees of freedom except the photon and gluons. Consequently, due to the Abelian nature of g_1 , all couplings except g_3 approach a non-trivial fixed point as $\bar{\mu}^2$ reaches $(\bar{\mu}^2)^* = 0$. This provides a UV complete picture where the SM emerges from an asymptotically free SU(3) Yang-Mills theory. This behaviour is depicted in Figure 3.19 above k_{inst} by dashed lines.

This unstable potential phase comes along with severe consequences that question the validity of the SM in this minimal approximation. First, the ground state of the Higgs field is not well-defined, and hence neither is the theory. Second, as $\bar{\lambda}$ does not return to positive values, this scenario leads to a ruled-out EW vacuum decay probability [30, 33, 171, 173, 175, 256–259]. Last, recalling that the potentials displayed in the top panel of Figure 3.19 can be conceptually understood as the thermal phase evolution, the transition from an unstable to a metastable potential cannot be realised without a $\bar{\lambda} \geq 0$. Altogether, these findings point out that the phenomenological and formal validity of the SM could be at stake at scales way below the Landau pole.

Just for illustrative purposes, we may assume that the Higgs field remains at $H = 0$ along the unstable regime, so that the flows can be continued towards higher energies. This phase spans over

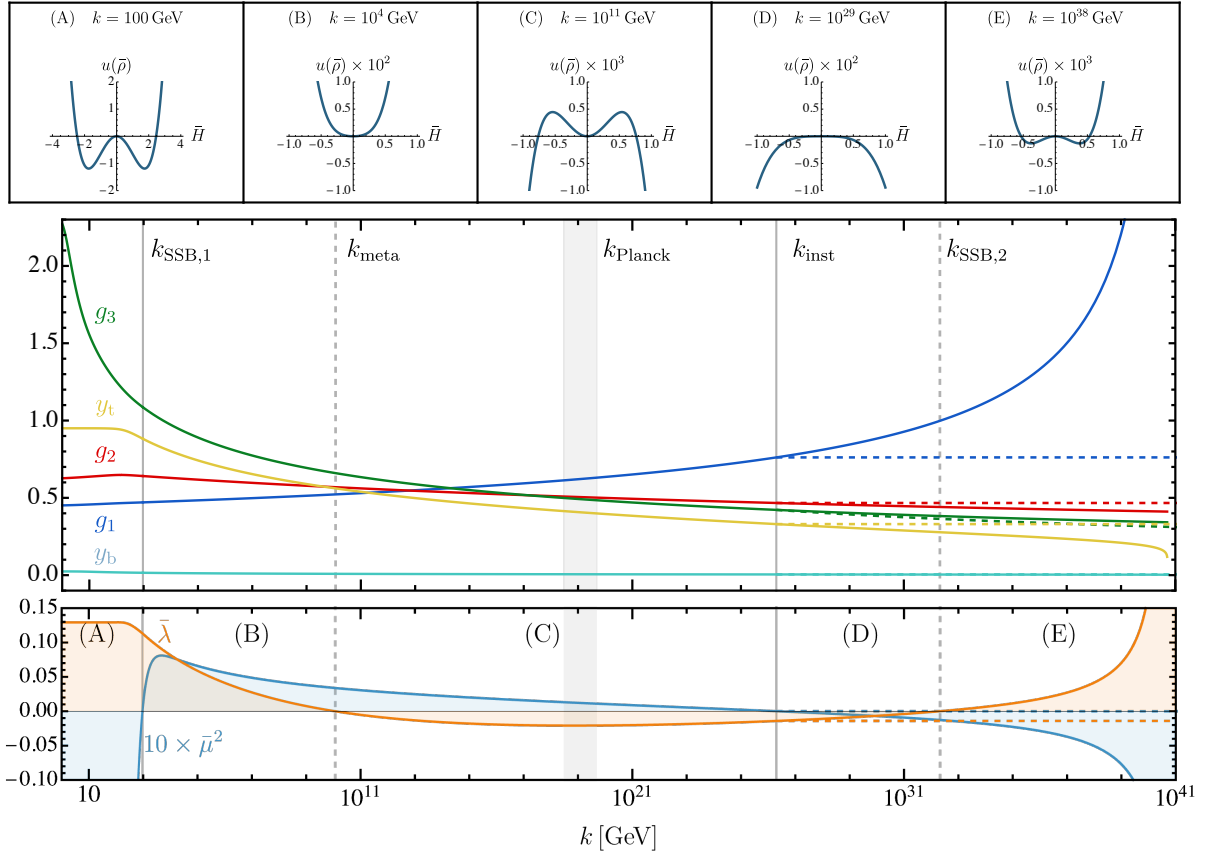


Figure 3.19: Extension of the physical trajectory for the SM couplings, shown in Figure 3.2, up to the Abelian Landau pole scale. Two additional UV phases of the effective average potential emerge above the instability scale (k_{inst}) and the second UV SSB scale ($k_{\text{SSB},2}$), as illustrated by the two right-most potentials at the top. The dashed trajectories above k_{inst} represent the decoupling scenario discussed below and in Subsection 3.3.3.

six orders of magnitude until the quartic coupling returns to positive values at $k_{\text{SSB},2} \approx 2 \cdot 10^{32}$ GeV rendering a stable shape with a non-trivial minimum (potential (E)). This non-trivial potential gives rise to gauge boson and fermion masses in a controlled and well-defined manner and as will be discussed later on in Subsection 3.3.3, could provide a cure for the Landau pole.

It is important to stress that for these results we have employed a polynomial Higgs potential as in (3.3) only containing renormalisable operators. We note that an unstable shape of the potential at large field values could be resolved by including higher order operators such as $\bar{\rho}^3, \bar{\rho}^4, \dots$ in the expansion [1, 85, 97, 143, 144]. In the absence of a UV completion, the respective couplings are free, leading to a highly complex analysis beyond the scope of this work and which will be addressed in an independent work. The addition of these is however not expected to change the behaviour of the well-converged small field curvature, given that this is dominantly determined

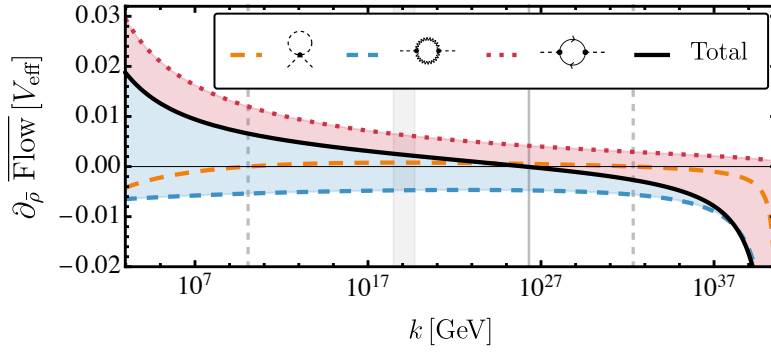


Figure 3.20: Total (solid black line) and partial (dashed lines) contributions in the power-law flow of the Higgs curvature mass, see (3.43). We separately show the contribution of scalar (orange), gauge (blue) and fermion (red) loops. Vertical lines delimit the different phases as in Figure 3.19.

by gauge and Yukawa effects, which is at the centre of our analysis. Beyond that, the potential relevance of higher terms highlights that the decoupling (dashed) and the illustrative $H = 0$ (plain) scenarios can be seen as two limiting cases of a well-behaved and stabilised setup, with large and small background value, respectively. Crucially, the presence of a new UV phase due to the zero in the quadratic term is a robust and new observation (independent of the presence of higher terms). As mentioned, further below we will show how the same physics, embedded in a slightly modified new physics scenario, leads to the emergence of the new stable phase below the Planck scale, without intermediate instabilities and thereby avoiding such subtleties at large field values.

3.3.1 Ultraviolet zero-crossings of the curvature mass

To understand what triggers a $\bar{\mu}^2$ zero-crossing which leads to the emergence of new phases, we analyse the contributions in the flow (3.12). These can be separated in two: the one proportional to the curvature mass and the one independent of it. The former is driven by the canonical scaling and the anomalous dimension of the Higgs field. Importantly, these corrections drive the curvature mass asymptotically towards zero but do not induce a zero-crossing. For this, a negative contribution independent of $\bar{\mu}^2$ must be present in (3.12). This, in fact, appears in the quadratic divergence of the Higgs two-point function flow which in (3.12) is shown as the $\bar{\rho}$ -derivative of the effective potential flow. The leading contributions in the Landau gauge read

$$\partial_{\bar{\rho}} \overline{\text{Flow}} [V_{\text{eff}}] \supset \frac{3 y_t^2}{8 \pi^2} - \frac{9 (g_1^2 + 5 g_2^2)}{320 \pi^2} - \frac{3 \bar{\lambda}}{8 \pi^2 (1 + \bar{\mu}^2)^2}. \quad (3.43)$$

Each term in (3.43) and its total are depicted in Figure 3.20. The Yukawa-mediated diagram contributes positively to supporting the symmetric phase while the gauge contributions drive

towards negative values to induce SSB. Given that the $\bar{\mu}^2$ -dependent contributions already drive $\bar{\mu}^2$ close to zero, at $k_{\text{inst}} \approx 10^{26}$ GeV the potential becomes unstable as the total $\bar{\mu}^2$ -independent contribution in (3.43) turns negative. We can determine the $\bar{\mu}^2$ -zero crossing scale $k_{\text{cross}} = \{k_{\text{inst}} | k_{\text{SSB}, 2}\}$ analytically via the condition

$$y_{t,k}^2|_{k_{\text{cross}}} \simeq \left[\frac{3}{40} (g_{1,k}^2 + 5 g_{2,k}^2) + \bar{\lambda}_k \right] \Big|_{k_{\text{cross}}} . \quad (3.44)$$

As the total gauge contribution is approximately constant over all scales and the top Yukawa coupling weakens, a $\bar{\mu}^2$ -zero crossing towards negative values is hardly avoidable. Importantly, this equality is generally satisfied at scales where all couplings are perturbative and hence the one-loop structure suffices to determine k_{cross} . For this reason, further zero crossings to positive $\bar{\mu}^2$ are unlikely to occur.

It is essential to stress that given the irrelevance of the threshold effects in (3.43) and (3.44) the results here discussed hold independently of the regulator choice and are common to any Callan-Symanzik-like scheme [247, 248], where the regularization procedure suppresses modes in a mass-like fashion.

3.3.2 New phases and need for new physics

We have discussed how a $\bar{\mu}^2$ -zero crossing is inevitable in the pure SM framework and therefore a negative quartic coupling not only leads to a metastable but also to a potentially severely problematic unstable potential. Therefore, addressing the meta/instability of the Higgs potential becomes a highly relevant task. The main negative contribution to the flow of the effective potential is sourced in the large top Yukawa coupling measured in the IR. As the fermionic loops screen $\bar{\lambda}$, decreasing y_t leads to a stabilisation of the potential. Given that the gauge coupling flows are insensitive to these variations, such an approach to a stabilisation of the Higgs potential causes a $\bar{\mu}^2$ -zero-crossing at much lower scales. This investigation is summarised in the left plot in Figure 3.21. Although the top quark pole mass measurement [260] shows the largest uncertainty in all SM parameter measurements, the physical trajectory leads confidently to an unstable regime. For a $m_{\text{top}}^{\text{pole}} = 169$ GeV (2% smaller than the measured value) the quartic coupling is always positive and the $k_{\text{SSB}, 2}$ scale has decreased four orders of magnitude. Moreover, for a $m_{\text{top}}^{\text{pole}} = 166$ GeV (4% smaller than the measured value) $k_{\text{SSB}, 2} < k_{\text{Planck}}$ and of order of inflationary scales.

New physics beyond the SM can provide an explanation to standing open problems and additionally stabilise $\bar{\lambda}$ [33, 137, 169, 181, 259, 261–263]. As an example, we may consider a minimal extension with a real singlet scalar S , coupled only to the Higgs field via a portal coupling [264–266]

$$\Delta u(\bar{\rho}, \bar{\rho}_S) = \bar{\lambda}_{\text{HS}} \bar{\rho} \bar{\rho}_S , \quad (3.45)$$

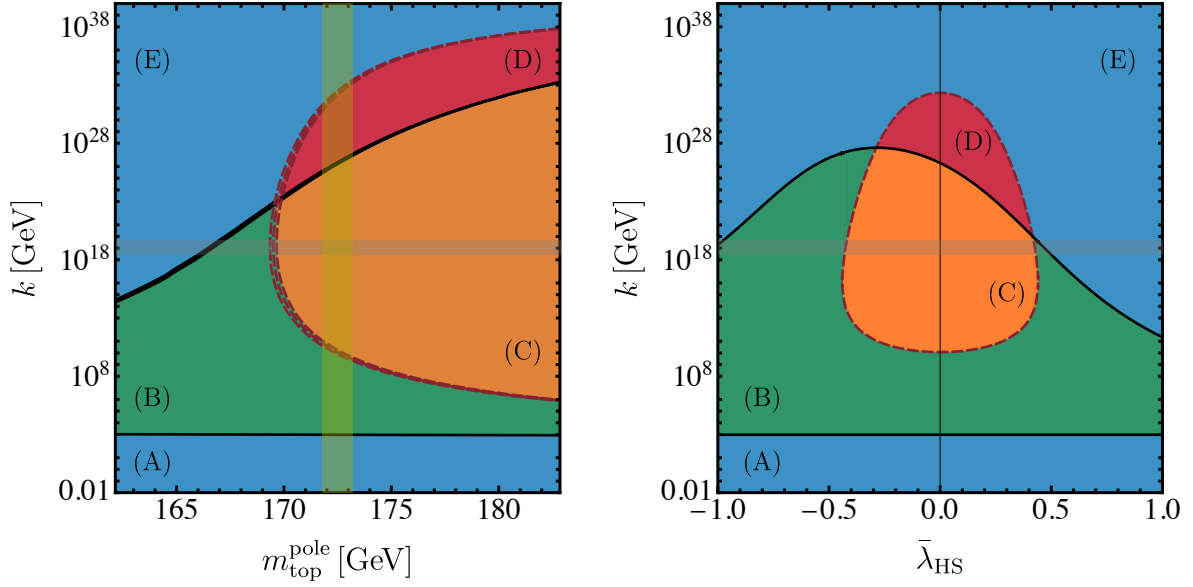


Figure 3.21: Left plot: phase diagram of the SM Higgs potential as a function of the cutoff scale and for different top quark pole masses. The plain black (dashed red) lines mark the scales at which the curvature mass (quartic coupling) vanishes. The line thickness accounts for $\pm 3\%$ variations in the Higgs pole mass. The blue (green) region depicts the regime in which the average effective Higgs potential is stable and has a non-vanishing (vanishing) minimum. The red (orange) region denotes the unstable (metastable) regime. The Planck scale is marked by an horizontal grey-shaded region and the experimental uncertainty of the top quark pole mass [260] with a vertical yellow-shaded band.

Right plot: analogous phase diagram but considering an additional singlet scalar field coupled to the Higgs via a portal coupling ($\bar{\lambda}_{\text{HS}}$) as in (3.45).

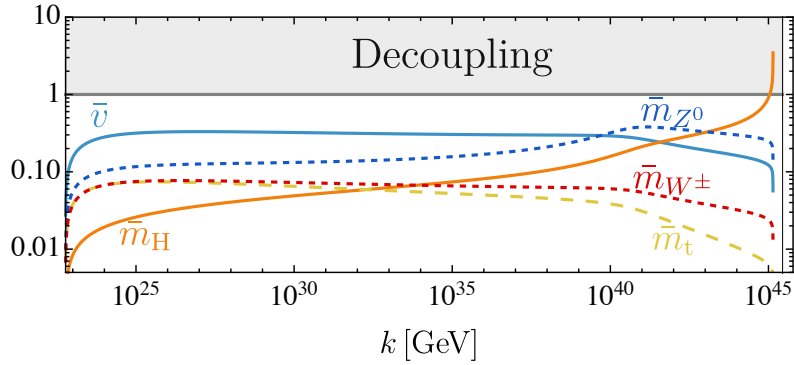


Figure 3.22: Scale dependence of the dimensionless $\bar{v}$ (plain blue) and Higgs (plain orange), Z^0 (blue dotted), $W^\pm$ (red dotted) and top quark (yellow dashed) Euclidean masses above the k_{cross} scale for a stable SM-like trajectory with a slightly lighter top quark pole mass of 169.4 GeV. The grey shaded region depicts the decoupling condition in (3.46) necessary for UV completeness.

where $\bar{\rho}_S = Z_S S^2/2$. Being agnostic about the singlet field potential, we set different values of the portal coupling above the threshold of the singlet mass scale assumed to be of the EW scale. In Figure 3.21, we summarise the effect of this minimal extension on the phase structure of the Higgs potential. For $|\bar{\lambda}_{\text{HS}}| \gtrsim 0.4$ the positivity of $\bar{\lambda}$ is ensured at all energy scales. As the corrections to the Higgs 2-point function are linear in $\bar{\lambda}_{\text{HS}}$, there is no symmetric dependence in k_{cross} . For positive couplings, the new UV broken phase emerges at $k_{\text{SSB}, 2} < k_{\text{Planck}}$ and for negative portal couplings this is pushed to higher energy scales. Moreover, for $-0.4 \lesssim \bar{\lambda}_{\text{HS}} \lesssim -0.3$ trajectories with metastable but no unstable regions can be found.

3.3.3 Towards a fundamental Standard Model

We have shown that the SM trajectory leads to an unstable potential which could either mean that the theory is ill-defined, and hence, physically unviable or further minima exist but cannot be resolved in the current lowest-order polynomial approximation. In this last case and in stable SM trajectories as the ones discussed, the only impediment against formulating a fundamental theory valid at arbitrarily large energy scales is the presence of a Landau pole in the Abelian coupling. This divergence is core-rooted in the quantum corrections to g_1 mediated by massless fermions and bosons. This strong regime will now be approached in the presence of a non-trivial $\bar{v}$ and where, as in the IR, the relevant degrees of freedom are properly described by a massless photon and massive Z^0 , $W^\pm$ and fermion fields.

The UV non-trivially stable regime discussed in this work provides a natural way to cure the Landau pole and formulate the SM as a fundamental theory. To avoid this divergence, the quantum corrections to the electric coupling must vanish or become negative, achieving

asymptotic safety or freedom. The first case is realised with the decoupling of all electrically charged fermionic fields as the quantum fluctuations of momenta k being integrated out are smaller than the Euclidean masses of the off-shell particles propagating, $m_{\psi_i} > k$. As the fermion masses are generated through the Higgs mechanism, this constraint translates into the UV fixed point condition

$$\bar{v}^* y_i^* / \sqrt{2} > 1. \quad (3.46)$$

In Figure 3.22 we display as an example, the scale dependence of the dimensionless masses and the flowing minimum for a stable SM-like trajectory where $m_{\text{top}}^{\text{pole}} = 169.4 \text{ GeV}$. Although the potential develops a $\bar{v} \approx 0.3$, this trajectory does not suffice to trigger the decoupling of any of the fermionic fields as all Yukawa couplings also remain small. However, the presence of masses over many orders of magnitude leaves an imprint on the couplings which is noticeable in the Landau pole scale being delayed by five orders of magnitude from the scenario with $\bar{v} = 0$. Nevertheless, for a conclusive resolution of the evolution along the strong Abelian regime and its impact on the other sectors, an improved truncation is necessary. Besides, (3.46) is trivially satisfied if $\bar{v} \rightarrow \infty$ as shown in Figure 3.19. In the case of further hidden minima unresolvable in the current polynomial approximation, the question remains open.

3.3.4 Wrap-up of the Higgs ultraviolet phases

In this Section, we have studied the SM trajectory at all accessible energy scales using the fRG. The treatment of the Higgs mass parameter with this Wilsonian RG allows the evolution of the full effective Higgs potential along the RG flow and to trace occurrences as SSB.

In the extrapolation of the SM trajectory from measured values, the Higgs potential undergoes previously unknown phases: an unstable and a stable with a non-trivial minimum. The scale where the curvature mass shows a zero crossing is triggered by the gauge corrections overtaking the top Yukawa's and can be determined by a simple equality (3.43). This crossing occurs where $\bar{\lambda} < 0$ and leads to an unstable potential. While this finding seems to invalidate the SM trajectory at higher energy scales and signals the need for new physics, the possibility of further minima unresolvable in the current polynomial approximation of the Higgs potential exists.

After the appearance of [6], a similar kind of phase transition has been reported in [199], where theories involving two types of scalar fields with $O(N) \times \mathbb{Z}_2$ symmetries in $2 + 1$ dimensions, including finite-temperature effects, have been studied. It is shown that the transition from the symmetric to the broken phase, analogous to the one discussed in Subsection 3.3.1, is driven by the interplay between scalar contributions and power-law corrections to the flow of the curvature mass. These findings align well and support the discussion in the present Section.

We have studied the SM phase diagram for different top quark pole masses and considering new physics in the form of a Higgs-scalar portal coupling. The new UV non-trivially stable phase

appears below the Planck scale in scenarios seeking stable trajectories. This phase provides a potential UV-complete formulation to the SM and similar theories via the decoupling of fermions. These findings have a great impact on existing approaches to new physics and suppose a new look at the high-energy structure of the SM.

3.4 Beyond the Planck scale:

The Asymptotically Safe Standard Model

This Section contextualises and collects publication [1].

So far in this Chapter, we have explored the SM as a LEFT and targeted its deep UV behaviour in Section 3.3, attempting to formulate its completeness. Nonetheless, we have ignored one of the major shortcomings of the SM: it lacks to include the remaining fundamental force known to be present in nature, gravity. From the measured gravitational strength at low energies, namely the Newton coupling $G_N = M_{\text{Planck}}^{-2}$, quantum gravitational effects are expected, merely from canonical scaling, to become relevant at energy scales near the Planck mass, $k_{\text{Planck}} \approx M_{\text{Planck}} \approx 10^{19}$ GeV.

However, addressing the quantum nature of gravity is a particularly challenging task. Observational prospects to illuminate this issue are not expected soon due to the manifestation of sizeable quantum effects at extremely short length scales unreachable by present or near future experiments. Therefore, various approaches which consider vastly different foundational grounds are currently under investigation, ranging from string theory, loop quantum gravity to asymptotic safety.

On the other hand, when assuming that quantum gravity can be described by upgrading its classical action to its quantum analogue, additional technical challenges arise. The theory is not perturbatively renormalizable, meaning that standard techniques fail to yield predictive results. However, this does not imply that the theory is fundamentally ill-defined but rather that it requires a non-perturbative treatment to uncover its consistent formulation.

Asymptotically safe quantum gravity (ASQG) [56–58] is a promising approach to quantum gravity which formulates the theory as a QFT with a dynamical metric. It relies on the existence of a UV fixed point that respects diffeomorphism and quantum scale invariance but renders the theory UV complete, fundamental, renormalizable, and predictive. Although gravity is power-counting non-renormalizable due to the canonical dimensionality of the Newton coupling, the existence of a finite number of relevant directions at the UV fixed point demonstrates its non-perturbative renormalizability [66, 76].

Determining the UV fixed point requires non-perturbative techniques, such as the provided in the fRG approach widely employed in this thesis. The number of relevant directions, namely the dimensionality of the critical surface, has been extensively studied in various approximations of the full gravitational action [58, 267–276], as well as in considering various kinds of matter content [100, 103, 103, 228, 277–290]. Currently, there is solid evidence for only three relevant directions.

We close this Chapter with the publication [1] where the complete SM coupled to ASQG was self-consistently derived and studied for the first time, realising the *Asymptotically Safe Standard*

Model (ASSM). This work collects several results across different sectors. In the following we provide a summary of the main findings and refer the interested reader to the manuscript printed below in Appendix A.

Matter sector: This work presents the first complete implementation of the SM within the fRG, collecting the following noteworthy features:

- ◊ We derived the flows of all SM couplings, excluding the CKM matrix parameters and the QCD θ_{scP} angle, within a vertex expansion truncation and resolved its trajectory from deep IR to transplanckian scales. This self-consistent framework incorporates existing functional results from $N_f = 2 + 1$ flavour QCD [97, 291] with the respective colour confining and chirally breaking dynamics.
- ◊ We studied in detail the different sectors of the SM, including the identification of Yukawa (Goldstone and Higgs) couplings, the consistent implementation of the strong gauge sector and their influence on the Higgs metastability scale.
- ◊ We provided, for the first time, a detailed scale-setting analysis for the SM theory in the fRG scheme. In particular, we derived the top-quark pole mass which plays a critical role in the UV profile of the SM flows.

Gravity-matter sector: The study of gravity-matter systems is one of the most important directions to test the validity of ASQG. One of the unique and noteworthy features of this quantum gravity approach is its ability to be benchmarked against QFT of matter known to be present in nature. This work significantly advances in this direction providing new technical and conceptual developments:

- ◊ First, we computed the gravitational corrections to all matter sectors of the SM. This was achieved with a vertex expansion truncation within the fluctuation approach, which allows for a systematic improvement of the results.
- ◊ With this computational setup, we studied the explicit momentum dependencies of gravitational corrections to gauge and Yukawa couplings. This is a significant qualitative and quantitative improvement over previous existing results which incorporated effects via the explicit consideration of higher-dimensional operators.
- ◊ Beyond the momentum dependencies, we also analysed the regulator dependence as well as the consideration of relative cutoff scales in the gravity and matter sectors. With these, we support the delicate existence of the ASSM.
- ◊ We studied in detail the graviton contributions to the Higgs potential. To this end, we considered a polynomial expansion up to order H^{34} and also derived a global analytic solution of the field dependence of the respective UV fixed-point potential.

- ◇ Our results indicate that the UV Higgs potential is non-trivially flat. It is parametrized by two relevant directions and exhibits a flat shape which at asymptotic field values is unbounded from below. This peculiar solution has negative implications in terms of the stability of the present EW vacuum [176]. Further investigation is then required to clarify the severity of this issue which could point out towards the need of new physics along this quantum gravity scenario.
- ◇ Furthermore, we investigated all possible solutions of the UV fixed-point potential, including those with unstable and non-trivially stable shapes as well as those with increased predictivity.
- ◇ The number of relevant parameters in the ASSM matches that of the SM plus the three additional ones associated with the gravitational sector.

This study represents a significant advancement in the quantitative analysis of gravity-matter systems, which is essential for progressing the asymptotic safety programme. Nevertheless, two critical open questions necessitate further investigation: the compatibility of the Standard Model Yukawa sectors with the gravitational fixed point, and the ultraviolet fixed point Higgs potential alongside the lifetime of the electroweak vacuum

See Appendix A for the published version.

NON-PERTURBATIVE NEW PHYSICS

This Chapter compiles three works which exemplify the applicability of functional methods in the study of non-perturbative extensions of the Standard Model. The first Section presents a first-principles study of the landscape of gauge-fermion quantum field theories, which are the foundation of many popular strongly interacting extensions. The second Section addresses the flavour puzzle in Composite Higgs models. Here, we estimate the generation of the fermion masses directly from the fundamental properties of the modes by using the versatile emergent composites approach. The last Section explores the phase structure of extra-dimensional gauge theories, which are of particular interest in addressing the hierarchy problem and the unification of forces.

In the previous Chapter, we studied the SM with the fRG, demonstrating how this non-perturbative approach not only provides a novel and convenient way for examining the deformations of the phase diagram of a multi-parameter QFT but also a framework within which non-perturbative extensions can be tested self-consistently.

This Chapter collects three works that exemplify the exceptional power of the fRG in addressing various aspects relevant for non-perturbative SM extensions. We focus on two of the most challenging directions: strong dynamics and non-perturbatively renormalisable theories. Despite their theoretical intricacy, these directions have been widely employed in different BSM contexts. For instance, strong dynamics such as those present in physical QCD have been used to generate EWSB. Notable examples are CH [44–48] and TC [49–52] models, which replace the Higgs mechanism and boson by phenomena analogous to the emergence of condensates and bound states in QCD-like theories. Additionally, strong sectors are particularly attractive as they provide many interesting DM candidates. This unresolved problem could be explained by the existence of a strong dark sector, either fully or partially disconnected from the SM, whose bound state spectrum (i.e., hadron- or glueball-like states) accounts for the observed DM density in the Universe. These directions have been extensively studied in the context of asymmetric DM [292–301], partially motivated by the observed *coincidence* between baryon and DM densities, $\Omega_{\text{DM}}/\Omega_{\text{b}} \approx 5$. One particularly beneficial feature of such theories is their potential to display phase transitions during the thermal evolution of the Universe, which could source gravitational wave signatures. This opens exciting possibilities for testing the nature of these candidates in current or future gravitational wave observatories.

In summary, many SM extensions involving strong dynamics are built on gauge theories, often including fermions. These are attractive due to their rich dynamics and well-behaved character at a fundamental level. However, they pose significant challenges as simple perturbative approaches fail to capture key features such as $d\chi\text{SB}$. As a result, these models are often constrained to QCD-like dynamics and less predictive low-energy effective descriptions are used for the phenomenology.

Nevertheless, the class of gauge-fermion QFTs offers a vast landscape of theories with diverse dynamics and spectra. In Section 4.1, we will employ the fRG approach to chart the landscape of gauge-fermion systems with different number of colours and flavours for the first time, exploring their phases, spectra, and properties in a complete manner. This provides new insights into previously unexplored regions of theory space and establishes a framework for studying strongly coupled extensions of the SM beyond the QCD-like limit. Additionally, this Section proposes a simplified approach to deriving confining correlation functions, which can be employed in other strongly coupled theories (e.g., gauge-scalar systems) of significant phenomenological interest, see e.g. [302, 303].

While the study in Section 4.1 is general and focuses on foundational theoretical questions, in Section 4.2 we address the more specific issue of flavour hierarchies in CH models. This is done using the emergent composites formalism, inherent to the fRG and introduced in Section 2.3.

Finally, Section 4.3 delves into a different direction, also prominent in BSM model building. Here, we set aside strong dynamics to deepen the understanding of gauge (and gauge-fermion) theories in extra dimensions, where perturbative renormalisability is lost. In five space-time dimensions, gauge theories exhibit an analogous behaviour to gravity in four dimensions, with the canonical dimension rendering them asymptotically safe. Extra-dimensional settings have been widely employed in various contexts of model building, including the unification of forces [53, 304, 305] and the flavour [306–310] and naturalness [54, 55, 219] puzzles. This Section particularly studies the phase structure of extra-dimensional theories and their connection to the four-dimensional limit. This task is crucial for the development of theoretically coherent and phenomenologically viable models.

4.1 Charting the landscape of gauge-fermion theories

This Section is based on [5] and only the main novel findings are presented. For an introduction to the functional approach to gauge-fermion systems as well as details on the truncation we refer the interested reader to the publicly available manuscript.

In this Section we chart the landscape of gauge-fermion theories employing the first-principles fRG approach to gauge-fermion systems quantitatively developed in the context of QCD. See [43] for a review and [81, 82, 97, 98, 168, 311, 312] for quantitative studies of the physical theory. The results here presented generalise this well established formalism to an arbitrary number of flavours and colours. To that end, novel developments in the treatment of confinement have been performed. Here we provide a brief introduction to the approach and discuss the novel findings. Many details of the underlying formalism as well as on the truncation employed are omitted here and we refer the interested reader to the details in [5].

General gauge-fermion QFTs are described by the classical Euclidean action

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} \gamma_\mu D_\mu \psi + \mathcal{L}_{\text{gauge-fixing}} + \mathcal{L}_{\text{ghosts}}, \quad (4.1)$$

where four space-time dimensions are assumed and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c, \quad (4.2)$$

is the field strength tensor of the gauge field A_μ^a with the gauge coupling g and the structure constant f^{abc} of the gauge group. Moreover,

$$D_\mu = \partial_\mu - ig T^a A_\mu^a, \quad (4.3)$$

is the covariant derivative acting on the Dirac fermion field charged under a specific representation of the gauge group with generator T^a where the colour indices of the fermion fields have been kept implicit.

So far, only the interaction term has been defined and the local gauge and the global chiral symmetries remain unspecified. For the gauge group, we could choose between a multitude of symmetries, but for simplicity, in the following we restrict ourselves to $SU(N_c)$ groups as a plethora of quantitative functional and lattice results exist in the QCD limit with smaller numbers of colours and flavours. Nonetheless, the results presented below are straightforwardly generalisable to any other gauge group.

The last two terms in (4.1) account for the gauge-fixing procedure and the ghost part of the action necessary for the subtraction of the redundant modes and the correct quantisation of the theory. Approaches in the continuum limit require a gauge fixing procedure and in this Section we will employ the Landau gauge, see [5] for the explicit form of the full effective action.

On the other hand, for the global flavour symmetry we consider N_f copies of each Dirac pair of fermion and anti-fermion field. As the classical action (4.1) does not include any fermion mass term,

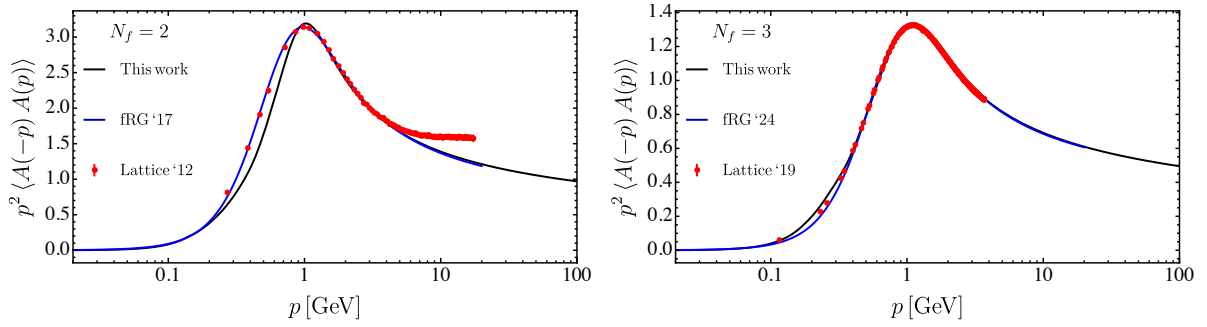


Figure 4.1: Dressing function of the gluon propagator. We compare results for a $SU(3)$ gauge theory with $N_f = 2$ (left plot) and 3 (right plot) flavours in the approximation put forward in [5] (black lines and labelled as “This work”), results from quantitative fRG truncations, [168] ($N_f = 2$) and [98] ($N_f = 2 + 1$) (blue lines) as well as with lattice simulations, [313] ($N_f = 2$) and [314] ($N_f = 2 + 1$) (red data points).

this renders it symmetric under chiral transformations where the respective chiral components can be rotated independently. Altogether, the fermionic sector enjoys a $SU(N_f)_R \times SU(N_f)_L$ chiral flavour symmetry at the classical level.

Upgrading the classical action to its effective analogue introduces the appearance of non-classical dynamics and grants access to correlation functions encoding the physical information of the theory. Gauge-fermion QFTs in four dimension display two characteristic intricate phenomena which require special attention: colour confinement and $d\chi$ SB.

4.1.1 Colour confinement

Confinement is characterised by the absence of coloured asymptotic states and the existence of a mass gap for them (e.g. through the glueballs in QCD). This is well described by a linear potential between colour charges with colour flux tubes respectively connecting them. For a quark–antiquark pair, this linear potential is derived from the expectation value of the closed Wilson loop operator that follows from the operator of a gauge invariant quark–antiquark state. Moreover, it can be shown that confinement is tantamount to an area law [315, 316] for asymptotically large distances with a string tension related to the confinement scale, k_{conf} (Λ_{QCD} in physical QCD), and the gauge field mass gap.

The question of how this picture of confinement can be extracted from correlation functions within a gauge-fixed approach has been extensively studied over decades, both in the vacuum and at finite temperature, see [82, 317–321] for examples. Most importantly, it has been shown that the underlying mechanisms of confinement—including the existence of the mass gap m_{gap} , the linear potential, and the area law—as well as the quantitative values of the respective observables, can be directly deduced from the properties of the propagators of gapped gauge fields. In the

Landau gauge the gluon propagator can be written as

$$G_{A\mu\nu}^{ab}(p, q) = \langle A_\mu^a(p) A_\nu^b(q) \rangle = \frac{(2\pi)^4 \delta(p+q)}{Z_A(p) p^2} \delta^{ab} \Pi_{\mu\nu}^\perp(p), \quad (4.4)$$

with the transverse projection operator $\Pi_{\mu\nu}^\perp(p) = \delta_{\mu\nu} - p_\mu p_\nu / p^2$. The gauge field mass gap is conveniently defined as the *inverse correlation length* or *screening mass* of the two-point function whose information is stored in the dressing $Z_A(p)$. While the propagator is not renormalisation group invariant, the respective correlation length is. The existence of a mass gap in the gluon propagator leads to the behaviour

$$\lim_{p^2/\Lambda_{\text{QCD}}^2 \rightarrow 0} \frac{1}{Z_A(p)} = 0, \quad (4.5)$$

for low exchange momenta which implies a peak of the dressing as it also perturbatively decays (logarithmically) for $p^2/\Lambda_{\text{QCD}}^2 \rightarrow \infty$. While the mass gap is not obtained from the value of the gluon propagator at $p = 0$, it can be determined by its singularity structure in the complex plane, namely the distance of the closed singularity of the gluon propagator to the Euclidean frequency axis (not counting possible cuts at $p = 0$) see e.g. [320, 322, 323]. Moreover, it can be shown that it is gauge-independent, even though it is extracted from a gauge-variant correlation function. In Figure 4.1, the dressing function in the approximation employed here and put forward below (namely that in [5]) is displayed in comparison with lattice and quantitative functional renormalisation group results for $N_f = 2$ and $N_f = 2 + 1$ (3 in the isospin and chirally symmetric limit) flavour QCD. The dressing function peaks at scales approximately 1 GeV and is closely related to Λ_{QCD} . Therefore this feature serves as an easy-access proxy for the gluon mass gap and the confinement scale which we will refer to as k_{conf} .

4.1.1.1 Bootstrap approach to confinement

In modern fRG approaches to gauge theories, confinement and the extraction of confining correlation functions is often treated within a bootstrap approach. While gauge theories have been widely explored since the first derivation of the flow equation [69, 72, 83, 324, 325], this class of theories requires a careful treatment as a mass-like regulator term is introduced which manifestly breaks *apparent* gauge invariance. However, this is not concerning given that apparent breaking is also common to the standard gauge fixing term. Furthermore, gauge symmetry in non-Abelian theories is ensured via the fulfilment of the Slavnov-Taylor identities (STIs) [326, 327], which are equivalent to the generalised Ward-Takahashi identities [328, 329] in Abelian theories. In the presence of a regulator, this contribution can be properly subtracted from the STI as an additional source of gauge-symmetry breaking, leading to the set of modified Slavnov-Taylor identities (mSTIs), see e.g. [69, 72]. Then, in the physical $k \rightarrow 0$ limit, the regulator vanishes, rendering the mSTIs the original STIs. This means that if a solution satisfies the mSTIs at a finite k , at $k \rightarrow 0$ will satisfy the STIs. Actually, this is not always the case given it is not

possible to solve the flow equation exactly. In other words, truncations may induce a deviation from the mSTIs along the integration of the flow. Nonetheless, there exist different approaches to control the deviation from the mSTIs such as optimisation [72, 330], adequate regulator choices, checking for the mSTI deviation along the RG-flow [331], or accounting for physical explicit momentum dependencies. It is also worth noting that the flow equation can be modified in such a way that a gauge-invariant regularisation procedure is performed, leading to a gauge-invariant flow equations. This has been the topic of investigation in numerous studies [332–337], but the resulting expressions appear too complicated for practical uses.

To make this point explicit and illuminate the physical mass gap, let's look at the quadratic part of the gauge effective average action,

$$\Gamma_k = \frac{1}{2} \int_p A_\mu^a(p) \left[Z_{A,k} (p^2 + m_{\text{gap},k}^2) \Pi_{\mu\nu}^\perp(p) + \frac{1}{\xi} Z_{A,k}^\parallel (p^2 + m_{\text{mSTI},k}^2) \frac{p_\mu p_\nu}{p^2} \right] A_\nu^a(-p). \quad (4.6)$$

Here, we have explicitly written the momentum and cutoff dependencies of wave functions and dressings, ξ is the gauge fixing parameter, $Z_{A,k}^\parallel$ the wave function renormalisation of the longitudinal mode. The longitudinal gauge field mass $m_{\text{mSTI},k}^2$ is induced by the mSTIs in the presence of a momentum cutoff and in the physical limit $k \rightarrow 0$ these approach the standard STIs of Landau gauge and the longitudinal mass vanishes, $m_{\text{mSTI},k=0}^2 = 0$. As discussed above, this makes apparent as discussed above how $m_{\text{mSTI},k}^2$ does not signal a gauge-variant massive gauge theory, but the modification of the Landau gauge STIs in the presence of the IR regulator. For a detailed discussion of this subtlety see e.g. [72, 82] and the review [43]. In turn, the transverse gauge field mass gap is finite, signalled by $m_{\text{gap},k=0}^2 \neq 0$ and as discussed carries physical information on confinement. Given the structure of the transversal part of the propagator, it is apparent how the mass gap here can be absorbed into the gauge field wave-function renormalisation.

Moreover, for cutoff scales above the confinement scale k_{conf} the two masses agree

$$m_{\text{gap},k}^2 = m_{\text{mSTI},k}^2, \quad \text{for scales} \quad k \geq k_{\text{conf}}, \quad (4.7)$$

as their difference can only be triggered by irregular vertices underlying the dynamical mechanism of confinement. In other words, as the confining dynamics kick in, the longitudinal vertices develop massless modes, either through the Schwinger mechanism [338–342] or the BRST quartet mechanism [343]. Note, that $m_{\text{gap},k \rightarrow \infty}^2$ can be negative or positive, depending on the choice of regulators and the matter content of the theory. This makes apparent how the $m_{\text{gap},k > k_{\text{conf}}}^2$ carries no physical meaning and is just a remnant of the breaking of gauge-invariance at $k \neq 0$ removed at $k = 0$.

In [82, 344] a bootstrap approach was put forward within the fRG that circumvents the necessity of dynamically generating the confining mass gap via dynamical process such as the

Schwinger or BRST quartet mechanisms. At the core of this approach lies the fact, that it is the dynamically generated massless excitations in the longitudinal part of the ghost-gluon, three-gluon and quark-gluon vertices, that lead to a non-trivial difference between the transverse mass gap and the longitudinal mass. This suggests that the flow of the mass function with regular *uniform* vertices is a good approximation for the flow of the transverse mass gap m_{gap}^2 . The second crucial ingredient of the bootstrap approach is the *assumption* of an IR closure of the gauge fixing which admits BRST charges, an assumption that underlies the Kugo-Ojima confinement scenario [345]. This solution in particular implies the existence of a BRST charge and not only that of infinitesimal BRST transformations, for a detailed discussion see [82, 344]. In this confinement scenario in the Landau gauge, the ghost and gauge field propagators display the same IR scaling limit with,

$$\lim_{p^2 \rightarrow 0} Z_A(p^2)(1 + m_{\text{gap},k}^2/p^2) \propto (p^2)^{\kappa_A}, \quad (4.8)$$

and a scaling coefficient $\kappa_A \approx -1.2$ in Yang-Mills. This is achieved by a domination of the IR dynamics by the ghost modes over the gapped gauge ones.

In the fRG approach, the precise scaling in (4.8) is arranged by a unique combination of the UV relevant parameters at a large cutoff scale. In other words, the initial condition for the transverse mass gap simply has to be tuned to *infrared scaling*. Moreover, other initial conditions for $m_{\text{gap},k=\Lambda_{\text{UV}}}^2$ lead to different IR solutions such as the *decoupling* or even gauge-variant *massive* Yang-Mills ones, see [5, 82] for details on the tuning and dialling of different solutions. Remarkably, the assumption of the Kugo-Ojima confinement scenario provides a unique solution with no additional freedom of choice.

The reservation of gauge symmetry along the RG flow can be checked with a self-consistency evaluation of the results. Varying the regulator matrix R_k triggers vastly different scale dependences of the mass gap parameter and the initial condition. These regulator dependences should be absent in the final result for the gluon propagator at $k = 0$ only if the above assumption for the fully physical nature of the flow of the mass gap is correct. This analysis has been done in [82] for Yang-Mills theory and in [168] for two-flavour QCD, and the gluon propagator as well as all other correlation functions showed no sensitivity to sizeable variations of R_k . Note however, that this insensitivity was only present for the fully quantitative approximation with full momentum dependences for propagators and vertices. Indeed, these momentum dependences and the respective momentum transfer arrange for also the cancellation of momentum-dependent cutoff dependences in the flows, triggered by the sizeable mass gap $m_{\text{gap},k}^2$ at $k \neq 0$.

In this Section we will not go in depth into the discussion of the confinement solution but we summarise that the method here employed allows for the dynamical treatment of confinement and the related observables. Here, we care about the qualitative features such as the confinement scale in comparison to others. Lacking a full analysis, we will employ the knowledge obtained from the gauge field dressing whose momentum peak roughly diagnoses the appearance of confining

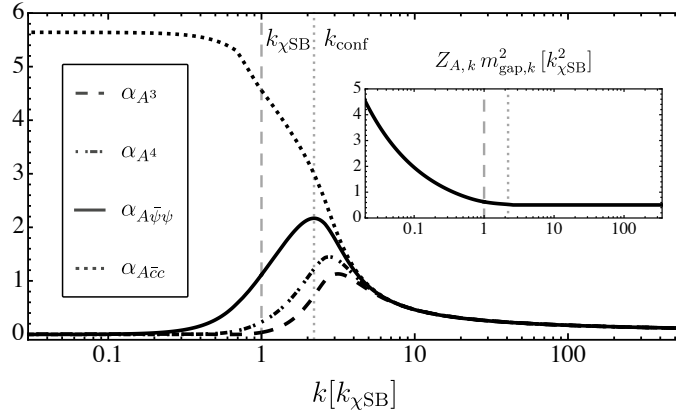


Figure 4.2: Three-gauge (dashed), four-gauge (dashed dotted), ghost-gauge (dotted) and fermion-gauge (plain) exchange couplings in an SU(3) gauge theory with $N_f = 2$ fundamental flavours. The momentum or cutoff scale is measured in units of the chiral symmetry breaking scale $k_{\chi\text{SB}}$, defined in (4.20) and depicted by the vertical grey dashed line. The confinement scale, k_{conf} is marked with a vertical grey dotted line. In the inlay plot, we show the gauge field two-point function at vanishing external momenta as a function of the cutoff scale.

dynamics and a gauge mass gap. Moreover, this feature also manifests in any gauge correlation function such as the gauge couplings, obtained from the three-gauge, four-gauge and ghost-gauge dressings,

$$\begin{aligned} \alpha_{A^3} &= \frac{\lambda_{A^3}^2}{4\pi} \frac{1}{(1 + \bar{m}_{\text{gap}}^2)^3}, & \alpha_{A^4} &= \frac{\lambda_{A^4}}{4\pi} \frac{1}{(1 + \bar{m}_{\text{gap}}^2)^2}, \\ \alpha_{c\bar{c}A} &= \frac{\lambda_{c\bar{c}A}^2}{4\pi} \frac{1}{(1 + \bar{m}_{\text{gap}}^2)}, & \text{and} & \quad \alpha_{\psi\bar{\psi}A} = \frac{\lambda_{\psi\bar{\psi}A}^2}{4\pi} \frac{1}{(1 + \bar{m}_{\text{gap}}^2)}. \end{aligned} \quad (4.9)$$

Here, λ_i with $i = A^3, A^4, c\bar{c}A$ and $\psi\bar{\psi}A$ are the dressings of the respective vertices in the effective average action which include the couplings and the respective dressings, and $\bar{m}_{\text{gap}} = m_{\text{gap}} k^{-2}$ is the dimensionless version of the mass gap and we have suppressed the k -dependence in the couplings. Note that both (4.9) and the λ_i 's can be understood as avatars of α_g or the gauge coupling g . The difference is the RG-invariant decoupling factor due to the gauge field mass gap, which is present for exchange couplings that occur in the flow diagrams. In any case, the gauge avatars α_i with $i = A^3, A^4, c\bar{c}A$ and $\psi\bar{\psi}A$ agree for large cutoff scales due to perturbative universality and are genuinely different in the strongly correlated IR regime. The parametrisation of the different α_i 's makes the mass gap factor apparent which induces the low energy decoupling of gauge fluctuations. We will see how the mass gap tends to zero, as the perturbative conformal window is approached and the IR decoupling disappears leading to a quantum scale invariant theory.

As has been shown in Figure 4.1 for the gauge field propagator for two and 2+1 flavours with

$N_c = 3$, there exist simple pure cutoff truncations [5] which compare very well to quantitative results from functional two and 2+1 flavour QCD and from lattice simulations. This identification $k \approx p$ can also be used for the exchange couplings α_i , which are shown in Figure 4.2. We display them over four orders of magnitude in momentum, from the asymptotic UV regime with perturbative universality to the strongly coupled IR. At high scales, all avatars α_i agree due to the STIs and $\bar{m}_{\text{gap}, \Lambda_{\text{UV}}}^2 \approx 0$. As the gauge dynamics strengthen towards the IR, the deviations between the avatars of the gauge coupling start growing due to the different decoupling factors and the genuine differences between the respective couplings λ_i . The exchange coupling strength increases until the confinement scale k_{conf} is reached. For practical and illustrative purposes, we approximate the peak of the wave function renormalisation and identify this scale with the peak of the fermion-gauge exchange coupling $\alpha_{\psi\bar{\psi}A}$. This exchange coupling decays at lower scales compared to the pure glue ones, as the loop corrections are not equally suppressed by the same powers as in the pure gauge case. The introduced confinement scale k_{conf} , serves as an approximate proxy for physical observables, such as the confinement-deconfinement critical temperature, and may also be identified with the peak of other pure-gauge exchange couplings. Notably, $\alpha_{c\bar{c}A}$ freezes towards the IR in the scaling solution. In the inlay plot, the two-point function at vanishing momenta is shown which is proportional to the gauge-field mass gap. This increases asymptotically for low cutoff and momentum scales in the scaling confinement limit.

At $k_{\chi\text{SB}}$, (4.20), $d\chi\text{SB}$ occurs, leading to the emergence of a chiral condensate and constituent masses for the fermions. This triggers the quick decoupling of the fermion loops and will be discussed in the following Section.

4.1.2 Dynamical chiral symmetry breaking

The second relevant phenomenon in gauge-fermion QFTs is dynamical chiral symmetry breaking ($d\chi\text{SB}$). While the classical action in (2.48) is invariant under global and chiral transformations, dynamics driven by quantum effects in the effective action generate a chiral condensate (and consequently a fermion mass), which breaks this symmetry to its diagonal vector subgroup

$$\text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V. \quad (4.10)$$

This mechanism plays a central role in the fermionic spectrum of the theory and has been the topic of discussion in Section 2.3.

At an asymptotically large cutoff scale, the full effective action tends towards the classical one as all quantum fluctuations are suppressed. Strictly speaking, this is only true in asymptotically free theories, given that in asymptotically safe ones quantum effects are decisive. Moreover, we have already discussed that the UV effective action consists of all UV-relevant terms that are compatible with the symmetries of the theory (namely the classical action). The gauge-fermion interaction term in (2.48) generates an infinite number of pure fermionic and fermion-gauge

interactions that are compatible with the flavour and colour symmetries of the theory. As all of them are UV-irrelevant in the asymptotically free regime, they can be set to zero at the initial cutoff scale, and they are generated as quantum corrections are accounted.

From the infinite tower of operators, the particularly relevant ones are those which affect the off-shell dynamics. An example are the four-Fermi operators $(\bar{\psi} \mathcal{T}_i \psi)^2$, where $\mathcal{T}_i$ are dimensionless tensor structures. Additional terms involving higher powers of fermions and/or gauge bosons, or momentum-dependent tensor structures, have higher canonical dimensions and can be therefore, in principle, neglected. These terms would require very strong dynamics to become IR relevant and are expected to have a negligible effect. A complete set of point-like four-Fermi operators constitute the part of the full effective action

$$\Gamma_{4\psi, k} [\bar{\psi}, \psi] = - \int_x Z_\psi^2 \left[\lambda_{\text{SP}} \mathcal{T}_{(\text{S-P})} + \lambda_+ \mathcal{T}_{(\text{V+A})} + \lambda_- \mathcal{T}_{(\text{V-A})} + \lambda_{\text{VA}} \mathcal{T}_{(\text{V-A})^{\text{adj}}} \right], \quad (4.11)$$

where the global factor Z_ψ^2 carries the (inverse) RG-scaling of the fields and hence the couplings $\lambda_{\text{SP}}, \lambda_\pm, \lambda_{\text{VA}}$ are RG-invariant. Moreover, we have omitted the assumed explicit k dependence of the couplings and dressing. The tensors $\mathcal{T}_i$ define different channels of the full four-Fermi interaction with given quantum numbers and constitute a Fierz-complete basis compatible with the full flavour symmetry. These include the vector and pseudovector tensor structures

$$\begin{aligned} \mathcal{T}_{(\text{V-A})} &= (\bar{\psi} \gamma_\mu T_f^0 \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 T_f^0 \psi)^2, \\ \mathcal{T}_{(\text{V+A})} &= (\bar{\psi} \gamma_\mu T_f^0 \psi)^2 - (\bar{\psi} \gamma_\mu \gamma_5 T_f^0 \psi)^2, \\ \mathcal{T}_{(\text{V-A})^{\text{adj}}} &= (\bar{\psi} \gamma_\mu T_f^0 T_f^a \psi)^2 + (\bar{\psi} \gamma_\mu \gamma_5 T_f^0 T_f^a \psi)^2, \end{aligned} \quad (4.12)$$

As will be discussed in the following Subsection 4.1.2.2, in the presence of anomalous $\text{U}(1)_A$ -breaking, the remaining dominant scalar pseudo-scalar (S-P) channel can be written as a sum of tensor structures of the $(\sigma - \pi)$ and $(\eta - a)$ modes,

$$\mathcal{T}_{(\text{S-P})} = \mathcal{T}_{(\sigma-\pi)} + \mathcal{T}_{(\eta-a)}, \quad (4.13a)$$

with

$$\begin{aligned} \mathcal{T}_{(\sigma-\pi)} &= (\bar{\psi} T_f^0 \psi)^2 - (\bar{\psi} \gamma_5 T_f^a \psi)^2 \\ \mathcal{T}_{(\eta-a)} &= (\bar{\psi} T_f^a \psi)^2 - (\bar{\psi} \gamma_5 T_f^0 \psi)^2. \end{aligned} \quad (4.13b)$$

4.1.2.1 Emergent composites

As previewed in Section 2.3, as the gauge dynamics strengthen towards the IR, the dressings of UV-irrelevant operators in (4.11) grow big as well. In fact, the dressing of the (S-P) channel becomes singular which signals the breaking of the chiral symmetry and the respective modes turning critical. This also entails that higher order fermion scatterings in this channel cannot

be neglected any more. The divergence of λ_{SP} as well as the higher order scatterings are very efficiently and reliably accommodated within the fRG with the *emergent composite* formalism first introduced in [123].

In this approach, the four-Fermi interaction in a particular momentum channel (here the scalar-pseudoscalar channel in the t -momentum channel), is rewritten as the exchange of a composite bosonic degree of freedom carrying the respective Lorentz, colour and flavour structure of the corresponding fermionic bilinear, see Figure 2.1 in Section 2.3. This field transformation is done at the level of the flow equation, and is reminiscent of a momentum-scale dependent Hubbard-Stratonovich transformation [116, 117]. In its implementation in the flow equation, it is an identity transformation, which has the important consequence, that, in contradistinction to the standard Hubbard-Stratonovich transformation, it is not troubled by potential double counting problems in *any* approximation to the effective action.

The emergent composites formulation has several beneficial consequences:

- ◇ First, it provides access to the physics of the broken regime starting from the symmetric classical limit, thereby achieving results from first principles. Unlike effective approaches (a.k.a. LEFTs), this method does not impose an upper UV scale that limits the validity of the low-energy theory and introduces no additional free parameters, as the theory is treated fundamentally.
- ◇ This framework allows for the straightforward computation of physically and phenomenologically relevant quantities, such as the chiral condensate and the fermion and bound state masses.
- ◇ From a technical point view, given the emergent composites formulation redefines the fermionic interactions in the selected channel as bosonic interactions, this enables the easy inclusion of higher-order fermionic interactions in a much more computationally simple manner via bosonic self-interactions. This is particularly crucial for capturing non-perturbative effects and achieving quantitative precision.
- ◇ On the same footing, the fRG approach with emergent composites accommodates a momentum-dependent four-Fermi coupling in a specific momentum channel, thereby surpassing the limitations of the cutoff-dependent approximation in the four-Fermi form (4.11).

To access the broken chiral regime, it suffices to bosonise the resonant channel which turns critical. In this case, we particularise the bosonisation on the $(\sigma - \pi)$ channel in the (S-P) tensor structure. This is absorbed entirely in the meson fields

$$\phi = (\sigma, \boldsymbol{\pi}) \quad \text{with} \quad \boldsymbol{\pi} = \pi^a = (\pi_1, \dots, \pi_{N_f^2-1}), \quad (4.14)$$

with the chiral invariant

$$\rho = \frac{\sigma^2 + \boldsymbol{\pi}^2}{2}. \quad (4.15)$$

The σ -mode is the radial mode in the order parameter potential and in physical QCD it is related to the $f_0(500)$ resonance with quantum numbers 0^{++} . Its resonance mass is about 500 MeV and in the chiral limit in two-flavour QCD within the fRG approach its mass is about 250 MeV, see [98]. This entails that its off-shell importance in the loops is subleading. As the radial mode of chiral symmetry breaking, it acquires an expectation value which is our expansion point of the effective action. This is one about the EoM, which is an optimal expansion point. On the other hand, in the present study of gauge-fermion theories in the chiral limit, π^a correspond to the pion-like modes which here are exact NGBs of the global symmetry.

After bosonisation (recall the discussion in Section 2.3) the four-Fermi part of the effective action (4.11) appears as

$$\Gamma_{4\psi, k} [\bar{\psi}, \psi, \phi] = \Gamma_{4\psi, k} [\bar{\psi}, \psi] \Big|_{\lambda_{\text{SP}}^{(\sigma-\pi)}=0} + \int_x \frac{1}{2} Z_\phi (\partial_\mu \phi)^2 + V(\rho) + Z_\phi^{1/2} Z_\psi h_\phi \bar{\psi} (T_f^0 \sigma + i\gamma_5 T_f^a \pi^a) \psi. \quad (4.16)$$

Here, T_f^0 is a normalised and diagonal tensor in flavour space defined in (2.37) and T_f^a are the generators of the $\text{SU}(N_f)$ flavour group. The second line replaces the $(\sigma - \pi)$ four-Fermi channel by the insertion the dynamical scalar modes as depicted in Figure 2.1. The Yukawa coupling h_ϕ is RG-invariant as the RG-scaling of the Yukawa term is captured by the prefactor $Z_\phi^{1/2} Z_\psi$. In the simplest version of the emergent composites approach, deep in the perturbative regime, the effective potential is Gaussian and is given by a mass term

$$V(\rho) = m_\phi^2 Z_\phi \rho, \quad (4.17)$$

with the RG-invariant mass m_ϕ . Chiral invariance in the symmetric regime sets the σ and pion mass functions in (4.17) to be the same. This is equivalent in the classical reparametrisation as done in (2.40). Going beyond the Gaussian approximation by considering non-trivial forms of the effective potential allows to include non-perturbative effects in the bosonised channel during the flow. This is one of the advantages of the bosonised formulation previously mentioned. For example, here we have included an effective potential term for the composite fields which we consider to have a polynomial form

$$V(\rho) = \sum_{n \geq N_{\min}}^{N_{\max}} \frac{\lambda_{\phi, n}}{n!} [Z_\phi (\rho - \rho_0)]^n. \quad (4.18)$$

The powers ρ^n effectively include multi-scattering terms $(\bar{\psi} \mathcal{T} \psi)^{2n}$ of fermionic interactions, which is readily seen in an iteration of the solution to the EoM with a general potential about the solution (2.39) for the Gaussian potential (4.17). In (4.18), $\lambda_{\phi, n}$ are the expansion coefficients and $\rho_0 = \langle \sigma \rangle^2 / 2$ is the flowing global minimum of the potential. In the broken phase ($\rho_0 \neq 0$) we have $N_{\min} = 2$, while in the symmetric phase ($\rho_0 = 0$) we have $N_{\min} = 1$ being the first term in the expansion the Gaussian one, (4.17). Moreover, we employ in both limits $N_{\max} = 5$, as this already provides well converged results even in the many flavour limit.

With the bosonised formalism, we can identify the cutoff scale $k_{\chi\text{SB}}$, below which $\langle\sigma\rangle \neq 0$ in the chiral limit, as the scale of $d\chi\text{SB}$. In other words,

$$\langle\sigma\rangle = \begin{cases} = 0 & k > k_{\chi\text{SB}} \\ \neq 0 & k < k_{\chi\text{SB}} \end{cases}, \quad (4.19)$$

with

$$k_{\chi\text{SB}} = \max\{k \mid \langle\sigma\rangle_k \neq 0\}. \quad (4.20)$$

Together with k_{conf} in (4.20), these scales define good proxies for the two dynamical scales in gauge-fermion systems. We shall use them from now on, and in particular $k_{\chi\text{SB}}$, as our reference scales.

Along with a condensate for the σ -mode, the interaction term with fermions in (4.16) generates a $\langle\sigma\rangle$ -dependent fermionic mass term with the RG-invariant mass

$$m_\psi = \frac{h_\phi \langle\sigma\rangle}{\sqrt{2N_f}}, \quad (4.21)$$

which leads to the constituent fermion mass with $d\chi\text{SB}$ proportional to the order parameter. One of the aims of this work is to connect with the quantum scale-invariant limit of gauge-fermion theories. For this particular reason we have focused this study on classically scale-invariant theories, where no explicit mass scale is present at the classical level (e.g., no current fermion mass as in physical QCD) and (4.21) is purely given by the chiral condensate contribution.

Furthermore, we can parametrise the strength of the bosonised channel using the relation obtained as a solution of the EoM, as done in Section 2.3. The mesonic part of the effective action reduces part of the scalar-pseudoscalar four-Fermi term with the momentum-dependent and RG-invariant dressing

$$\lambda_{\text{SP}}^{(\sigma-\pi)}(p) = \frac{1}{2} \frac{h_\phi^2}{p^2 + m_\phi^2}, \quad (4.22)$$

with $\phi \in \{\sigma, \pi^a\}$. At $p = 0$, the cutoff analogue of (4.22) can be written as the mesonic exchange coupling

$$\alpha_\phi = \frac{1}{4\pi} \lambda_{\text{SP}}^{(\sigma-\pi)}(p=0) = \frac{1}{8\pi} \frac{h_\phi^2}{(1 + \bar{m}_\phi^2)}. \quad (4.23)$$

The RG-invariant curvature masses of the mesonic modes are given by the second ϕ -derivatives (curvature) of the effective potential

$$m_\sigma^2 = k^2 [u'(\bar{\rho}_0) + 2\bar{\rho}_0 u''(\bar{\rho}_0)], \quad \text{and} \quad m_\pi^2 = k^2 u'(\bar{\rho}_0), \quad (4.24)$$

where we have used the more convenient form of the renormalised and dimensionless quantities, $u(\bar{\rho}) = V(\rho)/k^4$ and $\bar{\rho} = Z_\phi \rho/k^2$.

4.1.2.2 Axial anomaly

So far we have only discussed the $(\sigma - \pi)$ -part of the scalar-pseudoscalar channel in (4.12) and ignored the rest. At this point it is relevant to note that in gauge-fermion systems, axial $U(1)_A$ symmetry is broken anomalously [346–349]. This anomaly induces fermionic breaking terms known as the 't Hooft determinant [350], and in the chiral limit the lowest order term is a $2N_f$ -fermion operator [351]. Nonetheless, this operator feeds back into all interaction terms via the flow and lifts the degeneracy of the scalar–pseudoscalar four-Fermi sector. Also, the size of the Fierz-complete four-Fermi basis increases rapidly with N_f . Already in physical QCD with $N_f = 2$, the basis contains ten tensor structures, see e.g. [81, 98, 312, 352], and for $N_f = 2 + 1$ the basis contains 26 elements, see e.g. [352]. In the presence of $U(1)_A$ symmetry breaking, the scalar-pseudoscalar tensor structure can be written in terms of two different ones,

$$\lambda_{\text{SP}} \mathcal{T}_{(\text{S-P})} \longrightarrow \lambda_{\text{SP}}^{(\sigma-\pi)} \mathcal{T}_{(\sigma-\pi)} + \lambda_{\text{SP}}^{(a-\eta)} \mathcal{T}_{(a-\eta)}, \quad (4.25)$$

where the subscripts and superscripts $(\sigma - \pi)$ and $(a - \eta)$ for the tensors and the dressings respectively are given from the two-flavour case. There, the 't Hooft determinant is a four-Fermi term and its tensor structure can be used directly in the four-Fermi basis. Moreover, for $N_f = 2$ and $N_f = 2 + 1$ the degeneracy of the mass spectrum is lifted considerably by a couple of hundred MeV. In turn, the very nature of the axial $U(1)_A$ -breaking term as a $2N_f$ -fermion term entails that its impact is getting successively irrelevant with larger flavour number. Accordingly, in the large flavour limit we expect the anomalous $U(1)_A$ -breaking to be absent effectively in the four-Fermi terms, leading to

$$\lambda_{\text{SP}} = \lambda_{\text{SP}}^{(\sigma-\pi)} = \lambda_{\text{SP}}^{(a-\eta)}. \quad (4.26)$$

In the present work we refrain from computing the flow of the 't Hooft determinant via [351] and its increasingly irrelevant impact with larger N_f on the four-Fermi terms. In particular, this operator partly sources from topological contributions which are particularly challenging to implement in the present setup. Instead we use a generous estimate for the size of the axial $U(1)_A$ -breaking or the lack thereof: in the latter case full chiral symmetry is present in the symmetric regime and only the σ -mode as the radial mode is massive in the chirally broken regime. Accordingly, we identify the $(a - \eta)$ part of the multiplet, or rather its couplings and masses, with that of the NGBs. In the former case we estimate the effect of the $U(1)_A$ -breaking with identifying the couplings and masses of the $(a - \eta)$ part of the multiplet with that of the σ -mode. For larger values of N_f this is a very conservative upper bound. Moreover, these different choices reduce to identifying $(\eta - a)$ channel with that of the $(\sigma - \pi)$ one

$$\lambda_{\text{SP}}^{(\eta-a)}(p) = \lambda_{\text{SP}}^{(\sigma-\pi)}(p) = \frac{1}{2} \frac{h_\phi^2}{p^2 + m_\phi^2}. \quad (4.27)$$

and properly choosing the threshold function. In the chirally symmetric regime this certainly is a quantitative approximation for larger N_f and we will use it throughout. In the broken regime

we can assume that the $U(1)_A$ symmetry is effectively restored, which renders all the mesons in the $(\eta - a)$ channel Goldstone modes: $m_\phi^2 \rightarrow m_\pi^2 = 0$ in (4.27). Alternatively, we can assume maximal anomalous $U(1)_A$ breaking, that is explicit breaking (without NGBs). Then, for large N_f the mass of the σ -mode serves as a conservative upper value for the mass in the $(\eta - a)$ channel: $m_\phi^2 \rightarrow m_\sigma^2$. The results presented in the following Sections are based on the former identification with $m_\phi^2 \rightarrow m_\pi^2$, as we are most interested in the large N_f regime. We have used the identification $m_\phi^2 = m_\sigma^2$ for the systematic error estimate. We have seen results that the difference between these approximations is small which sustains the argument of the subleading effect of the $U(1)_A$ -breaking on the physics of many-flavour gauge-fermion systems.

4.1.2.3 $d\chi$ SB in a two flavour chiral-QCD example

We close this Section exemplifying how $d\chi$ SB and the related observables appear in the particular case of two-flavour QCD in the chiral limit, which can be compared with the results obtained for the physical theory.

In the top panel of Figure 4.3 we show the curvature of the chiral potential around the flowing global minimum ρ_0 as a function of the cutoff scale. In the deep UV, the curvature of the potential around the trivial minimum is very large, implying highly suppressed fermionic self-interactions. As the gauge dynamics strengthen, the chiral potential flattens until at $k_{\chi\text{SB}}$ (vertical dashed line) the potential develops a non-trivial minimum, $\langle\sigma\rangle \neq 0$. In the bottom panel, we show the evolution of this parameter (blue line) which additionally is the chiral phase transition order parameter. In the same plot, we also show the fermion constituent mass (m_ψ , in green, see (4.21)). It is worth noting in this context that the sharp onset of $\langle\sigma\rangle$ with the cutoff scale does not signal a second order-type behaviour, in terms of a physical momentum scale it is smooth, see e.g the recent discussion of the fermion mass function in [98]. Still, it mirrors the second order phase chiral transition at finite temperature.

In the middle panel of Figure 4.3, the mesonic exchange couplings are shown in comparison to the gauge ones (light blue lines). In the symmetric regime, chiral symmetry renders the σ and π modes indistinguishable, resulting in both exchange couplings being identical (orange line). In the broken phase, this degeneracy is lifted as the σ -mode (purple line) decouples from the off-shell dynamics, while the π -mode (pink line) remains massless and dominates the IR dynamics.

Without the emergent composites formulation, $d\chi$ SB is signalled by a divergent four-Fermi coupling given that $\lambda_{\text{SP}} \propto h_\phi^2/u'(\rho)$ as in (4.22). While this approach has been made quantitative in QCD studies [311, 312], it is significantly more challenging when studying properties in the chirally broken sector. In particular, the access to the broken regime in theories in the chiral limit is restricted for the four-Fermi formulation due to a singularity in the flow. Moreover, this strategy was traditionally employed to trace whether $d\chi$ SB occurs or not [353, 354] in the

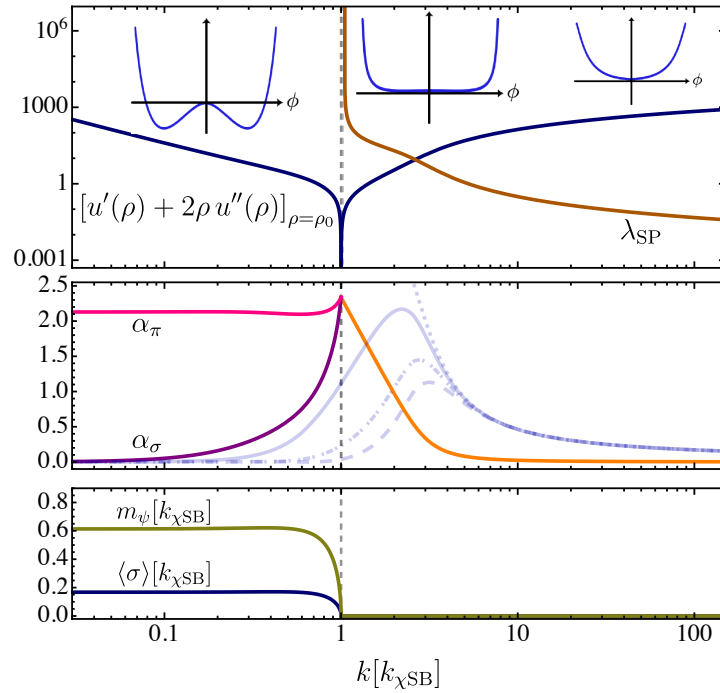


Figure 4.3: Top plot: for an $SU(3)$ gauge theory with $N_f = 2$ fundamental flavours, the curvature of the dimensionless renormalised effective potential evaluated at the minimum (dark blue) is shown along with the scalar-pseudoscalar four-Fermi coupling (brown) as defined in (4.11).

Middle plot: the two mesonic exchange couplings, α_σ and α_π , as defined in (4.23), are displayed in the symmetric phase (orange) and the broken phase (purple and pink, respectively).

Bottom plot: the evolution of the non-trivial minimum of the effective chiral potential (blue) and the constituent fermion mass (green) are presented. The scale $k_{\chi SB}$ is indicated by vertical dashed lines.

many-flavour limit of gauge-fermion QFTs. In the top panel of Figure 4.3, we show the four-Fermi coupling for the (S–P) channel (brown line), which is the relevant one to trace the development of a non-trivial scalar condensate. It becomes apparent how the bosonised picture is more convenient as it straightforwardly allows to flow into the broken phase of the potential. We note that each independent computation leads to a slightly different $k_{\chi SB}$, see vertical dashed lines in the top panel of Figure 4.3.

The curvature of the mesonic potential displayed in Figure 4.3 is equivalent to the mass of the scalar degrees of freedom, see (4.24). In the UV, the curvature is very large leading to very large masses and hence a decoupling of these modes from the dynamics. In Figure 4.4 we show the euclidean masses of the bosonised channels at scales approximately and below $k_{\chi SB}$. For

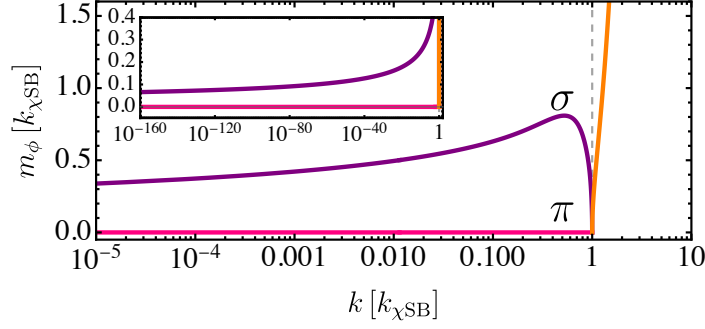


Figure 4.4: σ and π^a mode masses (m_σ and m_π) in the symmetric (orange) and broken (purple and magenta respectively) phases of the chiral potential for a SU(3) gauge theory with $N_f = 2$ fundamental flavours in the chiral limit. In the inlay, we zoom into the deep IR scales.

$k > k_{\chi\text{SB}}$, the symmetry is not broken and the NGBs and radial modes are indistinguishable. In the broken phase, the NGBs are exactly massless and the σ -mode behaves analogous to the Higgs boson. Its mass in the chiral limit grows until the fermionic loops decouple as $k \sim m_\psi$. For lower scales, the only remaining dynamical degrees of freedom are the massless pions. The mass of the σ -mode is subject to discussion and is an open case of research [98, 355–359]. In the present truncation the bosonic contributions to the flow of the curvature of the potential lead to a mild change which over many orders of magnitude finally freezes around $m_\sigma \approx 0.07 k_{\chi\text{SB}}$. The precise determination of this quantity is delicate and subject to the truncation employed on the effective chiral potential (4.18). Similar computations employing a more advance non-polynomial type of potential have been performed in [98] and display a freeze-out if the mass at $m_\sigma \approx 0.53[k_{\chi\text{SB}}]$.

On the other hand, the results here obtained are well understood in the non-chiral limit and physical QCD, see e.g. [98]. The consideration of explicit fermion masses breaks chiral symmetry at the classical level. This leads to pions as pNGBs of the global chiral symmetry and hence massive. Consequently, their contribution decouples at a finite cutoff scale $k \sim m_\pi$ in the broken chiral phase, leading to a heavier m_σ . For example, in the case of $m_\pi \sim 0.1 k_{\chi\text{SB}}$ this will lead to $m_\sigma \sim 0.6 k_{\chi\text{SB}}$.

4.1.3 Caswell-Banks-Zaks fixed points and conformal gauge-fermion theories

The dynamical phenomena of confinement and $d\chi\text{SB}$ with their emergent mass scales constitute the non-perturbative character of gauge-fermion theories. However, this class of theories displays a scale-invariant limit where both of these phenomena are absent by definition. This regime is approached by softening the growth of the gauge coupling towards the IR via the enhancement of anti-screening fermionic corrections by incrementing N_f . This is well-captured by the beta function β_g of the gauge coupling. As we aim at a description of physics in the conformal regime

with $\beta_g \rightarrow 0$ with a relatively small fixed point coupling, we approach this problem within a perturbative expansion. In the chiral limit and hence in the absence of any explicit mass scales,

$$\beta(g, N_c, N_f) = -g \sum_{n=1}^{\infty} \left(\frac{g^2}{16\pi^2} \right)^n \beta^{(n)}(N_c, N_f) \quad (4.28)$$

with

$$\begin{aligned} \beta^{(1)}(N_c, N_f) &= \left(\frac{11}{3} C_A - \frac{4}{3} T_F N_f \right), \\ \beta^{(2)}(N_c, N_f) &= \left(\frac{34}{3} C_A - \left[4C_F + \frac{20}{3} C_A \right] T_F N_f \right), \end{aligned} \quad (4.29)$$

being the one and two loop terms in the expansion. Here, C_A and C_F are the Casimir operators of the colour group and of the representation under which the fermions transform, respectively, and T_F is the Dynkin index of the latter. Roots of the beta function correspond to fixed-point solutions g^* ,

$$\beta(g^*, N_c, N_f) = 0. \quad (4.30)$$

For sufficiently small number of flavours, $N_f < (11C_A) / (4T_F)$, the existence of a UV Gaußian fixed point g^* in (4.30) is ensured by the one loop term in (4.29), leading to asymptotically free theories as in the physical QCD case. In this regime with sufficiently small N_f the theory also features d χ SB and confinement. In turn, for large number of flavours, $N_f > (11C_A) / (4T_F)$, the UV Gaußian fixed point turns into a Gaußian IR fixed point with a strongly correlated UV regime, similarly to the situation in quantum electrodynamics with its UV Landau pole in the chiral limit.

In an intermediate regime with $N_f^{\text{crit}} < N_f < (11C_A) / (4T_F)$, non-trivial roots with $g_* \neq 0$ of the beta function exist. For example, considering up to two loops, one can find (N_c, N_f) -pairs for which (4.30) is satisfied for a non-vanishing value g^* of the gauge coupling, which signals an interacting IR fixed point known as Caswell-Banks-Zaks (CBZ) fixed point [360, 361]. In this regime, correlation functions are invariant under scale transformations at the fixed point and enjoy quantum scale invariance, elevating that of the classical gauge-fermion action to the quantum effective action.

In the absence of threshold effects and for mass-independent RG schemes, the beta functions of marginal couplings are known to be universal up to two loops [208], meaning that their form is scheme independent. As the existence of the perturbative CBZ fixed point is present at the two-loop level and beyond, it is expected to be universal in the weak coupling limit.

Perturbative approaches have proven to be very useful in the determination of this fixed point as they account in a systematic manner for the higher-order contributions in powers of the gauge coupling. This allows to properly account for colour and flavour factors which cancellation

between loop orders leads to the CBZ fixed point. Currently, $\overline{\text{MS}}$ results are available up to five-loop orders [362] and re-summation techniques have been developed to improve the highest order results, see e.g. [363].

While the upper boundary of the CBZ conformal window in the $N_f - N_c$ plane of theories is well-known, the lower boundary towards the QCD-like regime is given by the loss of quantum scale invariance by the dynamical emergence of a scale. Due to the non-perturbative nature of these phenomena, the location of the lower boundary cannot be exactly computed and remains a topic of research. Generally, the disappearance of the CBZ fixed point (4.30) can occur in three ways [364]:

- (i) **Fixed point goes to 0:** This is the case of the upper boundary of the conformal window where asymptotic freedom is lost into the non-Abelian-QED regime. This is caused by a change of sign in the one-loop beta function term (4.29).
- (ii) **Fixed point goes to ∞ :** The gauge fixed point coupling disappears into the strong limit where the perturbative expansion breaks down before the fixed-point solution disappears. This is the scenario found in the lower boundary of the CBZ window for two-, three- and four-loop $\overline{\text{MS}}$ results. Perturbative approaches fail to properly pin-point the precise value given that they cannot describe the dynamical appearance of scales. We will add on this point for the remaining of this Section and in Subsection 4.1.5.
- (iii) **Fixed-point merger into the complex plane:** This scenario is given as the CBZ and QCD* fixed points merge, disappearing in the complex plane [365–367]. This is found only for five-loop $\overline{\text{MS}}$ results where the fixed point disappears at weak coupling values.

Despite their high success and practical use, perturbative approaches lack the capability to track the emergence of dynamical phenomena such as χSB . In other words, such approaches can provide strong coupling fixed-point solutions which are unviable given the prior appearance of dynamics.

A common approach employed to estimate whether $\text{d}\chi\text{SB}$ occurs at perturbatively computed fixed points is to analyse the magnitude of the fermion mass operator anomalous dimension [368–377],

$$\gamma_m = \gamma_m^{(1)} + \gamma_m^{(>1)} = -\frac{3\alpha_g C_F}{2\pi} + \dots \quad (4.31)$$

where we have expanded in powers of α_g singling out the one-loop contribution $\gamma_m^{(1)}$. Commonly, $\text{d}\chi\text{SB}$ is said to occur in theories where $\left|\gamma_m^{(1)}\right|_{g^*} \gtrsim 1$ providing an approximate methodology to determine the boundary of the conformal window [378–383].

On the other hand, non-perturbative approaches have investigated self-consistently the close-conformal regime of theory space. Fully self-consistent computations employing the DSE in the many flavour limit have been performed [384, 385] and the boundary of the conformal window is

found at a rather low number of flavours $N_f^{\text{crit}} = 4.67 \pm 0.02$ for $N_c = 3$ [385]. This is known to be caused by too weak gauge-fermion dynamics. Additionally, see [386] for a DSE scaling analysis at large N_f and [387] for computations with an IR cutoff.

Monte Carlo lattice simulations have also investigated the many-flavour region, see e.g. [388–391] and [392–398] for studies in the context of BSM physics. However, even contemporary state-of-the-art lattice simulations only consider gauge-fermion systems significantly away from the chiral limit (hence with sizeable fermion masses). This certainly complicates the interpretation of lattice results in the walking regime and conformal window.

There have been various efforts studying $N_f = 12$ [399–401], showing strong numerical evidence for the existence of an IR fixed point. Additionally, there are studies which suggest $N_f = 10$ is inside the conformal window [402–407]. For lower number of flavours well within the non-conformal regime, the gauge beta functions have been determined for $N_f = 6, 4$ [408] and $N_f = 8$ [409–411].

Within the fRG approach, the appearance of $d\chi\text{SB}$ can be clearly traced, as has been demonstrated in the previous Subsection 4.1.5. This phenomenon can be diagnosed using the four-Fermi framework through a singular coupling [353, 354, 412, 413], or within the bosonised formulation [72, 123] by the emergence of a non-trivial minimum in the chiral potential. Consequently, the fRG approach offers distinct advantages in determining the onset of a dynamical scale and, thus, the boundaries of the conformal window. In fact, this approach has been employed for this purpose in [354]. In Subsection 4.1.5, we address this task in more detail and present quantitative and qualitative improvements in estimating the boundary of the CBZ window.

In the vicinity of the conformal window, but below the boundary curve $N_f^{\text{crit}}(N_c)$, gauge-fermion theories are expected to exhibit close conformal scaling where $d\chi\text{SB}$ and confinement are still present. Specifically, they will show a very slowly growing coupling along a near conformal regime, commonly known as *walking regime*. This phenomenon has been widely employed in BSM phenomenology, for example CH realisations require of such walking to generate flavour hierarchies in the fermionic sector, see e.g. [2, 296, 397, 414–419]. Walking dynamics have been studied with conformal perturbation theory [420], effective field theories [421] and Monte Carlo lattice simulation [409]. We will study in detail such particular scenarios in Subsection 4.1.4.3, analysing the interplay of dynamics and the spectrum of the theories from first principles.

4.1.4 Phases of gauge-fermion quantum field theories

In this Section we discuss the interplay between the chiral and confining dynamics of gauge-fermion systems for different number of flavours including the QCD-like and the vicinity to the conformal window. The results in the previous Section employing multiple avatars of the gauge coupling confirmed the semi-quantitative nature of the present approximation of the effective

action and the flow even in the strongly correlated QCD-like regime. On the basis of this structure we improve the approximation for the fermion-gauge coupling $\alpha_{\psi\bar{\psi}A}$ to capture higher orders in perturbation theory: this is relevant if running over several orders of magnitudes in the perturbative and semi-perturbative regimes and connecting to the conformal limit. We accompany this improvement with a simplification of the approximation that reduces the fine-tuning problem that increases with flowing over successively more orders of magnitude. Moreover, we identify all other avatars of the gauge coupling with $\alpha_{\psi\bar{\psi}A}$,

$$\alpha_i = \alpha_g, \quad \text{with} \quad \alpha_g = \alpha_{\psi\bar{\psi}A}, \quad (4.32)$$

and $i = A^3, A^4, c\bar{c}A$. This approximation was put forward in [5] where details and the unique gauge coupling flow are given. Effectively, the improved approximation incorporates the full perturbative multi-loop resummations, while still accommodates the non-perturbative IR dynamics discussed in Subsections 4.1.1 and 4.1.2. In summary, the approximation employed here governs both regimes well, the perturbative and semi-perturbative running for $k \gtrsim k_{\text{conf}}, k_{\chi\text{SB}}$, and the strongly correlated IR regime with $k \lesssim k_{\text{conf}}, k_{\chi\text{SB}}$.

The interplay between confinement and $d\chi\text{SB}$ can be divided into three sub-regimes in N_f for all N_c . Here, we will restrict our analysis to $N_c = 3$ with N_f fundamental flavours, where the regimes are the following:

- (i) **QCD-like regime:** small flavour numbers $N_f \lesssim 4$, where confinement, signalled by the gauge field mass gap, emerges at a larger momentum scale than the chiral symmetry breaking one, given by the onset of the chiral condensate. In this regime the new single avatar approximation lacks quantitative precision given the very different dynamics are accounted by a single gauge avatar.
- (ii) **Locking regime:** intermediate flavour numbers $5 \lesssim N_f \lesssim 8$. In this regime, confinement and chiral symmetry breaking are *locked* and are triggered at similar momentum scales for all flavours in this regime. This is a novel phenomenon, and to our best knowledge it has not been observed before [5]. Since the confining and chiral dynamics are fully intertwined, we expect that the current approximation is only semi-quantitative.
- (iii) **Walking regime:** large flavour numbers $8 \lesssim N_f \lesssim N_f^{\text{crit}}$ close to the conformal window signalled by the absence of confinement and $d\chi\text{SB}$. At the interface with the regime (ii) the locking is overcome by the dynamics leading to a rapid decay of the ratio before settling in the scaling in the walking regime. The walking is signalled by the close to vanishing gauge flow and $d\chi\text{SB}$ occurs at successively higher scales, and confinement follows suit at an increasing relative distance. We expect quantitative validity of the approximation given its perturbative character and the separation of the dynamical scales.

In the following, we evaluate the onset of the different dynamical sectors with the scales k_{conf} and $k_{\chi\text{SB}}$. We emphasise that these scales should be understood as approximate proxies for

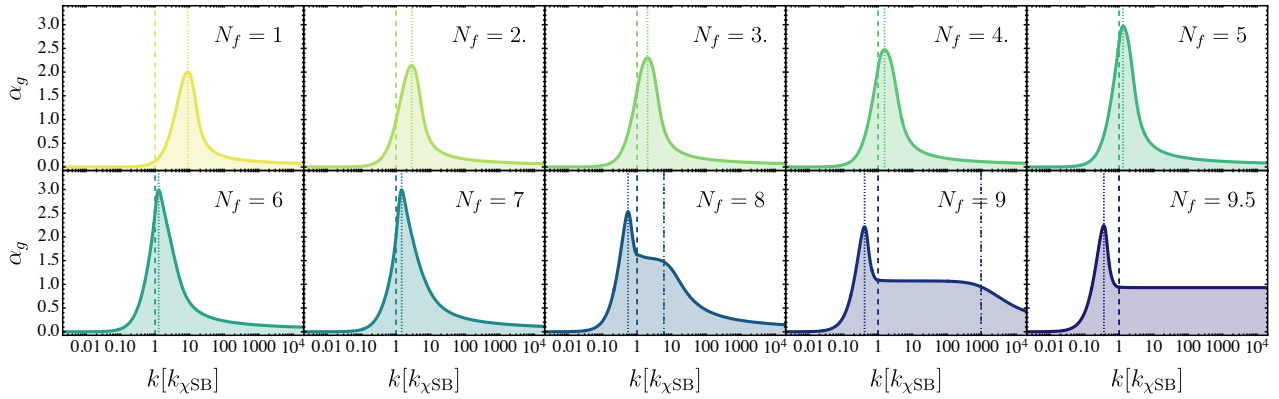


Figure 4.5: Gauge exchange couplings for a $SU(N_c = 3)$ gauge theory with N_f fundamental flavours. In each panel, the respective confinement (k_{conf} dotted) and chiral symmetry breaking ($k_{\chi\text{SB}}$ dashed) scales are marked by vertical lines. For theories in the close-conformal regime with $N_f \gtrsim 8$, the approximate onset of the walking regime k_{walk} is marked by a vertical dot-dashed line.

the physical ones, even though they are directly related to the physical confinement and chiral symmetry breaking scales.

In Figure 4.5 we display the fermion-gauge exchange coupling for an $SU(N_c = 3)$ theory coupled N_f fundamental flavours with $N_f = 1, \dots, 9.5$. For more flavours the gauge coupling enters the conformal regime analysed in Subsection 4.1.5. The confinement and chiral symmetry breaking scales are depicted as vertical lines, where k_{conf} is a dotted line and $k_{\chi\text{SB}}$ is a dashed one. We can clearly distinguish the three regimes (i-iii) which are discussed independently in the following subsections.

4.1.4.1 QCD-like regime for $N_f \lesssim 4$

For few $N_f \lesssim 4$, the confining mass gap proportional to k_{conf} is sizeably larger than $k_{\chi\text{SB}}$, see the upper left panels of Figure 4.5. In this regime the theory is clearly QCD-like and we see an approximately linear decrease with N_f ,

$$k_{\text{conf}}/k_{\chi\text{SB}} \approx (\text{const.} - \gamma^{(i)} N_f) \quad \text{with} \quad \gamma^{(i)} \approx 0.6. \quad (4.33)$$

This ratio is depicted in Figure 4.6 which also illustrates the clear separation of the three regimes (i-iii).

The large value of the gluon mass gap effectively switches off the glue dynamics and α_g decays towards the IR as $k \lesssim k_{\text{conf}}$. Conceptually, this picture allows for an scenario without chiral symmetry breaking if α_g peaks too early with a relatively small peak value. This intricacy is responsible for the fact that few-flavour theories are very sensitive to the value of the coupling

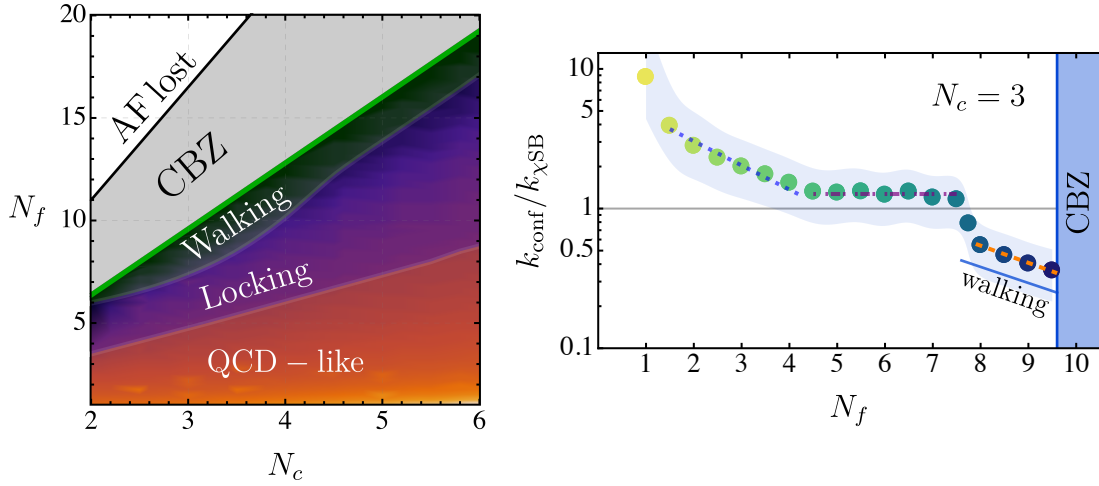


Figure 4.6: Left plot: regimes of $SU(N_c)$ gauge theories with N_f fundamental flavours. Above the top black line at $N_f \approx 5.5N_c$, asymptotic freedom is lost. The CBZ conformal window is depicted by a shaded grey region. Below the green line at $N_f \approx 3.2N_c$, confining and chirally breaking dynamics appear. In the close-conformal region, a regime with near conformal scaling is present where $k_{\text{conf}}/k_{\chi\text{SB}} \lesssim 1$ and for fewer flavours, dynamics dominate in the locking and QCD-like regimes.

Right plot: ratio of the confinement scale, k_{conf} , to the chiral symmetry breaking scale, $k_{\chi\text{SB}}$, is shown for an $SU(3)$ gauge theory with N_f fundamental flavours. The fitted lines show the different scaling behaviours: the QCD-like regime (i) following (4.33) (dotted blue), the regime where confinement and $d\chi\text{SB}$ are locked (ii) following (4.34) (dot-dashed purple), and the walking regime (iii) following (4.35) (dashed orange). The shaded blue bands indicate a 40% relative variation, with a slight enhancement in the few-flavour limit. The vertical blue line marks the onset of the conformal CBZ window, as discussed in Subsection 4.1.5.

and its momentum dependence. In short, physical QCD and even more the $N_f = 1$ case, are *living on the (chiral) edge*, see [43].

Furthermore, in terms of an absolute UV scale, the onsets of confinement and chiral symmetry breaking are delayed with an increasing number of flavours. This can be seen in Figure 4.7 where we have depicted the relevant scales in units of an absolute UV scale for different N_f . We show the results in the single (diamonds) and multi-avatar (stars) approximations. For $N_f = 1$, the single-avatar approximation displays a dip in $k_{\chi\text{SB}}$ in comparison to the multi-avatar truncation. This is an artefact of the latter truncation which displays a too low $k_{\chi\text{SB}}$ in the $N_f \lesssim 2$ limit. This is a straightforward consequence of only accounting for a single avatar (the fermion-gauge coupling) which is demanded to describe both, qualitatively different confining and chiral dynamics. As shown in Figure 4.2, the pure gauge avatars are the responsible for

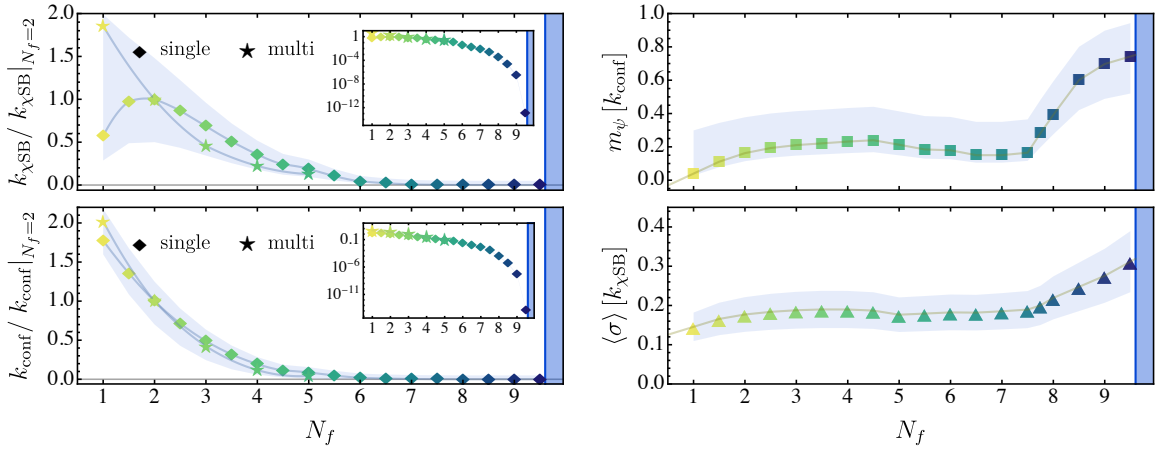


Figure 4.7: Left panel: N_f dependence of the $k_{\chi\text{SB}}$ (top) and k_{conf} scales (bottom) in units of an absolute UV scale (where all couplings are initiated with $\alpha_g \approx 0.08$) for a SU(3) gauge theory with N_f fundamental flavours. Values are normalised to those in the two-flavour case. We show the results accounting for a single (diamonds) and multiple avatars (stars) of the gauge coupling. The shaded areas depict the estimated errors discussed in [5]. The inset plots show the same quantities in a logarithmic scale.

Right panel: N_f dependence of the constituent fermion mass $m_\psi[k_{\text{conf}}]$ (squares in the top plot) and the chiral condensate $\langle\sigma\rangle[k_{\chi\text{SB}}]$ (triangles in the bottom plot). The blue bands show the estimated error.

confinement and peak much lower than the gauge-fermion one, responsible for $d\chi\text{SB}$. Identifying all to the same, requires a longer exposure to the gauge dynamics in order to trigger $d\chi\text{SB}$ which causes a delay in the appearance of $k_{\chi\text{SB}}$. Moreover, this defect propagates into $k_{\chi\text{SB}}$ -dependent quantities such as the constituent fermion mass or the chiral condensate, which both show a smaller magnitude in units of k_{conf} . Consequently, it is well understood why the single-avatar truncation in the few flavour limit only provides a qualitative description. On the other hand, we point out the remarkable agreement in k_{conf} between both truncations. In shaded blue in Figure 4.7, we depict the error estimate on these quantities accounting for the knowledge from both approximations each successful in a different regime. For a detailed discussion we refer to the original manuscript [5].

4.1.4.2 Locking regime for $5 \lesssim N_f \lesssim 7$

In this regime we encounter a novel phenomenon, the *locking* of confinement and chiral symmetry breaking, see Figure 4.6: the ratio of the confinement and $d\chi\text{SB}$ scales is approximately unity for all N_f in this regime,

$$k_{\text{conf}}/k_{\chi\text{SB}} \approx 1. \quad (4.34)$$

This is in stark contrast to the decrease of the ratio in the QCD-like regime and the scaling in the walking regime, discussed in the next Subsection 4.1.4.3, see (4.35). Its detection in the present work is closely linked to the differential scale resolution inherent to the fRG. In this strongly correlated regime, fermion self-interactions are more enhanced than in the QCD-like regime, causing typically subdominant channels to become relevant in the chiral dynamics. In fact, in this regime, we find evidence for potential critical channels at $k < k_{\chi\text{SB}}$, particularly the second-leading $(V-A)_{\text{adj}}$ channel. A finer resolution of the dynamics is provided in the right plot of Figure 4.7, where we show the constituent fermion mass in units of the confinement scale, $m_\psi[k_{\text{conf}}]$ (squares), and the chiral condensate in units of the chiral symmetry breaking scale, $\langle\sigma\rangle[k_{\chi\text{SB}}]$ (triangles). Both quantities are approximately constant for $3 \lesssim N_f \lesssim 7$, also providing further evidence for our definitions of k_{conf} and $k_{\chi\text{SB}}$ as good proxies for the confinement and $d\chi\text{SB}$ scales.

To our knowledge, this phenomenon was not observed before [5] and this exciting locking of the confining dynamics and that of $d\chi\text{SB}$ deserves a detailed analysis as it certainly leads to new insights in both of them. In this context we remark that a similar phenomenon has been observed in finite temperature QCD [422, 423], where it was shown that the order parameters of both phase transitions, the Polyakov loop and the chiral condensate, are dynamically locked as are the phase transitions.

4.1.4.3 Walking regime for $8 \lesssim N_f < N_f^{\text{crit}}$

As discussed before, at the interface between the locking and walking regime (iii), the chiral dynamics overpowers the confining one. Hence, the locking is released and the ratio $k_{\text{conf}}/k_{\chi\text{SB}}$ changes rapidly within less than one additional flavour until it settled in the walking regime, see Figures 4.6 and 4.7. Indeed, this behaviour illustrates the locking itself.

We see in Figure 4.5, showing the momentum-scale dependence of the fermion-gauge coupling α_g , that a growing momentum window with close-conformal scaling emerges in the regime $8 \lesssim N_f < N_f^{\text{crit}}$. The scaling window starts at a momentum scale k_{walk} where the coupling growth slows down. This scale is approximately determined by the minimum of the gauge coupling flow in the regime $k > k_{\chi\text{SB}}$ and its marked by a dot-dashed vertical line in Figure 4.5. This near-root in the gauge flow is approached before the onset of $d\chi\text{SB}$ and indicates the appearance of *walking* spanning over a range of scales. The walking window extends rapidly towards the IR as the number of flavours approaches the critical one. As well, $k_{\text{conf}}/\Lambda_{\text{UV}}$ and $k_{\chi\text{SB}}/\Lambda_{\text{UV}}$ in terms of the UV cutoff scale Λ_{UV} , rapidly decay. The critical flavour number where conformality appears is $N_f^{\text{crit}} \approx 9.6$ for $N_c = 3$ and its precise determination will be the topic of Subsection 4.1.5.

The emergence of walking triggers a particular scaling of fundamental parameters which is shown in Figure 4.6. This is directly caused by a progressive separation of the dynamical scales

$k_{\text{conf}}/k_{\chi\text{SB}} \ll 1$ given by the walking regime emerging at weaker values of α_g as N_f increases. Moreover, after fermions decouple at $k \lesssim m_\psi \sim k_{\chi\text{SB}}$, the gauge dynamics are approximately those of a pure gauge theory. Hence, α_g grows until the Yang-Mills-like gluon mass gap appears. Then, in the regime $k \lesssim k_{\chi\text{SB}}$, the difference between close-conformal theories is given by the boundary condition of the gauge coupling at $k_{\chi\text{SB}}$. For increasing N_f at a fixed N_c , we observe an exponential decay (dashed orange line in Figure 4.6)

$$k_{\text{conf}}/k_{\chi\text{SB}} \propto e^{-\gamma^{(\text{iii})}N_f} \quad \text{with} \quad \gamma^{(\text{iii})} \approx 0.28, \quad (4.35)$$

in the ratio of the two dynamical scales.

The decay of the confinement scale relative to that of $d\chi\text{SB}$ suggests that the spectrum of resonances in the close-conformal limit features glueballs that are increasingly lighter than the mesonic and baryonic states, except for the massless NGBs modes, in contradistinction to QCD-like theories. Being short of a complete analysis that would necessitate solving bound state equations in the current setup, we use the current proxies for the dynamical mass scales: to begin with, the mass scale of chiral symmetry breaking is the chiral condensate, which is proportional to $\langle\sigma\rangle$ and the constituent fermion mass is proportional to the product of the chiral condensate and the Yukawa coupling, see (4.21). If we use it as a proxy for the characteristic hadron mass scale, its ratio with the confinement scale provides an insight in the development of the mass ratios of glueballs and, say, baryons. Surprisingly, this ratio freezes in close to the conformal window, even though the ratio of the chiral condensate and the confinement scale rises even stronger than the ratio of $k_{\chi\text{SB}}$ and the latter. This hints at a quite non-trivial dynamics in the vicinity of the conformal window, which deserves further investigation.

Finally, we investigate the scaling of the walking region with N_f . In Figure 4.8 we show $k_{\chi\text{SB}}$ in units of the UV scale (lower blue circles) zooming into the conformal regime. Additionally, we display the onset scale of the walking regime (k_{walk} , top squares) in absolute units of the Λ_{UV} scale. The shaded regime shows the orders of magnitude over which α_g walks. For $\Delta N_f^{\text{crit}} = N_f^{\text{crit}} - N_f \lesssim 10^{-2}$, that is close to the conformal window, we find

$$\ln[k_{\chi\text{SB}}/\Lambda_{\text{UV}}] \sim \text{const} + 2.78 \ln \Delta N_f^{\text{crit}}, \quad (4.36)$$

see the orange dashed line in Figure 4.8. This is in very good agreement with Miransky scaling [186, 372, 373, 412, 413, 424] and equivalent to a Berezinskii-Kosterlitz-Thouless transition in two dimensions [425]. Notably, in the approach to the conformal window, the size of the walking regime expands towards infinity characterising the nature of the conformal phase transition.

Determining the size of the walking regime is pivotal for beyond the SM model building. It requires an approach sensitive to the appearance of fixed points and capable of describing the non-perturbative emergence of $d\chi\text{SB}$, as the employed in the present computation. The right plot in Figure 4.8 displays the size of the walking regime, $\Delta k_{\text{walk}} = k_{\text{walk}}/k_{\chi\text{SB}}$, as a function of

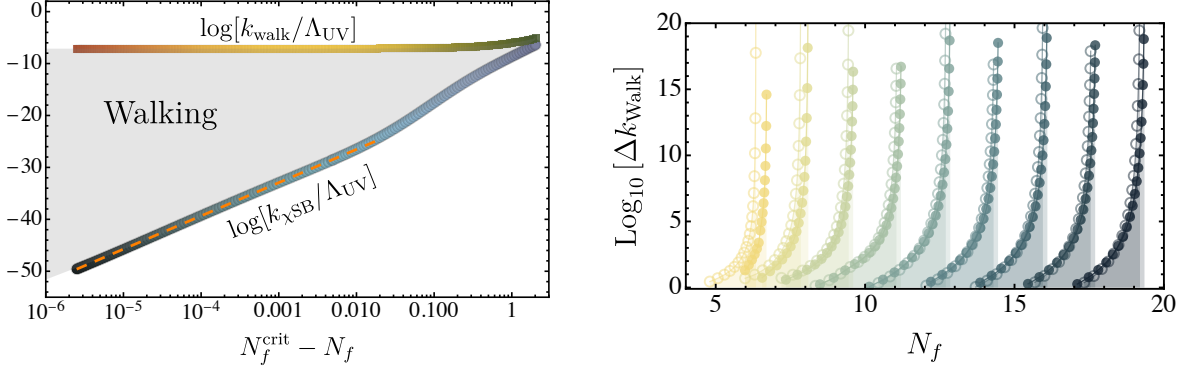


Figure 4.8: Left plot: near-conformal scaling of the onset of the dχSB ($k_{\chi\text{SB}}$, bottom circles) and walking (k_{walk} , top squares) scales in units of an absolute UV scale (Λ_{UV}). The grey-shaded region indicates the length of the walking regime. The dashed orange line shows the fit in (4.36), describing near-conformal scaling.

Right plot: walking regime size (Δk_{walk}) for different N_f and SU(N_c) gauge theories with numbers of colours from $N_c = 2$ (left-most light orange points) to $N_c = 6$ (right-most and dark blue points) in half integer steps. We show estimates employing three- (circles) and four-loop (filled circles) $\overline{\text{MS}}$ beta functions.

the number of fundamental flavours and colours. We emphasize the novelty of this first-principles result and its importance for existing models [2, 296, 397, 416–419].

A detailed determination of the critical exponents associated to the conformal transition and the size of the walking regime for different gauge groups and fermionic representations will be provided in an independent work.

4.1.5 Dissecting the conformal window

In this Subsection we address the conformal limit of gauge-fermion QFTs introduced in Subsection 4.1.3. In particular, we provide a quantitative estimate for the lower boundary of the conformal window $N_f^{\text{crit}}(N_c)$, improving qualitatively and quantitatively on previous estimates from functional approaches. This improvement is based on the emergent composites formulation which allows us to systematically incorporate resonant massless and close-massless composites. Importantly, this also includes the multi-scattering effects in these channels that are known to be important in the vicinity of phase transitions, or more generally, in scaling regimes.

In Section 2.3 we have discussed a necessary condition for the emergence of dχSB: the gauge coupling must exceed the critical coupling strength $\alpha_{\chi\text{SB}}^{\text{crit}}$ in (2.52), satisfying (2.53). For a sufficient criterion it is crucial to analyse all scenarios that entail the disappearance of the CBZ fixed point for small enough N_f . We distinguish two scenarios which also encompass those introduced in Subsection 4.1.3: In the first scenario, the CBZ fixed point disappears at values

larger than $\alpha_{\chi\text{SB}}^{\text{crit}}$, either towards infinite or via a merger in the strong limit. In the second scenario, the CBZ fixed point disappears at values smaller than $\alpha_{\chi\text{SB}}^{\text{crit}}$, most likely via a merger. Each of these two scenarios leads to a qualitatively different dynamical behaviour of theories with $N_f \lesssim N_f^{\text{crit}}$.

In the case where the CBZ fixed point disappears at values larger than $\alpha_{\chi\text{SB}}^{\text{crit}}$, a walking regime necessarily appears at $N_f \lesssim N_f^{\text{crit}}$, signalled by close-zero β -functions for the gauge coupling and the dimensionless four-Fermi coupling, to wit,

$$\beta_{\alpha_g}(\alpha_{\text{walk}}) \rightarrow 0^- \quad \text{and} \quad \beta_{\bar{\lambda}_{\text{SP}}}(\alpha_{\text{walk}}) \rightarrow 0^- . \quad (4.37)$$

Note that in this scenario the walking regime of the gauge coupling is achieved first and the four-Fermi coupling is dragged towards its walking regime in a finite flow time. The smallness of the β -functions in the walking regime is inversely proportional to its lengths, and $d\chi\text{SB}$ occurs at a finite flow time as the β -function of the dimensionless four-Fermi coupling is negative for all $\bar{\lambda}_{\text{SP}}$, see Figure 2.3 in Subsection 2.3.2 and the related discussion. The occurrence of $d\chi\text{SB}$ also terminates the walking of the gauge coupling and confinement follows suit.

As N_f^{crit} is infinitesimally approached from below, the walking coupling α_{walk} tends to the critical one from above, $\alpha_{\text{walk}} \rightarrow \alpha_{\chi\text{SB}}^{\text{crit}}$. This opens the limit where an infinite walking regime occurs as both couplings approach the critical one, $\alpha_{\text{walk}} \rightarrow \alpha_g^* \rightarrow \alpha_{\chi\text{SB}}^{\text{crit}}$. This leads to the disappearance of the $d\chi\text{SB}$ scale, k_{SSB} , towards zero and an infinite size walking regime as the conformal window is approached, as depicted in the left plot in Figure 4.8. The described mechanism explains the smooth and continuous disappearance of dynamics and the appearance of quantum scale invariance.

In conclusion, in this scenario the introduced condition (2.53) is both, necessary and sufficient. Hence, the boundary between dynamics and conformality is given by the theories satisfying

$$\alpha_{\chi\text{SB}}^{\text{crit}} = \alpha_g^* : \quad \beta_{\bar{\lambda}_{\text{SP}}} = 0 = \beta_{\alpha_g} . \quad (4.38)$$

A plausible but unlikely sub-scenario within the present class may be given when the $d\chi\text{SB}$ scale disappears at finite values as the conformal window is reached. This occurs if the CBZ fixed point profile displays a discontinuity in N_f . Then for $N_f < N_f^{\text{crit}}$, α_{walk} does not cross $\alpha_{\chi\text{SB}}^{\text{crit}}$ but remains at larger values. Then, the discontinuity at $N_f \geq N_f^{\text{crit}}$ continues with a fixed point at $\alpha_g^* < \alpha_{\chi\text{SB}}^{\text{crit}}$, realising conformality. Here, the conformal-non-conformal phase transition is of first order.

The second possibility involves the CBZ fixed-point solution disappearing at values smaller than $\alpha_{\chi\text{SB}}^{\text{crit}}$, most likely via a fixed-point merger. This mechanism, reviewed in Subsection 4.1.3, is indeed present in the perturbative five-loop $\overline{\text{MS}}$ β -function for α_g [362, 426, 427]. Up to this order, and without further improvement of the perturbative series, the CBZ fixed point disappears at weak coupling values, much smaller than $\alpha_{\chi\text{SB}}^{\text{crit}}$. In this scenario, the lower boundary of the conformal window is determined by the merger rather than the appearance of a dynamical scale.

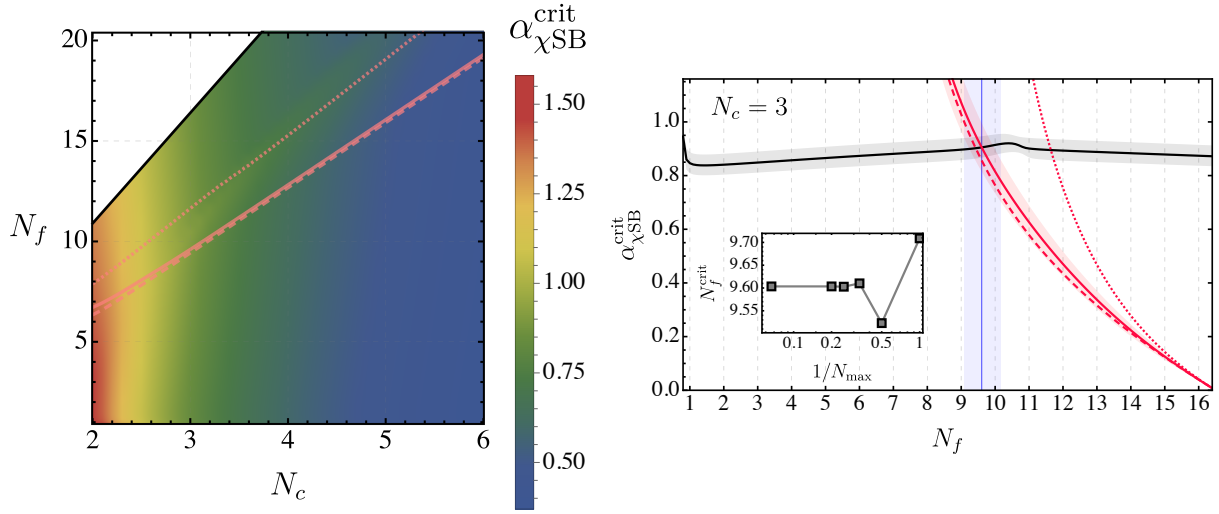


Figure 4.9: Left plot: critical exchange gauge coupling ($\alpha_{\chi\text{SB}}^{\text{crit}}$) for $\text{SU}(N_c)$ gauge theories with N_f flavours in the fundamental representation. The upper black line marks the loss of asymptotic freedom and the pink ones the lower boundary of the CBZ window considering two- (dotted), three- (dashed) and four-loop (plain) $\overline{\text{MS}}$ results.

Right plot: $\alpha_{\chi\text{SB}}^{\text{crit}}$ as a function of N_f projected onto $N_c = 3$. The grey-shaded area depicts the error in the $\alpha_{\chi\text{SB}}^{\text{crit}}$ determination and the red-shaded one, the error estimated on the fixed-point value. The vertical blue line marks N_f^{crit} and the shaded area its estimated uncertainty. In the inlay plot, we display N_f^{crit} as a function of the highest-order fermionic operators accounted for in (4.39).

In this case, theories below the conformal window do not exhibit a quasi-root in the flow of the gauge coupling, leading to the absence of walking for $N_f < N_f^{\text{crit}}$. Then the locking regime previously found up to $N_f \sim 7.5$, extends up to N_f^{crit} . The transition between the conformal and dynamical phases in this cases matches that of the latter sub-scenario and is of first-order type.

While this scenario cannot be entirely ruled out, we consider it unlikely. Notably, the fixed-point merger is absent after a (Padé or Borel) resummation [363] and does not appear in lower-loop orders of the coupling. The fRG inherently generates such a resummation via the $[\Gamma_k^{(2)} + R_k]^{-1}$ structure, which supports the resummation results. Moreover, lattice computations do not observe this scenario, instead finding indications of walking regimes. In summary, this suggests that fixed-point mergers are artefacts of the polynomial expansion of the β -function in powers of α_g , where it is well-known that such expansions may not only introduce fixed-point mergers but also additional fixed points.

4.1.5.1 Boundary of the conformal window

Having discussed the sufficient criteria for determining the conformal regime in various scenarios we now focus on the likely scenario where the CBZ fixed point disappears at larger values than the critical coupling $\alpha_{\chi\text{SB}}^{\text{crit}}$. The boundary between the conformal and dynamical regimes is then given by the equality in (4.38). For a quantitative determination of $N_f^{\text{crit}}(N_c)$, where this relation is satisfied, we require two pieces of information: the precise dependence of $\alpha_{\chi\text{SB}}^{\text{crit}}$ and the CBZ fixed point α_g^* on the number of flavours and colours.

In the present approach with emergent composites, the absence of $d\chi\text{SB}$ (signalled by $\beta_{\lambda_{\text{SP}}} = 0$) appears as a globally positive curvature of the effective chiral potential (4.18) at $k = 0$. More precisely, $\partial_{\bar{\rho}} V|_{\bar{\rho}=0} > 0$, or equivalently, the first expansion coefficient $\lambda_{\phi,1,k=0} > 0$. The gauge dynamics strength that nullifies these quantities at $k = 0$ precisely determines $\alpha_{\chi\text{SB}}^{\text{crit}}$.

In particular, for the first time we consider higher order mesonic scattering effects. These are captured in the effective potential including high-order interactions in the mesonic fields up to order N_{max} . These terms accommodate key non-perturbative effects encoded in fermionic self-interactions up to power

$$(\bar{\psi} \mathcal{T}_{(\text{S-P})} \psi)^{2N_{\text{max}}}, \quad (4.39)$$

in the resonant and dominant scalar-pseudoscalar channel $\mathcal{T}_{(\text{S-P})}$. We emphasise that non-perturbative contributions are accounted for when $N_{\text{max}} > 1$. These effects are crucial for a quantitative determination of N_f^{crit} , see the inlay in the right panel of Figure 4.9, and are one of the novel advances of the present work. In the following analysis we employ $N_{\text{max}} = 5$ which shows well converged results.

The second ingredient for the determination of N_f^{crit} with (4.38) is the CBZ fixed-point values as a function of N_f . For a quantitative determination, we include higher loop orders of the α_g beta function derived in perturbative computations. Accordingly, we infuse this running into the hybrid non-perturbative approach discussed in the previous Subsection that includes both, the multi-loop perturbative running for small couplings and the emergence of $d\chi\text{SB}$ and confinement for larger values. This heuristically amounts to substituting the anomalous dimensions and β -function prefactors in the diagrams by those of higher-order perturbation theory. This procedure can be applied systematically and here we do a first step in this direction directly using perturbative β -functions in the $\overline{\text{MS}}$ scheme, available up to several loop orders [362, 428, 429, 429–431]. This approximation ignores the relative RG scales, which manifest themselves, amongst other effects, in different Λ_{QCD} in the $\overline{\text{MS}}$ scheme and that used (implicitly) in the fRG , the MOM^2 scheme, see [432]. A detailed discussion of the respective errors, the systematic improvement of this scheme, and the incorporation of results from other perturbative approaches, such as the mMOM scheme [433, 434], super-YM-inspired beta functions [435–438], and others, as well as the application of improvement techniques like Padé and Borel resummations [363, 437], will be addressed in a

separate work.

With these preparations, we are set up to predict the boundary of the conformal window. In the left panel of Figure 4.9, we depict $\alpha_{\chi\text{SB}}^{\text{crit}}$ for arbitrary $\text{SU}(N_c)$ gauge theories with N_f fundamental fermions. Its profile decreases with N_c and is approximately constant in N_f . We mark the line of theories satisfying the critical condition (4.38) employing two- (dotted), three- (dashed) and four-loop (plain) $\overline{\text{MS}}$ results in the determination of α_g^* . On the right panel, we show the projection on to the $N_c = 3$ plane (black line) and display the fixed-point values for each of the perturbative beta functions studied (red lines). The critical coupling derived in the best approximation of the bosonised system matches the fixed-point value of the four-loop $\overline{\text{MS}}$ scheme at

$$N_f^{\text{crit}}(N_c = 3) = 9.60_{-0.53}^{+0.55}, \quad (4.40)$$

providing a non-perturbative and quantitative prediction for the boundary of conformality. A detailed analysis of the error estimate determination as well as a comparison to different truncations and previous computations can be found in the Appendices of [5]. In the following, we briefly discuss the two main sources of error.

First, the present approach uses input from high-order perturbative computations for the beta-function coefficients and fixed-point values which are scheme dependent and subject to an error. For the employed beta functions we estimate an error of 10% in the magnitude of the fixed-point value (red shaded area in the right plot in Figure 4.9). We find this to be a good estimate given the relative difference of the four- and three-loop ones is less than 3%.

The second source of error in (4.40) comes from the determination of $\alpha_{\chi\text{SB}}^{\text{crit}}$ in the fRG. A detailed analysis as well as numerous checks of the truncation were performed concluding that the results presented are very stable and all the aspects of the truncation are well converged. However, we consider a 5% error (grey band) for unaccounted tensor structures in the gauge-fermion vertex.

We close this Section with a discussion on the commonly used approach for diagnosing the appearance of $\text{d}\chi\text{SB}$. It is based on a perturbative computation of the anomalous dimension γ_m of the fermion mass, see (4.31) in Subsection 4.1.3. It is suggestive that $\text{d}\chi\text{SB}$ occurs for a fixed-point value $|\gamma_m^*| \sim 1$. The non-perturbative fRG approach here used allows us to assess the accuracy and reliability of such a determination of the boundary of the conformal regime. The details of the computation of γ_m^{fRG} are presented in the Appendices of the original manuscript [5]. This anomalous dimension effectively incorporates higher orders of perturbation theory as well as the dynamics of chiral symmetry breaking absent in the standard purely perturbative setup. We quote the result on the four-loop $\overline{\text{MS}}$ fixed-point values for $N_c = 3$ and N_f^{crit} ,

$$\left| \gamma_m^{\text{fRG},*} \right| (N_f^{\text{crit}}) = 1.49. \quad (4.41)$$

In Figure 4.10, the anomalous dimensions are evaluated at the four-loop $\overline{\text{MS}}$ fixed point for different N_f . The functional results lie closely to the three-loop anomalous dimensions for

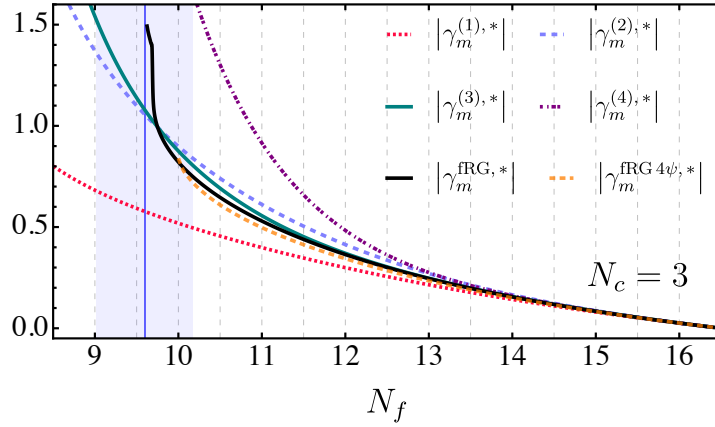


Figure 4.10: Comparison between the non-perturbative fRG fermion mass anomalous dimension (γ_m^{fRG} , black line), the obtained in the pure four-Fermi form ($\gamma_m^{\text{fRG},4\psi}$, orange dashed) and the results with the one- ($\gamma_m^{(1)}$, red dotted), two- ($\gamma_m^{(2)}$, light blue dashed), three- ($\gamma_m^{(3)}$, green plain) and four-loop ($\gamma_m^{(4)}$, dark purple dot-dashed) $\overline{\text{MS}}$ expressions [430]. All are evaluated on the four-loop $\overline{\text{MS}}$ fixed-point values. The vertical blue line marks N_f^{crit} and the shaded area its estimated uncertainty.

$N_f \gtrsim 10$, before non-perturbative effects become significant. Indeed, in the non-perturbative $|\gamma_m^{\text{fRG},*}|$ we observe an incremental growth as the critical flavour number is approached. This effect is driven by high-order mesonic interactions leading to $\bar{m}_\phi^* \lesssim 10$ and manifests itself in the threshold functions. In summary, this highlights the importance of a non-perturbative treatment of gauge-fermion systems even within the conformal window. Moreover, it provides non-trivial evidence for the potential existence of strongly interacting conformal field theories at the lower boundary of the CBZ window, which may have interesting phenomenological consequences.

This result has to be contrasted with the perturbative one-, two-, three-, and four-loop $\overline{\text{MS}}$ results [430] where

$$|\gamma_m^{(i),*}|(N_f^{\text{crit}}) = \{0.58, 1.05, 1.08, 2.64\}, \quad (4.42)$$

respectively. Interestingly, there are large differences between (4.42) and (4.41), if comparing approximations that accommodate the same order of perturbation theory deep in the conformal regime. We conclude from Figure 4.10 that this difference stems from the rapid rise of γ_m^{fRG} close to the lower boundary of the conformal regime. This rise originates in the strong chiral dynamics close to the boundary which cannot be captured within perturbation theory due to its non-perturbative nature. While this emphasises the lack of quantitative precision of the perturbative estimate, the present functional determination also shows that the perturbative estimate employing the two- and three-loop expressions provides a successful rough estimate.

4.2 The flavour puzzle in Composite Higgs models

This Section contextualises and collects publication [2].

In this Section we slightly change gears and move from the previous general analysis to a particular but relevant question in model building. One of the most popular applications of strong dynamics in new physics are CH models [44–48]. These exploit chiral and gauge dynamics alike the charted in the previous Section 4.1, to construct extensions of the SM in which a Higgs-like bound state exists, typically as a pNGB. Historically, CH models were motivated by the hierarchy problem which is resolved as the Higgs mass is dynamically generated and hence absent at the fundamental level of the theory. Furthermore, these models have shown to provide suitable DM candidates and mechanisms for generating the matter-antimatter asymmetry observed in the Universe, among other solutions to the present SM puzzles, see e.g. [439–441] for a review.

A common challenge in CH models is the generation of fermion masses. While in the SM, Yukawa couplings between fermions and the elementary Higgs are free parameters of the theory, in these composite realisations, such interactions arise from higher-dimensional operators which at low energies, manifest as fermion-meson interactions. Moreover, fermion masses are generated from linear mixings of SM-like fermions of each flavour with composite fermionic operators consisting of fundamental fields bound by a new confining interaction. Below the condensation scale (around the TeV scale), these mixings render the fermion mass eigenstates as superpositions of elementary SM-like fermions and composite resonances. This mechanism is referred to as *partial compositeness* [415].

Realising the correct mass hierarchy in this framework translates into the challenging task of solving strongly coupled dynamics and designing the appropriate fundamental theory with the adequate symmetries. Canonical dimensional analysis suggests that a straightforward choice is to construct composite fermions as bound states of three fundamental fermionic degrees of freedom [414, 442–444]. However, such constructions face severe challenges: the scaling dimensions of composite operators must significantly deviate from their canonical values to generate the heavy fermion spectrum, such as for the top quark mass. Lattice results indicate that large deviations from the canonical scaling (i.e., large anomalous dimensions) are unlikely for three-fermion bound states [445–448]. Thus, these approaches may not substantially improve upon TC models, where fermion masses are generated through bilinear couplings to the EWSB condensate [449]. Consequently, exploring alternative mechanisms for realising partial compositeness becomes a priority.

The work [2], central to this Section, studies the mechanism of fermion mass generation in CH scenarios from a functional approach. We contributed to this task by computing the anomalous dimension of the composite fields with which the SM fermions mix. For this, we employed the fRG approach to emergent composites introduced in Section 2.3 and generalized it to accommodate

more exotic bound states. This first principles approach grants access to the bound state features directly from the fundamental properties of the theory, namely its symmetries and dynamics. As a first example, we examined a particular class of models in which SM fermions mix with bound states of new fermions and scalars (which could be bound-states of other fermions), known as *fundamental partial compositeness* [416, 450, 451]. From canonical counting arguments, this model is expected to reproduce, in a less challenging manner, the whole spectrum of SM fermion masses. Moreover, given it had not previously been treated with non-perturbative methods we chose it as the subject of this first study and provided estimates for its minimal incarnation.

The use of functional techniques to investigate the viability of fermion mass generation is highly advantageous. This approach provides a convenient framework where modifications can be easily implemented, offering a significant advantage over costly lattice simulations. Additionally, the same methodology allows for the study of other composite states, such as fermion bilinears or baryon-like bound states, with different UV completions—an aspect currently under investigation. Moreover, these models require the presence of walking regimes (see Subsection 4.1.4.3), which can be treated from first principles within this framework.

The conclusions obtained from [2] highlight that functional methods are a powerful tool for investigating whether CH models can realize the fermion mass hierarchies observed in the SM and address the flavour puzzle. For the specific case of the minimal fundamental partial compositeness model, we find that these hierarchies could be realized within certain ranges of the walking regime. Furthermore, the functional approach is highly versatile, enabling the exploration of other models with different matter content and composite realizations. This makes the present approach tailor-made for this task and similar applications.

See Appendix B for the published version.

4.3 Phase structure of extra-dimensional gauge(-fermion) theories

This Section contextualises and collects publication [4].

Another interesting direction towards resolving open puzzles is guided by the principles of minimality and unification, aiming to formulate the SM in a more compact form with fewer parameters at a more fundamental scale. These ideas were pioneered long ago with the proposal of unifying the SM gauge groups within a GUT [40, 452, 453]. Since then, various extensions and refinements have been proposed which additionally solve a broad range of puzzles, see e.g. [128, 454] for modern reviews.

An intriguing scenario is provided by the Hosotani mechanism [53, 455, 456], originally formulated for the unification of the EW gauge and scalar sectors of the SM in a five-dimensional spacetime setting. Here, the Higgs and Goldstone bosons should be interpreted as the five-dimensional components of the EW gauge fields. At short length scales, nature exhibits an additional flat extra dimension in which the SM is formulated in a more minimal manner. Then, the transition from five to four dimensions occurs at a certain critical length or energy scale, analogous to a finite temperature system. For length scales shorter than the radius of the extra-dimension, the effective four-dimensional description of the SM is recovered, explaining the great success of the model.

Five-dimensional gauge theories have also been widely explored in other contexts, such as the hierarchy problem. A particularly popular approach involves warped extra dimensions [54] which effectively lower the Planck scale to near the EW scale.

Commonly to all model-building directions formulated in higher dimensions, a key theoretical question arises. The fundamental nature of gauge theories, with or without fermions, in five dimensions remains an open question. For dimensions greater than four, gauge theories lack a perturbative UV Gaussian fixed point, resulting in the absence of asymptotic freedom. These theories are perturbatively non-renormalisable as exhibit a gauge coupling with negative canonical dimension ($[g^2] = -1$ in five dimensions), analogue to the Newton coupling in four dimensional gravitational theories. However, evidence suggests the existence of non-trivial UV fixed points [3, 4, 84, 457] potentially rendering these theories non-perturbatively renormalizable and asymptotically safe. Given their exotic nature, understanding their phase structure in higher dimensions is not only of phenomenological interest but also of theoretical significance.

The work [4] central to this Section, contributes to this task by studying the phase structure of gauge-fermion theories in five space-time dimensions and its connection to the well-known four-dimensional limit. Here, we focus on the properties of the gauge vacuum and the fixed-point structures. The former, is particularly relevant for understanding gauge phase transitions along dimensions and the phenomenological realisation of the Hosotani mechanism. To investigate this extra-dimensional setting we employ the fRG which has been widely developed in the context of

gauge theories (e.g. QCD) as well as non-perturbatively renormalisable ones (e.g. ASQG in four dimensions).

This work [4], is a continuation of a previous one in which we studied pure Yang-Mills theories in a five-dimensional setting [3]. Here we extend the analysis to gauge-fermion theories, incorporating both fundamental and adjoint Dirac flavours. The work collects several noteworthy results:

- ◊ It explores the IR and UV fixed-point structure in the five-dimensional limit and its evolution to the four-dimensional landscape of gauge-fermion theories. We show how the boundaries for the loss of asymptotic safety (equivalent to asymptotic freedom in the four-dimensional case) emerge as a function of the fermion content.
- ◊ The formulation in terms of the compactification radius, analogous to temperature in 3+1 dimensions, combined with a mass-dependent RG scheme, enables the integration of RG trajectories emanating from the five-dimensional limit to the four-dimensional one. This setup led to the identification of interesting trajectories which are UV and IR non-trivially finite.
- ◊ Regarding the gauge vacuum, we study its dependence on fermion masses with different representations. We analyse the phase evolution as the dimensional transition occurs and find good agreement with results from effective theory computations. So far, in the range of theories investigated, we do not find viable realisations of the phenomenologically relevant scenarios, potentially pointing towards the need of more realistic embeddings with chiral content.

There are natural follow-ups to this work. First, it would be worth revisiting the question of renormalisability and aim to prove apparent convergence in the number of relevant operators for the five-dimensional theory. Additionally, in the pursuit of more phenomenological embeddings, it will be necessary to include chiral fermions, which require alternative spacetime constructions (e.g. orbifold) as they cannot be consistently defined in odd dimensions. Lastly, the embedding used in this study, along with the employment of a mass-dependent RG scheme, appears highly convenient for realistic phenomenological studies. In particular, theoretically viable models could ultimately be self-consistently implemented with functional realisations of the SM introduced in Chapter 3.

See Appendix C for the published version.

CONCLUSIONS AND OUTLOOK

So far, experimental tests of all Standard Model (SM) sectors have confirmed the remarkable accuracy of theoretical predictions. While no inconsistencies within the model forecast its breakdown below the Planck scale, further experimental evidence demonstrates the necessity of new physics beyond the SM (BSM). This calls for the theoretical effort of extending the SM beyond its current formulation to provide consistent and verifiable explanations for long-standing problems and maybe to puzzles.

In this thesis, we have contributed to addressing unresolved questions in fundamental physics with a novel approach based on the application of advanced quantum field theoretical techniques. Specifically, we have employed the functional Renormalisation Group (fRG), which offers a significant qualitative upgrade through a non-perturbative treatment compared to the standard methods used in high-energy physics. This quantitative and versatile framework allows us to study a wide class of theories, chart the theory space and uncover novel dynamics in a reliable manner, shedding light on the underlying nature of fundamental physics.

This thesis lays a foundation for integrating functional approaches into the domain of realistic high-energy particle physics applications beyond QCD, offering new insights on the SM and its extensions. We have studied a broad range of topics, each collecting several developments. To keep this summary concise, we overview the main novelties and refer to the respective Sections for the details.

In [Chapter 3](#), we studied the SM using the fRG framework, which served both to contextualise existing knowledge and to highlight beneficial features of the approach with respect to standard perturbative methodology:

- ◊ The mass-dependent nature of this formalism proved especially powerful for continuously connecting physics across diverse energy scales, from Planckian down to sub-MeV regimes.

In particular, it opens the door to threshold-sensitive quantities of experimental relevance, such as the Weinberg angle.

- ◊ We investigated the phase structure of the SM class of theories. This includes the (partial) fixed-point structures as well as the properties of the Higgs quantum phase transition.
- ◊ Exploiting the Wilsonian nature of the approach, we examined the sensitivity of infrared (IR) observables to ultraviolet (UV) scales, seeking, in a model-independent way, new physics directions guided by principles of naturalness and predictivity. In this context, we highlighted the criticality of the SM as a theory on the boundary to no electroweak symmetry breaking (EWSB).
- ◊ Furthermore, we showed the appearance of new non-trivially stable and unstable phases of the Higgs potential in the deep UV. These novel phases could introduce inconsistencies to the theory or provide a potential mechanism to render the theory UV complete. Moreover, the inclusion of new physics tends to decrease the scale at which the new phases appear, increasing their phenomenological relevance.
- ◊ Finally, we confronted the SM with the Planck scale and studied its minimal coupling to asymptotically safe quantum gravity (ASQG), resulting in the Asymptotically Safe Standard Model (ASSM). We explored the properties of this unified embedding and the conditions required for a viable UV completion.

The analysis in this Chapter provides a detailed view of the class of theories containing the SM. This new treatment not only represents a qualitative improvement in the assessment of the model but also provides a framework in which non-perturbative new physics extensions can be self-consistently implemented and tested alongside the SM.

In [Chapter 4](#), we shifted the focus to extensions of the SM, particularly to those exploring popular and compelling non-perturbative scenarios that are technically challenging:

- ◊ We extended the fRG approach to gauge-fermion QFTs, generalizing results from physical QCD to arbitrary theories. This study charted the landscape of gauge-fermion theories, examining the interplay between confinement and chiral symmetry breaking dynamics. From QCD-like regimes to the walking region, we uncovered a novel locking regime and provided quantitative estimates for the boundary between dynamical and conformal theories. This work unveils the dynamics of gauge-fermion theories and opens the door to model building in previously unexplored regions of theory space.
- ◊ Additionally, we investigated the more specific question of fermion mass generation within Composite Higgs (CH) models. This pressing question is a long-standing challenge for the viability of these kinds of extensions. Assessing it requires studying, in a non-perturbative

fashion, the strongly coupled dynamics in different models. Using dynamical bosonisation techniques, we provided an example of the derivation of the relevant quantities as well as direct estimates for the fermion masses. This exercise offers a new versatile methodology to assess this pressing question in other models with various dynamical mechanisms.

- ◊ Motivated by the existence of phenomenological models, we studied non-perturbative renormalisable gauge-fermion QFTs in five-dimensional spacetimes. This analysis focused on the phase structure of the theory space, the configurations of the gauge vacuum, and the connection to the four-dimensional limit. A solid understanding of the theoretical properties of this class of extradimensional theories is crucial for ensuring viable extensions.

The qualitative upgrade provided by a non-perturbative approach opens new phenomenological and theoretical avenues to be explored for BSM physics.

Throughout this work, we have developed new techniques within the fRG approach that can be exploited in other contexts. For instance, we presented a novel and simplified method to obtain confining correlation functions in gauge-fermion theories that can be applied to many other confining systems. Such examples are phenomenologically compelling gauge-scalar theories, studied in the context of naturalness, dark matter, and cosmological phase transitions.

The flexibility of the fRG enables exploration of the strongly interacting limit of theories typically avoided by phenomenological models. This limitation comes from standard methods being restricted to perturbative regimes with weak coupling. Furthermore, these omitted regions of theory space are particularly intriguing, as they can exhibit rich dynamics and may reveal new mechanisms. The approach used in this thesis lifts artificially imposed perturbative assumptions on new physics and allows for the exploration of all possible scenarios.

Future work may focus on incorporating realistic models involving strong dynamics or extra dimensions into the functional framework of the SM. This will enable a self-consistent analysis of the theoretical and phenomenological validity of such proposals.

In this thesis, we have studied the properties of theories in the vacuum. However, most of the analyses (e.g. the SM and gauge-fermion QFTs) can be straightforwardly extended to finite-temperature settings, where the thermal properties of phase transitions can be extracted. This is particularly significant in light of cosmological implications and upcoming gravitational wave observatories, which are expected to provide measurements of stochastic signals from early Universe phase transitions. This opens the door to test the studied class of models not only at particle colliders but also at the cosmological level.

In an era where the resolution of fundamental problems faces both limited experimental input and the lack of clear theoretical guidance, re-examining the SM and its extensions with new

approaches is more important than ever. This thesis has integrated advanced QFT methods into high-energy physics, succeeding to provide a qualitative upgrade in the study of new physics. By overcoming the limitations of traditional approaches, this work lifts artificial constraints imposed on new physics and uncovered novel dynamics potentially realisable in nature. It is an exciting time for new ideas and techniques with which to chart the way towards the fundamental theory of nature.

THE ASYMPTOTICALLY SAFE STANDARD MODEL

This Appendix contains [1], one of the core papers of this thesis, contextualized in Section 3.4.

The Asymptotically Safe Standard Model: From quantum gravity to dynamical chiral symmetry breaking

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Abstract

We present a comprehensive non-perturbative study of the phase structure of the asymptotically safe Standard Model. The physics scales included range from the asymptotically safe trans-Planckian regime in the ultraviolet, the intermediate high-energy regime with electroweak symmetry breaking to strongly correlated QCD in the infrared. All flows are computed with a self-consistent functional renormalisation group approach, using a vertex expansion in the fluctuation fields. In particular, this approach takes care of all physical threshold effects and the respective decoupling of ultraviolet degrees of freedom. Standard Model and gravity couplings and masses are fixed by their experimental low energy values. Importantly, we accommodate for the difference between the top pole mass and its Euclidean analogue. Both, the correct mass determination and the threshold effects have a significant impact on the qualitative properties, and in particular on the stability properties of the specific ultraviolet-infrared trajectory with experimental Standard Model physics in the infrared. We show that in the present rather advanced approximation the matter part of the asymptotically safe Standard Model has the same number of relevant parameters as the Standard Model, and is asymptotically free. This result is based on the novel UV fixed point found in the present work: the fixed point Higgs potential is flat but has two relevant directions. These results and their analysis are accompanied by a thorough discussion of the systematic error of the present truncation, also important for systematic improvements.



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1 Introduction

One of the most challenging open tasks in contemporary high energy physics is its ultraviolet (UV) closure including quantum gravity. In the past three decades, asymptotically safe gravity [1–3] has been established as a viable option for this endeavour, for reviews see [4–14].

Asymptotically safe matter-gravity systems potentially include the Asymptotically Safe Standard Model (ASSM), the UV-complete unification of the Standard Model (SM) with gravity. This minimal set-up for the fundamental theory of matter and gravity with an absence of new physics implies that our current understanding of particle physics may hold up to the Planck scale. Hence, all masses and couplings converge at an interacting asymptotically safe UV fixed point. The solidified existence of such a minimal set-up would also allow for a systematic and well-controlled extension toward UV-complete Beyond Standard Model scenarios including the potential exclusion of such UV-safe embeddings in specific models.

This highly interesting programme has been set-up and pursued in a number of works concerning the matter content compatible with asymptotic safety [15–23], as well as the relation to the IR physics [23–34], and beyond the SM physics [35–39]. The running of SM couplings and full Higgs potentials has also been studied without the inclusion of gravity in similar frameworks [40–49]. While the physical mechanisms in the UV and potential scenarios have been well understood, the control over the mechanisms is still lacking. For example, it is necessary to consider higher-order curvature invariants such as R^2 in order to reliably determine a bound on the field content compatible with asymptotic safety [23]. Also the gravity contributions to the matter couplings are only partially under control: while it has been well understood that gravity supports asymptotic freedom of the gauge coupling or is vanishing at leading order [23, 25, 50–54], the gravity contribution to the Yukawa coupling carries more intricacies [28, 55]. Moreover, so far, the full ASSM flows were investigated in perturbative threshold-free RG flows for the matter fields, in particular for sub-Planckian physics, as well as a (classical) ϕ^4 -approximation of the Higgs effective potential. For reliable predictions on the nature of the ASSM, a consistent RG flow including physical threshold effects is needed.

The ASSM covers physics at vastly different momentum scales ranging from trans-Planckian momenta with asymptotic safety in the UV to sub-Fermi momenta with strongly correlated quantum chromodynamics (QCD) in the infrared (IR). Its parameters or rather the specific UV-IR trajectory are fixed by their experimental values measured in the IR, which also fixes the respective UV fixed point. Importantly, the UV landscape includes several physically distinct fixed-point classes. They range from fixed points with stable asymptotically free matter parts (or shifted Gaußian fixed points), fixed points with fully interacting matter parts over unstable FPs to regimes without fixed points. These differences not only concern the existence, stability and interaction nature of the UV regime, but also the number of relevant directions. This can lead to predictive trajectories with fewer parameters in the matter sector of the ASSM than in the SM. Trivial examples are boundary trajectories between the stable and unstable FP classes as well as the Gaußian FP class with a flat Higgs potential with only one relevant parameter, the Higgs mass.

While many of the SM parameters have little impact on the UV fixed point class, if varied largely about the experimental value, there are few whose precise value and correct IR-UV trajectory has a huge and qualitative impact on both, high energy physics below the Planck scale and the trans-Planckian UV physics. In particular, the values and correct scale dependence of the top quark and Higgs masses have a qualitative impact on the UV fixed point class of the ASSM, as well as the high energy stability of the Higgs potential.

In short, reliable access to even the *qualitative* physics properties of the ASSM require *quantitative* control of its physics at all scales. Accordingly, this task requires a systematic, self-consistent approach able to treat non-perturbative physics both in the UV and IR. In the present work, we add to this task by computing the UV-IR flow of the ASSM self-consistently within the functional renormalisation group (fRG) approach. Here, self-consistency refers to two important aspects: Firstly, all coupling parameters, whose flows are computed are fed back to the flow, which leads to full resummations. This property is required for the rapid convergence of physics results and is mandatory for a precise determination of the IR parameters of the ASSM. Secondly, all flows are computed within the renormalisation scheme inherent to the fRG approach. In particular, this scheme incorporates physical threshold effects naturally and self-consistently due to its Wilsonian nature. In terms of standard RG schemes this can be phrased as follows: the fRG scheme leads to independence of the physics results from the RG-point already in relatively simple approximations.

In the present work, we add substantially to this endeavour by considering for the first time a fully consistent fRG system of all couplings of the ASSM. In particular, the physical thresholds of the respective matter and gravity dynamics are included. This is required for reliable and quantitative access to the physics of electroweak spontaneous symmetry breaking and strong chiral dynamical symmetry breaking. Furthermore, we take into account that the top mass parameter is related but not equivalent to the experimentally measured pole mass. We compute the pole mass of the top quark in the present non-perturbative setting and adjust the top mass parameter such that the pole mass takes its experimental value. This is essential for a correct estimate of the metastability scale of the Higgs potential.

Finally, we compute the full effective Higgs potential in the trans-Planckian regime within a high order of the Taylor expansion in the Higgs field. This allows us to reveal the presence of a *novel* UV fixed point for the Higgs potential that is present for a large parameter range: while the full effective potential is *flat*, it features *two* relevant directions. One of them is aligned with the Higgs mass operator, but the operator of the most relevant direction is non-polynomial in the Higgs field. In the current approximation, it is this novel non-trivial fixed point that is connected to the physical SM in the IR where the parameters are fixed by experimental observables.

In contradistinction, the standard Gaußian fixed point potential features *one* relevant direction, the Higgs mass. Naturally, for a vanishing coupling of the most relevant non-polynomial operator this FP is embedded in the novel FP as a one-dimensional sub-manifold. Parameter values in this sub-manifold are not connected to the physical SM in the IR: the resulting Higgs and top mass are roughly 3 GeV from the central experimental values. Whether this distance is close or feasible is subject to the evaluation and interpretation of the systematic error of the present approximation. This is but one of the reasons for the rather detailed description of the systematic error estimates in the present work.

The physical thresholds and the dynamics of spontaneous symmetry breaking are specifically important for two interrelated reasons: Firstly, in the absence of any thresholds the IR sector of the SM is not accessible, and the SM parameters have to be chosen for cutoff values at least above the electroweak scale. Such a procedure requires the identification of momentum and cutoff scales, which typically work qualitatively but not quantitatively. Secondly, the fRG renormalisation scheme differs significantly from the standard $\overline{\text{MS}}$ and MOM schemes used

in particle physics, and the use of the respective perturbative β -functions may only work qualitatively. In particular, the naive identifications of momentum and cutoff scales for physical thresholds such as the electroweak symmetry breaking scale may not be suitable. Indeed, we shall observe in the present computation, that it is nearly off by one order of magnitude. This has a significant impact on the parameter choices and hence on the selected UV-IR trajectory as well as potentially on the fixed-point class.

To wrap up, the present work includes for the first time the sub-Planckian physics of spontaneous symmetry breaking, both in the electroweak sector and in QCD, including all physical threshold effects. This is necessary (but not sufficient) for a precision determination of the matter parameters of the ASSM. We also determine the pole mass of the top quark instead of estimating it by Euclidean running masses. This precision computation is of qualitative importance for the stability of the Higgs sector at high energies, as well as the UV fixed point physics. For the latter, we find *novel* fixed point properties: the matter fixed point is asymptotically free (Gaussian or shifted Gaussian), but with a flat Higgs potential with *two* relevant directions. Our analysis includes a thorough error analysis, which suggests that the present findings have to be corroborated within systematic extensions of the truncation.

This work is organised as follows. In [Section 2](#) we discuss the fRG approach to the ASSM and detail our approximation. In [Section 3](#), we discuss the existence, stability, and physics properties of the UV fixed point and the ASSM. This also includes a part of the systematic error analysis. In [Section 4](#), we present our results on the full phase structure of the ASSM ranging from the asymptotically safe regime with the Reuter fixed point to the deep IR with QCD and chiral symmetry breaking, summarised in [Figure 3](#). The novel property of a flat effective Higgs potential at the UV fixed point with two relevant directions is discussed in [Section 5](#). There we also discuss the fixed-point landscape for general values of the gravity fixed-point values. In [Section 6](#), we evaluate the predictivity of the UV fixed point and perform an analysis of the main sources of the systematic error. In [Section 7](#), we conclude with a summary of our results, including a short discussion of some consequences. Most of the technical details have been deferred to Appendices.

2 Asymptotically Safe Standard Model

In the Asymptotically Safe Standard Model (ASSM), the coupled SM–quantum-gravity system is fully described as a quantum field theory. The underlying classical action consists of the classical gauge-fixed action of the SM and the gauge-fixed Einstein-Hilbert action of General Relativity,

$$S_{\text{ASSM}}[\phi] = S_{\text{SM}}[\phi] + S_{\text{gravity}}[\phi_{\text{grav}}], \quad (1)$$

for the explicit form see [Appendix B.1](#) (SM) and [Appendix B.2](#) (gravity).

The field content ϕ is given by the gauge fields $\mathcal{A}_\mu$ of the SM gauge groups, $SU(3)_C \times SU(2)_L \times U(1)_Y$, as well as the matter fields, leptons and quarks, $l, \bar{l}, q, \bar{q}$ and the scalar doublet, Φ . The metric field $g_{\mu\nu}$ is split into a flat Euclidean background metric $\bar{g}_{\mu\nu} = \delta_{\mu\nu}$ and a fluctuation field $h_{\mu\nu}$, which carries the dynamics of the metric,

$$g_{\mu\nu} = \delta_{\mu\nu} + \sqrt{16\pi G_N} h_{\mu\nu}. \quad (2)$$

We define the dynamical fluctuation superfield, which also includes the auxiliary ghost fields stemming from the gauge-fixing sector,

$$\phi = (\phi_{\text{grav}}, \phi_{\text{SM}}), \quad (3a)$$

with

$$\begin{aligned}\phi_{\text{grav}} &= (h_{\mu\nu}, c_\mu, \bar{c}_\mu), \\ \phi_{\text{SM}} &= (\mathcal{A}_\mu, \mathcal{C}, \bar{\mathcal{C}}, l, \bar{l}, q, \bar{q}, \Phi).\end{aligned}\quad (3b)$$

The matter superfield ϕ_{SM} comprises the gauge and ghost fields of the SM gauge group $U(1)_Y \times SU(2)_L \times SU(3)_C$,

$$\mathcal{A}_\mu = (B_\mu, A_\mu^a, G_\mu^b), \quad \mathcal{C} = (c^a, c^b), \quad (3c)$$

with the hypercharge gauge field B_μ , the weak gauge fields A_μ^a with $a = 1, 2, 3$, and the gluons G_μ^b with $b = 1, \dots, 8$. The field $\mathcal{C}$ contains the respective ghost fields of the weak and strong gauge groups. ϕ_{SM} also contains the three families of quarks q and leptons l ,

$$q = (d, u, s, c, b, t), \quad l = (e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau). \quad (3d)$$

The full quantum effective action of the ASSM, $\Gamma[\phi]$ contains all interaction terms that are compatible with the symmetries of the ASSM.

2.1 Functional RG for the ASSM

We are interested in the full UV-IR phase structure of the ASSM, ranging from the trans-Planckian asymptotically safe UV regime ruled by gravity, to the electroweak (EW) scale with EW symmetry breaking, and finally to the deep IR with strongly correlated QCD including strong chiral symmetry breaking and confinement. For covering this vast range of scales and different physics we use the functional renormalisation group (fRG) approach, which already has proven its applicability in all these regimes. In this approach, an IR regulator is introduced in the path integral, which suppresses quantum fluctuations below a given IR cutoff scale k , leading to the respective scale-dependent effective action $\Gamma_k[\phi]$. Then, the full effective action $\Gamma[\phi] = \Gamma_{k=0}[\phi]$ is obtained by successively integrating out momentum fluctuations at the scale k . The respective flow equation for Γ_k , the Wetterich equation [56–58], reads

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right], \quad (4a)$$

where

$$\Gamma_{\phi_{i_1} \dots \phi_{i_n}}^{(n)}[\phi](p_1, \dots, p_n) = \frac{\delta \Gamma[\phi]}{\delta \phi_{i_1}(p_1) \dots \delta \phi_{i_n}(p_n)}. \quad (4b)$$

Equation (4) provides us with a full non-perturbative setup, that enables us to access the full UV-IR phase structure of the ASSM.

2.2 Approximations of the quantum effective action

In general, (4) cannot be solved for the full effective action of an interacting theory, leaving aside the ASSM. Hence, we have to approximate the full effective action and its flows. In the present section, we summarise the approximations and the reliability considerations behind them, more details can be found in [Appendices B and C](#).

To begin with, we aim at an accurate description of the full dynamics of the gauge and scalar sectors, and specifically that of the EW sector. Hence, the scale dependence (average momentum dependence) of all primitively divergent vertices in this sector is taken into account. Within this setup, we can, for the first time, reproduce the EW transition $U(1)_Y \times SU(2)_L \rightarrow U(1)_{\text{EM}}$ including all threshold effects, starting from the asymptotically safe initial conditions.

2.2.1 Momentum symmetric point approximation

Below the Planck scale, quantum gravity effects quickly decouple and we are left with the quasi-perturbative quantum dynamics of the SM. Its effects are captured well by only considering the average momentum running (with k) of the primitively diverging vertices of the SM. To that end, we consider general vertices $\Gamma_k^{(n)}(p_1, \dots, p_n)$ defined at symmetric points with

$$p_i^2 = p^2, \quad \left| \frac{p_i^\mu p_j^\mu}{p^2} \right| = \cos \theta. \quad (5)$$

It has been shown both for strongly correlated QCD [59–62], and in gravity [14, 63, 64], that the system of flow equations of symmetric point vertices is well-approximated by only feeding back these vertices in the diagrams: effectively these vertices represent a close system of flow equations, and define a good approximation $\Gamma_k^{(\text{SP})}[\phi]$ of the full effective action if only being interested in symmetric point physics. Note however, that it can be shown in QCD, that this approximation is limited to vertices and regimes without resonant momentum channels in vertices such as the scalar and pseudo-scalar meson channels in QCD for momentum and cutoff scales $k, p \lesssim 1 \text{ GeV}$.

We also emphasise that $\Gamma_k^{(\text{SP})}[\phi]$ does not suffice to give access to important physics information such as S -matrix elements for generic scattering momenta related to the full vertices $\Gamma_k^{(n)}(p_1, \dots, p_n)$. However, the latter can be obtained in a systematic way from $\Gamma_k^{(\text{SP})}[\phi]$ by their flows. Importantly, feeding back the full scattering vertices to the flow of $\Gamma_k^{(\text{SP})}[\phi]$ leads to subleading effects.

In a final step, we introduce a further approximation to the symmetric-point vertices: they only depend on the symmetric-point momentum, the mass gaps $\vec{m}$ of the ASSM, and the cutoff scale k . For cutoff scales above the mass gaps, the (dimensionless) dressings can only depend on the ratio p^2/k^2 and we can trade one for the other. This property is also at the root of mass-independent RG-schemes, where (in asymptotic regimes) one can read off the momentum dependence of couplings from the RG-scale dependence. In turn, for cutoff scales below the mass threshold of specific fields, their contribution to the flows and the physics is suppressed, while the remaining flow still only depends on p^2/k^2 . In consequence, for symmetric-point vertices, the cutoff dependence (at $p = 0$) reflects well the momentum dependence with

$$k = \alpha p, \quad (6)$$

and hence supports a low-order derivative expansion with vertices $\Gamma_k^{(n)}(p = 0)$ with an approximate effective action $\Gamma_k^{(\text{av})}[\phi]$ of these average vertices. As for the symmetric-point vertices, this expansion holds in the absence of resonant vertex structures or more generally strong momentum and angular dependences of vertices. It is well-tested by now both in the asymptotically safe UV regime of gravity-matter systems [10, 16, 63–66] and also holds true in the SM including QCD [59, 61, 67, 68]. We remark that within such an expansion the mass parameter extracted at $p = 0$ corresponds to the Euclidean curvature mass and not to the pole mass. However, it has been shown that in the absence of strong momentum dependences of the wave-function renormalisation or anomalous dimensions the two agree well, see [69]. This intricate topic is discussed further in [12–14].

2.2.2 Couplings of the ASSM

In our approximation, all primitively divergent vertices, as well as all wave functions of the SM, are taken into account with scale-dependent dressings. Moreover, we consider an effective

Higgs potential $V_{\Phi,\text{eff}}$ in a high-order Taylor expansion about the flowing minimum. In short, we parametrise

$$\Gamma_k^{(n)}(p_1, \dots, p_n) = \prod_{i=1}^n \sqrt{Z_{\phi_i,k}} S^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k), \quad (7a)$$

where $S^{(n)}$ are the n th derivatives of the classical ASSM action in (1), augmented with higher-order Higgs-self interactions. In (7a) the classical couplings are substituted by their running quantum analogues,

$$\vec{\lambda}_k = (\vec{g}_{1,k}, \vec{g}_{2,k}, \vec{g}_{3,k}, \vec{y}_{q,k}, \vec{y}_{l,k}, \vec{\lambda}_{\Phi,k}, \vec{G}_k, \vec{\Lambda}_k). \quad (7b)$$

Here the vector on the coupling indicates that they represent a set of avatars of this coupling originating from different vertex functions, which will be explained below. The g_i with $i = 1, 2, 3$ are related to the hypercharge gauge coupling with $g_1 \equiv \sqrt{5/3} g_Y$, the weak gauge coupling g_2 , and the strong gauge coupling g_3 . The classical dispersions are augmented with wave functions,

$$Z_{\phi,k} = (Z_{A,k}, Z_{C,k}, Z_{l,k}, Z_{q,k}, Z_{\Phi,k}, Z_{h,k}, Z_{c,k}). \quad (7c)$$

The wave-function factors in (7c) encode the running of the attached fields, and the coupling parameters in (7b) are indeed running couplings (and masses), and not only vertex factors. From here on we drop the subscript k on all couplings and wave functions and their scale dependence is implied.

In the Higgs sector, we go significantly beyond the approximation in (7a), and consider the full Higgs potential within a high-order of a Taylor expansion about the flowing minimum. We parametrise the Higgs doublet Φ as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ v + H + i\mathcal{G}_3 \end{pmatrix}, \quad \rho = \text{tr } \Phi^\dagger \Phi, \quad (8)$$

where v is the flowing minimum, and H is the (radial) fluctuation Higgs field with a vanishing expectation value $\langle H \rangle = 0$, as the latter is explicitly carried by v . The $\mathcal{G}_i$ with $i = 1, 2, 3$ are the Goldstone modes. Then, the Higgs potential is a function of $\rho - v^2/2$. In the symmetric regime with a vanishing minimum of the potential, $v = 0$, we use the parametrisation

$$V_{\Phi,\text{eff}}(\rho) = V_0 + \sum_{n=1}^{N_{\max}} \lambda_{\Phi,2n} Z_{\Phi}^n \rho^n, \quad (9)$$

where $V_0 = \Lambda/(8\pi G_N)$ is related to the cosmological constant, see (B.16). The running mass parameter $\mu_{\Phi} > 0$ and the quartic Higgs self-coupling, already present in the classical Higgs potential, are

$$\mu_{\Phi} = \lambda_{\Phi,2}, \quad \lambda_{\Phi} = \lambda_{\Phi,4}, \quad (10)$$

and the expansion starts at $n = 1$ with the mass term.

In turn, in the broken regime with $v > 0$, we parametrise

$$V_{\Phi,\text{eff}}(\rho) = \tilde{V}_0 + \sum_{n=2}^{N_{\max}} \lambda_{\Phi,2n} Z_{\Phi}^n \left(\rho - \frac{v^2}{2} \right)^n, \quad (11)$$

where $\tilde{V}_0$ is V_0 minus the series, evaluated at $\rho = 0$. The lowest term in the series with $n = 2$ encodes the classical Higgs potential. At the *spontaneous symmetry breaking* cutoff scale k_{SSB} ,

below which EW symmetry breaking kicks in, the flow is switched from (9) for $k > k_{\text{SSB}}$ with a flowing μ_Φ to (11) for $k < k_{\text{SSB}}$ with a flowing ν_k .

In the following, we refer to both μ_Φ, ν^2 as $\lambda_{\Phi,2}$, and hence the set of coupling parameters of the Higgs potential is always given by $\vec{\lambda}_\Phi = \{\lambda_{\Phi,2n}\}$ with $n = 1, \dots, N_{\text{max}}$. It is understood that $\lambda_{\Phi,2} = \nu^2$ is taken in the broken regime while $\lambda_{\Phi,2} = \mu_\Phi$ is taken in the symmetric regime. More details on the EW symmetry breaking and the respective scales can be found in Section 4.2.

In the UV regime, we consider a high-order Taylor expansion with the maximal monomial power

$$N_{\text{max}} = 17. \quad (12)$$

This rather high-order of the Taylor expansion allows us to discuss the convergence of the potential at the UV fixed point. For the sake of simplicity, we do not consider such a general potential for scales below $k \leq 10^{17}$ GeV. Such non-trivial potentials at the EW and metastability scales will be discussed in detail elsewhere.

The flows of the couplings and wave functions in the approximation are extracted at vanishing momentum, which captures the average momentum dependence as discussed in detail at the beginning of this section. For the details of the projection onto the flow equations for all couplings, see Appendix C.

We close this section with a discussion of the gauge and gravity couplings. The set of coupling parameters (7b) comprises different avatars of gauge and gravity couplings as required for the flow of different gauge and gravity vertices. For gravity, we consider vertex couplings of the n -graviton vertices proportional to the classical tensor structure,

$$\Gamma_{h\dots h}^{(n)} \simeq Z_h^{n/2} S_{h\dots h}^{(n)}(G_n, \Lambda_n). \quad (13)$$

Note that G_n and Λ_n are the vertex couplings of the classical tensor structure. While they should not be confused with the Newton coupling G_N and cosmological constant Λ , they have the scale and momentum running of couplings. The different G_n and Λ_n are related by modified Slavnov-Taylor identities, see [14]. In the classical limit or rather in classical regimes, the effective action reduces to the Einstein-Hilbert action, and all these couplings agree,

$$G_n = G_N, \quad \Lambda_n = \Lambda. \quad (14)$$

Within our approximation, we use (14) throughout.

We also emphasise that the vertex gauge couplings defined in (7b) are subject to two-loop universality (in mass-independent RG schemes). However, in the presence of threshold effects and/or non-perturbative physics they differ genuinely. Specifically, we then have to consider avatars of the gauge couplings for different vertices. In the present approximation to the ASSM, this concerns the quark-gluon couplings as well as the matter-gauge couplings in the EW sector. For example, the quark-gluon couplings are

$$\vec{g}_3 = (g_{3,u}, g_{3,d}, g_{3,s}, g_{3,c}, g_{3,b}, g_{3,t}), \quad (15)$$

where the field subscripts $q_i = u, d, s, c, b, t$ indicate the quarks in the respective quark-gluon vertex. We consider the strong isospin symmetric approximation with

$$g_{3,u} = g_{3,d}, \quad (16)$$

for all scales. Moreover, for momentum scales beyond the top threshold, all avatars converge towards a unique strong vertex coupling, $g_{3,q_i} = g_3$. In turn, in the presence of non-perturbative physics and threshold effects the two-loop universality of gauge couplings (in mass-independent RG schemes) does not hold anymore, and the matter-gauge couplings do not necessarily agree anymore, and also differ from the pure gauge couplings.

2.3 Strongly coupled UV & IR regimes

In the IR regime for $k, p \lesssim 10 \text{ GeV}$, the approximation (7) has to be improved both in the gluonic sector for including confinement, as well as in the matter sector for including dynamical spontaneous chiral symmetry breaking, see [68]. Here, the epithet *dynamical* refers to the fact, that the quasi-goldstone modes of strong spontaneous chiral symmetry breaking in QCD are effective IR degrees of freedom, the pions, and not a fundamental field such as the Higgs field Φ in the ASSM. In turn, for more quantitative access to the asymptotically safe regime, momentum dependences for the pure gravity sector are required, see [64]. The two asymptotic regimes are discussed below in Sections 2.3.1 and 2.3.2.

2.3.1 Confinement and strong chiral symmetry breaking

At momentum scales $p, k \lesssim 10 \text{ GeV}$, strongly correlated IR-QCD starts getting relevant and even two-loop perturbative approximations successively lack reliability, for a detailed discussion see [70]. However, in this regime, we can resort to results in functional approaches for 2 and 2+1 flavour computations that meet lattice 2 and 2+1 flavour benchmarks in the IR regime of physical QCD, see in particular [61, 68, 70]. Hence, below an *interface scale* k_{inter} with

$$5 \text{ GeV} \lesssim k_{\text{inter}} \lesssim 15 \text{ GeV}, \quad (17)$$

we use IR-QCD flows as an external input in our system of ASSM flow equations: it has been shown in [70] that below $k \approx 10 \text{ GeV}$ two-loop resummed approximations successively lose their reliability, a -weak- limit being given by $k \approx 5 \text{ GeV}$. In turn, for $k \gtrsim k_{\text{inter}}$, the approximation to the ASSM flows discussed here suffices. Consequently, we use the stability of the results under a variation of k_{inter} as a consistency and reliability check of the interface flows, see Figure 12 in Appendix C.5.

Specifically, we utilise results from [68] as the approximation of the respective computation resembles most that used here, and the results can be used directly. This concerns the running of the quark masses including dynamical chiral symmetry breaking, as well as the running of the dressing of the gluon propagator, whose mass gap is related to the physical mass gap of QCD.

In terms of the correlation functions considered here, this amounts to using the gluon dressing Z_A or rather the anomalous dimension

$$\eta_A = -\frac{\partial_t Z_A}{Z_A}, \quad (18)$$

from [68]. This implements the physical gapping of glue interactions for small momentum scales and suffices to describe the respective decoupling of the glue sector from the matter sector semi-quantitatively. It has been shown in [68, 71] that the anomalous dimension is well-described as a function of α_s and the mass gaps of the theory. This allows us to write the full anomalous dimension as

$$\eta_A^{(\text{ASSM})} \approx \eta_A^{(2+1)}(\vec{\alpha}_s^{(\text{ASSM})}, \vec{m}^2) + \eta_A^{(c,b,t)}, \quad (19)$$

where $\vec{\alpha}_s^{(\text{ASSM})}$ is the vector of all strong fine structure constants derived from different vertices, see (C.21) in Appendix C.5. The vector $\vec{m}^2$ is the vector of all mass gaps in the QCD sector, see (C.19). The second term on the right-hand side, $\eta_A^{(c,b,t)}$, comprises the contributions from the heavier quarks c, b, t . The relation (19) works quantitatively for mapping the anomalous dimension from pure glue to two-flavour QCD, and from two flavour QCD to 2+1 flavour QCD, see [68]. The approximation becomes better for larger N_f and heavier quarks.

While the use of the relation in (19) is required for a quantitative description of the full IR dynamics of QCD, for the applications in the present work it suffices to use a semi-quantitative approximation derived in Appendix C.5, see (C.27),

$$\partial_t Z_A^{(\text{ASSM})} \approx \partial_t Z_A^{(2+1)} - \eta_A^{(c,b,t)} Z_A^{(\text{ASSM})}, \quad (20)$$

where $Z_A^{(\text{ASSM})}$ is determined by a UV-IR consistency condition at the interface scale k_{inter} in the range (17), see (C.30). In Appendix C.5 it is also discussed, that the results show a small dependence on k_{inter} within this range, see Figure 12.

Spontaneous chiral symmetry breaking requires the inclusion of the resonant scalar–pseudo-scalar four-quark interaction channel as in [68]. Effectively this leads to an additional mass contribution to the quark masses,

$$m_q = \frac{y_q v}{\sqrt{2}} \rightarrow \frac{y_q v}{\sqrt{2}} + \Delta M_q^{\text{const}}. \quad (21)$$

The term $\Delta M_q^{\text{const}}$ is taken from the results for the u, d quarks in [68], computed in a strong isospin symmetric approximation: $y_u = y_d$. Then, the QCD contribution is given by the full physical constituent quark mass M_q^{const} without the current quark contribution, $\Delta M_q^{\text{const}} = M_q^{\text{const}} - y_u v$, where $y_u v$ is evaluated at $k = 0$. We also use that the resonant interaction channel does not directly enter the flows of the other parameters considered in our approximation.

At large cutoff scales $k \gtrsim 10 \text{ GeV}$, the approximation used in [68] matches that used here, so the QCD part of the present ASSM investigation flows into strongly correlated QCD. In combination this allows us to flow the ASSM down to $k = 0$, including confinement and strong chiral symmetry breaking, leading to a semi-quantitative IR closure of the ASSM.

2.3.2 Asymptotically safe UV regime

In the asymptotically safe regime for $k \gtrsim M_{\text{pl}}$, we utilise the results of [63, 64] where pure gravity flows with momentum dependences of propagators and vertices at a symmetric point have been considered. We emphasise that the momentum dependences of the vertices imply the inclusion of higher-order curvature invariants and also the difference between background and fluctuation correlation functions is resolved.

Here, we consider the momentum-dependent couplings

$$g_{h,n}(p) = G_{h,n} k^2, \quad \lambda_{h,n} = \Lambda_n / k^2, \quad (22)$$

of the curvature tensor structures $(\sqrt{g} \mathcal{R})^{(n)}$ and the volume-form tensor structures $(\sqrt{g})^{(n)}$ respectively. They are part of the n -point vertices $\Gamma_{h \dots h}^{(n)}$ of the fluctuation graviton h , and the present approximation is built upon the assumption of their dominance over other tensor structures in the vertices. While reminiscent of Newton coupling and the cosmological constant, these coupling parameters are vertex couplings and should not be confused with the former physical observables, see [14].

Moreover, for the fixed-point analysis, we take into account the momentum-dependent wave functions of the fluctuation graviton, $Z_h(p)$, and the gravity ghost, $Z_c(p)$.

We identify the avatars of the vertex couplings, see (14), and compute the flow of

$$Z_h, Z_c, \quad g_h = g_{h,3}, \quad \mu_h = -2\lambda_{h,2}, \quad (23)$$

where all the avatars of the Newton coupling are identified with that of the three-point vertex. In [64, 72, 73], it has been shown that they are of similar size. The flow of g_h and the anomalous dimensions are evaluated bilocally between $p = 0$ and $p = k$. The couplings of the volume form

tensor structures are of sub-leading importance and we use $\lambda_{h,n \geq 3} \approx 0$. This approximation has been chosen due to their small fixed-point values for $n = 3, 4, 5$.

In [21, 23, 72, 73], it has been shown that matter-gravity systems admit an *effective universality* and a *close perturbative* behaviour. This entails that we can (approximately) identify the different gravity-matter vertex couplings of the n matter fields $\Phi_{i_1}, \dots, \Phi_{i_n}$ with m gravitons h with the uniform one in pure gravity, g_h ,

$$g_{\Phi_1 \dots \Phi_i h \dots h} = g_h, \quad (24)$$

also used for the back coupling of matter to gravity. We only consider gravity-matter couplings that arise from the dispersion of the matter fields (kinetic and mass term), that is two identical or conjugate matter fields and one to three fluctuating gravitons.

In summary, this approximation is informed by the results on the momentum dependence of vertices and propagators in [64] and the effective universality and close perturbative behaviour of matter-gravity couplings. It hence reflects the state-of-the art approximations for matter-gravity systems in the literature, except for gravity-induced higher order couplings, see, e.g., [21, 28, 55, 74–81]. The respective extension will be considered elsewhere.

A final important novel ingredient is the non-trivial dimensionless Higgs potential u considered in the fixed-point analysis. It derives from (9) by rescaling all dimensionful quantities by respective powers of the cutoff scale k . Restricting ourselves to the parametrisation of the Higgs potential in the symmetric regime, (9), we are led to

$$u(\bar{\rho}) = \frac{V_{\Phi, \text{eff}}}{k^4} = u_0 + \sum_{n=1}^{N_{\max}} \bar{\lambda}_{\Phi, 2n} \bar{\rho}^n, \quad (25a)$$

where $u_0 = \lambda_{h,0}/(8\pi g_h)$ with $\lambda_{h,0}$ as defined in (22), and $N_{\max} = 17$, see (12). The dimensionless radial Higgs field $\bar{\rho}$ and couplings $\bar{\lambda}_{2n}$ are given by

$$\bar{\rho} = Z_{\Phi} \frac{\text{tr } \Phi^{\dagger} \Phi}{k^2}, \quad \bar{\lambda}_{\Phi, 2n} = \frac{\lambda_{\Phi, 2n}}{k^{4-2n}}. \quad (25b)$$

In (25b), we have also absorbed the wave-function factors Z_{Φ}^n into the definition of $\bar{\rho}$, for more details see Appendix C.3. This allows us to map out the non-trivial UV phase structure of the ASSM with unstable potentials, trivial Gaussian potentials with one relevant direction, non-trivial flat potentials with two relevant directions, and stable as well as semi-stable potentials in Section 5.

2.4 Intermediate high energy regime

For momentum scales above $k_{\text{inter}} \approx 10 \text{ GeV}$, the fermionic matter sector hosts all three families of quarks and leptons. We use the fact that for cutoff scales $k \gtrsim 10 \text{ GeV}$ only the t quark experiences a threshold effect due to its mass, while the other quarks, $q_l = d, u, s, c, b$, and all leptons l can be assumed to be quasi-massless. Hence, for the sake of simplicity, we do not consider the running of the gauge couplings separately as they agree for $k \gtrsim 10 \text{ GeV}$, but identify all couplings with

$$g_{1, \phi_i} = g_{1, e}, \quad g_{2, \phi_i} = g_{2, e}, \quad g_{3, q} = g_{3, u}. \quad (26)$$

The approximation (26) ensures that all fields have the required universal running above the respective thresholds, and settle at their correct IR value at $k = 0$. Below the mass threshold m_{ϕ_i} of a given field ϕ_i , the running of its couplings is approximated with that of the lightest fields of the same type. This guarantees that the light fields have the correct running above the threshold, and is of sub-leading consequence for the dynamics triggered by the heavier field

ϕ_i : all diagrams with internal lines of the field ϕ_i are suppressed with powers of the running scale k over its mass, $k^2/m_{\phi_i}^2 \rightarrow 0$. Consequently, these contributions are suppressed.

A similar approximation could be applied to the set of Yukawa couplings. We have refrained from using it here, as quantitative access to the top Yukawa coupling is chiefly important for an accurate determination of the top pole mass to the per cent level. In conclusion, we include the running of all Yukawa couplings separately.

2.5 Wrap-up of the approximation

In summary, the approximation of the full effective action of the ASSM, is given by the classical one with running couplings and wave functions and a full effective Higgs potential within a high-order Taylor expansion in the trans-Planckian regime. In the asymptotic IR and UV regimes, we also take into account momentum-dependences for the gravity part in the asymptotic safety regime and for QCD in the strongly correlated IR regime. Then, the flow of the dynamical fluctuation parameters is solved self-consistently, the running parameters are fed back into the flow and importantly, we consider all physical threshold effects. The flow solved here is RG-consistent [82, 83], and can be systematically improved.

3 The ASSM: Existence & stability

A highly non-trivial aspect of general gravity-matter systems and the ASSM, in particular, is the reliability of predictions in the asymptotically safe regime. A necessary but not sufficient reliability criterion is the requirement that results are stable under changes of the regularisation and improvements of the approximation and also pass benchmark tests in simpler subsystems. It has been argued in [23] that the Reuter fixed point is always present for minimally coupled matter systems. Therefore, any approximation to the full system of flow equations that do not accommodate this property lacks full predictive power. In [23], this was illustrated in the example of Yang-Mills gravity system. It was shown that the presence of the required Reuter fixed point including asymptotically free Yang-Mills theory seemingly depends on the regulator choice. Moreover, the sign of the graviton contributions of the gluon anomalous dimension $\eta_a(p)$ depends on the value of the graviton mass parameter as well as the momentum p .

In the current work, we extend this analysis to the full ASSM and provide a reliability analysis of the UV regime. We are interested in the scenario where the marginal matter couplings run into the asymptotically free UV fixed points and hence pure matter contributions are subleading in the scaling regime around the fixed point. Then, the system shows competing effects between graviton tadpole diagrams and diagrams with more vertices. These competing effects originate in the inherent momentum dependences of the matter-graviton vertices and they complicate the determination of the sign of the β -functions due to the non-trivial momentum dependence. The latter also implies that a derivative expansion about vanishing momentum has to be done with care. For works including momentum dependences see [21, 23, 63–66, 72, 73, 84], and the recent review [10]. An expansion in powers of curvature invariants as is done in most applications of heat-kernel methods is tantamount to a derivative expansion at vanishing momenta, see [10]. Consequently, the same reliability arguments apply and in order to access the full momentum dependence, the inclusion of invariants with full covariant momentum dependence is required, see [85–89]. Most heat-kernel computations additionally use the background-field approximation which is lifted in the present fluctuation approach.

Most expansion schemes at work in asymptotically safe gravity explicitly or implicitly use expansions about specific backgrounds, in most cases the flat background. An optimal choice for such a background would be the (minimal) solution of the equations of motion, leading to

an *on-shell* expansion. Notably, the flat background, or equivalently an expansion in powers of curvature invariants in the background field approximation, is an *off-shell* expansion. Off-shell expansion schemes may show an unphysical behaviour of the system at least in low orders of the expansion scheme if the off-shell expansion point is too far away from the on-shell background. A further intricacy is the fact, that commonly used expansions about a given background have a finite radius of convergence and one may see convergence towards an unphysical behaviour. Finally, the theory may not exist for all possible metric backgrounds. A respective analysis requires the evaluation of the fixed point and the flow for all backgrounds. In the fluctuation approach, the computation of background-dependent vertices was initiated in [22, 90], for respective works in the background field approximation see e.g. [91–94].

In the present case, we do not expect the approximation to provide stable results for general background fields but only in the vicinity of the equations of motion (on-shell). Even though the present approximation together with the accompanying stability checks is qualitatively more elaborate than those used in previous works, we expect instabilities for far off-shell configurations.

3.1 Beta functions of marginal matter couplings

In the following, we restrict our analysis to the flow of the three-point couplings $\lambda_i = (g_1, g_2, g_3, \gamma)_i$ in the vicinity of their Gaussian fixed point. The UV properties of the other avatars of the gauge couplings as well as the other Yukawa couplings follow similarly. We drop the subleading pure matter contributions, and we arrive at

$$\partial_t \lambda_i(\mathbf{p}) = \beta_{\lambda_i}^{\text{grav}}(\mathbf{p}), \quad (27a)$$

where $\mathbf{p} = (p_1, p_2)$ are two of the three independent four-momenta of a three-point function. The gravity contributions to these β -functions have been previously computed. For example, the gravity contribution to the gauge coupling was computed within the fRG in [23, 25, 52, 77, 95–97] and within perturbation theory in [51, 53, 98]. Here we go beyond previous fRG works since we derive this contribution for the first time from the gauge-quark vertex. The gravity contribution to the Yukawa coupling was computed in [17, 28, 55, 99, 100] and we are in agreement with the results from, e.g., [55]. The gravity contribution to the quartic scalar coupling was studied in [30, 32, 99–104]. The right-hand side of (27a) reads

$$\beta_{\lambda_i}^{\text{grav}}(\mathbf{p}) = \eta_{\lambda_i}^{\text{grav}}(\mathbf{p}) \lambda_i(\mathbf{p}) + F_{\lambda_i}^{\text{grav}}(\mathbf{p}), \quad (27b)$$

with

$$\eta_{\lambda_i}(\mathbf{p}) = \frac{1}{2} \sum_{j_i} \eta_{\phi_{j_i}}(p_{j_i}). \quad (27c)$$

Equation (27c) comprises the sum of (1/2 of) the momentum-dependent anomalous dimensions of the fields $\phi_{j_i}(p_{j_i})$ whose scattering is described in the vertex at hand, and $\eta_{\lambda_i}^{\text{grav}}(\mathbf{p})$ is the matter-gravity part.

The second term in (27b), $F_{\lambda_i}^{\text{grav}}$, stands for the diagrams of the vertex flow, see also Appendix F. We can distinguish two classes of diagrams, the class of matter graviton tadpoles / polarisation diagrams (tp), and genuine three-point function triangle diagrams (tri). The vertices in these diagrams depend on more momenta than the original couplings, for example, the two-graviton–two-quark–gluon vertex depends on four independent four-momenta. In the present approximation, these momentum dependences cannot be fed back properly. In the tadpole diagrams, the two graviton legs have the momenta q and $-q$ and therefore we identify

$\lambda_i(\mathbf{p}, q, -q) = \lambda_i(\mathbf{p})$, and similarly for the polarisation diagrams where always two matter legs have the momenta p_1 and p_2 . This leads us to

$$F_{\lambda_i}^{\text{grav}}(\mathbf{p}) = F_{\lambda_i=1}^{\text{tp}}(\mathbf{p}) \lambda_i(\mathbf{p}) + F_{\lambda_i}^{\text{tri}}(\mathbf{p}), \quad (28)$$

where the tadpole and polarisation diagrams only include the dressings $\lambda_i(\mathbf{p})$ as a prefactor in the involved graviton-matter vertices. In turn, the triangle diagrams include the genuine matter three-point vertices, and the loop momentum runs through the coupling, and hence this coupling cannot be pulled out in front of the diagram. In these diagrams, we can for example choose to identify the momenta of the coupling with the internal momenta $q_1 = q$, $q_2 = -p_1 - q$, leading to

$$\lambda_i(q_1, q_2)|_{\text{tri}} \xrightarrow{\mathbf{p} \rightarrow 0} \lambda_i(q, -q). \quad (29)$$

This also entails that the coupling at an exceptional momentum with $p_1 = 0$ is relevant for this investigation, and not that at the symmetric point.

The coupling $\lambda_i(\mathbf{p})$ in (27) is the renormalisation group invariant dressing of the vertex at hand. In the flow equation approach, respective couplings are typically evaluated at a symmetric point: feeding back only this dressing in an average momentum approximation can be shown to lead to quantitative results in the absence of resonant interaction channels, for more details see [10, 12, 13]. Further interesting couplings are defined at exceptional momenta with soft momenta for one of the fields.

In order to investigate the scaling regime in the vicinity of the UV fixed point for $k \rightarrow \infty$, we evaluate (27) at a fixed momentum $\mathbf{p}$. The dimensionless quantities in (27) are functions of $\mathbf{p}/k \rightarrow 0$ and we can safely use this limit for all fixed momenta, and hence the right-hand side of (27a) reduces to $\beta_{\lambda_i}^{\text{grav}} = \beta_{\lambda_i}^{\text{grav}}(\mathbf{p} = 0)$, with

$$\beta_{\lambda_i}^{\text{grav}} = \left(\eta_{\lambda_i}^{\text{grav}} + F_{\lambda_i=1}^{\text{tp}} + \gamma^{\text{tri}} F_{\lambda_i=1}^{\text{tri}} \right) \lambda_i, \quad (30a)$$

where

$$F_{\lambda_i=1}^{\text{tp/tri}} = F_{\lambda_i}^{\text{tp/tri}} \Big|_{\lambda_i(\mathbf{p})=1} (0), \quad \gamma^{\text{tri}} = \frac{F_{\lambda_i}^{\text{tri}}(0)}{\lambda_i F_{\lambda_i=1}^{\text{tri}}}. \quad (30b)$$

The first two terms in the parenthesis on the right-hand side of (30a) only involve the anomalous dimension at vanishing momentum $\eta_{\lambda_i}^{\text{grav}} = \eta_{\lambda_i}^{\text{grav}}(0)$, the tadpole and polarisation diagrams $F_{\lambda_i=1}^{\text{tp}}$ and the coupling $\lambda_i = \lambda_i(0)$ at vanishing momentum. In contradistinction, the coupling $\lambda_i(q_1, q_2) \rightarrow \lambda_i(q, -q)$ in the flow term $F_{\lambda_i}^{\text{tri}}(0)$ is integrated over loop momenta q^2 . Hence, it is sensitive to all momenta smaller than the cutoff scale as $q^2 \lesssim k^2$. This is taken into account with the prefactor γ^{tri} which is cutoff-independent in the scaling regime. This relative prefactor changes the balance between the triangle diagrams and the tadpole and polarisation diagrams. It may increase or suppress the contribution of the triangle diagram, subject to the momentum dependence of the coupling $\lambda_i(q, -q)$ in the vicinity of the fixed point. The integrand of the triangle diagram involves a factor q^3 from the radial momentum integration and a factor q^4 from the vertices, leading to the factor $(q/k)^7$ in the integrand. This suppresses very efficiently the small loop momentum regime of the vertex dressing $\lambda_i(q, -q)$ in comparison to the regime with $q^2 \approx k^2$ in the triangle diagram.

Equation (30a) is readily solved which leads us to

$$\lambda_{i,k_1} = \lambda_{i,k_2} \exp \left(\int_{k_2}^{k_1} \frac{dk}{k} \left[\eta_{\lambda_i}^{\text{grav}} + F_{\lambda_i}^{\text{tp}} + \gamma^{\text{tri}} F_{\lambda_i}^{\text{tri}} \right] \right), \quad (31)$$

where both cutoff scales k_1 and k_2 are in the scaling regime. The present Gaussian fixed point scenario for the matter couplings λ_i is sustained for

$$\eta_{\lambda_i}^{\text{grav}} + F_{\lambda_i}^{\text{tp}} + \gamma^{\text{tri}} F_{\lambda_i}^{\text{tri}} < 0. \quad (32)$$

This concludes the derivation of the UV flows of marginal matter couplings in the ASSM.

3.2 Variations of the regulator and cutoff scales

We are now in the position to access the stability of the Gaussian fixed point as well as the reliability of the results in the present approximation. A necessary condition is the independence of UV fixed-point nature from the chosen regulator. A given approximation typically only works for a given class of regulators. In turn, extreme choices of regulators potentially invalidate any approximations. In this context, we have to take into account that in the present approximation minimally coupled systems do not show the Reuter fixed point for all cutoff choices [23]. Therefore we only expect to see a stable Gaussian fixed point for specific choices of regulators.

We analyse the fixed-point scenario under specific variations of the regulators. For the sake of simplicity, we only consider relative changes of the cutoff scales k_{grav} for pure gravity and k_{mat} for matter fields. To that end, we introduce

$$k = \min(k_{\text{grav}}, k_{\text{mat}}), \quad k_{\text{mat}} = \gamma_{\text{mg}} k_{\text{grav}}. \quad (33)$$

The standard choice is $\gamma_{\text{mg}} = 1$, but the qualitative features of the system should be independent of it. Its choice can be constrained by optimisation arguments in a given approximation: in the present approximation, we have dropped momentum dependences of the effective action. Then, optimal regulators are those that minimise the momentum transfer in diagrams as such a transfer (strong momentum dependences) cannot be captured by the present approximation that relies on $p^2 \approx k^2$ for all momenta. While this implies $\gamma_{\text{mg}} = 1$ for different fields of the same kind, this is by far not obvious within a system that either contains both fermions and bosons or shows vastly different anomalous dimensions. By considering a variation of the relative cutoff scale between the matter and gravity sector, we provide a first exploratory study of the stability of the system.

Technically, we proceed as follows: the fixed point analysis is done in dimensionless variables. These variables are defined by multiplying the dimensionful ones with appropriate powers of the gravity cutoff scale k . This implies that for $\gamma_{\text{mg}} < 1$ the only change in our system of flow equation originates in the matter shape functions and vice versa for $\gamma_{\text{mg}} > 1$. For $\gamma_{\text{mg}} < 1$, the matter shape functions are given by

$$r_{\text{mat}}(x) = r_{\text{flat}}\left(\frac{1}{\gamma_{\text{mg}}^2}x\right), \quad x = \frac{p^2}{k^2}, \quad (34)$$

and

$$r_{\text{flat}}(x) = \left(\frac{1}{x} - 1\right) \Theta(1 - x). \quad (35)$$

In a rough first estimate, this change introduces relative factors γ_{mg}^4 or even higher powers in the diagrams with a scale-derivative $\partial_t R_{\text{mat}}$, if evaluated at vanishing external momentum. The power four is obtained for momentum-independent couplings, the higher power is obtained for momentum-dependent couplings, triggered by

$$\int_0^{\gamma_{\text{mg}}} dq q^3 (q^2)^n = \frac{1}{4 + 2n} \gamma_{\text{mg}}^{4+2n}, \quad (36)$$

with the highest power $n = 2$ for the triangle diagrams in (30a). For this estimate we have neglected the subleading effect that the graviton and matter propagators have a non-trivial momentum dependence for $q^2 > 1$ and $q^2 > \gamma_{\text{mg}}^2$. In summary, the main effect of the choice (34) is the suppression of diagrams with $\partial_t R_{\text{mat}} (\gamma_{\text{mg}} < 1)$ or $\partial_t R_{\text{grav}} (\gamma_{\text{mg}} > 1)$. If diagrams are contributing with different signs, this may introduce a change in the sign of given β -functions, entailing a qualitative change of the nature of the fixed point.

We note that such an enhancement or suppression can also be obtained by varying the graviton mass parameter μ_h : large positive masses $\mu_h \rightarrow \infty$ suppress diagrams with $\partial_t R_{\text{grav}}$ as they contain one further graviton propagator. In turn, for $\mu_h \rightarrow -1$, these diagrams are enhanced. As discussed in [105], an on-shell analysis requires a non-trivial metric solution which compensates effects of the mass parameter but also changes the vertices. This implies that the physical consequences of a suppression or enhancement that originates in the value of the mass parameter, should be taken with a grain of salt: they are based on an off-shell background that can easily destroy the fixed-point properties. On-shell backgrounds with constant curvature were calculated in [22, 90] within the vertex expansion and the on-shell background curvature turned out to be $\mathcal{O}(1)$ in units of the cutoff scale. This suggests that a natural estimate for the on-shell analysis is given by small values of the graviton mass parameter.

3.3 Stability of the Gaußian matter fixed point

We proceed with the stability analysis of the Gaußian matter fixed point. The pivotal part of the full system of flow equations is the matter-gravity part of the β -functions: for graviton couplings this part may destabilise the Reuter fixed point, leaving us with no UV closure. In turn, for matter couplings, the matter-gravity part is required for the existence of respective UV fixed points. Therefore the following discussion is restricted to the matter-gravity part as this is already conclusive.

Now we use that in the UV regime the matter-gravity parts of all matter-gauge couplings are identical as are that of all Yukawa couplings

$$\beta_{g_i}^{\text{grav}} \rightarrow \beta_g^{\text{grav}}, \quad \beta_{y_{q,l}}^{\text{grav}} \rightarrow \beta_y^{\text{grav}}, \quad (37)$$

and we can restrict our analysis to the pair $(\beta_g^{\text{grav}}, \beta_y^{\text{grav}})$. For the non-Abelian gauge couplings, the pure matter part of the β -function is negative, $\beta_{g_{2,3}}^{\text{mat}} \propto g_{2,3}^3$, which entails asymptotic freedom with $g_{2,3} \rightarrow 0$. Thus, the gravity part $\beta_{g_{2,3}}^{\text{grav}} \propto g_{2,3}$ dominates the asymptotically safe UV regime and is required to be negative for supporting the Gaußian fixed point with asymptotic freedom.

The pure matter parts of the U(1) coupling is positive $\beta_{g_1}^{\text{mat}} > 0$ and hence the matter-gravity parts has to be negative, $\beta_{g_1}^{\text{grav}} < 0$. In the Yukawa beta function, the Yukawa interaction contributes positively $\beta_y^{\text{mat}} \propto y^3 > 0$ while the Yukawa-gauge interaction contributes negatively $\beta_y^{\text{mat}} \propto y g^2 < 0$. For the Gaußian matter fixed point, the matter-gravity parts have to be negative, $\beta_y^{\text{grav}} < 0$. In summary, in all cases negative matter-gravity parts are required, $\beta_g^{\text{grav}}, \beta_y^{\text{grav}} < 0$.

The analysis is facilitated further as we focus on the leading order contribution and neglect the anomalous dimensions stemming from regulator insertions in the diagrams. Then the dimensionless Newton coupling, g_h , is simply an amplitude factor in β_g, β_y . In the limits $\gamma_{\text{mg}} \rightarrow 0$ and $\gamma_{\text{mg}} \rightarrow \infty$, also the graviton mass parameter only changes the amplitude of the flow as all diagrams left have the same number of graviton propagators. For all other values of γ_{mg} , we have a competition of diagrams with one and two graviton propagators and the value of the graviton mass parameter can change the sign of the contribution. For the analysis, we use $\mu_h = 0$. The results for other values μ_h can be inferred by a simple rescaling of γ_{mg}

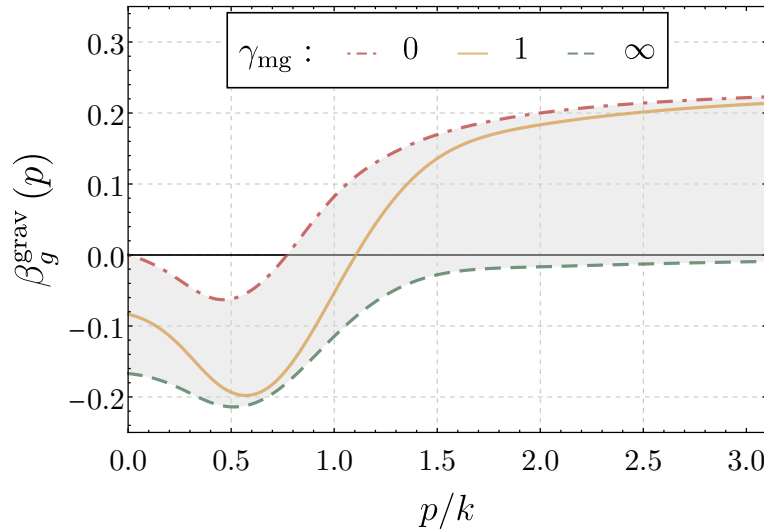


Figure 1: Matter-gravity contributions to the gauge beta functions β_g^{grav} derived from the gauge-fermion vertex for $g = 1$, $g_h = 1$, $\mu_h = 0$, and different values of γ_{mg} . For $\gamma_{\text{mg}} = 0$, β_g^{grav} vanishes due to an exact cancellation between the fermion anomalous dimension and the three-point diagrams.

via $\gamma_{\text{mg}, \mu_h=0} = (1 + \mu_h)\gamma_{\text{mg}}$. Accordingly, the fixed-point values of the pure gravity sector are irrelevant for the analysis.

We consider relative rescalings of matter and gravity cutoffs with γ_{mg} defined in (33) in the regime

$$\gamma_{\text{mg}} \in [0, \infty), \quad (38)$$

which indirectly allows us to also access variations of the shape functions. As the present approximation does not allow for momentum transfers over large momentum scales, the limits $\gamma_{\text{mg}} \rightarrow 0, \infty$ can only support the UV fixed point if matter fluctuations dominate the matter-gravity part ($\gamma_{\text{mg}} \rightarrow 0$), or gravity fluctuations dominate the matter-gravity part ($\gamma_{\text{mg}} \rightarrow \infty$). These are two physically different UV scenarios whose existence is accessed in these limits:

- (i) For $\gamma_{\text{mg}} \rightarrow 0$ discussed in Section 3.3.1, the matter sector is not IR regularised and quantum fluctuations of the matter fields are included at all scales. The matter propagators lack the IR mass introduced by the regulator and hence are enhanced relative to the graviton propagator. Therefore, this limit is a natural one for systems that are dominated by matter fluctuations (*matter matters* [15]).
- (ii) For $\gamma_{\text{mg}} \rightarrow \infty$ discussed in Section 3.3.2, the gravity sector is not IR regularised and quantum fluctuations of the graviton are included at all scales. The graviton propagator lacks the IR mass introduced by the regulator and hence is enhanced relative to the matter propagators. Therefore, this limit is a natural one for systems that are dominated by gravity fluctuations (*gravity rules* [16, 23]).

The results of [23] entail that minimally coupled gravity matter systems should show a gravity-dominated Reuter fixed point apart from other fixed points. Evidently, an ASSM fixed point with a Gaussian or shifted Gaussian matter sector falls into the gravity-dominated class of fixed points, and we shall see in Section 3.3.2 that the ASSM fixed point with a Gaussian matter sector is supported in this limit, while the Gaussian matter fixed point is not present in the matter-dominated limit of the matter-gravity part as shown in Section 3.3.1. Note also, that

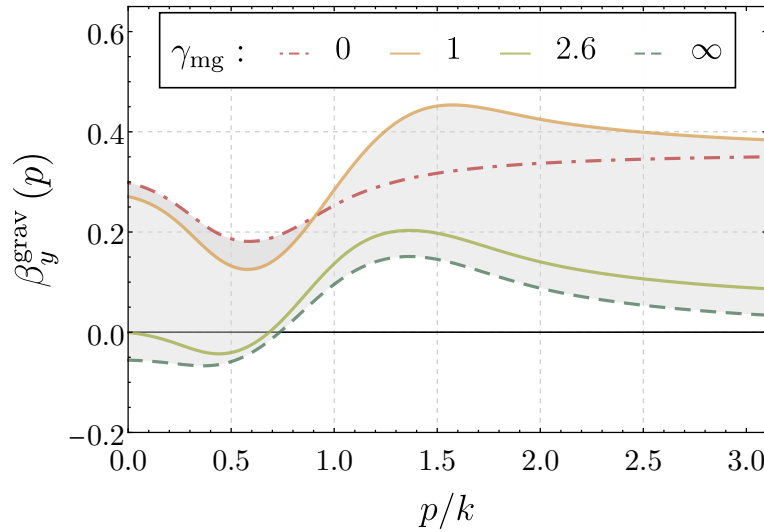


Figure 2: Matter-gravity contribution to the Yukawa beta function β_y^{grav} for $y = 1$, $g_h = 1$, $\mu_h = 0$, and different values of γ_{mg} . For $\gamma_{\text{mg}} \approx 1.6$, β_y^{grav} turns negative at $p/k \sim 0.5$, and for $\gamma_{\text{mg}} \approx 2.55$ we observe a sign change at $p = 0$. Larger/smaller values of g_h increase/decrease the amplitude of β_y^{grav} .

this limit cannot work for the full system as it accesses the UV sector of the ASSM without UV fluctuations of gravity. This is simply the UV sector of the SM which is UV unstable.

This leaves us with the question in which regime for γ_{mg} we lose the ASSM fixed point. In [Section 3.3.3](#) we show that this happens for $\gamma_{\text{mg}} \approx 1$, which in our opinion is non-trivial support for the Gaussian fixed point scenario dominated by gravity fluctuations: stability only in one of the extreme limits $\gamma_{\text{mg}} \rightarrow 0, \infty$, while it may show the correct physical behaviour also cast serious doubts on the approximation used. This is discussed in [Section 3.4](#).

3.3.1 Matter matters: no matter cutoff with $\gamma_{\text{mg}} \rightarrow 0$

All diagrams with $\partial_t R_{\text{mat}}$ drop out. This restates diffeomorphism and gauge consistency in the matter sector of the ASSM in terms of standard matter parts of the Slavnov-Taylor identities. Interestingly in this limit, the β -function of the minimal background gauge couplings vanishes as has been shown in [\[25\]](#): the non-trivial cancellation of the graviton tadpole diagram and three-point function diagram in the graviton contribution to the background propagator is based on a kinematic identity rooted in diffeomorphism invariance of the gauge sector in this limit as well as the transversality of the kinetic term of the gauge field.

It is a highly non-trivial result of the present work, that this non-trivial kinematic identity at work in the flow for the gluon two-point function in [\[25\]](#) extends to the same result for the gauge-fermion coupling,

$$\beta_g^{\text{grav}}(p=0) = 0, \quad \text{for} \quad \gamma_{\text{mg}} = 0, \quad (39)$$

see also the *red dot-dashed* curve in [Figure 1](#). As for the two-point functions of the gauge fields the identity (39) is rooted in diffeomorphism invariance of the gauge sector for $\gamma_{\text{mg}} = 0$ and transversality of the gauge field.

We emphasise that it is the above combination of diffeomorphism and gauge symmetry leading to this non-trivial result. For the Yukawa couplings, this cancellation between tadpole, polarisation and triangle diagrams is not present and the graviton tadpole dominates. Its

contribution to the β -function is positive and we are led to

$$\beta_y^{\text{grav}} > 0, \quad \text{for} \quad \gamma_{\text{mg}} = 0, \quad (40)$$

see also the *red dot-dashed* curve in [Figure 2](#).

In conclusion, the gauge-gravity part β_g^{grav} , (39), supports the asymptotically free Gaussian fixed point of the non-Abelian gauge couplings $g_{2,3}$ and allows for a stable Gaussian as well as unstable interacting fixed point for the U(1) coupling g_1 . Conversely, the matter-gravity part β_y^{grav} , (40), is positive and no combined fixed point is present. One may speculate that the absence of the kinematic identity in the Yukawa coupling is a truncation artefact that originates in the lack of (modified) diffeomorphism invariance of the present approximation, also missing in other approximations used in the literature.

3.3.2 Gravity rules: no gravity cutoff with $\gamma_{\text{mg}} \rightarrow \infty$

All diagrams with $\partial_t R_{\text{grav}}$ drop out. This reinstates diffeomorphism consistency in the pure gravity sector of the ASSM in terms of standard gravity parts of the Slavnov-Taylor identities. In this limit, the graviton tadpole is not present and the other diagrams lead to negative contributions to the β -function. In summary, we are led to

$$\beta_{g,y}^{\text{grav}}(0) < 0, \quad \text{for} \quad \gamma_{\text{mg}} = \infty, \quad (41)$$

see also the *green dashed* curves in [Figures 1](#) and [2](#). This entails, that in the gravity-dominated UV scenario we encounter a Gaussian fixed point for all gauge and Yukawa couplings.

It is important to note that for $\gamma_{\text{mg}} \neq 0$ the absence of the graviton tadpole can also be simulated by a sufficiently large positive graviton mass parameter $\mu_h \gg 0$ or a sufficiently negative cosmological constant $\Lambda \ll 0$ in the background field approximation. This limit suppresses the graviton propagator and leads to an *artificial* suppression of gravity: *on-shell* a large graviton mass parameter or large negative cosmological constant does not lead to a suppression of gravity fluctuations, and already classically gravitons are massless in the presence of large curvatures. It is suggestive that this option is a mere truncation artefact, but this has to be investigated in better approximations.

As in the matter-dominated limit discussed in [Section 3.3.1](#), a change of the graviton mass parameter only leads to a change of the amplitude of $\beta_{g,y}$, but does not change the sign. In short, the stability of the Gaussian fixed point in the gravity-dominated scenario persists off-shell.

3.3.3 Crossover regime at $\gamma_{\text{mg}} \approx 1$

So far we have analysed two limits $\gamma_{\text{mg}} = 0, \infty$ and differences are expected between these extreme choices. In the gravity-dominated limit with $\gamma_{\text{mg}} \rightarrow \infty$, the Reuter fixed point for the gravity couplings exists and *all* matter couplings have a stable Gaussian fixed point, while the Reuter fixed point is absent in the matter-dominated limit for $\gamma_{\text{mg}} = 0$ and only *some* matter couplings have a stable Gaussian fixed point.

It is now decisive to investigate the crossover from $\beta_y > 0$ to $\beta_y < 0$. Remarkably, it takes place in the regime with $\gamma_{\text{mg}} = \mathcal{O}(1)$: This hints at the fact, that the γ_{mg} -dependence is caused by the low order of the approximation rather than being an artefact of an extreme choice of regulators.

For γ_{mg} larger than $\gamma_{\text{mg}} \approx 1.6$ the β -function first dips below zero at momenta $p \approx k/2$, before it turns negative for $p = 0$ for $\gamma_{\text{mg}} > \gamma_{\text{mg}}^{\text{stab}}$ with

$$\gamma_{\text{mg}}^{\text{stab}} \approx 2.55, \quad (42)$$

see [Figure 2](#). The value $\gamma_{\text{mg}}^{\text{stab}}$ is very close to unity. This also entails that sign change may also be obtained by an appropriate change of the shape function of gravity and matter fields while keeping $\gamma_{\text{mg}} = 1$. In our opinion, this gives a hint for the viability of the stable Gaußian fixed point scenario by the fact that the crossover between the two regimes takes place for $\gamma_{\text{mg}} = \mathcal{O}(1)$.

3.4 Existence of the ASSM

In short, the highly relevant question of the UV stability of the ASSM and its fixed-point properties cannot be answered conclusively in the present approximation. Future analyses should include a discussion of the on-shell or off-shell properties or instabilities. This entails, that we are in high demand for quantitative and qualitative upgrades, and a comprehensive fixed-point analysis should include all possible scenarios. In our opinion, despite the deficiencies of the present analysis, it still provides non-trivial hints for the existence of the ASSM with a stable Gaußian fixed point for all gauge and Yukawa couplings.

The computation of the running Yukawa coupling should be improved in several aspects: firstly, the momentum dependences of the n -point correlation functions should be taken into account, in particular, those associated with the Yukawa coupling as discussed in [Section 3.1](#). Secondly, one should include tensor structures of higher-order curvature invariants and form factors that contribute to the flow, such as, $R\phi\bar{\psi}\psi$ and $Rf(\Delta)R$. Improving the flow of the Yukawa coupling is one of the most pressing issues of the ASSM.

In the remainder of this work, we discuss the UV-IR and the fixed-point properties of the ASSM in the present existence scenario. This entails a sub-leading behaviour of the graviton tadpole contributions to the flows of the matter couplings in the vicinity of the UV fixed point. Below the Planck scale, these contributions are also suppressed, and we can safely drop the respective diagrams. Then, the ASSM fixed point persists for all γ_{mg} and we choose $\gamma_{\text{mg}} = 1$ for the sake of simplicity. This approximation can be summarised as

$$\gamma_{\text{mg}} = 1, \quad \text{and} \quad F_y^{\text{grav,tad}} = F_g^{\text{grav,tad}} \approx 0, \quad (43)$$

i.e., we set the gravity-tadpole contributions to the running of the gauge and Yukawa couplings to zero.

4 Sub-Fermi to trans-Planckian ASSM

As a first application of the fRG set-up of the ASSM we compute its UV-IR phase structure. This includes the determination of the fundamental parameters by the respective experimental observables that give access to a comprehensive error analysis including both, experimental and systematic errors.

4.1 Fixing the coupling parameters of the ASSM

The physics trajectory of the ASSM is determined by matching its fundamental parameters to their experimental IR values. These observables have to be determined from S -matrix elements at $k = 0$ for the respective scattering momenta. We need to fix the following SM and gravity couplings

$$(g_1, g_2, g_3), \quad (\vec{y}_q, \vec{y}_l), \quad \vec{\lambda}_\Phi, \quad (g_h, \mu_h), \quad (44)$$

where we have paired the parameters of the different sectors of the ASSM, the gauge couplings, Yukawa couplings, parameters of the Higgs potential, and the Newton coupling. All SM

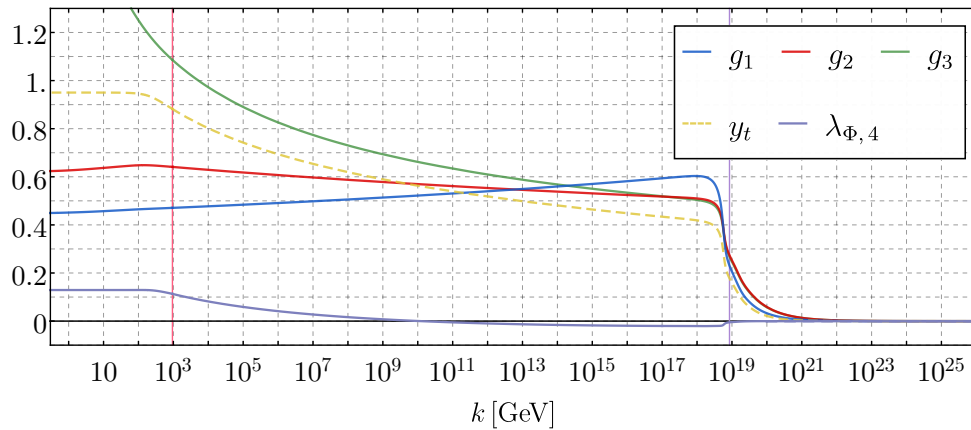


Figure 3: Scale dependence of the gauge couplings, the top Yukawa coupling and the Higgs self-coupling in the ASSM. We have omitted here the flow of the remaining Yukawas and the flow of the curvature mass and vacuum expectation value. In the UV regime beyond the Planck scale (indicated by a *violet* vertical line), all matter couplings approach the Gaußian fixed point, including a flat Higgs potential with two relevant directions. Below the k_{SSB} scale (indicated by a *red* vertical line), the top threshold is clearly visible in the y_t , $\lambda_{\Phi,4}$ and g_2 flows. Moreover, for the latter two the Higgs, $W^\pm$ and Z^0 bosons decoupling also play a quantitative role. For scales above $k = 10^{17}$ GeV the flow of $\lambda_{\Phi,4}$ is considered within the full Taylor expanded potential.

parameters except the strong coupling g_3 and the top Yukawa coupling y_t are determined at $p = 0$ for the physical cutoff scale $k = 0$.

For the EW gauge couplings, we use the experimental values for $p \rightarrow 0$ to fix the electron avatars $g_1 = g_{1,e}$ and $g_2 = g_{2,e}$ at vanishing momentum

$$g_1 = 0.446, \quad g_2 = 0.618. \quad (45a)$$

For the masses (except the top mass) we ignore the difference between the Euclidean curvature masses and the experimental pole masses, $M_{\phi_i}^{(\text{pole})}$. This leads us to

$$\lambda_{\Phi,4} = 0.129, \quad v = 246 \text{ GeV}, \quad (45b)$$

for the Higgs self-coupling and Higgs vacuum expectation value, and

$$\begin{aligned} y_b &= 0.0240, & y_c &= 0.00734, \\ y_s &= 5.46 \cdot 10^{-4}, & y_u &= 1.32 \cdot 10^{-5}, \\ y_d &= 2.76 \cdot 10^{-5}, & y_\tau &= 0.0102, \\ y_\mu &= 6.06 \cdot 10^{-4}, & y_e &= 2.93 \cdot 10^{-6}, \end{aligned} \quad (45c)$$

for the Yukawa couplings at $p = 0$ except for the top Yukawa coupling y_t . With the vacuum expectation value v in (45b) and the Yukawa couplings in (45c), all curvature masses except the top mass match the experimental pole masses from [106].

The Newton coupling and cosmological constant are given by

$$G_N = (1.22 \cdot 10^{19} \text{ GeV})^{-2}, \quad \Lambda \approx 0. \quad (45d)$$

It is left to determine the remaining two parameters, y_t and g_3 . We first discuss the top Yukawa coupling y_t . While the identification of Euclidean curvature masses with the respective pole mass is a qualitatively reliable approximation for low-lying masses, see [69] for a case study, we cannot use it for the top quark mass or rather its Yukawa coupling y_t : the phase structure of the ASSM including its UV asymptotics is very sensitive to its precise value. The experimental pole mass from cross-section measurements is given by [106],

$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV}, \quad (45\text{e})$$

with a decay width of

$$\Gamma_{t,\text{pole}}^{(\text{exp})} = 1.42_{-0.15}^{+0.19} \text{ GeV}. \quad (45\text{f})$$

In [Appendix G](#), we analytically compute the time-like momentum dependence of the top mass function $M_t(p)$ at vanishing cutoff scale, $k = 0$. The access to the analytic momentum dependence is rooted in the simple one-loop exact structure of the flow of the top quark propagator. The present determination includes full resummations of diagrams due to the iterative structure of the flow, and hence the result is very stable under further improvements of the approximation.

This allows us to determine the pole mass of the top as a function of the Euclidean mass parameter m_t , and for the experimental value of the pole mass, (45e), we arrive at the Euclidean mass parameter at vanishing cutoff scale, $k = 0$,

$$m_t = 165.4_{-0.2}^{+0.9} \text{ GeV} \quad \leftrightarrow \quad y_t = 0.950_{-0.001}^{+0.005}, \quad (45\text{g})$$

see [Appendix G](#). Having fixed the Euclidean mass parameter, the decay width is a prediction and we find

$$\Gamma_{t,\text{pole}}^{(\text{theo})} = 1.72_{-0.41}^{+0.09} \text{ GeV}, \quad (45\text{h})$$

in quantitative agreement with the experimental value (45f) from the PDG. Note, that the error in (45g) constitutes a systematic error estimate while the error in (45h) only describes the relative weighting of QCD and non-QCD contributions, see [Appendix G](#) for details.

The error in m_t in (45g) is dominated by the uncertainty of the value of the strong coupling g_3 . To begin with, it cannot be determined at $k, p = 0$: In the deep IR, QCD is strongly correlated and the strong coupling cannot be defined by scattering experiments. For this reason experimental values are only present at and above the τ -scale $p = 1.77 \text{ GeV}$, see [106]. Moreover, the definition of the strong coupling in the IR is subject to threshold effects and non-universality beyond two loops, see the discussion in [Section 2.3.1](#). For this reason, we fix its value at the perturbative Z -scale. Here, its experimental value ($pp/p\bar{p}$) is given by [106]

$$g_{3,k=M_Z}^{\overline{\text{MS}}} \approx 1.22 \quad \leftrightarrow \quad \bar{\alpha}_s := \frac{\left(g_{3,k=M_Z}^{\overline{\text{MS}}}\right)^2}{4\pi} \approx 0.118, \quad (45\text{i})$$

matching the strong coupling in an $\overline{\text{MS}}$ or MOM-scheme. In the 2+1 flavour QCD computation in [68], it has been shown that the momentum-dependent (up-)quark-gluon coupling $g_3(p)$ at $k = 0$ agrees quantitatively with that at $g_{3,k}(p = 0)$ for $k = p$ for perturbative and semi-perturbative momenta, $k \gtrsim 4 \text{ GeV}$. This has been checked with lattice results as well as the momentum-dependent fRG and DSE results from [61, 70] and relates to the logarithmic running of the (up-)quark-gluon coupling in this regime far from the up-quark threshold.

It has been shown in [70], that the MOM² RG-scheme, used in the fRG, leads to an enhancement of the avatars of the running gauge coupling $\alpha_s(p)$. In pure Yang-Mills, the

respective enhancement factor for α_s is approximately $4/3$ [67], dropping to an enhancement factor of approximately 1.2 in the 2+1 flavour case (measured in the regime between 10–40 GeV). A linear extrapolation from these two values can be done for the ASSM quark content leading to a scaling factor of 1.07. This analysis suggests identifying

$$g_{3,k=M_Z} = \sqrt{1.07(4\pi\bar{\alpha}_s)} = 1.26. \quad (45j)$$

However, being short of a full analysis, we show results within an estimate of the systematic error of the choice in (45j) with

$$\alpha_{s,k=M_Z} \in [\bar{\alpha}_s, 1.10 \bar{\alpha}_s], \quad (46)$$

see also (F5) and the detailed discussion in Appendix F.3. There and in Appendix G further estimates and consequences are discussed.

In particular, the above scale setting of the strong coupling, and its uncertainty, has a sizeable impact on the top Yukawa coupling and therefore in the high energy development of the ASSM flows. For example, the seemingly small uncertainty of the strong coupling has a huge effect on the metastability scale k_{meta} at which the quartic Higgs-self coupling turns negative. We discuss this point in detail and perform a systematic error analysis in Appendices F.3 and G.

4.2 The ASSM trajectory

The scale-dependence of the ASSM parameters is depicted in Figure 3. We span momentum scales from $k \rightarrow 0$ and $p = 0$ in the deep IR with chiral symmetry breaking and confinement to $k \rightarrow \infty$ far beyond the Planck scale in the asymptotically safe UV fixed-point regime. While the cutoff-scale dependence is not directly a physical momentum dependence, it relates to the physical momentum dependence at the symmetric point. For a detailed discussion see Section 6 and [12–14].

For cutoff scales below roughly 1 TeV, the ASSM enters the symmetry broken phase, as the effective potential of the Higgs field, $V_{\Phi,\text{eff}}$, develops a non-trivial minimum,

$$\mu_{\Phi}(k < k_{\text{SSB}}) < 0, \quad \text{with} \quad k_{\text{SSB}} = 940 \text{ GeV}. \quad (47)$$

The non-trivial background of the Higgs renders all SM fields except photons and gluons massive. Consequently, this leads to a decoupling of the fluctuations of the respective fields below their mass thresholds, clearly visible in Figures 3 and 4.

The Euclidean masses of the SM fields read,

$$m_{W^\pm} = \frac{g_2 v}{2}, \quad m_Z = \frac{g_2 v}{2 \cos(\theta_W)}, \quad m_H = \sqrt{2 \lambda_{\Phi,4}} v, \quad (48)$$

for the EW and Higgs bosons. The quark/lepton fields with/without the constituent mass contribution are given in (21). The scale dependence of all Euclidean particle masses is shown in Figure 4. From this figure, the intricacy of the particle masses can be understood. These evolutions are important for the correct determination of observables at colliders, for example, the weak mixing angle is especially sensitive to those.

Finally, dynamical strong chiral symmetry breaking has a sizeable impact on the (constituent) quark mass function (21), in particular for the light flavours u, d, s . The respective u -quark-gluon exchange coupling, defined in (C.31), is depicted in Figure 12. It decays for cutoff scales below ~ 1 GeV as a consequence of the QCD mass gap. Note that the avatars of the quark-gluon exchange couplings for the different quarks vary slightly due to the different quark wave functions, see (C.21). Moreover, the strong exchange couplings defined in (C.21) carry the physical threshold effect of the respective gluon exchange. This threshold is not present in the

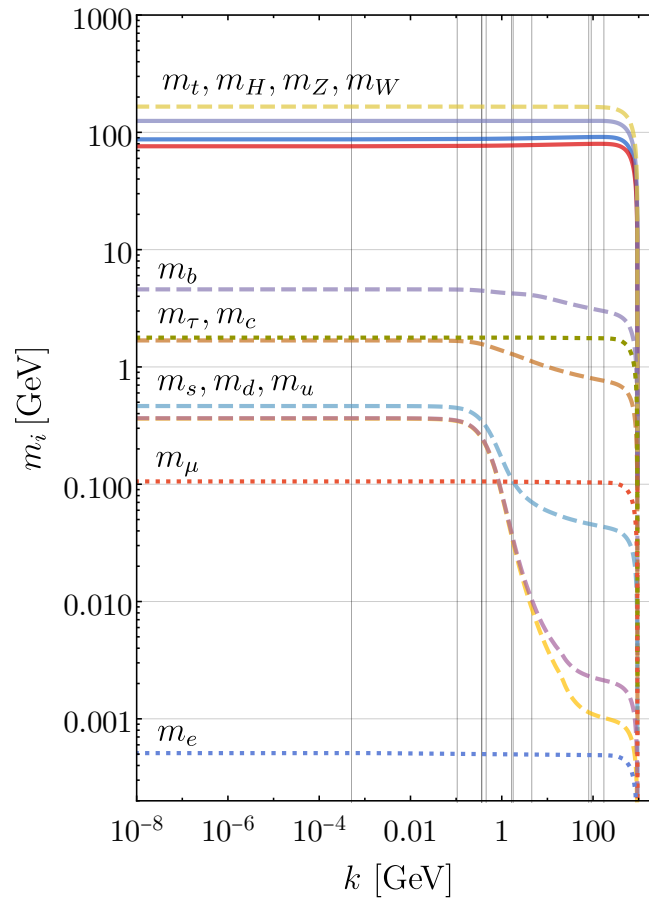


Figure 4: Flow of all SM Euclidean masses. Plain lines correspond to the EW and Higgs bosons, dashed to quarks and dotted to the leptons. The different particle decoupling scales are indicated by vertical lines. For quarks, the constituent masses as defined in (21) and obtained from [68] are also accounted. Above k_{SSB} , the vacuum expectation value of the Higgs and all masses vanish.

definition of the electroweak gauge couplings: the wave functions of $W^\pm, Z^0$ are the prefactors of the kinetic terms, while the masses originate from the interaction of the electroweak gauge bosons with the Higgs. These couplings are also called effective charges, e.g. [107, 108]. A related definition in QCD, that does not show the threshold of the mass gap in QCD, is given by the process-independent coupling or effective charge, see e.g. [109, 110]. In turn, the respective electroweak exchange couplings with the physical mass thresholds enter directly the ASSM flows, as do the quark-gluon exchange couplings, and directly show the decoupling of fluctuations below the mass thresholds. Their definition is provided in (C.31) in Appendix C.5. For an illustration of the threshold effect and their similarity with the quark-gluon exchange couplings, they are also depicted there in Figure 12.

The exchange gauge-fermion couplings show the decoupling of fluctuations in the flows of the fermion mass terms due to the mass gaps of the gauge bosons. A second source for the decoupling of fluctuations is given by the mass thresholds of the fermions themselves. Finally, the electroweak fluctuations are suppressed due to the small electroweak couplings. All these threshold effects and the dominant nature of the strong fluctuations below the electroweak scale are visible in Figure 4, where we show the flows of the ASSM fermion masses below $k \leq k_{SSB} = 940$ GeV, see (47). This regime covers all matter thresholds in the ASSM.

For $k \rightarrow k_{\text{SSB}}$ from below, the Higgs expectation value v drops sharply as a function of the cutoff scale, and all fields become massless. We note that while this is reminiscent of a second order phase transition, it should not be confused with it. This is a standard behaviour of RG flows with spontaneous symmetry breaking and, in particular, it does not carry directly the order of the thermal electroweak phase transition.

Above the EW cutoff scale, $k > k_{\text{SSB}}$, all fields are effectively massless and the β -functions and anomalous dimensions of all marginal (logarithmically running) parameters reduce to their universal form, and agree with that computed with perturbative methods, as do the respective trajectories, see e.g. [111]. In turn, the flow of the dimensionful Higgs mass parameter shows the expected quadratic running and hence is subject to quadratic fine-tuning in the UV. We emphasise that this is technically challenging but carries no conceptual intricacy.

At sufficiently large cutoff scales the quartic Higgs self-coupling changes sign and turns negative, see Figure 3. This indicates a metastable regime or the potential importance of higher-order couplings. In the present ASSM setup, this happens at

$$k_{\text{meta}} = 1.2 \cdot 10^{10} \text{ GeV}. \quad (49)$$

This value compares well with the values obtained in perturbative computations within the $\overline{\text{MS}}$ -scheme: As discussed before, a direct translation of the RG scales is not straightforward, as is their relation to momentum scales. However, a conservative estimate of these uncertainties is well below an order of magnitude, and hence we identify RG scales. This entails that (49) is well compatible with

$$10^9 \text{ GeV} \lesssim k_{\text{meta}} \lesssim 10^{11} \text{ GeV}, \quad (50)$$

in the $\overline{\text{MS}}$ -scheme, see e.g. [111]. A conclusive analysis requires the inclusion of higher-order couplings in this regime [40, 41, 43–45, 47]. Importantly, the metastability scale k_{meta} is very sensitive to the size of the Yukawa coupling and hence the determination of the top quark pole mass, discussed in detail in Appendix G. There it is shown that the largest systematic error in the determination of the pole mass can be attributed to the uncertainty in the determination of the running QCD coupling $g_{3,k=M_Z}$, that stems from the requirement of a self-consistent RG scheme for all scales.

In the present work, we use the estimate (45j) with a relative factor 1.07 in comparison to the $\overline{\text{MS}}$ -value. This is a linear extrapolation of the Yang-Mills and 2+1 flavour to the ASSM quark content with the error estimate (46), see also the discussion in Appendix F.3. Interestingly, for $\alpha_{s,k=M_Z}$ being roughly 15% larger than the $\overline{\text{MS}}$ -value, the metastability scale approaches the Planck scale $M_{\text{pl}}^2 = G_{\text{N}}^{-1}$. For the strong coupling $g_{3,k=M_Z}$, this entails a 7% enhancement,

$$g_{3,k=M_Z} = 1.31, \quad k_{\text{meta}} = M_{\text{pl}}. \quad (51)$$

An analysis of the respective systematic error is deferred to Appendix F.3, where we also show k_{meta} as a function of the strong gauge coupling $\alpha_{s,k=M_Z}$, see Figure 16.

Above the Planck scale, the ASSM quickly gets dominated by gravity fluctuations, as is the ASSM UV fixed point. This has been discussed in particular in [16, 23]. Moreover, matter contributions of the fixed-point equations of the pure gravity coupling and mass parameters g_h and μ_h are simply closed matter loops, and the external gravitons couple via avatars of the Newton coupling. Consequently, the fixed-point equations for these couplings are closed and only depend on the number of matter fields of a given kind. In the ASSM, this leads us to the gravitational fixed point

$$g_h^* = 0.147, \quad \mu_h^* = -0.656, \quad (52)$$

with the cosmological constant avatars $\vec{\lambda}_{h,n}^* \approx 0$ for $n \geq 3$. With the fixed-point values of the pure gravity parameters at hand, we can discuss the qualitative properties of the trans-Planckian asymptotic safety regime. For the physical trajectory, quantum gravity effects grow large at about the Planck scale, and the flow is dominated by gravity fluctuations, see [Figure 11](#) and [\[16, 23\]](#).

In the regime dominated by asymptotically safe quantum gravity, the gauge and Yukawa couplings are attracted towards the Gaußian fixed point. This is straightforward for the non-Abelian gauge couplings since they are asymptotically free without gravity and gravity is always supporting asymptotic freedom [\[23, 25\]](#). For the Abelian hypercharge coupling, the gravity fluctuations must exceed the matter fluctuations that drive the hypercharge coupling into a Landau pole, which indeed happens here. Other works have previously explored the possibility of a UV fixed point with non-vanishing hypercharge or Yukawa couplings. Such scenarios are of interest since then the hypercharge or Yukawa coupling belongs to an irrelevant direction and the respective parameter in the IR is predicted [\[26, 29, 96\]](#). In the present work, we also find these fixed points but they do not lead to a viable IR phenomenology since either the hypercharge coupling or the top mass is too large. Consequently, we focus on the UV fixed point where the hypercharge and Yukawa couplings are vanishing.

The UV Higgs sector requires special care, and it is discussed in detail in [Section 5](#). Depending on the pure gravity fixed-point values it shows different stability and relevance properties. For the values computed in the present approximation [\(52\)](#), we are left with a peculiar situation, which to our knowledge has not been discussed before: for increasing $N_{\max}$ we see a convergence of the full potential to a flat one for fields within the validity regime of the Taylor expansion, see [Figure 5](#). For each $N_{\max}$ we have two relevant parameters whose eigenvectors have maximal overlap with μ_Φ and $\lambda_{\Phi, \max}$. This is reminiscent of a standard Gaußian fixed point, but at the latter only μ_Φ is relevant while all $\lambda_{\Phi, 2n>2}$ are irrelevant. In [Figure 3](#), we show $\lambda_{\Phi, 4}$, which is obtained from the UV-IR flow of the potential [\(11\)](#) within the high (converged) order of the Taylor expansion with $N_{\max} = 17$, see [\(12\)](#). This flow is initiated in the vicinity of the UV fixed point and the full system is flown down to cutoff scales below the Planck scale: $k \sim 10^{17}$ GeV. Below the Planck scale, gravity fluctuations and the higher-order ($n \geq 3$) Higgs self-couplings decouple rapidly. We have checked this also numerically, and $k = 10^{17}$ GeV is more than one order of magnitude below the decoupling regime. Hence the higher-order couplings can safely be dropped for $k < 10^{17}$ GeV, which is done here for reducing the numerical effort. Matter effects on the expanded potential at cutoff scales close to the EW scale have already been studied [\[43, 46\]](#), and their investigation in the current framework will be done elsewhere. In this work, for cutoff scales $k \leq 10^{17}$ GeV, we continue within the Φ^4 -approximation.

In summary, at the ASSM UV fixed point, we have a flat Higgs potential

$$\vec{\lambda}_\Phi^* = 0, \quad (53)$$

and in particular the coupling parameters μ_Φ^* and λ_Φ^* of the classical Higgs potential, [\(B.8\)](#). Moreover, the ASSM has as many relevant parameters in the matter sector as the SM, and an additional three relevant couplings in the gravity sector, that overlap with G_N , μ_h , Λ_3 . This concludes our discussion of the physical trajectory of the ASSM. For the first time, we connect the asymptotically safe UV regime of the full SM coupled to asymptotically safe gravity to the deep IR regime of the SM with IR-QCD with confinement and dynamical strong chiral symmetry breaking.

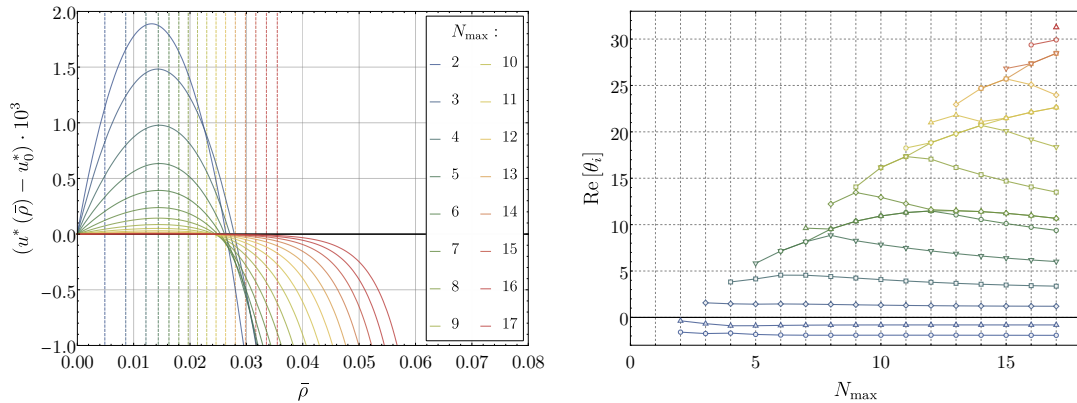


Figure 5: ASSM fixed point Higgs potential (left) and its critical exponents (right). The Higgs potential converges towards a flat potential within the reliability regime, which is indicated by the vertical lines as defined in (55). In contradistinction to a standard Gaußian Higgs potential with one relevant direction, we find two relevant directions related to the two negative eigenvalues (right) for all approximations, and the eigendirections have strong overlap with $\lambda_{\Phi,2}$ (blue triangle) and $\lambda_{\Phi,2N_{\max}}$ (blue circle), see Figure 6. The values rapidly converge with increasing $N_{\max}$, see (56). This rapid convergence is also seen for the lower lying positive eigenvalues, while the convergence of the large ones would require larger $N_{\max}$. The parameter space where such a non-trivial flat potential exist is indicated as the green regime in Figure 9, which includes the ASSM fixed point.

5 Fixed-point Higgs potential and the properties of the ASSM fixed point

In this section, we discuss the emergence of the full fixed-point potential of the Higgs field within a high-order Taylor expansion, see (25). This enables us to discuss the fixed-point properties of the ASSM, and in particular the number of relevant parameters and their stability. Moreover, we discuss the location of the ASSM fixed point in the fixed-point landscape.

In particular, the analysis in this section reveals a novel non-trivial fixed point of the ASSM that has not been seen before: the Taylor expansion of the Higgs fixed point potential $u^*(\bar{\rho})$ converges to a flat potential $u^*(\rho) \equiv 0$ with *two* relevant directions in contradistinction to a standard Gaußian fixed point with *one* relevant direction, see Figure 5. Such a potential cannot be identified within a low order of the Taylor expansion, and certainly, it cannot be seen within the standard Φ^4 approximation. Moreover, the two relevant directions cannot be revealed in a global analysis of such a fixed point without a full stability analysis including the computation of the relevant eigenperturbations, see Figure 7. In short, it is easily overlooked. Interestingly, a rather similar fixed-point pattern has been seen very recently for an effective potential $u((\partial_\mu \varphi)^2)$ in a shift-symmetric setting, see [112]. While there the novel fixed point has been attributed to shift symmetry, in the present case, the flat non-trivial fixed point is revealed for the ASSM with its explicit breaking of shift symmetry.

Besides being a novel fixed point, its relevance comes from the fact that in the current approximation this fixed point is indeed connected with the physical SM (with the parameters fixed by experimental observables) in the IR. In turn, the standard Gaußian fixed point is obtained with a vanishing coupling for the most relevant (non-polynomial) operator. However, it is not connected to the physical SM within the present approximation: the resulting Higgs and top mass are roughly 3 GeV from the central experimental values. Whether this distance is

close or feasible is subject to the evaluation and interpretation of the systematic error of the present approximation. This is but one of the reasons for the rather detailed description of the systematic error estimates in the present work.

We emphasise in this context that the stability aspects of such an analysis are only fully conclusive if the following three aspects are taken into account:

First, within an Einstein-Hilbert truncation, the gravity-matter fixed point with minimally coupled matter can be mapped to the pure gravity fixed point, see [23]. Any approximation that shows instabilities in this approximation, lacks credibility also in improved ones. It is the $\mathcal{R}^2$, $\mathcal{R}_{\mu\nu}^2$ and $\mathcal{C}_{\mu\nu\rho\sigma}^2$ tensor structures, that can trigger instabilities or qualitative changes similar to the Bank-Zaks in QCD as also argued in [23].

Secondly, a full Higgs potential is required. In fact, the Φ^4 truncation of the Higgs potential shows an unstable fixed point potential with $\mu_\Phi^* = 0.286$ and $\lambda_{\Phi,4}^* = -10.8$, which is qualitatively different from the solution it converges to, as we will show in this section.

Finally, one has to include gravity-induced matter interactions as discussed in [21, 28, 55, 77] that are relevant for the discussion of a potential weak gravity bound, see [77, 81, 113].

The current work takes the two first aspects partially into account: we consider a high-order Taylor expansion of the Higgs potential, and the gravity vertices used contain higher order momentum dependences that are generated by the higher order curvature invariants, for a respective discussion, see [14, 64], where it is argued that the $\mathcal{R}_{\mu\nu}^2$ is approximately absent at the fixed point.

5.1 Higgs fixed-point potential

In this section, we discuss the Higgs fixed-point potential as obtained within a high-order Taylor expansion. We will see that the potential rapidly converges towards a flat potential with two relevant directions. This analysis is augmented with a resolution of the eigenperturbation, the respective linear differential equation being of the Sturm-Liouville type. Hence, both the eigenperturbation as well as the fixed-point potential take the form of Kummer functions, and the coefficient of the fixed-point potential vanishes.

5.1.1 Taylor expansion of the fixed-point potential

We use a high-order Taylor expansion of the dimensionless Higgs potential $u(\bar{\rho})$ about vanishing field, see (25). At the UV fixed point, the other SM couplings are vanishing, see Figure 3, and the gravity couplings take their fixed-point values, see (52). The fixed-point equation for the Higgs potential $u(\bar{\rho})$ reads

$$\text{FP}_u(u^*) = 0, \quad \text{with} \quad \text{FP}_u(u) = \mathcal{D}u - \text{Flow}_u, \quad (54a)$$

where $\text{Flow}_u[u(\bar{\rho}), \bar{\rho}]$ comprises the flow diagrams for $V_{\Phi, \text{eff}}$ divided by k^4 , and subtracted at $\bar{\rho} = 0$. This subtraction leads to $u(0) = 0$ and eliminates the overlap of the Higgs potential with the cosmological constant. The scaling part is given by $\mathcal{D}u$ and reads

$$\mathcal{D}u = \left[4 - (2 + \eta_\Phi) \bar{\rho} \partial_{\bar{\rho}} \right] u(\bar{\rho}). \quad (54b)$$

The operator $\mathcal{D}$ generates full scalings on the potential u including the anomalous part, and hence $\mathcal{D}u$ comprises the scaling parts of the fixed-point equation. Note that the part 4 entails the canonical dimension of the potential, and in general we have $4 \rightarrow d_\varphi$ for $\mathcal{D}\varphi$, where d_φ is the full scaling dimension of the function $\varphi(\bar{\rho})$.

As the Yukawa and gauge couplings vanish at the fixed point, Flow_u only receives contributions from Higgs and graviton loops. In the present work, we use a high-order Taylor expansion of the potential. While this converges rather quickly for small $\bar{\rho}$, we have to carefully evaluate

the reliability regime and the radius of convergence of the expansion. Accordingly we define the reliability regime by $\bar{\rho} \in [0, \bar{\rho}_{\max}]$ with $\Delta u = u - u_0$,

$$\bar{\rho}_{\max} = \max_{\bar{\rho}} \left\{ \bar{\rho} \left| \frac{|\text{FP}_u(\Delta u^*)|}{\sqrt{(\mathcal{D} \Delta u^*)^2 + \text{Flow}_u^2(\Delta u^*)}} \leq 1\% \right. \right\}, \quad (55)$$

i.e., we aim at a relative accuracy of 1% in our fixed-point search. Furthermore, we only take into account potential fixed-point solutions for which the lower critical exponents display convergence, in particular the relevant (negative) ones. Naturally, the higher ones are less stable as they are sensitive to the dropped even higher-order couplings $\Lambda_{\Phi,n}$ with $n > N_{\max}$. Fixed-point potentials that do not satisfy these criteria are not considered.

At the ASSM fixed point, we find one non-trivial fixed-point solution, displayed in [Figure 5](#) for $N_{\max} = 2, \dots, 17$. We observe that the solution converges towards a flat potential within the reliability regime of the Taylor expansion and only outside of the regime we encounter an instability. Moreover, the reliability regime is growing with each order of the expansion. This is observed in the $N_{\max}$ -dependence of the ratio of the highest-order coupling and the second-highest one, $r_u(N_{\max}) = |\lambda_{\Phi,2(N_{\max}-1)} / \lambda_{\Phi,2N_{\max}}|$. For $\bar{\rho} = r_u$, the highest-order term is equal to the second-highest one, and we expect convergence for $\bar{\rho} \ll r_u(N_{\max})$. We find that r_u grows and reaches $r_u \approx 0.1$ at $N_{\max} = 17$. A simple extrapolation to $N_{\max} \rightarrow \infty$ leads us to $r_u \approx 0.23$. We expect the same tendency for the convergence radius of the Taylor expansion. Furthermore, we analysed the potential at a fixed field value $\bar{\rho} \in [0, \bar{\rho}_{\max}]$ as a function of the expansion order $N_{\max}$. The functional behaviour is roughly given by $1/N_{\max}$ for any $\bar{\rho}$ chosen and thus the potential value in the $N_{\max} \rightarrow \infty$ limit is compatible with zero. This provides further evidence that the fixed-point potential is indeed converging towards a flat potential.

The eigenvalues are discussed in detail in [Section 5.1.3](#) and are displayed in the right panel of [Figure 5](#). The eigenvalues converge well, in particular, the most relevant ones. Importantly, the fixed point potential has two relevant parameters with eigenvalues

$$\theta_1 = -1.93 \pm 2 \cdot 10^{-3}, \quad \theta_2 = -0.811 \pm 7 \cdot 10^{-5}. \quad (56)$$

Flat potentials with *two* relevant directions are *qualitatively* different from the standard Gaußian potential. The latter potential only has *one* relevant direction at the ASSM fixed point with the eigenvalue $\theta_{\Phi^2, \text{Gauß}} = -2 + \frac{3g_h^*}{(1+\mu_h^*)^2\pi} \approx -0.811$ equal to θ_2 in (56). Naturally, this FP is embedded in the two-dimensional critical surface of the non-trivial flat fixed point if using a vanishing coupling for the operator related to θ_1 .

This begs the question of how the solution in [Figure 5](#) can converge towards the flat potential but retain a stable second relevant direction. For that purpose, we investigate eigenvectors $v^{(i)}$ and eigenfunction $\varphi_i(\bar{\rho})$, which are related with

$$\varphi_i(\bar{\rho}) = c_i(N_{\max}) \sum_{l=0}^{N_{\max}} v_l^{(i)} \bar{\rho}^l, \quad (57)$$

with an $N_{\max}$ -dependent normalisation c_i . For our evaluations within the Taylor expansion, the coefficient of the constant part, $v_0^{(i)}$, drops out and hence we do not consider it here. In [Section 5.1.2](#), we will discuss the full (global) solution of the eigenfunctions including their constant parts. The constant part $u_0 = u(\bar{\rho} = 0)$ of the potential is related to the cosmological constant

$$u_0 = \frac{\lambda_{h,0}}{8\pi g_h}, \quad u_0^* = \frac{1}{16\pi^2} \left(24 - \frac{3}{1+\mu_h^*} \right), \quad (58)$$

with $\lambda_{h,0}$ as defined in (22). Note that $\lambda_{h,0}$ or rather its dimensionful version $\Lambda_{h,0} = k^2 \lambda_{h,0}$ is the observable cosmological constant at $k = 0$. It is an irrelevant coupling, as its dynamical role is taken by $\mu_h = -2\lambda_{h,2}$.

The normalisation with c_i is required as the fixed-point potential vanishes in the limit $N_{\max} \rightarrow \infty$, and so does the norm of all eigenvectors $v^{(i)}$. Accordingly the normalisation c_i has to be chosen such that the eigenvectors $\tilde{v}^{(i)}$ converge with a finite norm, leading also to converging eigenfunctions φ_i . Accordingly, a convenient choice for c_i is given by the inverse of a specific $v_{l_i}^{(i)}(N_{\max})$ with a fixed l_i . It is suggestive to choose $l_i = 1$ for all i . However, since potentially $v_1^{(i)} = 0$ for some eigenfunctions φ_i , this choice may not work globally. In the present case, we are only interested in $i = 1, 2$, where we indeed can choose

$$c_i(N_{\max}) = \frac{1}{v_1^{(i)}}, \quad \text{for } i = 1, 2. \quad (59)$$

We remark that the above construction only works if the respective eigenfunctions are well-defined in the limit $N_{\max} \rightarrow \infty$. We shall see that this is indeed the case, which provides further non-trivial evidence for the existence of the novel ASSM fixed point with a flat Higgs fixed point potential with two relevant directions.

5.1.2 Eigenperturbations and the full fixed-point potential

We briefly discuss the computation of the eigenfunctions from their respective fixed-point equations. Any potential can be expanded about the fixed-point solution in terms of the eigenfunctions,

$$u(\bar{\rho}) = u^*(\bar{\rho}) + \sum_i b_i \varphi_i(\bar{\rho}), \quad (60)$$

where the eigenfunctions carry the scaling of the respective eigenperturbation with the eigenvalue θ_i . Hence, the flow of the $\varphi_i(\bar{\rho})$ in the vicinity of $u^*(\bar{\rho})$ is given by

$$\partial_t \varphi_i(\bar{\rho}) = \theta_i \varphi_i(\bar{\rho}). \quad (61)$$

This entails that the relevant perturbations die out with $\exp(\theta_i t)$ when approaching the UV fixed point as $\theta_i < 0$. In turn, the irrelevant ones grow with $\exp(\theta_i t)$ as $\theta_i > 0$.

We have shown above how the eigenfunctions $\varphi_i(\bar{\rho})$ can be constructed from the $v^{(i)}$. They can also be derived from the respective fixed-point equation. For this derivation, we start with a potential that simply is the sum of the fixed-point potential $u^*(\bar{\rho})$ and a perturbation in a given eigendirection i ,

$$u(\bar{\rho}) = u^*(\bar{\rho}) + \varepsilon \varphi_i(\bar{\rho}), \quad (62)$$

with an infinitesimal parameter ε . This is inserted in the flow and expanded in ε . The zeroth order in ε is simply the fixed-point equation for the Higgs potential, leading to a potential converging to $u^*(\bar{\rho}) \equiv 0$. This implies that the fixed-point equation for the potential can also be expanded in powers of u^* and hence we can relate u^* to the eigenfunction φ_0 with vanishing eigenvalue in (61),

$$u^*(\bar{\rho}) = u_0^* + \varphi_0(\bar{\rho}), \quad \text{with } \theta_0 = 0, \quad (63)$$

with u_0^* related to the cosmological constant, see (58). In (63), we have used that $u(\bar{\rho}) \rightarrow u_0^*$ when approaching the UV fixed point.

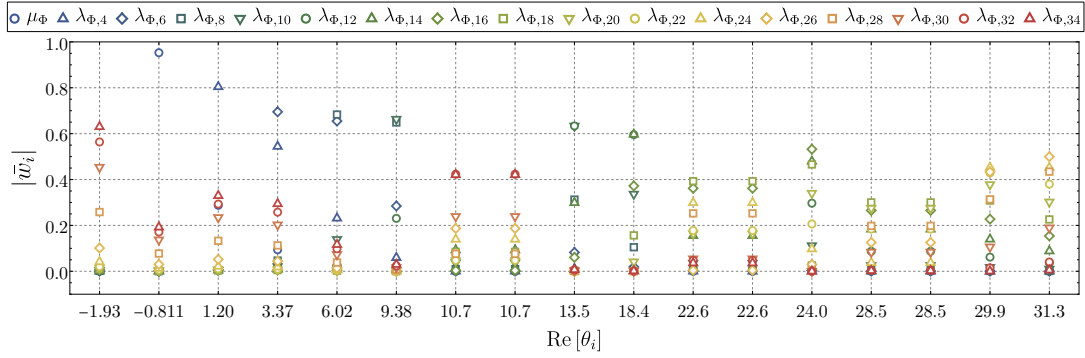


Figure 6: Weighted and normalised eigenvectors at order $N_{\max} = 17$. The most relevant eigenvalue $\theta_1 = -1.93$ has the largest overlap with the highest order coupling $\lambda_{\Phi,34}$. Otherwise we observe a clear correspondence between order of the coupling and relevance, i.e., $\theta_2 = -0.811$ has the largest overlap with μ_Φ , $\theta_3 = 1.20$ with $\lambda_{\Phi,4}$, etc.

The linear order of the flow equation of $u(\bar{\rho})$ gives the differential equation for the eigenperturbations $\varphi_i(\bar{\rho})$ with

$$\mathcal{D}\varphi_i(\bar{\rho}) - \text{Flow}_u(\varepsilon \varphi_i(\bar{\rho}))|_{\mathcal{O}(\varepsilon)} = 0, \quad (64)$$

where the flow term leads to further terms linear in φ_i , φ'_i , and φ''_i . The full scaling $\mathcal{D}\varphi_i$ of the eigenfunction φ_i reads

$$\mathcal{D}\varphi_i(\bar{\rho}) = \left[(4 + \theta_i) - (2 + \eta_\Phi) \bar{\rho} \partial_{\bar{\rho}} \right] \varphi_i(\bar{\rho}). \quad (65)$$

In the present case with a vanishing fixed-point potential $u^*(\bar{\rho}) \equiv 0$, the coefficients of the different $\bar{\rho}$ -derivatives of $\varphi_i(\bar{\rho})$ are at most linear in $\bar{\rho}$. After an appropriate redefinition of the eigenvalue, the scaling equation for the eigenperturbations reduce to that in a simple scalar theory,

$$(4 + \Theta_i) \varphi_i(\bar{\rho}) - (2 + \eta_\Phi) \bar{\rho} \varphi'_i(\bar{\rho}) + \frac{1}{16\pi^2} [2\varphi'_i(\bar{\rho}) + \bar{\rho} \varphi''_i(\bar{\rho})] = 0, \quad (66a)$$

with the shifted eigenvalues Θ_i ,

$$\Theta_i = \theta_i - \frac{3}{\pi} \frac{g_h^*}{(1 + \mu_h^*)^2}. \quad (66b)$$

In (66a), the full anomalous dimension of the Higgs field is proportional to $u'_0 = \partial_{\bar{\rho}} u^*(0)$,

$$\eta_\Phi = \frac{3g_h}{\pi} \frac{(2 + \mu_h)}{(1 + \mu_h)^2} \frac{u'_0}{(1 + u'_0)^2}, \quad (66c)$$

and hence vanishes on the fixed point $u^*(\bar{\rho}) = 0$. This is a peculiarity of the gravity gauge fixing used here, the de-Donder gauge with $\alpha = 0$ and $\beta = 1$ [16]. For general α and β , the Higgs anomalous dimension is present but this does not change the solution of (66) qualitatively.

In (66a), we have dropped the constraint $u^*(0) \equiv 0$ by simply omitting the subtraction of the flow at $\bar{\rho} = 0$. The latter would lead to $\varphi'(\bar{\rho}) \rightarrow \varphi'(\bar{\rho}) - \varphi'(0)$ in the square bracket. Apart from the slight simplification of the equation, dropping the subtraction also allows us to discuss the cosmological constant with the eigenvalue $\Theta_0 = -4$.

With (66b), the scaling equations (66a) are identical to those in a scalar SU(2) theory: the square bracket contains the contribution $3/2 \varphi'_i(\bar{\rho})$ from three Goldstone modes, while the rest, $1/2 \varphi'_i(\bar{\rho}) + \bar{\rho} \varphi''_i(\bar{\rho})$, stems from the radial Higgs field.

The scaling equations (66) are of the Liouville-Sturm type for general Θ , and the solutions $\varphi_i(\bar{\rho})$ are Kummer's function of the first kind [114–116],

$$\varphi_i(\bar{\rho}) = c_i M\left(-\frac{4+\Theta_i}{2}, 2, 32\pi^2 \bar{\rho}\right), \quad (67)$$

with free normalisations c_i . In (67), we have used $\eta_\Phi = 0$ for the present setup, the general solution is provided in Appendix D.

We are specifically interested in the $\bar{\rho} \rightarrow \infty$ behaviour of the eigenperturbations $\varphi_i(\bar{\rho})$. For $\Theta_i = -4 + 2n$ with $n \in \mathbb{N}$, Kummer's function M in (67) reduces to a polynomial of order $2 + \Theta_i/2$. In turn, for all other Θ 's the solutions grow exponentially with $\exp(32\pi^2 \bar{\rho})$ for $\bar{\rho} \rightarrow \infty$. Hence, the eigenfunctions and the potential are best expanded in Hermite polynomials, and the basis is square integrable with a unique solution for the potential. More details are provided in Appendix D.

The above analysis enables us to extend the fixed-point solutions $u^*(\bar{\rho}) = u_0^* + \varphi_0(\bar{\rho})$ with $c_0 \rightarrow 0$ and the eigenperturbations $\varphi_{i>0}$ to the full $\bar{\rho}$ -range. We concentrate on the fixed-point potential and fix the sign of the prefactor $c_0 \rightarrow 0$ with the first derivative $\varphi'_0(0) > 0$. This entails $\text{sign}(c_0) < 0$ and hence the fixed point potential diverges with $-\exp(32\pi^2 \bar{\rho})$ for large $\bar{\rho}$. Evidently, this cannot be the global solution, while $u^*(\bar{\rho}) = 0$ is. It is suggestive that as in Dilaton gravity the coupling of the Higgs to the curvature scale as well as a $Z_\Phi(\bar{\rho})$ is needed to arrive at a global solution. While this still suggests a potential with two relevant directions as the qualitative properties of the Taylor expansion remain unchanged, this global solution may also incorporate a non-trivial potential. While this is a highly relevant question, its resolution is beyond the scope of the present work and is left for future work.

5.1.3 Critical exponents and relevance

We introduce the weighted and normalised eigenvectors [117] of the critical exponents. This rescaling and normalisation are determinant to quantify the real overlap of each operator within each direction. The 'ad hoc' determined eigenvectors are known to be spuriously dominated by the highest orders of the expansion and therefore an equal weight of each operator must be imposed. The normalised eigenvector associated with the i -th critical exponent is defined as

$$\bar{w}_l^{(i)} = \frac{\zeta_l v_l^{(i)}}{\sqrt{\sum_{m=0}^{N-1} |\zeta_m v_m^{(i)}|^2}}, \quad (68)$$

where ζ_l are the rescaling vectors to the 'ad hoc' determined eigenvector $v_l^{(i)}$ with directions $l \in (\mu_\Phi, \lambda_{\Phi,4}, \lambda_{\Phi,6}, \dots)$, see [117] for a detailed derivation.

The coefficients of the weighted eigenvectors are depicted in Figure 6. We observe that the eigenvector $\bar{\omega}^{(2)}$ of the eigenvalue $\theta_2 = -0.811$ has the largest overlap with μ_Φ . Indeed, solving the fixed-point equation for the respective eigenfunction $\varphi_2(\bar{\rho})$ in (67) for the exact eigenvalue $\Theta_2 = -2$ with $c_2 = -1/(16\pi^2)$, we find the exact solution

$$\varphi_2(\bar{\rho}) = \bar{\rho} - \frac{1}{16\pi^2}. \quad (69)$$

The constant comprises the overlap with the cosmological constant which is not present in $u(\bar{\rho})$ with its normalisation $u(0) \equiv 0$. Accordingly, it should be subtracted. Thus, the eigenvector $v^{(2)}$ of the full solution points exactly in the μ_Φ direction, as does $\bar{\omega}^{(2)} = (1, 0, \dots)$. In turn, the

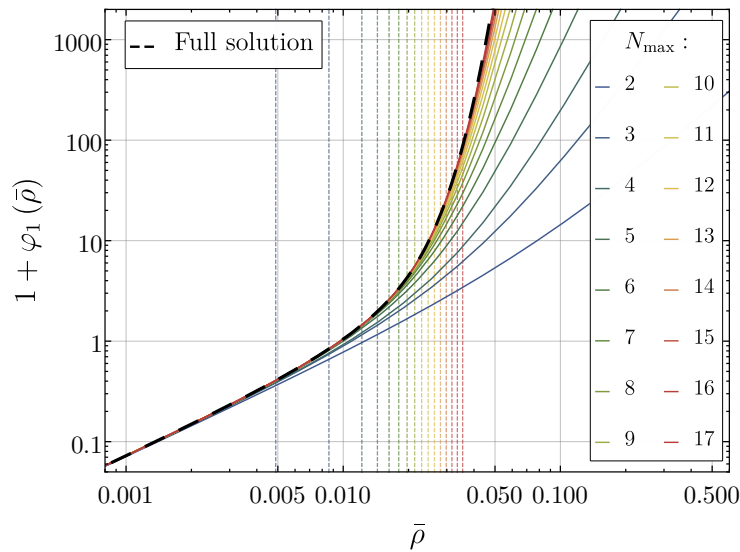


Figure 7: Eigenfunction $1 + \varphi_1(\bar{\rho})$ of the most relevant critical exponent θ_1 : full solution (dashed black), see (67), and different expansion orders $N_{\max} = 2, \dots, 17$. The properties of φ_1 are best seen if plotted on a double logarithmic scale, hence we show $1 + \varphi_1(\bar{\rho}) \geq 0$ with the normalisation $c_1 = -1$ in (68). The rapid convergence of the Taylor expansion towards the full solution is clearly seen. We also depict the reliability regime of the Higgs fixed-point potential u^* with the vertical lines as defined in (55).

numerical solution of the fixed-point equation for φ_2 and the eigenvector $\bar{\omega}^{(2)}$ derived from the fixed-point potential u^* and depicted in Figure 6 also includes higher-order terms. The latter originate in the subleading corrections in θ_2 in comparison to the anomalous dimensions in the fixed-point equation and give an estimate for the numerical accuracy of the θ_i . These terms grow large for large $\bar{\rho}$ and are an additional source for the finite validity range of the current Taylor expansion observed in the fixed-point potential, see the left panel in Figure 5.

The eigenvectors $\bar{\omega}^{(i)}$ of the higher eigenvalues $\theta_{i>2}$ are not exactly aligned with one coupling as it is the case for $\bar{\omega}^{(2)}$. Hence, the respective eigenfunctions φ_i are not monomials. However, the normalised eigenvector $\bar{\omega}^{(3)}$ of the eigenvalue $\theta_3 = 1.20$ has the (by far) largest overlap with $\lambda_{\Phi,4}$, the eigenvector $\bar{\omega}^{(4)}$ of the eigenvalue $\theta_4 = 3.37$ overlaps dominantly with $\lambda_{\Phi,6}$, and in general the normalised eigenvectors $\bar{\omega}^{(i)}$ of θ_i have the largest overlap with $\lambda_{\Phi,2i-2}$ for $i \leq 2$. This matches the expectation from the mass dimension of the operator.

The eigenvector $\bar{\omega}^{(1)}$ of the most relevant direction with the eigenvalue $\theta_1 = -1.93$ has the largest overlap with the highest-order coupling, that is with $\lambda_{\Phi,N_{\max}}$. We emphasise that in the present case with a flat effective potential this is expected and almost required: in a Taylor expansion this is arranged (for small enough fields) by a cancellation between all monomials, leading to relatively large Taylor coefficients which trigger the observed relevance ordering. Indeed, this is confirmed within a comparison of the global solution $\varphi_1(\bar{\rho})$ in (67), which is not a polynomial, and hence features growing Taylor coefficients. Indeed, the high-order Taylor expansion approaches the global solution (67) as depicted in Figure 7. There we also depict the reliability regime of the Higgs fixed-point potential with the vertical lines as defined in (55). We could have defined a reliability regime analogously to (55) by simply substituting $\Delta u^*(\bar{\rho}) \rightarrow \varphi_i(\bar{\rho})$ and $\mathcal{D}(\Delta u^*(\bar{\rho}))$ with (65). A more conservative estimate arises from using the reliability regime of the Taylor expansion of $\Delta u^*(\bar{\rho})$. Evidently, the latter leads to a smaller reliability regime since $\Delta u^*(\bar{\rho}) \rightarrow 0$ and the denominator in (55) is vanishing in the large $N_{\max}$

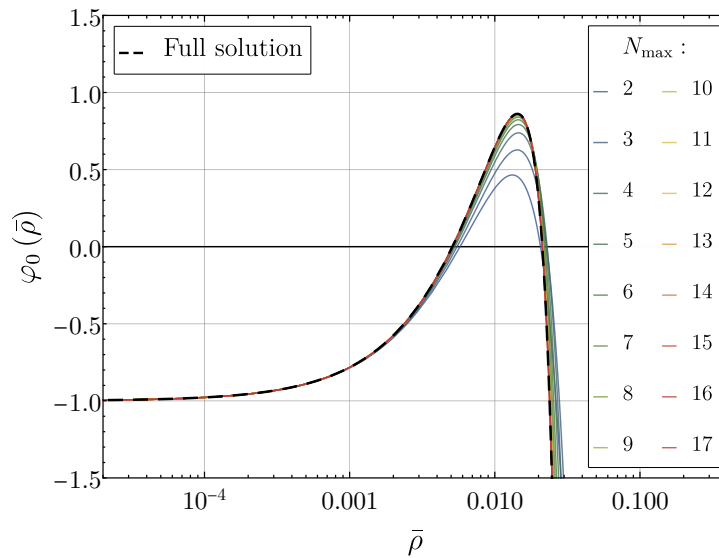


Figure 8: The fixed-point potential $\varphi_0(\bar{\rho})$ defined with (63). We compare the full solution (dashed black), see (67), and with different orders in the Taylor expansion $N_{\max} = 2, \dots, 17$. We choose a fixed normalisation with a negative $c_0 = -1$ which implies $\varphi'_0(0) = 221.9$. This captures the sign of the actual normalisation $c_0(N_{\max}) \rightarrow 0_-$: it goes to zero from below in agreement with the vanishing fixed-point potential $u^* \equiv 0$. The rapid convergence of the Taylor expansion towards the full solution is clearly seen.

limit, apart from the absolute convergence of the fixed-point equation. Hence, we resort to the reliability estimates from the potential throughout as a conservative estimate.

The same analysis can be done for the fixed-point potential $u^*(\bar{\rho})$ with the form (63): for $k \rightarrow \infty$ the potential converges towards its constant part $u^*(\bar{\rho}) = u_0^*$ which is related to the cosmological constant with (58). Accordingly, the $\bar{\rho}$ -dependent part converges towards $\varphi_0(\bar{\rho})$ with a prefactor $c_0(N_{\max}) \rightarrow 0$. This expectation is confirmed by the explicit comparison, see Figure 8. There, we show the Taylor expansion of the potential for various $N_{\max}$ in comparison to the full solution $\varphi_0(\bar{\rho})$. In Figure 8, we fix the normalisation c_0 of $\varphi_0(\bar{\rho})$ defined in (67) as $c_0 = -1$, implying $\varphi'_0(0) = 221.9$. A fixed negative c_0 is taken as the Taylor expansion implies that $c_0(N_{\max})$ approaches zero from below. As is evident from Figure 8, the Taylor expansion is converging rapidly towards $\varphi_0(\bar{\rho})$ with $c_0(N_{\max}) \rightarrow 0_-$. This fully confirms the structure of the solution already seen in the Taylor expansion.

Finally, this global analysis entails that for finite k the potential cannot approach the fixed-point potential in terms of the Kummer function. This is reminiscent to the situation in dilaton gravity where the effective potential had to be augmented with a field-dependent scalar wave functions and a field-dependent Newton coupling in order to access the global solution [118, 119]. The highly relevant question, whether the present solution can be extended to a global one, will be discussed elsewhere.

5.2 Higgs potential landscape

We now proceed with an analysis of the landscape of possible Higgs fixed-point potentials in dependence of the gravity parameters, g_h^* and μ_h^* . This analysis explores the stability of the Higgs fixed-point solution under the variation of gravity parameters given that there is a big uncertainty on the gravity fixed-point values. We investigate the regime $g_h^* \in [0, 1]$ and

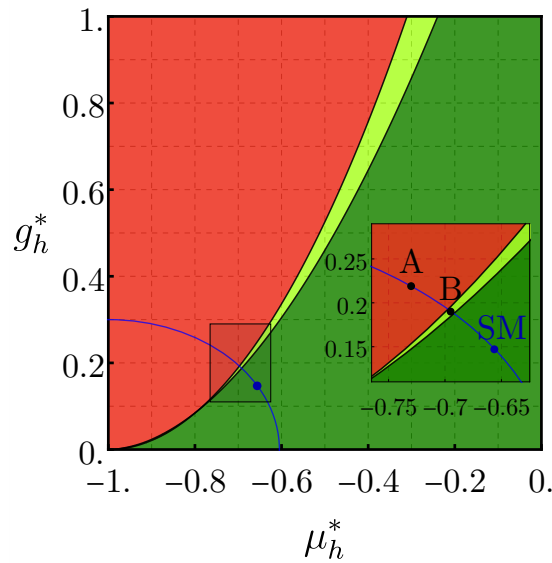


Figure 9: Fixed-point phase diagram of the scalar potential. In the green region, we observe the existence of a flat potential with two relevant directions, while in the lime region additionally a stable potential with zero relevant directions exists. In the red region, we find an unstable potential with one relevant direction. The boundary curves are described with (71) and (73). The effective gravity coupling is approximately given by $g_h^*/(1 + \mu_h^*)^2$, see (75a), and it grows from the green to the red region. The blue circle segment avoids this redundancy as it is close orthogonal to the boundary lines. The Higgs potential corresponding to (SM) is shown in Figure 5, while the potentials for (A) and (B) are shown in Figures 14 and 15 in Appendix E.

$\mu_h^* \in [-1, 0]$. We will argue later that g_h^* and μ_h^* can be combined into an effective gravity coupling and therefore the range of parameters gives us a complete view on the Higgs fixed-point solutions. The resulting phase structure of the UV fixed point is depicted in Figure 9.

Apart from the standard Gaussian fixed-point potential, that exists for all values of g_h^* and μ_h^* , we find three further non-trivial types of fixed-point potentials. The first is the non-trivial flat potential discussed in the previous section, see Figure 5, whose two-dimensional critical manifold naturally includes the standard Gaußian FP with one relevant direction mentioned above. This non-trivial FP with a flat fixed-point potential exists in a rather large parameter space, depicted in green in Figure 9. At the upper and lower boundary of the green region, one of the critical exponents approaches zero and it is suggestive that the fixed point vanishes due to a fixed-point collision. At the upper boundary, the second smallest eigenvalue θ_2 approaches zero, while at the lower boundary, the third smallest eigenvalue θ_3 approaches zero.

We also find a stable fixed-point potential with no relevant directions. This fixed-point potential exists only in the small lime-green band where it exists simultaneously with the non-trivial flat potential discussed before. We display the potential and the critical exponents of point (B) in Figure 15 in Appendix E. The upper boundary of this region coincides with the upper boundary of the green region: it is precisely the collision of these two fixed-point solutions that determine the end of these regions. At the lower boundary, the stable potential solution is lost, resembling a fixed point going to infinity.

The stable fixed-point potential has no relevant directions, and hence all parameters of the IR Higgs potential are fixed by the other couplings: It is a predictive fixed point, and specifically the ratio between Higgs and top mass as well as the ratio between EW and Planck scale are

predictions. However, we find that in the current approximation the potential does not enter a symmetry-breaking regime in the IR and can therefore not be connected to SM physics.

The third fixed-point potential is unstable with one relevant direction, present in the *red* region of Figure 9. We display the potential and the critical exponents of point (A) in Figure 14 in Appendix E. Due to the instability of the potential, we consider this solution to be unphysical.

In summary, in the present approximation only the non-trivial solution in the green region is physical and compatible with the SM in the IR. We remark that the stability analysis here only takes into account the convergence regime of the Taylor expansion, no conclusion about the global stability properties of the potentials can be drawn.

The boundary between the red and lime-green region is determined by a fixed-point collision and the eigenvalue corresponding to the Φ^2 direction approaching zero. This allows us to analytically determine the boundary. For a fixed order in the Taylor expansion, both, the non-trivial flat potential and the stable potential, become flatter and we can evaluate the eigenvalue of the Φ^2 direction for vanishing Higgs couplings. Then the eigenvalue is given by

$$\theta_{\Phi^2} = -2 + \frac{3 g_h^*}{\pi(1 + \mu_h^*)^2}, \quad (70)$$

where -2 is the canonical value and the second term is the shift due to gravity. Solving for $\theta_{\Phi^2} = 0$, we obtain the boundary

$$\frac{3}{2\pi} g_h^* = (1 + \mu_h^*)^2, \quad (71)$$

which is precisely the boundary line between the red and lime-green region in Figure 9.

The lower boundary of the green region is given by $\theta_{\Phi^4} = 0$, which we can again evaluate for vanishing Higgs couplings in the vicinity of the boundary. The eigenvalue is given by

$$\theta_{\Phi^4} = \frac{3 g_h^*}{\pi(1 + \mu_h^*)^2}. \quad (72)$$

Therefore the boundary extends up to vanishing Newton coupling. Note also, that the solution does not exist on the boundary, $g_h^* = 0$.

We could not obtain the lower boundary of the lime-green region analytically. In fact, we found that the boundary is well described by a fractional power

$$0.6 g_h^* = (1 + \mu_h^*)^{1.85}, \quad (73)$$

which indicates a non-trivial competition between diagrams with $g_h^*/(1 + \mu_h^*)^2$ and those with $g_h^*/(1 + \mu_h^*)$.

The above analysis emphasises, that Figure 9 is effectively a one-dimensional plot since typical diagrams contain

$$\frac{(g_h^*)^n}{(1 + \mu_h^*)^m}, \quad n, m \in \mathbb{N}. \quad (74)$$

Typical combinations are $(n, m) = (1, 1)$ and $(n, m) = (1, 2)$, a detailed analysis can be found in [23]. Accordingly, a smaller Newton coupling g_h^* is similar to a larger mass parameter μ_h^* . We define the effective fixed-point (Newton) coupling

$$\hat{g}^* \approx \frac{g_h^*}{(1 + \mu_h^*)^2}, \quad (75a)$$

which grows from the green to the red region

$$\hat{g}_{\text{green}}^* \lesssim \hat{g}_{\text{lime-green}}^* < \hat{g}_{\text{red}}^*, \quad (75b)$$

potentially implying a weak gravity bound $\hat{g}^* < \hat{g}_{\text{red}}^*$. Note that in an on-shell approximation the two-dimensional plot [Figure 9](#) collapses to a line as this approximation lacks any μ_h^* -dependence, and (75b) follows.

6 IR-predictivity & systematics

In this section, we discuss the physics of the UV fixed point that has been presented in [Section 5](#) and also evaluate the systematic error in the current truncation. In the present approximation, the ASSM lies in a regime with a non-trivial but flat Higgs potential with two relevant directions. Consequently, the matter sector of the ASSM has as many relevant parameters as the SM. Additionally, the gravity sector shows another two relevant directions: the cosmological constant and the scalar curvature (Ricci scalar). The number of relevant directions in the gravity sector is expected to increase to three, including the Ricci-scalar squared term, upon improving the truncation [[64](#), [117](#), [120–122](#)].

The non-trivial flat Higgs potential has two relevant parameters but nonetheless, we need to study which IR values can be reached from this fixed point. We observed in [Figure 5](#) that the fixed point is approached from below. This translates into the requirement of a negative quartic Higgs-self coupling around the Planck scale. Lowering the top mass leads to a larger metastability scale and eventually to a positive quartic coupling at the Planck scale. The values of the pole masses M_H and M_t that can be reached are displayed by the *green* regime in [Figure 10](#). The boundary of this green region is given by the *plain white* line which contains all points that connect to the standard Gaussian Higgs potential in the UV. The *white dashed* lines in [Figure 10](#) display our estimated error on this Gaussian line based on the uncertainty of the strong coupling. Additionally, for illustrative purposes, we show green and red shaded regimes for $\pm 1, 2, 3, 4, 5\%$ variations of the top mass determination. The standard Gaussian Higgs potential has a higher predictivity since it has one relevant parameter less and therefore it relates the values of the Higgs and top masses, as displayed in [Figure 10](#). The values of M_H and M_t in the *red* region connect neither to the non-trivial flat Higgs potential nor to the standard Gaussian Higgs potential. In fact, we did not find any viable UV completion for these points.

The *blue* ellipses in [Figure 10](#) display the experimental uncertainties of M_H and M_t at 1σ , 3σ , and 5σ . The ellipses are in the green region and the central values are at an approximate distance of 2.9 GeV to the standard Gaussian fixed point marked by a white line. Due to this vicinity, it is highly important to discuss the uncertainties going into the present computation.

We have identified four systematic error sources within our computation, and below we discuss their influence on the top-mass determination:

(i) The first systematic error source concerns the location of the UV gravity fixed point. We expect the errors on the values of the gravity fixed point to be rather large, and this influences the running of the SM couplings in particular through threshold effects around the Planck scale. We tested this error source by varying the UV fixed-point values and found a very subleading effect on the location of the standard Gaussian line in [Figure 10](#). The reason is that the threshold effects at the Planck scale are washed out through the long RG running to the IR. We estimate that this effect is smaller than 0.1 GeV. Importantly, the gravity fixed-point values determine the existence of the non-trivial but flat Higgs fixed-point potential, see [Figure 9](#), and thus the existence of a green regime in [Figure 10](#).

(ii) The second error source relates to the conversion from Euclidean curvature mass to pole mass discussed in detail in [Appendix G](#): the wave function $Z_t(p)$ of the top quark has a

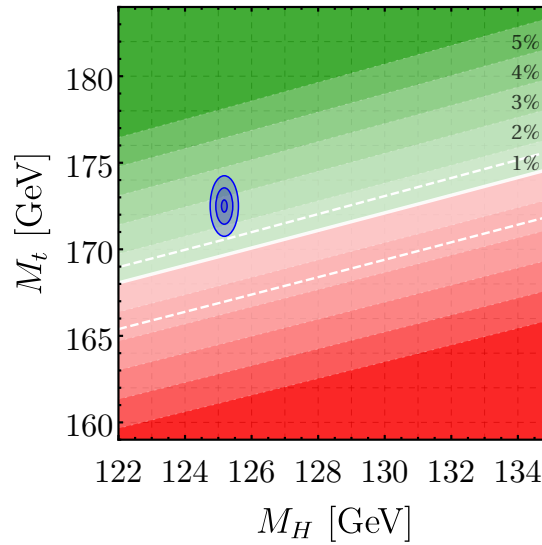


Figure 10: Values of M_H and M_t that can be reached from the non-trivial flat Higgs potential presented in Figure 5 (green region), that can be reached from the standard Gaussian fixed-point Higgs potential (white line), and that cannot be reached from either fixed point (red region). The white shadings show the 1%, ..., 5% error bands. The blue ellipsis displays the SM experimental measurement at 1σ , 3σ , and 5σ . The upper and lower dashed lines correspond to the Gaussian fixed-point trajectory for $\alpha_{s,k=M_Z} = 1.1 \bar{\alpha}_s$ and $\alpha_{s,k=M_Z} = \bar{\alpha}_s$ respectively.

negligible cutoff dependence for $k \leq k_{\text{SSB}} = 930 \text{ GeV}$, see Figure 18. This also entails a negligible momentum dependence for $p \lesssim 10^3 \text{ GeV}$, see Section 2.2.1 and Appendix G. Hence, we have approximated the wave function with a constant one in this regime also for timelike momenta of the same size, $Z_t(p) \approx 1$ for at least $|p^2| \leq (200 \text{ GeV})^2$. From the minimal variation of the cutoff dependence in Figure 18 in this regime, we estimate the respective systematic error with 0.5%, which translates into an error of roughly 0.8 GeV for the top pole mass determination, see also Section 2.2.1. An additional uncertainty stems from the strong coupling, as discussed in the next point. For the translation of Euclidean curvature mass to pole mass, this leads to an error of $^{+0.9}_{-0.2} \text{ GeV}$, see (45g).

(iii) The largest systematic error stems from the uncertainty on the strong coupling, due to the translation of the $\overline{\text{MS}}$ -value at $p = M_Z$, $\bar{\alpha}_s$, to the MOM^2 value used here, α_s , see Appendix E.3. From results in pure Yang-Mills and $2 + 1$ flavour QCD [67, 70], we extrapolated that the conversion factor is given by $\alpha_{s,k=M_Z} = 1.07 \bar{\alpha}_s$ in the ASSM, see (45j). This choice corresponds to the central white line in Figure 10. We use the interval $\alpha_{s,k=M_Z} \in [\bar{\alpha}_s, 1.10 \bar{\alpha}_s]$, see (46), to estimate our error in this identification. Using $\alpha_{s,k=M_Z} = \bar{\alpha}_s$ shifts the standard Gaussian line by -2.6 GeV in the top mass, see the lower white dashed line in Figure 10, while using $\alpha_{s,k=M_Z} = 1.1 \bar{\alpha}_s$ shifts the standard Gaussian line by $+0.9 \text{ GeV}$ in the top mass, see the upper white dashed line in Figure 10, which touches the 5σ band of the experimental error. For $\alpha_{s,k=M_Z} \approx 1.15 \bar{\alpha}_s$, the standard Gaussian line goes through the central experimental values since the metastability scale reaches the Planck scale, see (51), and the non-trivial flat potential needs to be approached from below, see Figure 5. A better control over this error would require a fully self-consistent computation of the strongly coupled IR-QCD sector with all SM flavours.

(iv) The last source is the remaining truncation error of the running couplings after taking into account the errors (i) to (iii). Here we neglected, for example, some momentum dependences and higher-order contributions due to the anomalous dimension, see the discussion in

Appendix F.1. We observed that the inclusion of the next order in the anomalous dimensions only had a subleading effect. Note that the matter couplings approach their Gaussian fixed point in the trans-Planckian UV regime with a power-law and the uncertainty of the gravity fixed point is already taken into account in (i). From this we conclude that the remaining truncation error comes predominantly from the sub-Planckian regime. In this regime, perturbation theory is mostly applicable, and the dominant errors are already taken into account with (ii) and (iii). Furthermore, our truncation includes all IR threshold effects, and therefore we believe that this source of error is subleading and smaller than 1 GeV.

In summary, the experimental values of the Higgs and top mass can be reached from the non-trivial flat Higgs potential with two relevant directions (56). The standard Gaussian fixed point has a distance of roughly 2.9 GeV from the central experimental values, which is similar but smaller than the sum of the errors listed above. This gives us confidence that the ASSM is not in the red regime, which we consider as non-trivial evidence for the *existence* of the ASSM. With the systematic errors listed above, we also consider it viable, while not most likely, to reach the SM values of M_H and M_t from the standard Gaussian Higgs potential.

In the present work, we have solely discussed the minimal ASSM. Hence, physics properties such as the stability considerations and the location and properties of the UV fixed points may change even qualitatively with the introduction of beyond SM degrees of freedom. These are relevant for a comprehensive discussion of open cosmological questions such as dark matter. A survey of asymptotically safe beyond SM theories, and in particular the impact on the stability of the Higgs potential and the physics properties of the UV fixed point is a natural and highly relevant extension of the present work. We hope to report on this in the near future.

7 Conclusions

In the present work, we have presented a comprehensive analysis of the phase structure of the Asymptotically Safe Standard Model (ASSM). The present analysis includes several qualitative improvements:

Firstly, RG-consistent flows have been derived and all flows carry the physical threshold effects required for a quantitative determination of the physical point in the parameter space of the ASSM. Secondly, in the deep IR the present approximation to the ASSM contains QCD with confinement and chiral symmetry allowing for a determination of most parameters at the physical cutoff scale $k = 0$ at vanishing momentum, $p = 0$. Thirdly, we have directly computed the pole mass of the top quark, which eliminates all ambiguities that go with mapping Euclidean curvature masses to the physical pole masses. Finally, our approximation includes a non-trivial Higgs potential for the transplanckian regime which is required for a predictive stability analysis of the Higgs fixed-point potential.

A detailed discussion of our results on the phase structure of the ASSM and the nature of the UV fixed points has been presented in Sections 3 to 5, culminating in an evaluation of the systematics in Section 6. In short, the ASSM lies in a regime of a novel UV fixed point with a non-trivial but flat Higgs potential with two relevant directions with the eigenvalues (56), see Section 5. Consequently, the matter sector of the ASSM has as many relevant parameters as the SM, and the gravity sector shows another three relevant directions with overlap to the cosmological constant, the scalar curvature (Ricci scalar) term and the Ricci-scalar squared term. The distance of the solution to the standard Gaussian fixed point regime with one relevant parameter less (also called ‘predictive regime’) is depicted in Figure 10. For a smaller top pole mass, we leave the non-trivial stable regime, cross the standard Gaussian line, and eventually enter the unstable regime. We remark, that while not excluded, we consider the distance of roughly 2.9 GeV as rather large. While the current approximation is not fully quantitative,

significant changes would be required for entering the ‘predictive’ regime. Note also, that this regime is but one step away from the unstable one, and we consider the relatively large distance from the latter as a positive outcome of the present analysis.

The current minimal ASSM does not provide an answer to fundamental open problems as for example the nature of dark matter or the fermion mass hierarchies. Nevertheless, the framework here presented shows great potential for beyond SM studies in the need of non-perturbative methods or mass dependent schemes. It provides an easily adaptable and RG-consistent playground in which the viability of models phenomenology can be tested. Moreover, this setup can be easily modified towards the study of effective field theories, different matter contents or other types of UV completions thanks to its versatility. Investigations of non-trivial Higgs potentials [43, 46] which follow this line have already been performed.

In conclusion, the present comprehensive analysis of the ASSM constitutes a further important step towards the quantitative construction of the ASSM. Moreover, this RG-consistent set-up can be systematically improved. Important next steps in the SM sector of the ASSM are the inclusion of the CKM matrix, the inclusion of the full Higgs potential at all scales, the resolution of further pole masses, as well as general momentum dependences. On the gravity side, the momentum dependences of the fixed-point solutions have to be used in the flow, gravity-induced matter vertices have to be added and a more complete tensor basis for the gravity vertices has to be used. All the above steps either have already been investigated in parts of the ASSM, or are work in progress. The present set-up also allows for a comprehensive study of scattering amplitudes giving access to the chiefly important question of unitarity in the ASSM. We hope to report on the respective progress soon.

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A Summary of Approximations

We summarise the approximations made in this work and refer to the sections where they are addressed.

General expansion scheme and diagrams

1. *Truncation of the effective action:* We employ a vertex expansion of the effective action and compute the flows of the two- and three-point functions in the symmetric momentum configuration. For the Higgs potential, we also compute higher-order n -point functions. See [Section 2](#) and [appendices B](#) and [C](#).
2. *Anomalous dimensions in diagrams:* We have neglected the sub-leading contributions from the anomalous dimensions in diagrams, that originate in the t -derivative of the regulator. See [Appendix F.1](#).

Gravity sector

1. *Gravity avatars*: We have identified all couplings of the different gravity modes with that of the transverse-traceless one. The flow of the Newton coupling is computed from the transverse-traceless three-point function, and all avatars of the Newton coupling in higher graviton n -point functions are identified with this one. The same procedure was applied to the avatars of the cosmological constant coupling. The flow of the graviton mass parameter is extracted from the transverse-traceless two-point function. See [Section 2.3.2](#) and [appendix B.2](#).
2. *Minimal coupling of gravity and matter*: We identify the gravity-matter couplings with the pure gravity Newton coupling discussed above. We neglect tensor structures from matter-field-curvature operators and higher-order derivative terms, hence implementing only the minimal gravity-matter coupling. See [Section 2.3.2](#) and [appendix B.2](#).
3. *Gravity tadpole contribution to matter three-point functions*: The contributions of the gravity tadpoles to the matter three-point functions are enhanced compared to computations with momentum-dependences of the vertices. We emulate these improved computations of the gauge and Yukawa couplings by simply dropping the tadpole contributions in their flows. For quantitative and qualitative reliability, the implementation of the full momentum dependences is one of the most urgent points to be addressed in the near future. See [Section 3.4](#).

Matter sector

1. *Higgs potential*: For scales above $k = 10^{17}$ GeV, the Higgs potential is expanded in a high-order Taylor expansion, which shows convergence. For lower scales, we use the quartic potential as the higher-order terms decouple rapidly below the Planck scale and hence are negligible. See [Section 2.2.2](#).
2. *Higgs and Goldstone Yukawa couplings*: In the symmetric phase, the flows are evaluated at the vanishing Higgs expectation value, and Higgs and Goldstone Yukawa couplings agree. In the EW broken phase with $k \lesssim 1$ TeV, the flows are evaluated at the running Higgs expectation value and the two sets of Yukawa couplings differ by higher-order derivatives in the Higgs field. We consider these terms to be subleading. We compute the Higgs Yukawa couplings and identify the Goldstone Yukawas with the Higgs Yukawa. See [Appendix F.4](#).
3. *Euclidean vs pole mass*: We compute the pole mass of the top quark. For all other particles, we have identified the pole mass with the Euclidean curvature mass. In the computation of the top pole mass, we assume a constant wave function renormalisation and a saturation scale for the strong coupling contributions. See [Section 4.1](#) and [appendix G](#).
4. *Strong coupling*: We compute the strong coupling in the MOM^2 RG-scheme, that is the common RG-scheme in the fRG approach, see [\[70\]](#). The MOM^2 coupling is slightly larger than that in the $\overline{\text{MS}}$ -scheme. Our results are based on that in pure Yang-Mills theory and 2+1 flavour QCD, [\[67, 68, 70\]](#), that are lifted to the SM value within a linear extrapolation in the number of flavours. See [Section 4.1](#) and [appendix F.3](#).
5. *Infrared QCD*: In the deep IR, the heavy quarks decouple successively and below an interface or decoupling scale, we use the 2+1 flavour results from [\[68\]](#). See [Section 2.3.1](#) and [appendix C.5](#).
6. *Phenomenological approximations*: We used a diagonal CKM matrix and massless neutrinos. See [Appendix B.1](#).

B Effective Action of the ASSM

We provide the gauge-fixed classical action (B.1) of the ASSM in Appendix B.1 (gravity-matter sector) and Appendix B.2 (pure gravity part). The classical gauge fixed-action of the ASSM can be split into a gravity-matter part and a pure gravity part

$$S_{\text{ASSM}} = S_{\text{SM}} + S_{\text{gravity}}. \quad (\text{B.1})$$

The gravity-matter part is the SM action in a given geometry defined by the metric $g_{\mu\nu}$. For the gauge fixing in the presence of a metric one usually introduces a linear split of the full metric into a background $\bar{g}_{\mu\nu}$ and a dynamical fluctuation $h_{\mu\nu}$ as shown in (2) which is then also taken for the expansion of the pure gravity action in terms of quantum fluctuations in a given background. The prefactor of the fluctuation term is chosen such that the fluctuation field $h_{\mu\nu}$ has momentum dimension one and its kinetic term has the canonical form of a spin-two field, a more detailed discussion of general splits can be found in [14].

B.1 Gauge-fixed Standard Model

The Euclidean SM action reads

$$S_{\text{SM}} = \frac{1}{4} \int_x F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{4} \int_x B_{\mu\nu} B_{\mu\nu} + S_{\text{gf}}^{\text{EW}} + S_{\text{ghosts}}^{\text{EW}} + \frac{1}{4} \int_x G_{\mu\nu}^b G_{\mu\nu}^b + S_{\text{gf}}^{\text{SU}(3)} + S_{\text{ghosts}}^{\text{SU}(3)} \\ + \int_x (D_\mu \Phi_i)^\dagger (D_\mu \Phi_i) + V(\Phi_i^\dagger \Phi_i) + \sum_{j=1,2,3} \int_x \bar{\psi}_{i,j}^{L/R} \not{D} \psi_{i,j}^{L/R} + S_{\text{Yukawa}}, \quad (\text{B.2})$$

where

$$\int_x = \int d^4x \sqrt{|g|}. \quad (\text{B.3})$$

The covariant derivative is defined in a concise form as

$$D_\mu = \partial_\mu - i \mathbf{g} \mathcal{A}_\mu, \quad \mathcal{A}_\mu = \mathcal{A}_\mu^i t^i, \quad (\text{B.4})$$

with the Lie algebra generators t^i ,

$$[t^i, t^j] = i f^{ijk} t^k, \quad t^i \in \mathfrak{u}(1)_Y \times \mathfrak{su}(2)_L \times \mathfrak{su}(3)_C, \quad (\text{B.5})$$

and the structure constants f^{ijk} of the SM gauge group $U(1)_Y \times SU(2)_L \times SU(3)_C$. The coupling matrix $\mathbf{g}$ in (B.4) is diagonal in the Lie algebra with entries g_Y, g_2, g_3 in the respective subspaces. Here, g_Y is the hypercharge coupling, g_2 is the weak coupling, and g_3 is the strong coupling. For the weak hypercharge gauge coupling we employ the normalisation

$$g_1 = \sqrt{\frac{5}{3}} g_Y. \quad (\text{B.6})$$

We compute $\vec{g}_1, \vec{g}_2, \vec{g}_3$ from the flow of gauge-matter vertices, where the vectors $\vec{g}_i$ comprise the couplings derived from the different gauge-matter vertices.

The field strength tensor $\mathcal{F}_{\mu\nu}$ reads

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - i \mathbf{g} [\mathcal{A}_\mu, \mathcal{A}_\nu]. \quad (\text{B.7})$$

The $\psi_{i,j}$ in (B.2) stands for a general fermion field representing quark $q_{i,j}$ and lepton $l_{i,j}$ doublets with their respective chiralities. The index j is the generation index, and i is the

isospin eigenvalue. We have included all three generations of leptons and quarks. We only consider interactions within fermions of the same generations, hence, a diagonal CKM matrix. This approximation is also used in the effective action discussed below.

The classical Higgs potential V_Φ in (B.2) is given by

$$V_\Phi(\rho) = \mu_\Phi \rho + \lambda_\Phi \rho^2, \quad (\text{B.8})$$

with the Higgs doublet Φ and its radial field ρ ,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ v + H + i\mathcal{G}_3 \end{pmatrix}, \quad \rho = \text{tr} \Phi^\dagger \Phi. \quad (\text{B.9})$$

Here, H is the Higgs field that acquires an expectation value and the $\mathcal{G}_i$ are the Goldstone modes.

The Yukawa terms of the SM read

$$S_{\text{Yukawa}} = \int_x \mathcal{Y}_u \bar{q}_{i,1}^L \tilde{\Phi}_i q_{1,1}^R + \mathcal{Y}_d \bar{q}_{i,1}^L \Phi_i q_{2,1}^R + \mathcal{Y}_{\nu_e} \bar{l}_{i,1}^L \tilde{\Phi}_i l_{1,1}^R + \mathcal{Y}_e \bar{l}_{i,1}^L \Phi_i l_{2,1}^R + \text{h.c.} + \dots \quad (\text{B.10})$$

where we have only shown explicitly one single generation of fermions and where

$$\tilde{\Phi}_i \equiv i \sigma_2 \Phi_i^*. \quad (\text{B.11})$$

The interaction terms between Goldstones and fermions will be used to read off the background field dependence of the Yukawa couplings. This subject is discussed within our systematics overview in [Appendix F.4](#). Moreover, we have checked that the Yukawa interaction and respective mass terms of the Neutrinos lead to negligible effects in the flow. Thus, we have dropped them for the results obtained here.

Finally, we have to specify the gauge fixings in the EW and QCD sectors. In the EW sector, we employ an R_ξ -gauge fixing which is adapted to the occurrence of massive modes in the broken phase

$$S_{\text{gf}}^{\text{EW}} = \frac{1}{2\xi} \int_{\bar{x}} \left(\partial_\mu A_\mu^a - i g_2 \xi t_{ij}^a (\Phi_i^\dagger v_j - v_i^\dagger \Phi_j) \right)^2 + \frac{1}{2\xi_Y} \int_{\bar{x}} \left(\partial_\mu B_\mu - \frac{i}{2} g_Y \xi_Y (\Phi_i^\dagger v_i - v_i^\dagger \Phi_i) \right)^2. \quad (\text{B.12})$$

Here,

$$\int_{\bar{x}} = \int d^4x \sqrt{|\bar{g}|}, \quad (\text{B.13})$$

refers to the volume factor of the background metric, which is taken to be the flat metric here. The associated Faddeev-Popov ghost action reads

$$S_{\text{ghosts}}^{\text{EW}} = \int_{\bar{x}} \left[-\partial_\mu \bar{c}_Y (\partial_\mu c_Y) - \xi_Y g_Y^2 (\bar{c}_Y c_Y) \phi_i^\dagger v_i \right] + \int_{\bar{x}} \left[-\partial_\mu \bar{c}^a \partial_\mu c^a - \xi g_2^2 t_{ij}^a t_{jl}^b (\bar{c}^a c^b) \Phi_i^\dagger v_l + g_2 A_\mu^c f^{abc} \partial_\mu \bar{c}^a c^b \right]. \quad (\text{B.14})$$

Here, v_i are the components of the Higgs background field, see (B.9).

For the high-energy regime of QCD, we use the gauge-fixing and ghost action

$$S_{\text{gf}}^{SU(3)} + S_{\text{ghosts}}^{SU(3)} = \frac{1}{2\xi_3} \int_{\bar{x}} \left(\partial_\mu G_\mu^{c_a} \right)^2 + \int_{\bar{x}} \bar{c}^{c_a} \partial_\mu D_\mu^{c_a c_b} c^{c_b}, \quad (\text{B.15})$$

in the Landau limit, $\xi_3 \rightarrow 0$.

B.2 Gauge-fixed gravity

The classical Einstein-Hilbert action is given by

$$S_{\text{EH}} = \frac{1}{16\pi G_N} \int_x (2\Lambda - R) + \frac{1}{2\alpha} \int_{\bar{x}} \bar{g}_{\mu\nu} F_\mu F_\nu + \int_{\bar{x}} \bar{c}_h^\mu \mathcal{M}_{\mu\nu} c_h^\nu, \quad (\text{B.16})$$

with the linear de-Donder gauge fixing

$$F_\mu = \frac{1}{16\pi} \left(\bar{D}^\nu h_{\mu\nu} - \frac{1}{2} \bar{D}_\mu h^\nu_\nu \right). \quad (\text{B.17})$$

Here, $\bar{D}$ is the covariant derivative only dependent on the background metric. The Faddeev-Popov operator for the gauge fixing (B.17) reads,

$$\mathcal{M}_{\mu\nu} = \bar{D}^\sigma (g_{\mu\nu} D_\sigma + g_{\sigma\nu} D_\mu) - \bar{D}_\mu D_\nu. \quad (\text{B.18})$$

C Flow equations in the ASSM

In this Appendix, we describe the projection procedure used to derive the flows for the couplings and anomalous dimensions from the functional flow (4) for the effective action. The anomalous dimensions are discussed in Appendix C.1, the flow of matter couplings except the Higgs self couplings is discussed in Appendix C.2. The flow of the Higgs potential is discussed in Appendix C.3, and that of the pure gravity parameters is discussed in Appendix C.4. Finally, in Appendix C.5 we discuss the flows in the strongly correlated IR sector of QCD.

In short, we perform field derivatives of the functional flow (4) to obtain flows for correlation functions $\partial_t \Gamma^{(n)}(p_1, \dots, p_n)$, and all flowing parameters are obtained by evaluating the flow of the respective correlation functions (or its momentum derivative in the case of anomalous dimensions) at vanishing momentum, $p_1 = \dots = p_n = 0$, in the spirit of the derivative expansion, with the exception of the gravity couplings. The symbolic expressions of the flows on n -point functions were derived using the Mathematica package DOFUN [123].

In the present work, we use RG-adapted [82] regulators

$$R_\phi(p) = Z_\phi R_\phi^{(0)}(p), \quad R_\phi^{(0)}(p) = P_\phi(p) r(p^2/k^2), \quad (\text{C.1})$$

with the classical dispersions $P_\phi(p)$, and the shape function of the Litim-type regulator [124, 125],

$$r(x) = (1-x)\Theta(1-x), \quad x = \frac{p^2}{k^2}, \quad (\text{C.2})$$

with the Heaviside step function Θ . The Litim-type regulator leads to analytic flows. It is also optimised for the 0th order derivative expansion [82, 124, 125], however, it is not the optimal regulator beyond this order. Optimal regulators can be constructed by functional optimisation [82]. Moreover, in the light of highly interesting further investigations of the resonance structure of the ASSM, scattering processes and the quantitative determination of (pole) masses there are further constraints on the analytic structure of the regulator also for timelike momenta, for related discussions in scalar theories, QCD and pure gravity see [105, 126–133].

C.1 Anomalous dimensions

The anomalous dimensions

$$\eta_{\phi_i} = -\frac{\partial_t Z_{\phi_i}}{Z_{\phi_i}} \quad (\text{C.3})$$

are obtained from the flow of the respective two-point functions $\partial_t \Gamma^{(2)}$. For leptons and quarks, we define

$$\eta_\psi = \frac{i \text{tr} [\not{p} \partial_t \Gamma_{\psi\bar{\psi}}^{(2)}(p)]}{Z_\psi p^2 \text{tr} \mathbb{1}} \Big|_{p=0}, \quad (\text{C.4})$$

where the trace sums over all internal and group indices. Note also that the evaluation at $p = 0$ is tantamount to a p^2 -derivative at $p = 0$ of the numerator.

For the Higgs field we use

$$\eta_\Phi = \frac{\text{tr} [\partial_t \Gamma_{\Phi\Phi}^{(2)}(p) - \partial_t \Gamma_{\Phi\Phi}^{(2)}(0)]}{Z_\Phi p^2 \text{tr} \mathbb{1}} \Big|_{p=0}, \quad (\text{C.5})$$

where the trace sums over all internal and group indices.

For the gauge fields, we use

$$\eta_A = \frac{\text{tr} \Pi^\perp(p) [\partial_t \Gamma_{AA}^{(2)}(p) - \partial_t \Gamma_{AA}^{(2)}(0)]}{Z_A p^2 \text{tr} \Pi^\perp(p)} \Big|_{p=0}, \quad (\text{C.6})$$

where the trace sums over all internal and group indices. In (C.6) we have used the transversal projection operator

$$\Pi_{\mu\nu}^\perp(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}. \quad (\text{C.7})$$

The anomalous dimensions of the auxiliary ghost fields are extracted with the flows

$$\eta_c = \frac{\text{tr} [\partial_t \Gamma_{CC}^{(2)}(p)]}{Z_c p^2} \Big|_{p=0}. \quad (\text{C.8})$$

Finally, we remark that the wave functions completely drop out of the diagrams as they are multiplicative factors in all vertices and (inverse) propagators. The global wave function factor Z_{ϕ_i} due to the external legs each carrying a $Z_{\phi_i}^{1/2}$ is cancelled by the $1/Z_{\phi_i}$ in the definition of η_{ϕ_i} . The only remnant dependence on Z_ϕ arises from the t -derivatives of the cutoff line $G_\phi \partial_t R_\phi G_\phi$ with propagators G_ϕ and regulators R_ϕ . We are led to

$$\frac{1}{Z_\phi} \partial_t R_\phi(p) = [\partial_t - \eta_\phi] R_\phi^{(0)}, \quad (\text{C.9})$$

more details on the structure of the equations for the anomalous dimension and possible systematic extensions have been deferred to [Appendix F](#).

C.2 Flows of matter couplings

All couplings considered here are computed from three-point functions. The only exceptions are the Higgs mass μ_Φ and self-couplings $\lambda_{\Phi,2n}$ in (10). The flow of the four-point function avatars of the gauge couplings are identified with that computed from the three-point function of the gauge fields coupled to fermions. In the present approximation, this identification is a consequence of gauge consistency and can be derived from the respective Slavnov-Taylor identities which collapse into Ward identities in the present approximation. However, the present projections also overlap with non-classical tensor structures hence the gauge couplings from different correlation functions differ, and the Slavnov-Taylor identities are more complicated in improved approximations.

The flows of the couplings of three-point functions can be obtained from (7a) with

$$\partial_t \frac{\Gamma_k^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k)}{\prod_{i=1}^n \sqrt{Z_{\phi_i, k}}} = S^{(n)}(p_1, \dots, p_n; \partial_t \vec{\lambda}_k). \quad (\text{C.10})$$

The left-hand side is simply given by

$$\partial_t \frac{\Gamma_k^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k)}{\prod_{i=1}^n \sqrt{Z_{\phi_i, k}}} = \frac{1}{2} \sum_{i=1}^n \eta_{\phi_i, k} S^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k) + \text{Flow}^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k), \quad (\text{C.11})$$

where the second line is simply the flow diagrams of the given n -point function divided by the wave functions $\sqrt{Z_{\phi_i}}$ for each of its external legs,

$$\text{Flow}^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k) = \frac{\partial_t \Gamma_k^{(n)}(p_1, \dots, p_n; \vec{\lambda}_k)}{\prod_{i=1}^n \sqrt{Z_{\phi_i, k}}}. \quad (\text{C.12})$$

As in the case of the anomalous dimensions this division, together with the use of RG-adapted regulators, eliminates all Z_ϕ -dependence from the diagrams at the expense of an η_ϕ -dependence.

Given the flow of a three-point function of the fields $\phi_{i_1}, \phi_{i_2}, \phi_{i_3}$ with the coupling $\lambda_{123} = \lambda_{\phi_{i_1} \phi_{i_2} \phi_{i_3}}$ we arrive at

$$\left(\partial_t - \frac{1}{2} \sum_{j=1}^3 \eta_{\phi_{i_j}, k} \right) \lambda_{123} = \text{tr} \left[P_{123}(\mathbf{p}) \text{Flow}_{\phi_{i_1} \phi_{i_2} \phi_{i_3}}^{(n)} \right] (\mathbf{p} = 0), \quad (\text{C.13})$$

where P_{123} is a projection operator or rather procedure such as used for the anomalous dimensions. We performed consistency checks to confirm the veracity of our setup. We found the expected universal results up to one loop in agreement with the literature [111].

In the solutions to the flows presented in Figures 3 and 4, we made different gauge fixing choices for each of the phases. In the symmetric phase, we chose the Landau gauge $\xi_Y = \xi = 0$. Note that in this regime the gauge dependency appears beyond one loop. In the broken phase, to facilitate a correct cancellation between the unphysical modes of the Goldstone bosons and EW ghosts and longitudinal polarisations we chose the Feynman gauge $\xi_Y = \xi = 1$.

C.3 Flow of the Higgs potential

In this section we derive the flow of the scalar field parameters: couplings, curvature mass and vacuum expectation value. Starting from the renormalised potential (9) and its dimensionless version (25a), the minimum $\bar{\rho}_0 = \frac{Z_H v^2}{2k^2}$ is defined by

$$\partial_{\bar{\rho}} u(\bar{\rho})|_{\bar{\rho}_0} = 0. \quad (\text{C.14})$$

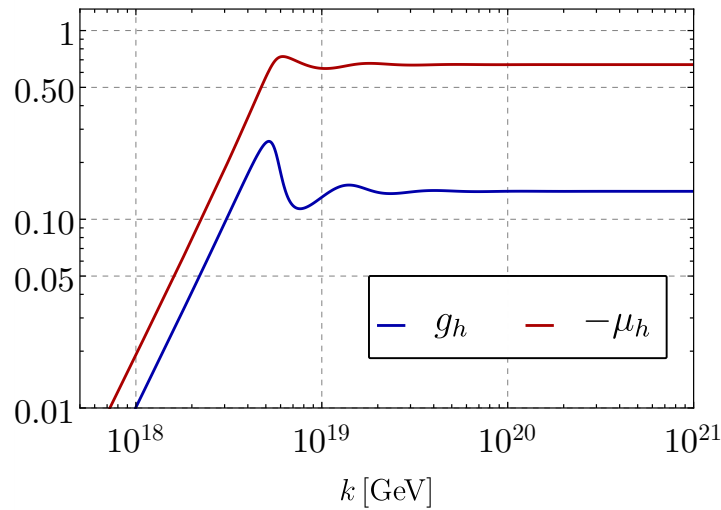


Figure 11: Dimensionless Newton coupling g_h and graviton mass parameter μ_h as a function of the RG scale k . Above the Planck scale, the couplings approach an interacting fixed point while in the IR both coupling decrease quadratically towards the Gaußian fixed point.

Taking a t -derivative, we obtain

$$\partial_t \bar{\rho}_0 = - \frac{\partial_{\bar{\rho}} \left(\partial_t |_{\rho} u(\bar{\rho}) - \partial_t |_{\rho} \bar{\rho} \partial_{\bar{\rho}} u(\bar{\rho}) \right)}{\partial_{\bar{\rho}}^2 u(\bar{\rho})} \Big|_{\bar{\rho}_0}. \quad (\text{C.15})$$

Introducing the definition of the diagrammatic flows

$$\partial_t |_{\rho} u(\bar{\rho}) + 4u(\bar{\rho}) = \frac{\text{Flow}[V_{\Phi, \text{eff}}]}{k^4}, \quad (\text{C.16})$$

where $\text{Flow}[V_{\Phi, \text{eff}}] = \partial_t |_{\rho} V_{\Phi, \text{eff}}(\rho)$ and

$$\partial_t |_{\rho} \bar{\rho} = -(2 + \eta_{\Phi}) \bar{\rho}, \quad (\text{C.17})$$

we reach the final expression for the flow of the vacuum expectation value of the Higgs field in the broken ($\mu_{\Phi} < 0$) regime.

The flow of the scalar coupling $\lambda_{\Phi, 4}$ and the curvature mass μ_{Φ} have been obtained from the flow of scalar two- and four-point functions. For the higher n -point functions, this becomes tedious due to the large number of diagrams and external legs. Instead, we derive the flow of the scalar couplings $\lambda_{\Phi, 2n}$ directly from (C.16). Considering a vanishing background for simplicity, the flow of any arbitrary $\lambda_{\Phi, 2n}$ can be derived by performing field derivatives of the flow of the effective potential,

$$\left(\partial_t |_{\bar{\rho}} \lambda_{2n} \right) \Big|_{\bar{\rho}=0} = \frac{1}{n!} \partial_{\bar{\rho}}^n \left[\frac{1}{k^4} \text{Flow}[V_{\Phi, \text{eff}}] \right] + \lambda_{2n} (n \eta_{\Phi} + (2n - 4)). \quad (\text{C.18})$$

This procedure gives identical results to the derivation from n -point functions. On the technical side, it is important to take properly into account the mixing between the Higgs field and the scalar modes of the graviton.

C.4 Flow of gravity parameters

We extract the flow of the Newton coupling g_h from the transverse-traceless graviton three-point function and the flow of the graviton mass parameter μ_h from the transverse-traceless graviton two-point function. All gravity flows and anomalous dimensions take into account momentum dependences and are evaluated bilocally between $p = 0$ and $p = k$. The flows are identical to [73] with the approximation $\lambda_{h,n \geq 3} = 0$, and the identification of all avatars of the Newton coupling with the avatar from the three-graviton vertex, see (24). This entails that the matter fields only contribute in their minimally coupled versions to gravity flows and in the end only enter via their field multiplicities. In the SM, these multiplicities are given by $N_s = 4$, $N_f = 45/2$, and $N_v = 12$. In the IR, we tune the Newton coupling and the graviton mass parameter such that we reach the physical value of the Newton coupling and a vanishing cosmological constant, see (45d). In Figure 11, we display the trajectories of g_h and μ_h .

C.5 Infrared regime of QCD

The IR regime of QCD with confinement and dynamical chiral symmetry breaking has been evaluated both qualitatively and quantitatively with functional approaches, for recent reviews see [13, 134]. We are solely interested in the running of the quark-gluon coupling, which is dominated by the gluon anomalous dimension $\eta_{A,k}$ and its IR flow below an *interface* scale k_{inter} defined in (17).

It has been argued and checked in [68, 71], that the gluon anomalous dimension $\eta_{A,k}$ is well described as a function of the running coupling α_s and the mass gaps of quarks and gluons,

$$\eta_{A,k} \approx \eta_A(\vec{\alpha}_{s,k}, \vec{m}_k^2), \quad \vec{m}^2 = m^2/k^2, \quad (\text{C.19})$$

with

$$\vec{m}_k^2 = (m_{A,\text{gap}}^2, \vec{m}_{q,k}^2). \quad (\text{C.20})$$

The gluon mass gap in (C.20) should not be confused with a gluon mass. The vector $\vec{\alpha}_{s,k}$ again comprises all avatars of the strong coupling; related to the primitively divergent part of the ghost-gluon, three-gluon, four-gluon and quark-gluon vertices. In the matter sector, we have avatars for each quark flavour, that differ below the mass thresholds of the quarks,

$$\alpha_{Aq_i\bar{q}_i} = \frac{1}{4\pi} \frac{\lambda_{Aq_i\bar{q}_i}}{Z_A^{1/2} Z_{q_i}}, \quad \text{with} \quad q = (d, u, s, c, b, t), \quad (\text{C.21a})$$

and in the pure glue sector we have three- and four-gluon couplings as well as the ghost-gluon coupling,

$$\alpha_{A^n} = \frac{1}{4\pi} \frac{\lambda_{A^n}}{Z_A^{n/2}}, \quad \alpha_{A\bar{c}c} = \frac{1}{4\pi} \frac{\lambda_{A\bar{c}c}}{Z_A^{1/2} Z_c}, \quad (\text{C.21b})$$

where the λ 's are the dressings of the classical tensor structures of the respective vertices [61, 68]. All the avatars of the strong fine structure constants agree at two loops (in mass-independent RG schemes), $(\alpha_s)_i = \alpha_s$, but run differently with the cutoff scale and momenta in the IR due to physical threshold effects introduced by the gluon and quark mass gaps $\vec{m}^2$, (C.20).

The above couplings are *exchange* couplings that describe the scattering of quarks ($\alpha_{s,Aq\bar{q}}$), gluons (α_{s,A^3}), and ghosts ($\alpha_{s,A\bar{c}c}$). These couplings occur directly in diagrams with the respective gluon lines, and carry directly the decoupling of gluons in the diagrams below the mass gap of QCD. In contradistinction, the electroweak g_i 's, depicted in Figure 3, do not include the

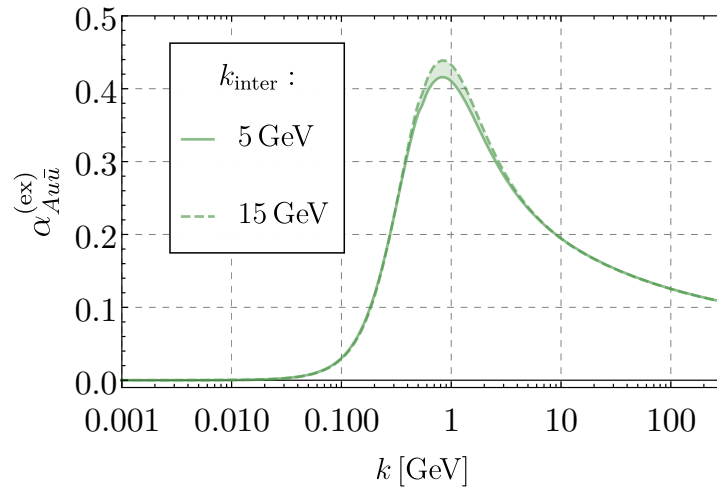


Figure 12: The quark-gluon exchange coupling $\alpha_{Au\bar{u}}^{(ex)}$ defined in (C.21) as a function of k for interface scales $k_{\text{inter}} = 5, 15$ GeV. The chosen k_{inter} does not affect severely the IR dynamics.

exchange propagators and are also called effective charges, see e.g. [107, 108]. For the definition and computation of a respective charge in QCD see [109, 110].

For the explicit computation, we utilise the 2+1 flavour IR-QCD results of [68]. For that purpose, we separate the 2+1 flavour part from the contributions $\eta_A^{(c,b,t)}$ of the heavier quarks

$$\eta_{A,k}^{(\text{ASSM})} = \eta_{A,k}^{(2+1)} + \eta_{A,k}^{(c,b,t)}, \quad (\text{C.22})$$

where the 2+1 flavour contribution in (C.22) depends on the full coupling and mass gaps of the ASSM,

$$\eta_{A,k}^{(2+1)} = \eta_{A,k}^{(2+1)} \left(\bar{\alpha}_{s,k}^{(\text{ASSM})}, \left(\vec{m}_k^{(\text{ASSM})} \right)^2 \right). \quad (\text{C.23})$$

The strong coupling is a multiplicative factor in all (QCD) diagrams and hence we have

$$\eta_{A,k}(\alpha_{s,k}, \vec{m}_k^2) \approx \eta_{A,k}(\vec{m}_k^2) \alpha_{s,k}, \quad (\text{C.24})$$

where $\alpha_{s,k}$ is the strong coupling avatar of the lightest field, the ghost,

$$\alpha_{s,k} = \alpha_{A\bar{c}c,k}. \quad (\text{C.25})$$

The identification of all avatars in (C.21) with the ghost-gluon coupling fails in the IR below the respective mass thresholds, see [61, 68]. There, however, the contributions to $\eta_{A,k}$ drop out. Furthermore, the quark masses in the 2+1 flavour computation are tuned to the physical ones. In summary, this supports the approximation

$$\eta_{A,k}^{(\text{ASSM})} \approx \eta_{A,k}^{(2+1)} \frac{\alpha_{s,k}^{(\text{ASSM})}}{\alpha_{s,k}^{(2+1)}} + \eta_{A,k}^{(c,b,t)}, \quad (\text{C.26})$$

also displayed in (19). Here, $\eta_{A,k}^{(2+1)}$ is now the genuine 2+1 flavour result and the difference between the ASSM strong fine structure constant and that in 2+1 flavour QCD is carried by the ratio of the couplings (C.25). The contribution from the heavier quarks, $\eta_{A,k}^{(c,b,t)}$, is simply that of the ASSM.

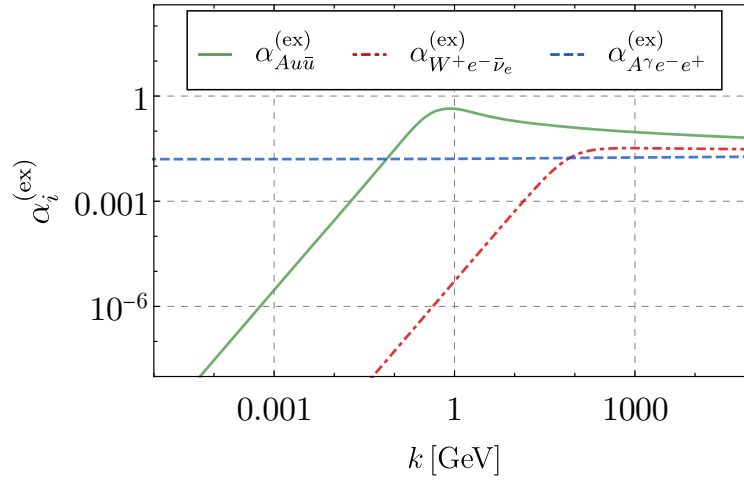


Figure 13: The exchange couplings of the quark-gluon coupling $\alpha_{Au\bar{u}}^{(ex)}$ (green, solid), the electron- $W^\pm$ coupling $\alpha_{W^\pm e^- \bar{\nu}_e}^{(ex)}$ (red, dash-dotted), and the photon-electron coupling $\alpha_{A\gamma e^- e^+}^{(ex)}$ (blue, dashed) as defined in (C.21) and (C.31). The decrease of the strong and weak coupling towards small momenta is triggered by the QCD mass gap and the mass of the W boson, respectively. The electromagnetic coupling does not decrease in the IR due to the absence of a photon mass.

Finally, we use that in leading order the ghost propagator dressing and ghost-gluon vertex dressing are insensitive to the number of quarks at all scales, and hence they drop out of the ratio. This leaves us with our final equation for the anomalous dimension,

$$\eta_{A,k}^{(ASSM)} \approx -\frac{\partial_t Z_{A,k}^{(2+1)}}{Z_{A,k}^{(ASSM)}} + \eta_{A,k}^{(c,b,t)}, \quad (C.27)$$

where $\partial_t Z_{A,k}^{(2+1)}$ is the flow of the wave-function renormalisation in 2+1 flavour QCD, not in the ASSM. This can be re-arranged as a flow equation for the gluon wave-function renormalisation,

$$\partial_t Z_{A,k}^{(ASSM)} \approx \partial_t Z_{A,k}^{(2+1)} - \eta_{A,k}^{(c,b,t)} Z_{A,k}^{(ASSM)}. \quad (C.28)$$

The right-hand side depends on the external input $\partial_t Z_{A,k}^{(2+1)}$ from 2+1 flavour QCD taken from [68], while all the other terms are that in the ASSM.

The flow (C.27) is now solved for IR-QCD scales below the interface scale k_{inter} in the range (17). The flow (C.27) requires as an input the gluon wave-function renormalisation $Z_{A,k}^{(ASSM)}$ at the interface scale. This input is fixed by the consistency condition that the anomalous dimension computed above and below k_{inter} have to agree at the interface scale,

$$\left(\eta_{A,k_{\text{inter}}^+}^{(ASSM)} - \eta_{A,k_{\text{inter}}^-}^{(ASSM)} \right) = 0, \quad k^\pm = \lim_{\epsilon \rightarrow 0} k \pm \epsilon. \quad (C.29)$$

Inserting (C.27) into (C.29) leads us to

$$Z_{A,k_{\text{inter}}^-}^{(ASSM)} = -\frac{\partial_t Z_{A,k_{\text{inter}}^-}^{(2+1)}}{\eta_{A,k_{\text{inter}}^+}^{(ASSM)} - \eta_{A,k_{\text{inter}}^+}^{(c,b,t)}}, \quad (C.30)$$

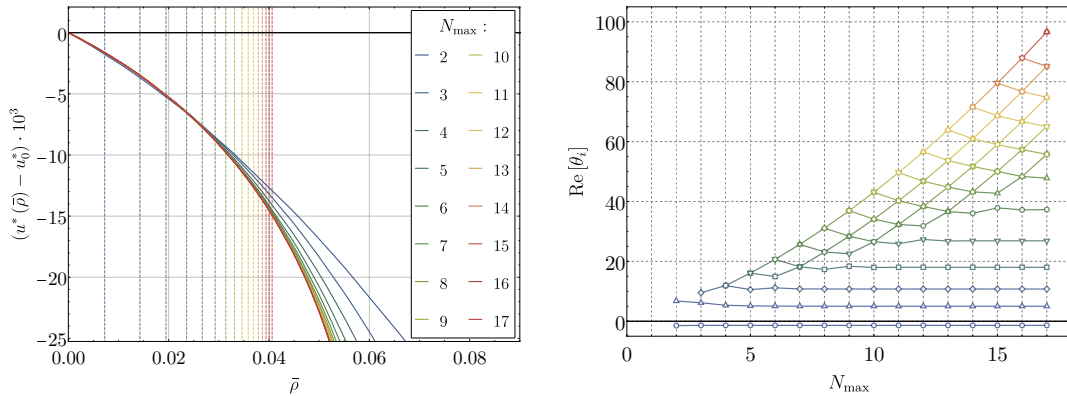


Figure 14: UV potential (left) and critical exponents (right) at different orders of the Taylor expansion for the gravitational fixed point (A) in Figure 9 with $g_h = 0.219$ and $\mu_h = -0.730$. The vertical dashed lines in the left panel show the reliability regimes at each order of the expansion with a relative error of 1% as defined in (55). The fixed point has one relevant direction with lowest critical exponent $\min[\theta_i^{(A)}] = -1.39$.

where the denominator is nothing but the 2+1 flavour part of the gluon anomalous dimension in the ASSM. This provides us with the initial condition for solving (C.27), also discussed in Section 2.3.1. For interface scales $5 \text{ GeV} \lesssim k_{\text{inter}} \lesssim 15 \text{ GeV}$, see (17), we find a small dependence of the IR coupling on the interface scale. This is depicted in Figure 12.

It is convenient to define *exchange* couplings not only for gluonic couplings but also for the other gauge fields,

$$\alpha_i^{(\text{ex})} = \frac{g_i^2}{4\pi} \left(\frac{1}{1 + m_i^2/k^2} \right), \quad (\text{C.31})$$

which account for the masses of the mediating field in the scattering process with the intermediate gauge field coupled via g_i : the prefactor $g_i^2/(4\pi)$ are the fine structure constants derived from the g_i 's shown in Figure 3. The factor $1/(1 + m_i^2/k^2)$ constitutes the dimensionless exchange propagator of the gauge field.

In Figure 13, we depict the strong, weak, and hypercharge exchange couplings from fermion-gauge vertices. For scales below the threshold of the propagating field, the exchange coupling decreases reflecting the suppression of the interaction in the diagrams.

D Eigenperturbations and the global UV Higgs potential

The scaling equations (66) are of the Liouville-Sturm type for general Θ_i , and the solutions φ_i are Kummer's function of the first kind. Here, we recall the solution (67) including the Higgs anomalous dimension, which is vanishing due to the de-Donder gauge (B.17) with $\alpha = 0$ and $\beta = 1$. The solution reads

$$\varphi_i(\bar{\rho}) = c_i M\left(-\frac{4 + \Theta_i}{2 + \eta_\Phi}, 2, 32\pi^2 \left(1 + \frac{\eta_\Phi}{2}\right) \bar{\rho}\right), \quad (\text{D.1})$$

with a free normalisation c_i . For $\Theta = -4 + (2 + \eta_\Phi)n$ with $n \in \mathbb{N}$, the Kummer function M reduces to a polynomial with the $\bar{\rho} \rightarrow \infty$ limit

$$\varphi_i \rightarrow \frac{1}{\Gamma\left(4 + \frac{\Theta_i}{2}\right)} (32\pi^2 \bar{\rho})^{\frac{4 + \Theta_i}{2 + \eta_\Phi}}. \quad (\text{D.2})$$

In turn, for all other Θ 's the solutions grow exponentially for $\bar{\rho} \rightarrow \infty$,

$$\varphi_i \rightarrow \frac{1}{\Gamma\left(-2 - \frac{\Theta_i}{2}\right)} \frac{1}{(32\pi^2 \bar{\rho})^{4 - \frac{\Theta_i}{2}}} e^{32\pi^2(1 + \frac{\eta_\Phi}{2})\bar{\rho}}, \quad (\text{D.3})$$

for

$$\Theta_i \neq -4 + (2 + \eta_\Phi)n, \quad n \in \mathbb{N}. \quad (\text{D.4})$$

Hence, the eigenfunctions and the potential are best expanded in Hermite polynomials $H_n(\bar{\phi})$,

$$H_n(\bar{\phi}) = (-1)^n e^{\bar{\phi}^2} \partial_{\bar{\phi}}^n e^{-\bar{\phi}^2}, \quad \bar{\phi}^2 = 32\pi^2 a \bar{\rho}, \quad (\text{D.5})$$

with $a < 1$. This basis is square integrable and orthogonal with

$$\int_{\mathbb{R}} d\bar{\phi} H_m(\bar{\phi}) H_n(\bar{\phi}) e^{-\bar{\phi}^2} = 2^n n! \sqrt{\pi} \delta_{nm}, \quad (\text{D.6})$$

and hence we can expand the φ_i in the Hermite polynomials. We arrive at

$$\varphi_i(\bar{\rho}) = \sum_{n=0}^{\infty} c_n^{(i)} H_n(\bar{\phi}), \quad (\text{D.7})$$

with

$$c_n^{(i)} = \frac{1}{2^n n! \sqrt{\pi}} \int_{\mathbb{R}} d\bar{\phi} \varphi_i(\bar{\rho}) H_n(\bar{\phi}). \quad (\text{D.8})$$

This concludes our analysis of the Higgs fixed-point potential and the eigenperturbations.

E UV Higgs potentials at different gravity parameters

In [Section 5](#), we have discussed the rich landscape of Higgs fixed-point potentials. Here, we study the potentials at the gravity parameters (A) with

$$g_h^* = 0.219, \quad \mu_h^* = -0.730, \quad (\text{E.1})$$

and (B) with

$$g_h^* = 0.190, \quad \mu_h^* = -0.695, \quad (\text{E.2})$$

marked in [Figure 9](#) which characterise each of the regimes found.

The potential at point (A) belongs to the red regime and is shown in [Figure 14](#). The scalar mass parameter and all scalar couplings $\lambda_{\Phi, 2n}$ are negative. In particular, for the lowest-order expansion, we find

$$\mu_\Phi^* = -0.230, \quad \lambda_{\Phi, 4}^* = -2.22. \quad (\text{E.3})$$

The critical exponents show a reasonable convergence pattern and in particular, the six lowest values are well converged. Remarkably, the higher-order critical exponents always form complex conjugate pairs. All critical exponents are strongly shifted towards irrelevance compared to their canonical value, for example, $\theta_{17} = \mathcal{O}(100)$ while the canonical value is 30. Moreover, a single relevant direction is found. Recall from the discussion in [Section 5.2](#) that the transition from

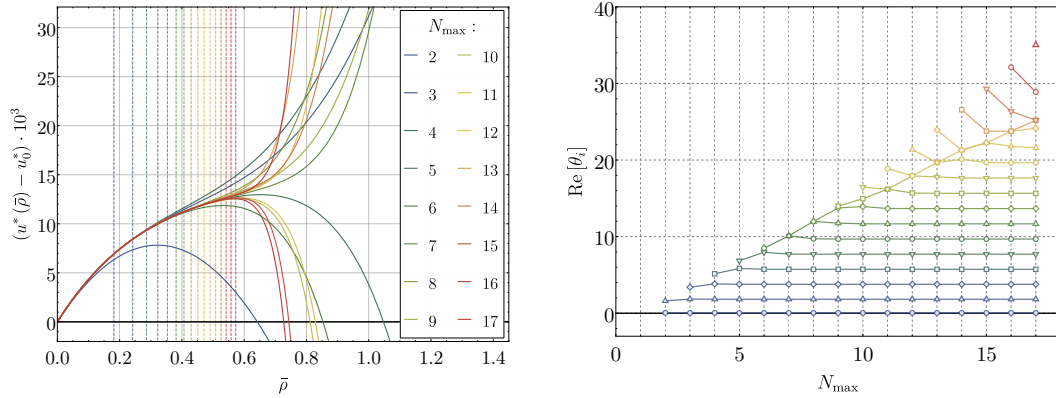


Figure 15: UV potential (left) and critical exponents (right) at different orders of the Taylor expansion for the gravitational fixed-point (B) in Figure 9 with $g_h = 0.190$ and $\mu_h = -0.695$. The vertical dashed lines in the left panel show the reliability regimes at each order of the Taylor expansion with a relative error of 1% as defined in (55). The minimum critical exponent is $\min[\theta_i^{(B)}] = 0.043$ and therefore, the fixed point (B) has no relevant direction and shows a maximal predictivity. As discussed in Section 5.1, the IR trajectory of this FP does not lead to a physical IR.

green to red regimes is given by the disappearance of a relevant direction. To further investigate this regime and in particular the boundary to the green region, it would be interesting to use a non-trivial background for the scalar field.

Point (B) falls in a regime where two fixed-point potentials exist. Additionally to the non-trivial flat potential discussed in Figure 5, a stable potential is found, shown in Figure 15. The fixed-point values for the lowest order expansion parameters read

$$\mu_{\Phi}^* = 0.0487, \quad \lambda_{\Phi,4}^* = -0.0758. \quad (\text{E.4})$$

Note their similar absolute magnitude. This property is preserved for higher orders and shows the peculiarity of this solution. This potential is stable within the reliability regime of the Taylor expansion but no statements can be made beyond this regime.

The critical exponents display a very quick convergence even at low orders of the Taylor expansion, see the right panel of Figure 15. Compared to the critical exponents of point (A), the critical exponents remain close to their canonical values. Interestingly, this potential has no relevant directions as the minimal critical exponent reads

$$\min[\theta_i^{(B)}] = 0.043. \quad (\text{E.5})$$

Therefore this fixed point shows maximal predictivity as all SM parameters related to the Higgs potential are predicted by the fixed point, namely, the ratio between the top and Higgs mass as well as the ratio between the Planck and EW scale. However, the trajectory emanating from this fixed-point potential does not lead to a physical IR limit in the present truncation. Particularly, this trajectory shows a negative quartic coupling for all scales below the Planck scale and does not generate a k_{SSB} scale.

F Systematics

In this section, we discuss the systematics of the present approximations as well as consistency checks and partial systematic error estimates. Specifically, we comment on the general structure

of the flows in the combined vertex and derivative expansion, and the systematic extension of the current approximation in [Appendix F.1](#). In [Appendix F.2](#), we discuss specifics of the gravity-matter intersection couplings and the embedding of this part in other works as well as the algorithmic effort, mainly due to this sector and pure gravity. In [Appendix F.3](#) we discuss the systematic error introduced by adjusting the present MOM² strong coupling to $\overline{\text{MS}}$ values. In [Appendix F.4](#), we discuss the effects of a non-trivial background on the Higgs- and Goldstone-type of Yukawa couplings.

F.1 Anomalous dimensions

In this expansion scheme the flows projected on the dimensionless couplings $\vec{\lambda} = (\mu, g_3, \lambda_3, \dots)$ of the gravity-SM system (including the mass parameters) read

$$\left(\partial_t + d_{\lambda_i} + \frac{1}{2} \sum_{n=1}^{n_i} \eta_{\phi_{i_n}}(0) \right) \lambda_i = F_{\lambda_i}^{(0)} + \sum_n \eta_{\phi_n}(k) F_{\lambda_i, n}^{(1)}, \quad (\text{F.1})$$

where d_{λ_i} are the canonical dimensions of the couplings λ_i and we have split the right-hand side into the contributions at vanishing anomalous dimensions $F_{\lambda_i}^{(0)}$ and proportional to the anomalous dimensions $F_{\lambda_i}^{(1)}$. The anomalous dimension on the left-hand side is evaluated at $p = 0$ and arises from the scale derivative of the wave-function renormalisation. On the right-hand side, the anomalous dimensions are evaluated at the cutoff momenta k and come from the regulator insertion in the diagrams. The latter is an approximation as the anomalous dimensions are integrated over the loop momentum q . Since the integrands of the flow peak at about $q^2 \approx k^2$, it is a well-working approximation to approximate $\eta_{\phi_i}(q^2) \rightarrow \eta_{\phi_i}(k^2)$.

The equation for the momentum-dependent anomalous dimensions has a similar structure,

$$\eta_{\phi_i}(p) = F_{\eta_{\phi_i}}^{(0)}(p) + \sum_n \eta_{\phi_n}(k) F_{\eta_{\phi_i}, n}^{(1)}(p). \quad (\text{F.2})$$

Due to the loop momentum approximation $\eta_{\phi_i}(q^2) \approx \eta_{\phi_i}(k^2)$ we only obtain a closed set of equations for $p = k$. Its solution can be obtained by inverting the resulting matrix from coupled system of anomalous dimensions. Given the large field content of the ASSM, the complete system of coupled equations cannot be solved exactly and we resort to an iterative process. Specifically, we replace $\eta_{\phi_n}(k) \rightarrow \eta_{\phi_n}(0)$ on the right-hand of [\(F.1\)](#) and, at lowest order, use [\(F.2\)](#) evaluated at $p = 0$ with $\eta_{\phi_n}(k) = 0$. At the next order, we do not set $\eta_{\phi_n}(k) = 0$ in [\(F.2\)](#) but instead replace $\eta_{\phi_n}(k) \rightarrow \eta_{\phi_n}(0)$ and use again [\(F.2\)](#) evaluated at $p = 0$ with $\eta_{\phi_n}(k) = 0$ for it. This defines the iterative process that we use. We used the first-order approximation of this iteration and found a good agreement with the zeroth-order approximation. This highlights the subleading role of the higher-order anomalous dimensions in the sub-Planckian regime of the ASSM, and in consequence we use the zeroth-order approximation for the results presented here.

F.2 Gravity-matter couplings

In our implementation of the interplay of gravity and matter, we distinguish the effects of matter on gravity from the gravity on matter. For the former we rely on particular studies of scalar-fermion- [\[16, 76\]](#) and gauge-gravity [\[23\]](#) systems to compute the effect of the SM matter content on the Newton coupling and the cosmological constant. The results there presented are easily extendable to arbitrary matter contents and provide a consistent RG-scheme with inclusion of mass thresholds.

For the effects of gravity on matter, we employed the same truncation as the cited literature and proceeded to compute the full entangled SM-gravity system. The number of n -point

functions relevant to the theory significantly increases and needs for automatization. For example, the SM alone presents ~ 130 n -point functions while coupled to gravity ~ 270 . For the derivation of the n -point functions we relied on the Mathematica package VERTEXPAND [135].

There are some subtle points in the inclusion of gravity along the EW sector and non-trivial backgrounds of the Higgs field that require further explanation. The gauge-fixing and ghost actions do not produce graviton interactions due to the background metric used in these terms. It is important to note that as this does not affect standard gauge-fixings (as the used in the strong sector), it does in the special R_ξ -gauge. Since the full metric is implicit in the contraction of the covariant derivatives applied on the scalar kinetic term, mixings between Goldstone, gauge and graviton fields do not cancel with the corresponding term generated from an R_ξ -gauge-fixing action containing the full metric. This naturally creates additional diagrams contributing to the self-energy of gauge and Goldstone bosons.

In general, these diagrams do not pose a severe problem as they are proportional to the background of the Higgs field. Moreover, in the EW broken phase, the gravitational coupling is negligible, and at Planckian scales, the flow of the curvature mass indicates that $\mu_\phi > 0$, e.g. a vanishing trivial minimum.

F.3 IR-QCD

In Section 4.1, we have discussed the determination of the strong coupling in the present approach: its value in the $\overline{\text{MS}}$ -scheme at $k = M_Z$ is given by

$$\bar{\alpha}_s := \alpha_{s,k=M_Z} = \frac{(g_{3,k=M_Z}^{\overline{\text{MS}}})^2}{4\pi} \approx 0.118. \quad (\text{F.3})$$

It has been shown in [70], that the MOM^2 RG-scheme used in the fRG, leads to an enhancement of the running gauge couplings $\alpha_s(p)$ compared to the $\overline{\text{MS}}$ value. In pure Yang-Mills, the respective enhancement factor for α_s is approximately $4/3$ [67], dropping to an enhancement factor of approximately 1.2 in the 2+1 flavour case (measured in the regime between 10-40 GeV). Moreover, in these works it is also shown that $\alpha_{s,k}(0) \lesssim \alpha_{s,k=0}(p=k)$, leading to a further (subleading) enhancement.

Being short of a full analysis, which requires the full implementation of IR-QCD, we use a linear extrapolation in flavour numbers, which leads to an enhancement factor of 1.07 in the ASSM compared to the $\overline{\text{MS}}$ value at the M_Z scale

$$\alpha_{s,k=M_Z} = 1.07 \bar{\alpha}_s. \quad (\text{F.4})$$

Furthermore, we use a variation of the strong MOM^2 coupling α_s at $k = M_Z$ as an error estimate for our computation. We vary the strong coupling in the range

$$\alpha_{s,k=M_Z} \in [\bar{\alpha}_s, 1.10 \bar{\alpha}_s]. \quad (\text{F.5})$$

The main impact of this variation is the shift of the zero crossing of the quartic Higgs coupling, commonly referred to as metastability scale k_{meta} . In Figure 16, we show k_{meta} as a function of the strong coupling (F.5). For our central value (F.4), we find a metastability scale slightly above 10^{10} GeV in good agreement with perturbative computation in the $\overline{\text{MS}}$ -scheme [111]. Interestingly, for $1.15 \bar{\alpha}_s$ this scale hits the Planck scale,

$$k_{\text{meta}}(1.15 \bar{\alpha}_s) = \frac{1}{\sqrt{G_N}}, \quad (\text{F.6})$$

where G_N is the physical value of the Newton constant at $p = 0$. The metastability scale is computed in a ϕ^4 approximation of the Higgs potential, which is a good approximation

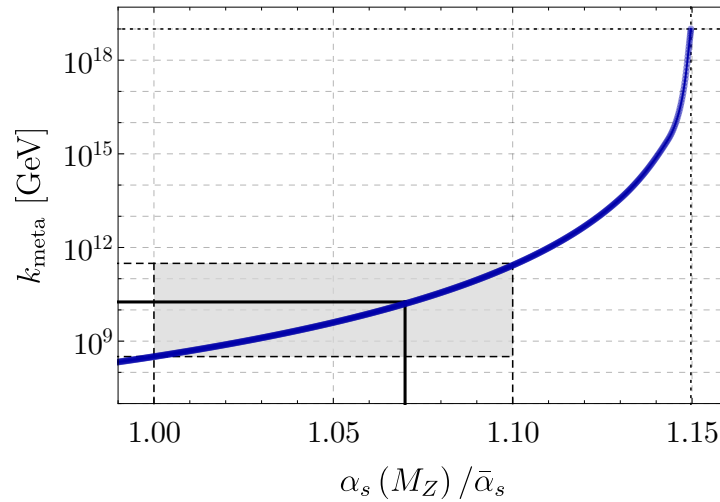


Figure 16: Metastability scale as a function of $\alpha_{s,k=M_Z}/\bar{\alpha}_s$. Here, $\bar{\alpha}_s \approx 0.118$ is the $\overline{\text{MS}}$ -value with at the EW scale $p = M_Z$ (and $k = 0$), see (F.3). For each value of $\alpha_{s,k=M_Z}$ the remaining parameters of the ASSM are set to reproduce the experimental values. Throughout this work, we use $\alpha_{s,k=M_Z} = 1.07 \bar{\alpha}_s$ marked by a plain line, see (F.4). The shaded interval, defined in (F.5), is used as a systematic error estimate.

below the Planck scale. Around the Planck scale, this approximation cannot be trusted but we nonetheless might expect the disappearance of a metastability scale for a large enough strong coupling. This concludes our systematic error analysis of the strong coupling.

F.4 Yukawa couplings

The Yukawa couplings in the fermion-scalar interactions are subject to a scalar-field dependence $\tilde{y}(\rho)$. Such is key to reproducing multi-scalar production from fermion scattering, see for example [45, 136]. Although this dependency is not explicitly considered in the Yukawa couplings defined in (B.10) the flow equation is known to generate such non-trivial contributions. As the flow is evaluated on the equations of motion, the scalar field dependency is only generated in the modes with a non-trivial background. To quantify such difference, we take advantage of the previous point and define two Yukawa couplings, the coupling to Higgs and Goldstones

$$\begin{aligned}\partial_t y_t^{(H)}(\bar{\rho}) &= \text{Tr}[P_{Ht\bar{t}}(p)\text{Flow}_{Ht\bar{t}}^{(3)}] + \frac{1}{2}(\eta_H + \eta_{t_L} + \eta_{t_R})y_t^{(H)}(\bar{\rho}), \\ \partial_t y_t^{(G)}(\bar{\rho}) &= \text{Tr}[P_{G^0t\bar{t}}(p)\text{Flow}_{G^0t\bar{t}}^{(3)}] + \frac{1}{2}(\eta_{G^0} + \eta_{t_L} + \eta_{t_R})y_t^{(G)}(\bar{\rho}).\end{aligned}\quad (\text{F.7})$$

While the former contains the complete dependency in the broken phase, by subtracting the latter we find

$$\partial_t y_t^{(H)}(\bar{\rho}) - \partial_t y_t^{(G)}(\bar{\rho}) = \bar{\rho} \partial_t|_{\bar{\rho}} \left(\partial_{\bar{\rho}} y_t^{(G)}(\bar{\rho}) \right). \quad (\text{F.8})$$

In fact, one single Yukawa coupling enters the theory but distinguishing as in (F.7) and computing two different flows allows us to obtain the differences caused by a non-trivial background. All flows in (F.7) and (F.8) are shown in Figure 17. The flow of the Higgs Yukawa coupling shows a maximum at $k \sim 100$ GeV, which is not present in the Goldstone coupling. In the symmetric phase $k > k_{\text{SSB}}$, both couplings agree due to equal vanishing background. This shows that there is a small qualitative difference between the two couplings. In the SM flows

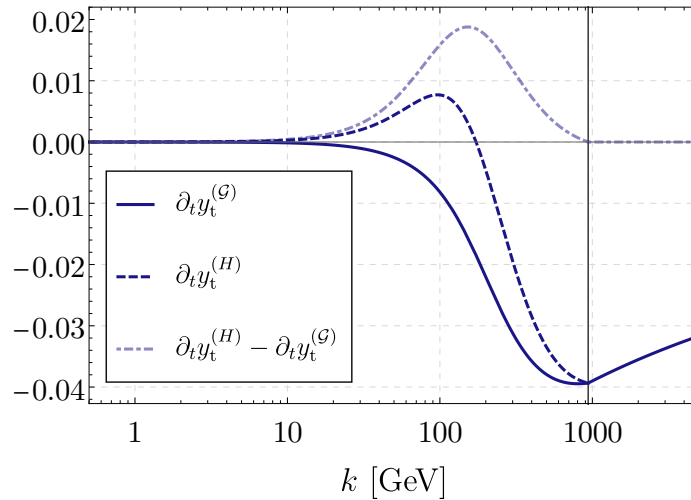


Figure 17: Flow of the top-quark Godstone (plain) and Higgs (dashed) Yukawa couplings defined in (E.7) and their difference (dot-dashed) (E.8). The difference is given by the scalar-field dependence of the Yukawa coupling. In the symmetric phase both couplings agree due to equal vanishing background.

in Figures 3 and 4, we did not include both couplings separately but assumed them to be equal. The Goldstone Yukawa couplings were chosen as their appearance in fermion-scalar vertices is predominant.

G Pole masses

For $k \rightarrow 0$, the relevant parameters of the ASSM can be determined by respective particle physics measurements and the classical (low energy) Newton coupling and the cosmological constant. The matter and gauge couplings can be determined by selected cross sections with a scattering momentum of the order $\lesssim 10^2$ GeV, typically chosen close to the EW scale. Most of these couplings and masses are not a very sensitive input, with the exception of the pole mass of the top quark: it has a relatively large experimental error, and its value (as the largest mass parameter) has a considerable influence on stability estimates of the Higgs potential.

We argue now, that its determination can be obtained with a very small systematic error from the current approximation: to begin with, the missing momentum dependence of the matter couplings and propagators discussed in Section 2.2 has a subleading impact on the flow of couplings, wave-function renormalisation and masses at vanishing momentum. This has been checked thoroughly in IR-QCD ($p \lesssim 10$ GeV), where the genuine momentum dependence is far stronger than here due to the significantly larger coupling and the resonant hadronic correlations, see in particular [68], utilising also the momentum dependent QCD correlation functions from [61].

However, while the feedback of the momentum dependences is subleading, they are required for a determination of the coupling strengths via scattering processes and the determination of the pole masses. Specifically, for the present task of computing the top-pole mass $M_{t,\text{pole}}$, the momentum-dependent mass function of the top quark is required. We parametrise the two-point function as

$$\Gamma_{t\bar{t}}^{(2)}(p) = Z_{t,k}(p) [i\not{p} + M_{t,k}(p)], \quad (\text{G.1})$$

with the cutoff- and momentum-dependent wave function $Z_{t,k}(p)$ and mass function $M_{t,k}(p)$. For momentum-independent dressing functions this reduces to the approximation used in the present work,

$$Z_{t,k} = Z_{t,k}(p=0), \quad m_{t,k} = M_{t,k}(p=0). \quad (\text{G.2})$$

At $k=0$, (G.1) leads us to the propagator

$$G_t(p) = \frac{1}{Z_t(p)} \frac{1}{p^2 + M_t(p)^2} [-i\not{p} + M_t(p)], \quad (\text{G.3})$$

with $Z_t(p) = Z_{t,k=0}(p)$ and $M_t(p) = M_{t,k=0}(p)$. The pole mass $M_t^{(\text{pole})}$ is extracted at $k=0$ from

$$[p^2 + M_t(p)^2]_{p^2 = -[M_t^{(\text{pole})}]^2} = 0, \quad (\text{G.4})$$

or equivalently from

$$[Z_t(p)p^2 + Z_t(p)M_t(p)^2]_{p^2 = -[M_t^{(\text{pole})}]^2} = 0, \quad (\text{G.5})$$

which follows from (G.4) by multiplying with $Z_t(p)$.

In the following, we determine the pole mass by using (G.5): The combination $Z_t(p)M_t(p)^2$ is obtained analytically from the integrated flow of the top two-point function

$$\partial_t [Z_t(p)M_t(p)^2] = \frac{1}{4} \text{tr}_D \partial_t \Gamma_{t\bar{t}}^{(2)}(p), \quad (\text{G.6})$$

with the Dirac trace tr_D . The analytic momentum dependence allows us to resolve the mass function for timelike momenta $p^2 > 0$.

The second ingredient in (G.5) is the wave function renormalisation $Z_t(p)$. Here we use that the momentum dependence of correlation functions $\mathcal{O}_{k=0}(p)$ with a mild cutoff dependence $\mathcal{O}_k(p=0)$ at vanishing momentum is typically well approximated by

$$\mathcal{O}_{k=0}(p) \approx \mathcal{O}_{k=\alpha p}(p=0). \quad (\text{G.7})$$

This has also been discussed in Section 2.2.1, and is well-tested in QCD, where quantitative correlation with the full momentum dependence as well as the cutoff dependence have been computed and compared, see in particular [59, 61, 67, 68]. In the present case of the wave function renormalisation of the top quark, its cutoff dependence is negligible in the symmetry broken phase for $k \leq k_{\text{SSB}} = 930 \text{ GeV}$, see (47). Indeed, the k -dependence of $Z_{t,k}$ is very small, as shown in Figure 18, where $Z_{t,k}$ is shown for $k \lesssim k_{\text{SSB}}$.

In summary, the above analysis suggests that a quantitative estimate is already obtained with $Z_t(p) = 1$ also for timelike momenta within the regime $|p^2| \leq (200 \text{ GeV})^2$. From the minimal variation of the cutoff dependence in Figure 18 in this regime, we estimate the respective systematic error with 0.5%, which translates into an error of roughly 0.8 GeV for the top pole mass determination. A direct computation will be presented elsewhere, together with further pole mass determinations.

Within this approximation we are led to

$$[p^2 + Z_t(p)M_t(p)^2]_{p^2 = -[M_t^{(\text{pole})}]^2} \approx 0, \quad (\text{G.8})$$

and it is left to compute the combination $Z_t(p)M_t(p)^2$ for timelike momenta $p^2 \leq 0$, in particular including the pole position. We utilise that the sub-Planckian structure of the flow facilitates

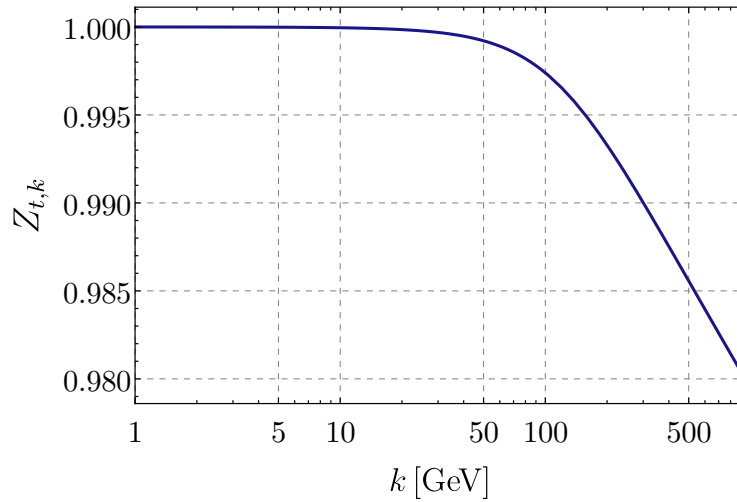


Figure 18: Wave-function renormalisation $Z_{t,k}$ of the top quark as a function of the cutoff scale k . The k -dependence in the broken phase is negligible, and is well approximated by $Z_{t,k} = 1$, leading to the estimate $Z_t(p) = 1$ in (G.8).

the Wick rotation. First, the mass function does not receive contributions for cutoff scales $k \geq k_{\text{SSB}} \approx 10^3$ GeV, see (47). For larger scales, the theory is chiral and hence $M_{t,k \geq k_{\text{SSB}}}(p) = 0$. Accordingly,

$$Z_t(p)M_t(p) = \int_{k_{\text{SSB}}}^0 \frac{dk}{k} \partial_t [Z_t(p)M_{t,k}(p)]. \quad (\text{G.9})$$

For scales $k \leq k_{\text{SSB}}$, the SM couplings quickly settle at their IR value and can be taken to be k -independent with the exception of the strong coupling, see Figure 3.

The strong coupling changes significantly between k_{SSB} and $k = 0$, see Figure 3. We now use, that the flow of the top mass quickly saturates, see Figure 19, and does receive negligible contributions below the top scale. More specifically, the QCD contributions in the flow are proportional to

$$\frac{\alpha_{s,k}}{\left(1 + \frac{M_{t,k}^2}{k^2}\right)^3}, \quad (\text{G.10})$$

and higher powers in the denominator. These terms drop rapidly for $k < M_{t,k}$ towards smaller cutoff scales despite the rising coupling, which is responsible for the saturation of the QCD part of the flow of $m_{t,k}$. Accordingly, we can estimate its contribution by using an average coupling at the saturation scale k_{sat} . We estimate this saturation scale k_{sat} very conservatively as

$$k_{\text{sat}} \in [170, 300] \text{ GeV}, \quad (\text{G.11})$$

and use a respective constant strong coupling $g_3(k_{\text{sat}})$ in the following computation of the pole mass. Varying this coupling for cutoffs in the range (G.11) provides us with a very conservative systematic error estimate. Moreover, the above argument implies that the minimal value

$$k_{\text{sat}} = 170 \text{ GeV}, \quad (\text{G.12})$$

is a good choice for the computation of the mass parameter m_t from the pole mass condition (G.8). Finally, we have to also take into account the uncertainty in the size of the coupling due

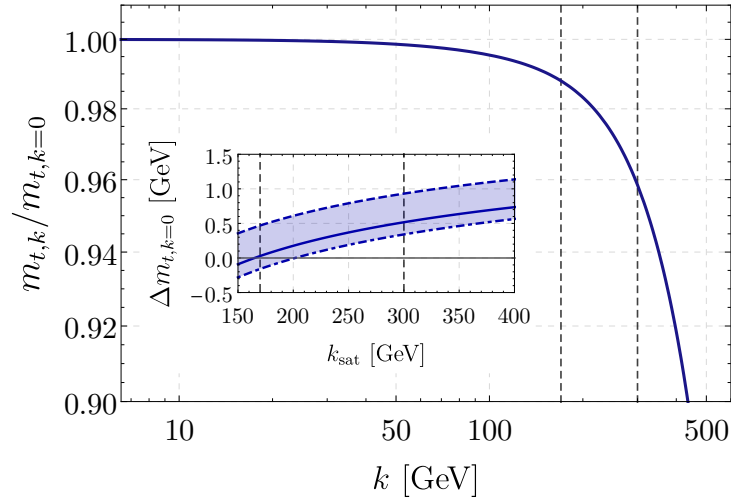


Figure 19: Top mass parameter $m_{t,k}$ measured in units of $m_t = m_{t,k=0}$ as a function of the cutoff scale. The mass parameter saturates at cutoff scales $k \approx 200$ GeV, and the vertical lines indicate the borders of the cutoff range (G.11) for k_{sat} . In the inlay, we depict the variation of m_t required for the experimental pole mass (45e) as a function of the saturation scale k_{sat} used for $g_{3,k=k_{\text{sat}}}$ in (G.21). The upper dashed line is obtained for $\alpha_s = \bar{\alpha}_s$, the lower dot-dashed line is obtained for $\alpha_s = 1.1 \bar{\alpha}_s$ (see (F.5)) and the central plain line for the ASSM flavour number extrapolation $\alpha_s = 1.07 \bar{\alpha}_s$. This covers the uncertainty in mapping the $\overline{\text{MS}}$ -value of the coupling to the MOM^2 value used in the fRG computation.

to mapping the $\overline{\text{MS}}$ -coupling to that in the MOM^2 scheme in the fRG. Hence, we additionally employ the conservative estimate (F.5) and vary the coupling between its $\overline{\text{MS}}$ -value $\bar{\alpha}_s$ and $1.1 \bar{\alpha}_s$.

In summary this leaves us with the explicit cutoff dependence of the regulator in the loops contributing to $\partial_t [Z_t(p)M_{t,k}(p)]$. Then, the flow is (explicitly) a total derivative w.r.t. to the cutoff scale,

$$\partial_t [Z_t(p)M_{t,k}(p)] = \frac{d}{dt} [Z_t M_t]_k^{\text{1loop}}(p), \quad (\text{G.13})$$

where $[Z_t M_t]_k^{\text{1loop}}(p)$ are the standard one-loop diagrams with all couplings and parameters taken at $k = 0$ and the only cutoff dependence in the loops comes from the regulators, either $R_{k=0} = 0$ or $R_{k=k_{\text{SSB}}}$ in the second term. Consequently, in this regime the flow can be integrated analytically, and leads to a difference of standard one-loop diagrams $[Z_t M_t]_k^{\text{1loop}}(p)$,

$$\Delta [Z_t M_t]^{\text{1loop}}(p) = [Z_t M_t]_{k=0}^{\text{1loop}}(p) - [Z_t M_t]_{k=k_{\text{SSB}}}^{\text{1loop}}(p). \quad (\text{G.14})$$

Now we use that for $p \ll k_{\text{SSB}}$ the second term is well approximated by its value at $p = 0$. This leads us to a simple one-loop expression with the full (fRG-resummed) couplings and vertices at $k \rightarrow 0$,

$$[Z_t M_t]^{\text{1loop}}(p \lesssim k_{\text{SSB}}) = Z_t m_t + \Delta [Z_t M_t](p), \quad (\text{G.15})$$

with the wave function $Z_t = Z_{t,k=0}(p = 0)$ and top mass parameter $m_t = M_{t,k=0}(p = 0)$ in our approximation, and

$$\Delta [Z_t M_t](p) = [Z_t M_t]^{\text{1loop}}(p) - [Z_t M_t]_t^{\text{1loop}}(0). \quad (\text{G.16})$$

The first term $Z_t m_t$ in (G.15) is a direct result from the ASSM flow. The second term, $\Delta[Z_t M_t](p)$ has the form of the one-loop gap equation for the t -quark, subtracted at $p = 0$; however with the full coupling and mass parameters of the SM at $k = 0$. It is a finite difference of loops and can be evaluated within dimensional renormalisation for computational convenience. Its momentum dependence can be read off from the scalar parts of the one-loop self-energy diagrams in (G.6). Generically, the momentum structure of the diagrams reads

$$\begin{aligned}\mathcal{I}(p, m_i) &= \mu^{2\varepsilon} \int \frac{d^d q}{(2\pi)^d} \int_0^1 dx \left[\frac{1}{[q^2 + \Delta(p)]^2} - \frac{1}{[q^2 + \Delta(0)]^2} \right] \\ &= -\frac{1}{(4\pi)^2} \int_0^1 dx \log \frac{\Delta(p)}{\Delta(0)},\end{aligned}\quad (\text{G.17})$$

where $d = 4 - 2\varepsilon$, μ is the RG scale in dimensional regularisation, and Δ is

$$\Delta(p) = m_i^2(1-x) + x m_t^2 + x(1-x)p^2. \quad (\text{G.18})$$

The right-hand side of (G.17) does not depend on μ as the integral does not require renormalisation in the first place and the use of dimensional regularisation was only for computational means.

In (G.17), $\Delta(p)$ with $m_i = 0$ occurs in the top self-energy diagrams with massless modes such as gluons and photons (at the relevant scales), and the general case with $m_i \neq 0$ is used in top self-energy diagrams with massive modes such as the intermediate vector bosons $Z^0, W^\pm$.

For general masses m_i , the self-energy diagrams are computed as

$$\begin{aligned}\mathcal{I}(p^2, m_i) &= 1 + \frac{1}{2} \left(\frac{m_t^2 + m_i^2}{m_t^2 - m_i^2} + \frac{m_t^2 - m_i^2}{p^2} \right) \log \frac{m_t^2}{m_i^2} \\ &\quad - \frac{\sqrt{m_i^4 + (m_t^2 + p^2)^2 + 2m_i^2(p^2 - m_t^2)}}{p^2} \\ &\quad \times \operatorname{arccoth} \left[\frac{m_t^2 + m_i^2 + p^2}{\sqrt{m_i^4 + (m_t^2 + p^2)^2 + 2m_i^2(p^2 - m_t^2)}} \right].\end{aligned}\quad (\text{G.19})$$

For massless modes, $m_i = 0$, (G.19) reduces to

$$\mathcal{I}(p^2) = 1 - \left(1 + \frac{m_t^2}{p^2} \right) \log \left(1 + \frac{p^2}{m_t^2} \right), \quad (\text{G.20})$$

where we have introduced $\mathcal{I}(p^2) = \mathcal{I}(p^2, 0)$. With (G.19) and (G.20), we arrive at the final expression for the momentum-dependent mass function,

$$\begin{aligned}\Delta[Z_t(p)M_t(p)] &= \frac{m_t}{16\pi^2} \left[4g_3^2 \mathcal{I}(p^2) + 3 \left(\frac{2g_2 \sin \theta_W}{3} \right)^2 \mathcal{I}(p^2) \right. \\ &\quad \left. - \frac{h_t^2}{2} \mathcal{I}(p^2, m_H) + \frac{h_t^2}{2} \mathcal{I}(p^2, m_Z) + h_b^2 \mathcal{I}(p^2, m_W) \right. \\ &\quad \left. - \frac{g_Y^2 (1 + 2 \cos 2\theta_W)}{9} \mathcal{I}(p^2, m_Z) \right],\end{aligned}\quad (\text{G.21})$$

computed in the gauge (B.12). The first two terms on the right-hand side stem from gluon and photon mediated diagrams, and the terms in the second line stem from Higgs, $\mathcal{G}^0, \mathcal{G}^\pm$,

and Z^0 . Vertices, wave functions and the mass function are given by the full couplings and wave functions at $k = 0$ and $p = 0$ except for the strong coupling $g_3(k_{\text{sat}})$ which is evaluated at $k_{\text{sat}} = 170 \text{ GeV}$, see (G.12).

Equation (G.15) with (G.21) allows us to resolve (G.8) for the pole position. This fixes the Euclidean mass parameter to

$$m_t = 165.4^{+0.9}_{-0.2} \text{ GeV}, \quad (\text{G.22})$$

for the PDG pole mass (cross section measurements) $M_t = 172.5 \text{ GeV}$, see (45e). The error in (G.22) is solely due to the error estimate of the value of the strong coupling used in (G.21). It is discussed below (G.9), and is summarised in the inlay of Figure 19: The upper variation comes from the variation of the saturation scale up to $k_{\text{sat}} = 300 \text{ GeV}$ with $\alpha_s = \bar{\alpha}_s(300 \text{ GeV})$, while the lower error is related to the estimate of the maximal MOM²-value of the strong fine structure constant in (F.5) with $\alpha_s = 1.1 \bar{\alpha}_s(170 \text{ GeV})$.

We consider the (systematic) error estimate in (G.22) a very conservative one. Note also that the theoretical systematic error is of the size of the experimental one.

A first prediction concerns the decay width Γ_t of the Higgs. We obtain

$$\Gamma_{t,\text{pole}}^{(\text{theo})} = 1.72^{+0.09}_{-0.41} \text{ GeV}, \quad (\text{G.23})$$

which agrees quantitatively with that in the PDG one, $\Gamma_{t,\text{pole}}^{(\text{exp})} = 1.42^{+0.19}_{-0.15} \text{ GeV}$ in (45f). Note, that while the error in (G.22) constitutes a systematic error estimate, the error in (G.23) only describes the relative weighting of QCD and non-QCD contributions. In a pure QCD system, our analysis would lead to a vanishing error on the decay width in (G.23).

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FLAVOUR HIERARCHIES FROM EMERGENT FUNDAMENTAL PARTIAL COMPOSITENESS

This Appendix contains [2], one of the core papers of this thesis, contextualized in Section 4.2.

Flavor hierarchies from emergent fundamental partial compositeness

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Composite Higgs extensions of the Standard Model provide an explanation for the large hierarchies between the Yukawa couplings. We study their realization in the context of fundamental partial compositeness where the Standard Model fermions mix linearly with bound states of the new sector, consisting of a fermion and a scalar. The properties of this composite are unraveled with the functional renormalization group approach using dynamically emergent composites. Specifically, we extract the scaling of correlation functions along a walking regime and provide indicative estimates for the minimal incarnation of the theory.

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I. INTRODUCTION

Understanding the hierarchical structure of fermion masses and mixings is one of the major open questions in fundamental physics. The concept of partial compositeness (PC) for fermions, first proposed in [1] and put forward in [2–5] in extradimensional duals of composite Higgs (CH) [6–8] models, offers a promising means to address this question. In this approach, the fermion mass terms of the Standard Model (SM) are generated from linear mixings of SM-like fermions of each flavor with composite fermionic operators $\mathcal{O}_B$, containing fundamental fields of a new sector that are bound together by a novel confining interaction. Below the condensation scale Λ_c , these terms lead to the light fermion mass eigenstates being a superposition of elementary SM-like fermions and composite resonances excited by the operators $\mathcal{O}_B$ in the infrared (which explains the denotation “partial compositeness”). The latter resonances provide the connection to the composite Higgs and thus to electroweak symmetry breaking (EWSB).

Small differences in the scaling dimensions of the composite operators translate to exponentially large differences in the strengths of the linear mixings at low energies due to the renormalization-group evolution from a large UV flavor scale Λ_{UV} , where the couplings of the SM

fermions with the strongly coupled sector are generated, down to the condensation scale Λ_c . Assuming a walking regime with an almost conformal behavior between those scales, the linear-mixing coefficient will scale as $\sim(\Lambda_c/\Lambda_{UV})^{d-5/2}$, with the dimension $d = [\mathcal{O}_B]$ of the composite operator, see, e.g., [9,10] for similar realizations. After integrating out the heavy states, these hierarchically different couplings to the bound states of the new strong sector lead to hierarchical mass eigenvalues of quarks and leptons.

While the initial focus in the literature was on effective low-energy descriptions of PC (see [9,11,12] for reviews), more recently UV-complete realizations have been explored [13–23]. These consider the fundamental degrees of freedom and the dynamics that lead to the bound states that mix linearly with the elementary SM-like fermions. Here, an obvious approach is to assume the composite fermions being composites of three fundamental fermionic degrees of freedom [13–15,17]. Such constructions however face severe challenges since the scaling dimension of the composite operators needs to deviate very significantly from the canonical value of $[\mathcal{O}_B]_{0,c} = 3[\mathcal{F}]_c = 9/2$. The latter would lead to fermion masses that are too small. The linear mixings are suppressed by $[\mathcal{O}_B]_{0,c} + 3/2 - 4 = 2$ powers of the flavor scale Λ_{UV} where they emerge as higher dimensional operators involving fundamental fermions (flavor bounds indicate $\Lambda_{UV} \gg \Lambda_c$). This issue is particularly severe in the case of the large top-quark mass.

In fact, lattice results suggest that large deviations from the canonical scaling dimension (i.e., large anomalous dimensions) are not possible for the straightforward realization with three-fermion bound states [24–27]. Thus, it is

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not obvious whether this completion improves the situation compared to the technicolor (TC) way of generating fermion masses via bilinear couplings to the EWSB condensate—where in general the masses are also suppressed too much (unless the scaling dimension of the Higgs-like bound state becomes as small as to reintroduce UV instabilities), see, e.g., [9]. In consequence, finding alternative mechanisms of realizing PC becomes a priority.

One of these alternatives is based on the assumption that the fermionic composites are not formed by three elementary fermions but by an elementary fermion $\mathcal{F}$ and a scalar $\mathcal{S}$ [18–20], see also [28–31]. These scenarios have been dubbed “fundamental partial compositeness” (FPC), and rely on renormalizable Yukawa-like couplings of the SM-like fermions to $\mathcal{S}$ and $\mathcal{F}$. The latter two form a bound state $\mathcal{B} \sim \mathcal{S}\mathcal{F}$ that mixes with the SM fermions in the infrared (IR). Here, $\mathcal{B}$ refers to the lightest resonance in a tower of states excited by $\mathcal{O}_B$ [21].

The inclusion of scalars and hence the resulting canonical dimension of $[\mathcal{O}_B]_{0,c} = [\mathcal{F}]_c + [\mathcal{S}]_c = 5/2$ carry the crucial advantage that the linear mixings are not necessarily suppressed. Thus, the hope is to obtain the heavy top quark mass, while the light fermion masses are still set via natural input parameters. The presence of elementary scalars may reintroduce a hierarchy problem, but the approach could be seen as an intermediate step to a full UV theory. In fact, an interesting construction circumventing naturalness problems could be two-step models of condensation where the scalars $\mathcal{S}$ would emerge from fundamental fermions in the UV.

In the original works on FPC, the question of a dynamical generation of the (hierarchically) light fermion masses was left open [18–20,32]. In principle, one can just generate such hierarchical fundamental Yukawa couplings around the condensation scale or from hierarchies in the fundamental scalar masses [21]; however, this would not address the flavor puzzle. Given the promising starting point discussed above, it is worthwhile to investigate whether a natural dynamical emergence of all fermion masses could be realized using FPC. To this end, in this work we explore the possible range of anomalous dimensions of the $\mathcal{O}_B$ operators, containing a scalar and a fermion, to see if an envisaged evolution over a sizeable almost-conformal regime could generate the sought flavor hierarchies.

Studying the dynamics of strongly coupled theories and the emergent formation of composites is a nontrivial task requiring nonperturbative methods. Usually, this is addressed with Monte Carlo simulations on the lattice. In the present work we use the functional renormalization group (fRG) [33–35] to study the anomalous scaling of the linear mixing coupling between SM fermions and composites. Functional methods based on the effective action allow for a systematic treatment of nonperturbative effects while being versatile enough to scan over the parameter and

model sets. This flexibility suits the needs for investigations of strong and nonperturbative new physics. Specifically, the fRG allows for the systematic inclusion of emergent composites, see [36–39]. By now this technique is widely used for very different phenomena with emergent composites, ranging from condensed matter systems to quantum chromodynamics (QCD), for recent reviews and developments see [39–41]. In contrast to effective field theories, the flows with emergent composites allow us to continuously interpolate between the physics described by different degrees of freedom. A prominent example is QCD, where flows with emergent composites take care of the dynamical formation of hadrons (dynamical hadronization) at low momentum scales or temperatures.

This work is organized as follows. In Sec. II, we introduce the details of PC and in particular, the model of FPC sought to generate the SM fermion masses. In Sec. II A, we discuss the relevant terms of the effective action of the fundamental theory, and in Sec. II B we present in detail the emergent composite approach for the composites of interest. In Sec. II C, we discuss the dimensionality of the composite and in Sec. II D derive the momentum scaling of the linear coupling of interest from the effective action. In Sec. II E, we discuss the role of the walking regime and estimate the properties of the bound states from the fundamental effective action. Subsequently, in Sec. III we provide a brief introduction to the fRG. The latter is used to compute the anomalous scaling of the relevant two-point functions, which allows us to derive an estimate for a particular minimal realization of FPC. Finally, in Sec. IV we conclude. Many of the technical derivations and discussions are deferred to the Appendix.

II. FPC AND THE EFFECTIVE ACTION

Fundamental CH models are usually constructed as extensions of the SM by introducing a new confining gauge group G_{TC} and TC-charged chiral fermions $\mathcal{F}^{a,a}$. As the TC-gauge coupling becomes strong towards the IR, it will give rise to the formation of composites. Given the TC fermions transform nontrivially under a global “flavor” symmetry $\mathcal{G}$, the fermionic condensate

$$\langle \mathcal{F}^{a,a} \epsilon_{\text{TC}} \mathcal{F}^{a,b} \rangle = \Lambda_c f^2 \Sigma_{\theta}^{ab}, \quad (1)$$

spontaneously breaks the global symmetry $\mathcal{G} \rightarrow \mathcal{H}$, realizing the Higgs as a pseudo-Nambu-Goldstone boson (pNGB) of the coset $\mathcal{G}/\mathcal{H}$. In (1), α corresponds to the TC-gauge group index while a and b are flavor indices of the group $\mathcal{G}$. The latter contains the SM group G_{SM} to ensure a viable phenomenology. Moreover, $\Lambda_c \sim 4\pi f$ denotes the TC condensation scale with f being the pNGB decay constant, ϵ_{TC} is the antisymmetric tensor of G_{TC} . Finally, Σ_{θ} is a matrix that parametrizes the alignment of the unbroken symmetry in the physical

vacuum, in particular with respect to the gauged subgroup of $\mathcal{G}$ (usually G_{SM}). A misalignment then leads to EWSB.

After the global symmetry $\mathcal{G}$ is explicitly broken, the pNGBs obtain a vacuum expectation value that shifts the unbroken symmetry $\mathcal{H}$ by a finite amount such that it does no longer include G_{SM} . The pNGBs $\Pi_{\hat{a}}$ of $\mathcal{G}/\mathcal{H}$ (including the Higgs boson) are parametrized as fluctuations around the vacuum via the Goldstone matrix

$$\Sigma(x) = \exp \left[i \frac{2\sqrt{2}}{f} \Pi_{\hat{a}}(x) T_{\theta}^{\hat{a}} \right] \Sigma_{\theta}, \quad (2)$$

where $T_{\theta}^{\hat{a}}$ are the broken generators of $\mathcal{G}/\mathcal{H}$. A common minimal choice in CH model building is $G_{\text{TC}} = \text{Sp}(N_{\text{TC}})$ with four Weyl fermions $\mathcal{F}^{a,a}$, $a = 1, \dots, 4$, per technicolor [19,42], resulting in the global symmetry-breaking pattern $\text{SU}(4)_{\mathcal{F}} \rightarrow \text{Sp}(4)_{\mathcal{F}}$ after fermion condensation. This leads to $\dim[\text{SU}(4)_{\mathcal{F}}/\text{Sp}(4)_{\mathcal{F}}] = 5$ pNGBs, with three of them, $\Pi_{1,2,3}$ being the electroweak Goldstone modes, Π_4 the Higgs boson, and Π_5 an additional electroweak singlet. In this case, the vacuum matrix reads

$$\Sigma_{\theta} = \cos \theta \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} + \sin \theta \begin{pmatrix} 0 & \mathbf{1}_2 \\ -\mathbf{1}_2 & 0 \end{pmatrix}, \quad (3)$$

with the electroweak vacuum expectation value $v = \sin \theta f$, such that $\sin \theta = 0$ ($\sin \theta = 1$) corresponds to unbroken (fully broken) electroweak symmetry. We note that for the more minimal coset of $\mathcal{G}/\mathcal{H} = \text{SO}(5)/\text{SO}(4)$, also preserving custodial symmetry, no fundamental four dimensional UV completion with elementary fermions exists [21].

The idea of PC provides a mechanism to reproduce the large hierarchies in the SM Yukawa couplings by introducing a linear mixing term between SM fermions and bound states of the composite sector. Focusing on the third-generation up-type quarks, the PC Lagrangian reads

$$\mathcal{L}_{\text{mix}} = \frac{\bar{\lambda}_q}{\Lambda_{\text{UV}}^{[\mathcal{O}_B^{q,t}]-5/2}} \bar{q}_L \mathcal{O}_B^q + \frac{\bar{\lambda}_t}{\Lambda_{\text{UV}}^{[\mathcal{O}_B^{q,t}]-5/2}} \bar{t}_R \mathcal{O}_B^t + \text{H.c.} \quad (4)$$

Here, q_L and t_R are the embeddings of the SM-like fields into irreducible representations of the global symmetry $\mathcal{G}$ of the composite sector. The couplings $\bar{\lambda}_{q,t}$ are dimensionless $\mathcal{O}(1)$ parameters at the Λ_{UV} scale and $[\mathcal{O}_B^{q,t}]$ are the dimensions of the composite-sector operators. As the TC-gauge coupling becomes strong, the composite operators can excite fermionic resonances

$$\langle 0 | \mathcal{O}_B^{q,t} | \mathcal{B}^{q,t} \rangle \neq 0, \quad (5)$$

which lead to the light fermion mass eigenstates becoming superpositions of the elementary fermions q , t and the resonances.

At an IR scale Λ_c , the linear mixing couplings will read as dictated by their renormalization group (RG) scaling

$$\bar{\lambda}_q(\Lambda_c) \simeq \bar{\lambda}_q(\Lambda_{\text{UV}})(\Lambda_c/\Lambda_{\text{UV}})[\mathcal{O}_B^q] - 5/2. \quad (6)$$

The large hierarchies between the SM Yukawa couplings are translated to the parameters $\bar{\lambda}_q$ and are naturally explained by their RG evolution over many orders of magnitude. While the linear mixing couplings are expected to be $\mathcal{O}(1)$ at Λ_{UV} , at the IR scale Λ_c they will split up such as to correctly reproduce the SM fermion masses. In PC, both scales are largely separated to avoid an unachievable large scaling of the coupling. This is accomplished when the gauge coupling enters a quasi-fixed-point regime and freezes the TC-gluon dynamics. This is commonly referred to as a walking regime.

In summary, these two key ingredients together, an adequate scaling of the linear mixing coupling, and the presence of the walking regime determine whether the SM Yukawa couplings can be reproduced. In the following, we will detail this mechanism for the case of FPC and derive in detail the scaling of the linear mixing operators from the effective action.

A. Effective action

To realize FPC, fundamental scalars $\mathcal{S}^{a,i}$ charged under the TC-gauge group are introduced in the fundamental CH action [18–21,43]. In the present renormalization-group approach based on the full effective action Γ , the strongly coupled TC part of Γ is parametrized as

$$\begin{aligned} \Gamma_{\text{CH}} = \int_x \bigg\{ & \frac{Z_A}{4} G_{\mu\nu} G_{\mu\nu} + \mathcal{L}_{\text{gf+ghosts}} \\ & + \frac{Z_S}{2} [(D_{\mu} \mathcal{S}^i)^{\dagger} (D_{\mu} \mathcal{S}^i) + \mathcal{S}^{i\dagger} m_S^2 \mathcal{S}^i] \\ & + Z_{\mathcal{F}} \bar{\mathcal{F}}^a (\sigma_{\mu} D_{\mu} + m_{\mathcal{F}}) \mathcal{F}^a \\ & + \sqrt{Z_{\Psi} Z_{\mathcal{F}} Z_S} \psi_{\text{TC}}^{i,a} \epsilon_{ij} \Phi^j \epsilon_{\text{TC}} \mathcal{F}^a + \text{H.c.} + \dots \bigg\}, \end{aligned} \quad (7)$$

where the dots $\dots$ in the last line stand for higher-order interaction terms in the fields. The TC-gauge field strength tensor reads

$$G_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - ig_{\text{TC}} [A_{\mu}, A_{\nu}], \quad (8)$$

with the covariant derivative

$$D_{\mu} = \partial_{\mu} - ig_{\text{TC}} A_{\mu}. \quad (9)$$

In (7) and from here on, we keep the color indices and generators of the TC-gauge group implicit.

In the following, we concentrate on the most minimal incarnation of $SU(4)_{\mathcal{F}}/Sp(4)_{\mathcal{F}}$ FPC. The TC fermions are accordingly assumed to form a weak (chiral) doublet $\mathcal{F}^{1,2}$ with vanishing hypercharge as well as two $SU(2)_L$ singlets $\mathcal{F}^{3,4}$ with hypercharges $Y = \mp 1/2$ (per technicolor). Beyond that, the addition of 12 complex scalar degrees of freedom, residing in three generations of color triplets S_q with hypercharge $Y = -1/6$, and corresponding color singlets S_l with $Y = 1/2$, allows for appropriate composite operators for all families of quarks and leptons. Both, TC-fermion and -scalar fields, transform under the fundamental representation of the TC-gauge group $SU(2)_{\text{TC}} \sim Sp(2)_{\text{TC}}$. This economic realization of the FPC idea is accordingly called “minimal fundamental partial compositeness” (MFPC) [18,19,42]. Further possible emerging coset structures in CH models have been widely investigated, both from an IR [11] and a UV [18,44,45] perspective. In the absence of fundamental fermion and scalar masses, $m_{\mathcal{F}} = m_S = 0$, the TC-fermions (scalars) exhibit a global $SU(4)_{\mathcal{F}}$ ($Sp(24)_S$) flavor symmetry, corresponding to transformations along the index a (i) in (7). We note that, in the following, we will neglect the mass term for the TC-fermion fields and assume a flavor-trivial scalar mass term for simplicity.

The last line of (7) comprises the Yukawa interaction between the SM fermions (shown here as a spurion field $\psi^{i,a}$ of the full global symmetry) and the fundamental TC fields. Here, $y_{\text{TC}}^{i,a}$ are the Yukawa couplings associated to each component of the spurion field, ϵ_{ij} is the antisymmetric tensor in $Sp(24)_S$ and the TC-scalar fields have been arranged as $\Phi = (S, -\epsilon_{\text{TC}} S^*)^T$, see [18,21,42,43,46].

We emphasize that the complete effective action Γ and also its TC part (7) should not be confused with their classical counterparts. The effective action Γ takes into account the full quantum dynamics of the theory. We would like to elucidate this important aspect with the example of physical running couplings of given scattering processes. The respective scattering vertices or rather their one-particle irreducible (1PI) part are given by the n th derivative of the effective action with respect to the fields involved in the scattering,

$$\Gamma_{\phi_{i_1} \dots \phi_{i_n}}^{(n)}[\phi](p_1, \dots, p_n) = \frac{\delta}{\delta \phi_{i_1}(p_1)} \dots \frac{\delta}{\delta \phi_{i_n}(p_n)} \Gamma[\phi], \quad (10)$$

where ϕ is a superfield that comprises all fields in the theory. In the present case it reads

$$\phi = (A_\mu, c, \bar{c}, S, \mathcal{F}, \bar{\mathcal{F}}, \dots), \quad (11)$$

with the ghost and antighost $c, \bar{c}$. In the following we shall also use the abbreviations $\Gamma^{(n)}$ and $\Gamma_{\phi_{i_1} \dots \phi_{i_n}}$ for the sake of readability.

The vertices (10) are not renormalization group invariant and carry the inverse RG scaling of the fields. Their RG-invariant core is defined by

$$\bar{\Gamma}_{\phi_{i_1} \dots \phi_{i_n}}^{(n)}[\phi](p_1, \dots, p_n) = \frac{\Gamma_{\phi_{i_1} \dots \phi_{i_n}}^{(n)}[\phi](p_1, \dots, p_n)}{\prod_{j=1}^n Z_{\phi_{i_j}}^{1/2}(p_j)}, \quad (12)$$

which is the nonperturbative version of the standard definition of running couplings in perturbation theory. The amputation of the external wave function reflects the amputation of external propagators in the LSZ formalism and (12) is simply the 1PI core of the respective scattering amplitude, for a recent discussion in QCD see [47]. Equation (12) carries the full momentum dependence of the given process, and hence directly provides the information about unitarity and momentum scaling. Finally, we note that $\bar{\Gamma}^{(n)}$ is simply obtained by derivatives with respect to

$$Z_\phi^{1/2}(p)\phi(p), \quad (13)$$

which is the RG invariant field.

The effective action (7) also includes higher-order interaction terms of fundamental TC fields. These are encoded in the $\dots$ and are generated from loop diagrams. A relevant example is the two-scalar-two-fermion scattering, which is generated from the gauge-mediated box diagrams depicted in Fig. 1. This leads to terms in the full effective action Γ , or rather its technicolor part Γ_{CH} ,

$$\Gamma_{S^2\mathcal{F}^2} = \int_x Z_S Z_{\mathcal{F}} g_{S\mathcal{F}} S^\dagger S \bar{\mathcal{F}} \mathcal{F} + \dots, \quad (14)$$

where $\dots$ indicate all further two-scalar-two-fermion terms in the effective action. Further, in (14) we have dropped the momentum dependence of the wave functions Z_S and $Z_{\mathcal{F}}$ as well as that of the coupling $g_{S\mathcal{F}}$. From (14) we are lead to the momentum-dependent and RG-invariant scattering coupling as defined in (12),

$$\mathcal{P}_{S^\dagger S \bar{\mathcal{F}} \mathcal{F}} \bar{\Gamma}_{S^\dagger S \bar{\mathcal{F}} \mathcal{F}}^{(4)}(p_1, p_2, p_3, p_4) \simeq g_{S\mathcal{F}}(p_1, p_2, p_3), \quad (15)$$

where $\mathcal{P}_{S^\dagger S \bar{\mathcal{F}} \mathcal{F}}$ indicates the projection on the two-scalar-two-fermion term in (14). In (15) we have omitted the momentum conservation $(2\pi)^4 \delta(p_1 + \dots + p_4)$ as well as the Dirac, flavor, and technicolor tensor structures as indicated by $\simeq$. While the discussion of a complete tensor basis for these terms goes beyond the scope of the present work, we provide a specific example with a vector tensor structure in the fermionic part: $S^\dagger S \bar{\mathcal{F}} \not{\partial} \mathcal{F}$. The respective RG-invariant scattering coupling $g_{S\mathcal{F}}^{(v)}$ is given by

$$\mathcal{P}_{S^\dagger S \bar{\mathcal{F}} \not{\partial} \mathcal{F}} \bar{\Gamma}_{S^\dagger S \bar{\mathcal{F}} \not{\partial} \mathcal{F}}^{(4)}(p_1, p_2, p_3, p_4) \simeq g_{S\mathcal{F}}^{(v)}(p_1, p_2, p_3) \not{p}_3, \quad (16)$$

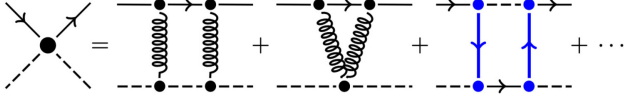


FIG. 1. Generation of the interaction between two TC fermions (plain black arrow line) and two TC scalars (dashed) from the TC gluons (curly) and SM fermions (plain blue arrow line) mediated box diagrams. All vertices $[\Gamma^{(n)}(p_1, \dots, p_n)$, black and blue dots] and propagators $(1/\Gamma^{(2)}(p_1, p_2))$ are full (inverse) correlation functions. The dots $\dots$ represent additional loop diagrams simultaneously containing TC-gluon and SM fermion propagators and/or four-field interactions. See text for details.

where $\mathcal{P}_{S^\dagger S \bar{F} \delta F}$ indicates the projection on the term $S^\dagger S \bar{F} \delta F$ in (14).

As can be inferred from Fig. 1, the scattering coupling strength is proportional to a combination of g_{TC}^4 and y_{TC}^4 , multiplied with inverse powers of the momenta p_i , flowing through the diagram. For illustration, we restrict ourselves to a symmetric point with $p_i^2 = p^2$. Then, the momentum scaling reads

$$g_{S\mathcal{F}}(p) \propto \frac{r_g g_{\text{TC}}^4(p) + r_y y_{\text{TC}}^4(p)}{\sqrt{p^2}}, \quad (17)$$

where $r_{g/y}$ encode the combinatorial factors of each box diagram. Accordingly, (14) is suppressed in the ultraviolet due to the asymptotic freedom of the TC-gauge group and the Yukawa interaction. In turn, in the infrared, the TC-gauge coupling grows strong, and (14), as well as further gauge-mediated interactions between the scalar and fermionic fields, become more relevant.

We also note that the right-hand side of Fig. 1 contains further diagrams, for example, a fish diagram with two two-scalar-two-fermion vertices and a mixed diagram with one TC-gluon exchange and a two-scalar-two-fermion vertex. Moreover, further one-loop diagrams such as box diagrams mixing TC-gauge fields and SM-fermions are also present. For small $g_{\text{TC}} \ll 1$ these diagrams are suppressed as they are proportional to g_{TC}^8 (fish diagram) and g_{TC}^6 (mixed diagram).

B. Emergent composites

Analogously to the formation of mesons in QCD from resonant four-quark scatterings, resonant two-scalar-two-fermion scatterings may give rise to the formation of the fermionic composites $\mathcal{B} \sim SF$. These resonant channels are then well described by the propagation of a new degree of freedom, a composite operator consisting of both fundamental TC fields. The respective composite operator is given by

$$\mathcal{O}_B^{a,i} \sim \epsilon_{ij} S^{a,j} \mathcal{F}^{a,a} \mathcal{T} \quad (18)$$

and may also include higher-order terms. Here, $\mathcal{O}_B^{a,i}$ carries the chirality of the TC fermion and transforms as a singlet under the confining gauge group. Moreover, all symmetries of the TC fields (containing the SM gauge symmetries) are encoded in the indices a and i . The undetermined operator $\mathcal{T}$ projects on the spin indices of the TC fields. Its consideration makes explicit the generality in the symmetry structure of $\mathcal{O}_B$ as well as a potential choice of elaborated spin configurations for excited states.

The nonperturbative phenomenon of the formation of composites will be treated by functional renormalization group (fRG) techniques, introduced in Sec. III A. With the adequate choice of external momentum, the effective four-field interaction can be exactly rewritten as the exchange of a composite of both fundamental external fields and a residual contribution. This is known in the fRG approach to QCD as dynamical hadronization and more generally it is taking into account emergent compositeness [36–39]. Here we follow [37,39,48], where the composite fields are simply introduced on the level of the path integral via the respective current term,

$$\exp \left\{ \int_x J_B \mathcal{B}(\varphi_f) \right\}, \quad (19)$$

where $\mathcal{B}$ is related to (18). In (19), the field φ_f is the fundamental super field that is integrated over in the path integral. Accordingly, the introduction of the composite fields in terms of a current does not signal a reduction from the fundamental to an effective theory but rather the convenient reparametrization of the fundamental theory in terms of emergent composite degrees of freedom. A well-known and well-studied example of such a reparametrization is the two-particle irreducible effective action, which is obtained by introducing a current for the two-point function.

Such a transformation allows us to rewrite a given scattering vertex in terms of an exchange of an emergent (resonant) composite field in a specific momentum channel and the remnant scattering, see Fig. 2 for the composite $\mathcal{B}$ discussed above. At a given momentum scale and for bilinear composites this is reminiscent of a Hubbard-Stratonovich transformation. However, the fRG approach

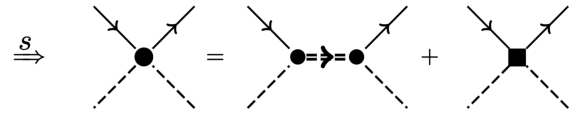


FIG. 2. Rewriting the full (fermion-scalar)² interaction in the s channel as the exchange of the composite $\mathcal{B}$ plus a remnant interaction accounting for the momentum dependence of the resonant tensor structure as well as the scattering in the other tensor structures. Double dashed lines with an arrow indicate the spin-1/2 composite propagators, plain lines with an arrow correspond to fundamental TC-fermion propagators and dashed lines to fundamental TC-scalar propagators.

with flowing dynamical composites [36–39,48] also incorporates general composites [37,39] not only bilinear ones. Further, it allows us to (re)do this transformation at each scale in a mathematically well-defined way without potential overcounting problems.

In the presence of the current term (19) in the path integral, the respective effective action is obtained via a Legendre transformation and contains additional terms that carry the dynamics of the emergent composite field,

$$\Gamma_B = \int_x \{ Z_B \bar{B} (\sigma_\mu \partial_\mu + m_B) B + h_B \sqrt{Z_B Z_S Z_F} [S(\bar{B}F) + S^\dagger(\bar{F}B)] + \dots \}, \quad (20)$$

where we have omitted flavor indices, chiral labels and the explicit consideration of $\mathcal{T}$ and the antisymmetric tensor ϵ_{ij} . We emphasize that, as in (14), all wave functions and the coupling h_B are momentum dependent, and the latter is simply the RG-invariant scattering coupling

$$\bar{\Gamma}_{S\bar{F}B}^{(3)}(p_1, p_2, p_3) \simeq h_B(p_1, p_2), \quad (21)$$

where we omitted the momentum conservation $(2\pi)^4 \delta(p_1 + p_2 + p_3)$ as well as the Dirac, flavor and TC tensor structure. Importantly, (20) encodes the diagrammatic relation in Fig. 2 in terms of the equation of motion of the composite field B . For s -channel momenta $s = (p_1 + p_2)^2$ this relates (20) to (14) with

$$\Gamma_B|_{B_{\text{EOM}}} = \Gamma_{S^2 F^2} \quad (22)$$

with the relation

$$\frac{h_B^2(s) m_B(s)}{s + m_B^2(s)} \propto g_{S\mathcal{F}}(s) \quad (23)$$

and $m_B(s) \propto \sqrt{s}$. It follows with (17) that $h_B^2(s) m_B(s) / (s + m_B^2(s)) \propto g_{TC}^4(s) / \sqrt{s}$, and hence decays rapidly in the UV. In turn, if the two-scalar-two-fermion s -channel scattering becomes resonant in the IR, this is well described in terms of the B exchange. Note, however, that the introduction of the emergent dynamical composite B only captures the respective momentum channel as well as further parts of the full scattering with $g_{S\mathcal{F}}(p_1, p_2, p_3)$.

We have mentioned below (18) that the field B or the operator $\mathcal{O}_B$ may also include higher order terms. Indeed, (20) only comprises the lowest order terms in the composite field: a dispersion term for the composite state and two interaction terms which link fundamental fields' interactions to the propagation of a composite state. Higher order terms are generated as well and are indicated with $\dots$. They contribute to the equations of motion of B , leading to higher order terms in the fundamental fields and in particular $\mathcal{S}, \mathcal{F}$.

The full effective action in the presence of the composites is given by $\Gamma[\phi]$ and includes (7) and (20), where the superfield ϕ now also contains the composite field B and potentially further composites,

$$\phi = (\phi_f, \phi_c). \quad (24)$$

The superfield ϕ_f is that of the fundamental fields, defined in (11). The superfield of all composites is denoted by ϕ_c , to wit,

$$\phi_f = (A_\mu, c, \bar{c}, \mathcal{S}, \mathcal{F}, \bar{\mathcal{F}}, \dots), \quad \phi_c = (B, \bar{B}, \dots). \quad (25)$$

The dots in (25) indicate that we may also introduce additional composite fields for further resonant interaction channels. This also entails the dynamics of such a system are well captured by the present approach as long as all channels are considered: the emergence of a dynamical composite field is signaled by a divergent or resonant interaction channel. This resonant channel can be treated with the emergent compositeness approach discussed very generically in [39].

We have already emphasized that the present approach describes the full theory. Its conceptual and technical advantage is the explicit appearance of the emergent dynamical degrees of freedom, that otherwise have to be described in terms of resonant interaction channels of higher-order correlation functions. The latter is technically more challenging and also has the disadvantage that higher-order scatterings of the emergent fields are far more difficult to describe in terms of the fundamental fields. An illuminating example is low energy QCD, which is dominated by (multi)pion exchange processes. While these processes are complicated multi-quark scatterings, they are far simpler taken into account by multipion diagrams. This structure is at the root of chiral perturbation theory.

Still, we can describe the theory in terms of the effective action of the fundamental degrees of freedom. The relation between the two effective actions $\Gamma[\phi_f]$ and $\Gamma[\phi]$ is given by

$$\Gamma[\phi_f] = \Gamma[\phi_f, \phi_c^{\text{EOM}}(\phi_f)], \quad (26)$$

where ϕ_c^{EOM} is the solution to the equations of motion (EOM) of the composites in the presence of a background ϕ_f of the fundamental fields,

$$\left. \frac{\delta \Gamma[\phi]}{\delta \phi_c} \right|_{\phi_c = \phi_c^{\text{EOM}}} = 0. \quad (27)$$

A solution of the EOM (27) for B is tantamount to $J_B = 0$, and hence (26). Its lowest order version is given by (22).

Equation (26) highlights the fact that the introduction of the composites does not entail the reduction of the full theory to an effective one. To make this even more explicit,

we close this discussion with the remark that the correlation functions of ϕ_f are given by

$$\begin{aligned} \Gamma_{\phi_{f,i_1} \dots \phi_{f,i_n}}^{(n)}[\phi_f] &= \frac{\delta \Gamma[\phi_f, \phi_c^{\text{EOM}}(\phi_f)]}{\delta \phi_{f,i_1} \dots \phi_{f,i_n}} \\ &= \prod_{j=1}^n \left[\frac{\delta}{\delta \phi_{f,i_j}} + \frac{\delta \phi_{c,r}^{\text{EOM}}(\phi_f)}{\delta \phi_{f,i_j}} \frac{\delta}{\delta \phi_{c,r}} \right] \Gamma[\phi_f, \phi_c] \Big|_{\phi_c = \phi_c^{\text{EOM}}} . \end{aligned} \quad (28)$$

Equation (28) entails, that a given correlation function of the fundamental fields is split into the part described by the exchange of the composites and the remaining part. For a detailed discussion of these parametrizations of correlation functions see [39,48].

We close this section with a short summary of the relevant properties and advantages of the present approach. It allows us to investigate the theory in a global and complete manner, taking into account fundamental and composite degrees of freedom simultaneously as well as their interplay. The effective action evolves from UV regimes dominated by the interplay of TC gauge and TC matter to a transition or interface regime where the rising gauge couplings lead to emergent dynamical composites. In the infrared below this transition regime, the dynamics are dominated by that of the emergent composites. The latter regime is typically described by means of low-energy effective field theories. Then, the respective low-energy couplings or parameters are tuned such that they accommodate the experimental low-energy measurements. In the present effective action approach, these couplings are determined consistently from those in the fundamental high energy theory, thus restricting the latter. The accuracy or rather predictive power of these constraints hinges on a quantitative understanding of the strongly correlated dynamics in the interface regime in which the composites emerge and become dynamical. These dynamics are well captured by the present approach whose technical implementation within the fRG is discussed in Sec. III A.

C. Dimension of the composite

In this section, we elaborate on the dimensionality of the composite and the convention used. To begin with, the effective action only depends on the dressed combination $Z_B^{1/2}(p)\mathcal{B}(p)$. This is also reflected by the fact that the RG-invariant scattering vertices (12), are the RG-invariant building blocks of S -matrix elements as are obtained by derivatives with respect to $Z_\phi^{1/2}(p)\phi$, see (13). The RG-invariant field $Z_B^{1/2}(p)\mathcal{B}(p)$ has the canonical dimension 3/2, which follows from the dispersion relation for fermionic fields (such as the hadronlike composite of interest),

$$[Z_B^{1/2}\mathcal{B}]_c = 3/2. \quad (29)$$

Evidently, observables are independent of the choice of the canonical dimension $[\mathcal{B}]_c$ of the field. Specifically, all choices for the canonical dimension of the composite field, and in particular the common choices $[\mathcal{B}]_c = 3/2$ or $[\mathcal{B}]_c = 5/2$, lead to the same RG-invariant (physical) vertices $\bar{\Gamma}^{(n)}$ defined in (12). Moreover, S -matrix elements are constructed directly from tree-level diagrams with the $\bar{\Gamma}^{(n)}$, and hence unitarity or its failure is derived from their momentum dependence.

From the above discussion it also follows that a convenient choice is provided by a dimensionless wave-function renormalization Z_B : if we would consider the canonical dimension of the composite to be the sum of the fermion and scalar canonical dimensions, $[\mathcal{B}]_c = [\mathcal{F}]_c + [\mathcal{S}]_c = 5/2$, the canonical dimension of $Z_B(p)$ in (29) would be minus two, in summary leading to (29). In fact, the former is the choice used for QCD with dynamical hadronization as well as in low-energy effective theories of QCD. There, composite baryons are described by fermionic operators with the canonical dimension 3/2, while composite mesons are described by bosonic operators with the canonical dimension 1. The respective dimension of the constituents is 9/2 for three quarks q^3 or 3 for a $q\bar{q}$ pair following from the dimension of the quarks $[q]_c = 3/2$.

D. Linear mixing operator and anomalous scaling

With the introduction of the dynamical composites in the previous sections, the respective effective action also comprises all terms whose generation is not prohibited by symmetries. One example is the linear mixing term

$$\Gamma_{\text{mix}} = \int_x \{ \lambda_t^L Z_q^{1/2} Z_B^{1/2} \bar{q}_L \mathcal{B}_R^q + \lambda_t^R Z_t^{1/2} Z_B^{1/2} \bar{t}_R \mathcal{B}_L^t + \text{H.c.} \}, \quad (30)$$

key for the realization of the SM fermion masses in FPC. In (30), we have restricted ourselves to the linear mixing term with the third generation of quarks. Moreover, all wave functions in (30) and the RG-invariant couplings or mixing masses $\lambda_t^{L/R}(p)$ are momentum dependent.

Terms like (30) carry part of the quantum fluctuations of the scatterings of the fundamental fields. The latter correlations (or rather their 1PI part) are obtained by taking the n th field derivative of the effective action of the fundamental fields $\Gamma[\phi_f]$ in (26), see (28). Evidently, this correlation function can also be obtained by taking ϕ_f derivatives of $\Gamma[\phi_f, \phi_c^{\text{EOM}}(\phi_f)]$ on the right-hand side of (26). In our case we only consider one composite structure, $\phi_c = \mathcal{B}$. The solution of its EOM in the present simple approximation with the kinetic term in (20) and the linear mixing term (30) is given by

$$\mathcal{B}^{\text{EOM}} = \sqrt{\frac{Z_S Z_{\mathcal{F}}}{Z_B}} \frac{h_B^2}{\sigma_\mu \partial_\mu + m_B} \mathcal{S}\mathcal{F} + \sqrt{\frac{Z_f}{Z_B}} \lambda_f f, \quad (31)$$

where f is the respective SM fermion with adequate chirality. For constant $\mathcal{B}$ the composite is simply proportional to $\mathcal{S}\mathcal{F}$ and a linear term f in the SM fermion. When inserted on the composite effective action (30) we recover the fundamental Yukawa interaction in (7) and a mass term for the SM fermion field f . The latter term reflects already the masslike character of the linear mixing couplings in (30).

We now elucidate the important relation between the couplings in the fundamental and composite actions within a relevant example: the fundamental Yukawa interaction in the last line of (7). In the present approximation, its strength is given by the Yukawa couplings $y_{\text{TC}}^{i,a}$ in the effective action $\Gamma[\phi_f]$. Hence it is obtained from the ψ , $\mathcal{S}$, $\mathcal{F}$ -derivative of $\Gamma[\phi_f]$. In turn, we can also obtain it from the ψ , $\mathcal{S}$, $\mathcal{F}$ derivative of $\Gamma[\phi_f, \mathcal{B}^{\text{EOM}}(\phi_f)]$. However, the effective action in the presence of composites also contains a Yukawa term with a coupling $y_{c,\text{TC}}^{i,a} \neq y_{\text{TC}}^{i,a}$. The relation between y_{TC}^f , $y_{c,\text{TC}}^f$, and λ_f is obtained from the ψ , $\mathcal{S}$, $\mathcal{F}$ derivative of (26), using (31) and (30),

$$y_{\text{TC}}^f = y_{c,\text{TC}}^f - \lambda_f \frac{h_B^2 m_B}{p^2 + m_B^2}. \quad (32)$$

Equation (32) entails that the introduction of a composite redistributes contributions of the quantum fluctuations to a given correlation function. Moreover, only part of the quantum fluctuations is still carried by the quantum fluctuations of the fundamental fields and the rest is carried by the mixed correlations of composites and fundamental fields.

We emphasize that while it seems to be suggestive to identify y_{TC}^f with $y_{c,\text{TC}}^f$, it clearly amounts to a double-counting of fluctuations as is evident from (32). Importantly, in the present emergent composite approach, this double counting problem is absent as the correct relations between the different couplings such as (32) are implemented by definition. We further elaborate on (32) and the distribution of fluctuations in Sec. III B.

The canonical momentum dimension of any arbitrary fermion f linear mixing coupling λ_f is one, as follows from (32). It is also easily inferred from the canonical dimension of the dressed fields,

$$[\sqrt{Z_f Z_B} \bar{f} \mathcal{B}]_c = 3, \quad (33)$$

which follows readily from the dimensional analysis of the kinetic terms of quarks f and composite $\mathcal{B}$. Hence, the canonical dimension of λ_f is given by $[\lambda_f]_c = 1$.

We have already discussed that all parameters in the effective action are momentum dependent, which comprises the information about unitarity as well as the evolution of masses and couplings in the theory. If all parameters evolve according to their canonical momentum dimension, then the theory is in the scaling regime of a fixed point. For the mass mixing parameter, this entails $\lambda_f(k) \propto k$, where k is an average momentum scale.

The anomalous momentum scaling of couplings and masses is best accessed from the scaling of their dimensionless counterparts. In the case of λ_f , the dimensionless coupling $\bar{\lambda}_f$ reads

$$\bar{\lambda}_f = \lambda_f k^{-1}. \quad (34)$$

The average momentum scale k is also used as a cutoff scale in the fRG approach, employed for explicit computations in the present work. This is detailed in Sec. III A.

Here we note that in the absence of an anomalous scaling, the dimensionless couplings and parameters do not scale with k . Conversely, the anomalous dimension is given by

$$\partial_t \bar{\lambda}_f = \gamma_{\lambda_f} \bar{\lambda}_f, \quad (35)$$

where $\partial_t \equiv k \partial_k$. We also note that the anomalous scaling of the wave functions Z_ϕ is given by

$$\partial_t Z_\phi = \gamma_\phi Z_\phi. \quad (36)$$

Equation (36) entails that the anomalous scaling of the fields is given by $-\gamma_\phi$. For that reason, the common definition in fRG applications contains a minus sign in (36).

We define the full momentum dimension $[\mathcal{O}]$ of an operator $\mathcal{O}$ with its k scaling. For the mixed mass term (30) it follows that

$$[\bar{\lambda}_f k Z_f^{1/2} Z_B^{1/2}] = \gamma_{fB}. \quad (37)$$

Then, the full dimension of the mixing coefficient $\bar{\lambda}_f$ is obtained by subtracting the scaling of the external fields and the canonical dimension carried in the explicit momentum dependence in k ,

$$[\bar{\lambda}_f] \equiv \gamma_{\lambda_f} = \gamma_{fB} - 1 - \frac{\gamma_f}{2} - \frac{\gamma_B}{2}. \quad (38)$$

Moreover, given $\bar{\lambda}_f$ at a UV scale $k = \Lambda_{\text{UV}}$, its value at a measurable IR scale $k = \Lambda_c$ is given by the momentum scaling

$$\bar{\lambda}_f(\Lambda_c) = \bar{\lambda}_f(\Lambda_{\text{UV}}) \left(\frac{\Lambda_c}{\Lambda_{\text{UV}}} \right)^{\gamma_{\lambda_f}}. \quad (39)$$

The function γ_{λ_f} encloses all nonperturbative information on the composite and its dynamics. Equation (38) is

agnostic to the particular composition of the fermionic bound-state $\mathcal{B}$ and hence holds for any type of fermionic composites (e.g., $\mathcal{B} \sim \mathcal{F}\mathcal{F}\mathcal{F}$). Different natures of $\mathcal{B}$ will give rise to different quantum corrections in each of the scaling functions in (38).

Considering γ_f to be negligible, the magnitude and sign of γ_B and γ_{fB} will lead to a more or less enhanced scaling of the coupling $\bar{\lambda}_f$. In the case where $\gamma_{fB} - \gamma_B/2 \lesssim 0$ the coupling $\bar{\lambda}_f$ scales as that of a relevant operator (e.g., as a fermionic mass term) and grows towards the IR. If $\gamma_{fB} - \gamma_B/2 \approx 1$, then the scaling is like for a marginal operator and will show a conformal scaling at the fixed point if $\gamma_{\lambda_f} = 0$. Last, if $\gamma_{\lambda_f} > 0$, the coupling scales as an irrelevant operator growing towards the UV and decreasing towards the IR. This is explicitly necessary in the FPC framework in order to generate the masses of the lightest SM fermion generations.

We close this section by introducing some useful notations for the upcoming sections. It is instructive to define the degree of compositeness at a scale Λ_c (following the notation of [46,49]), which parametrizes the strength of the mixing of the SM-like fermions with the composite resonances below the condensation scale

$$\epsilon_f^L \equiv \frac{\bar{\lambda}_f^L(\Lambda_c)}{g_B(\Lambda_c)} = \frac{\bar{\lambda}_f^L(\Lambda_{UV})}{g_B(\Lambda_c)} \left(\frac{\Lambda_c}{\Lambda_{UV}} \right)^{\gamma_{\lambda_f}^L}. \quad (40)$$

Here, g_B is the coupling strength of the resonances and Λ_c is the IR condensation scale, reasonably expected around ~ 10 TeV. For naturalness reasons, the dimensionless coupling $\bar{\lambda}_f^L(\Lambda_{UV})$ is expected to be of $\mathcal{O}(1)$.

Generalizing to three generations of quarks and leptons, the hierarchical pattern of SM fermion masses (and mixings) can now be argued to arise from small differences in the anomalous dimensions γ_λ via RG flow between the largely separated scales Λ_c and Λ_{UV} . In fact, the resulting hierarchical degrees of compositeness enter the fermion masses, after integrating out the heavy resonances (see, e.g., [12,46,49]), which read

$$m_f \sim g_B(\Lambda_c) \frac{v}{\sqrt{2}} \epsilon_f^L \epsilon_f^R = \frac{\mathcal{Y}_f v}{\sqrt{2}} \left(\frac{\Lambda_c}{\Lambda_{UV}} \right)^{(\gamma_{\lambda_f}^L + \gamma_{\lambda_f}^R)}, \quad (41)$$

with

$$\mathcal{Y}_f = \bar{\lambda}_f^L(\Lambda_{UV}) \bar{\lambda}_f^R(\Lambda_{UV}) / g_B(\Lambda_c), \quad (42)$$

which gathers all $\mathcal{O}(1)$ quantities.

E. Emergent composites in the walking regime

In FPC, a walking regime is expected to emerge around a scale Λ_{UV} at which the beta function of the TC gauge coupling vanishes. As discussed in the previous sections, this is required to generate the SM fermion masses through

the momentum scaling of λ_q along many orders of magnitude, see (39).

Walking regimes are generated from IR quasi-fixed-point solutions in the TC-gauge coupling. These are purely determined by the structure of the gauge group and the charged matter content. Depending on the number of fields and the representation under which they transform, this conformal scaling regime arises at different gauge coupling values and is related to Banks-Zaks (BZ) fixed points. The properties of the conformal window, such as the magnitude of the fixed point coupling and its boundaries, have been studied with perturbative methods [50–53], and nonperturbative functional methods [54,55] as well as lattice simulations [56,57].

In FPC, dynamical chiral symmetry breaking is required to take place in order to form pNGBs which realize the Higgs boson. Through this mechanism, the TC fermions become massive and decouple from the contributions to the TC-gauge beta function leading to an IR Landau pole and terminating the walking regime [58–60]. Therefore, it is argued [18,21] that the different fundamental incarnations should be found outside (below) the conformal BZ window [52].

In this work, we do not investigate further the existence of viable walking regimes within different FPC models but assume the scenario in which dynamical chiral symmetry breaking triggers at the end of the walking regime. While we delegate the analysis of the viable models with the fRG to an independent work, here we focus on the qualitative properties of the bound states of interest along the walking regime.

In order to address whether the scaling γ_λ reproduces the correct SM fermion masses, the parameters of the composite sector along the quasiconformal regime need to be analysed. By rewriting the four-TC-field interaction term in (22) as the exchange of a composite, the Yukawa coupling h_B and the mass of the composite m_B in (20) can be estimated from the quantum corrections giving rise to the four TC-field interaction. In the case at hand, the leading quantum corrections are provided by the diagrams shown in Fig. 1.

The first box diagram is mediated by TC gluons and will dominate as the gauge coupling enters a strong walking regime, allowing the formation of bound states and later on, triggering chiral symmetry breaking. Following, the Yukawa coupling between TC fields and SM fermions may become strong due to the effect of TC-gauge quantum corrections. In this case, the second SM fermion-mediated box diagram may contribute substantially to the formation of the bound state. The TC-gauge corrections on y_{TC} are subject to the group structure and the matter content in the theory and may lead to different qualitative results depending on the model [18,21]. One particularly interesting scenario is where the formation of bound states is dominated by the exchange of SM fermions. In this case,

$y_{\text{TC}} \gg g_{\text{TC}}$ via the TC-gluon enhancement in the quantum corrections to the Yukawa coupling between SM and TC fields. This scenario would help to relax the magnitude of the fixed point value g_{TC}^* , reaching perturbative values, while still providing the formation of composites, purely dominated by Yukawa interactions.

In the walking regime, the momentum scaling of the different couplings is given by their canonical momentum dimension. Marginal couplings and dimensionless parameters (as the composite mass $\bar{m}_B$) are hence fixed to a constant value, leading to the relation

$$\frac{(h_B^*)^2 \bar{m}_B^*}{1 + (\bar{m}_B^*)^2} \propto r_g (g_{\text{TC}}^*)^4 + r_y (y_{\text{TC}}^*)^4. \quad (43)$$

Here, $r_{g/y}$ encodes the diagrammatic combinatorial factors as well as the TC-color and spin structures of the respective box diagrams. This simplified relation allows us to estimate the composite sector parameters from the couplings of the fundamental theory.

It is important to stress that in principle $\bar{m}_B$ and h_B could be precisely determined from the flow of two- and three-point functions of composites and fundamentals. However, this is beyond the scope of the current analysis and here we just estimate them via the relation (43), whose validity is backed up by knowledge from $2+1$ flavors QCD [48,61–63]. The equivalent h_B for pions is known to be approximately constant even though the gauge coupling runs strongly. The respective coupling to QCD composites shows an IR attractive quasifixed point behavior in which the only variation is sourced by chiral symmetry breaking. This is caused by all the momentum scaling of g_{TC} being translated to the momentum scaling of the composite mass. In fact, this is the dynamical explanation for the emergence of chiral symmetry breaking. The strong interactions drive the meson masses to zero triggering spontaneous symmetry breaking of the chiral potential, see [48,61–63] for more details. In the current scenario, the gauge coupling reaches a fixed point and hence no variation is propagated to the composite parameters further supporting the estimate in (43).

III. ANOMALOUS SCALING AND FLAVOR HIERARCHIES

In the present section, we compute the anomalous scaling of different two-point functions necessary to determine whether a successful reproduction of the fermion mass hierarchies of the Standard Model particles is achievable within MFPC.

The fluctuation analysis of dynamics of the composite (as well as that of the other fields) is done within the functional renormalization group approach, and in Sec. III A we briefly summarize the fRG approach to theories with emergent composites. This approach is used in Sec. III B to compute the anomalous dimensions of the composite sector, and as an

example, we provide an estimate for a particular scenario. This is done on a conceptual level and a more quantitative study will be presented elsewhere.

A. Functional renormalization group

For the computation of the momentum scaling of the linear mixing coupling λ_f and future analysis of the dynamics of CH sectors, we will employ the functional renormalization group [34,35,64], which already has proven its applicability in very similar strongly coupled regimes. For a general review see [40], for IR QCD studies see, e.g., [48,62,63,65] and for SM applications [66–70]. In this approach, an IR regulator quadratic in the fields is introduced at the level of the classical action,

$$\Delta S_k[\phi] = \frac{1}{2} \int_x \phi(-q) R_k(q) \phi(q). \quad (44)$$

Equation (44) implements the Wilsonian idea of progressive integration of momentum shells by suppressing quantum fluctuations of momenta p below a given IR cutoff scale k . $R_k(q)$ is a matrix in field space and introduces masslike terms for the different fields ϕ_i in the infrared. In turn, for large momenta $R_k(q)$ decays rapidly.

The regulator insertion leads to the definition of an RG-cutoff dependent effective action $\Gamma_k[\phi]$. The full effective action is obtained after all quantum fluctuations have been integrated out at $k=0$ with $\Gamma[\phi] = \Gamma_{k \rightarrow 0}[\phi]$. The cutoff scale dependence of $\Gamma_k[\phi]$ is described by the Wetterich equation [33],

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \partial_t R_k^{ij} \right], \quad (45)$$

where $\Gamma_k^{(2)}$ is the second functional derivative of the scale-dependent effective action Γ_k with respect to the fields ϕ , see (10). The subscript ij indicates the different species ϕ_i . Note that the regulator matrix R_k^{ij} is diagonal in its bosonic components and symplectic in its fermionic components. The derivative $\partial_t = k \partial_k$ is the logarithmic scale derivative with respect to the infrared cutoff scale, which may be interpreted as an average momentum scale.

The nonperturbative flow equation (45) is a one-loop exact equation that gives us access to the nontrivial strongly coupled sector in CH scenarios, while remaining versatile to investigate large theory and parameter space. Moreover, the dynamical emergence of bound states or more generally dynamical composites can be incorporated within a modified version of (45), [36–39,48]. This allows us to consider the effective action in the presence of emergent composites in a mathematically sound way. This approach is known as dynamical condensation or the fRG approach with emergent composites [39], the latter literally describing its core. It has been widely employed in the areas of condensed

matter and QCD. In the latter, it is commonly called dynamical hadronization.

Its general version has been derived in [37], see also [71] for a formulation for the Wilsonian effective action. In [39] its application to a successive tower of composites is explained in the example of QCD (scalar-pseudoscalar mesons, vector mesons, diquarks, and baryons), and this application is similar to the present use. Here we briefly discuss the generalization in [37] at the example of one emergent scale-dependent composite at lower momentum scales. While ϕ_c is an independent emergent field in the effective action, its flow is described by a functional

$$\dot{\phi}_c[\phi], \quad (46)$$

with $\phi = (\phi_f, \phi_c)$ in (24). Equation (46) takes care of the rotation of the field basis due to the scale dependence of the composite. Typically it is chosen such that the entire dynamics of the respective resonant channel is stored in the propagation of the composite. This choice optimizes the field basis and hence stabilizes expansion schemes of the effective action. In short, it leads to the most rapid convergence of the diagrammatic expansion of the full theory.

In the presence of the composite, the flow (45) is modified by additional terms that take care of the scale-dependence of the emergent dynamical degree of freedom ϕ_c . It reads

$$\begin{aligned} & \left(\partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c} \right) \Gamma_k[\phi] \\ &= \frac{1}{2} \text{Tr} \left[\left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \left(\partial_t \delta^{jn} + 2 \frac{\delta \dot{\phi}_n}{\delta \phi_j} \right) R_k^{ni} \right], \end{aligned} \quad (47)$$

where $\dot{\phi}_i = \delta_{ic} \dot{\phi}_c$ in a slight abuse of notation. The additional term on the left-hand side of (47) simply accounts for the (field-dependent) anomalous dimension of the emergent composite, while the additional term on the right-hand side accounts for the rotation of the composite field axis in the field basis of the fundamental fields.

B. Anomalous dimensions

We have discussed in Sec. II that the presence of a walking regime and an adequate scaling of the coupling between composites and SM fermions is necessary for reproducing the SM masses via FPC. In this section, we compute the momentum scaling of an arbitrary linear mixing coupling $\bar{\lambda}_f$ between a SM fermion and a composite field $\mathcal{B}$. Following from the derivation in (38), the momentum scaling of the coupling reads

$$\gamma_\lambda = -1 + \gamma_{f\mathcal{B}} - \gamma_f/2 - \gamma_{\mathcal{B}}/2. \quad (48)$$

As discussed in Sec. II E, the magnitude of the fundamental Yukawa couplings y_{TC}^* is considered to be subdominant

with respect to the gauge dynamics. This restricts ourselves to the QCD-like scenario in which the composite dynamics is driven by the TC-gauge dynamics. In this scenario, no sizeable contributions aside from the SM ones are present in γ_f . As these are known to be small and negligible [66], the SM fermions anomalous dimension can be approximated as

$$\gamma_f \approx 0. \quad (49)$$

Nonetheless, it is important to note that scenarios where the Yukawa coupling becomes strong along with the TC-gauge coupling may lead to further important corrections to the SM fermion phenomenology, which would be interesting to investigate further.

The momentum scaling of the vertex (37) and the anomalous dimension of the composite $\gamma_{\mathcal{B}}$ remain to be computed. We obtain the first from the linear-mixing two-point function

$$\Gamma_{\mathcal{B}\bar{f},k}^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \mathcal{B}(p) \delta \bar{f}(-p)} = \lambda_f \sqrt{Z_f Z_{\mathcal{B}}}, \quad (50)$$

by performing two functional derivatives of the full effective action [see (10)] with respect to $\bar{f}$ and $\mathcal{B}$. By applying a scale derivative on both sides of (50), we obtain

$$\gamma_{f\mathcal{B}} = \partial_t \Gamma_{\mathcal{B}\bar{f},k}^{(2)} / \Gamma_{\mathcal{B}\bar{f},k}^{(2)}. \quad (51)$$

In the first line of Fig. 3, the diagrammatic flow $\partial_t \Gamma_{\mathcal{B}\bar{f},k}^{(2)}$ is depicted. As the linear mixing couplings carry a scalar-tensor structure (like a fermionic mass term) and there is no vector structure in any of the Yukawa couplings, the only nonvanishing contribution in the loop diagrams will be proportional to the scalar mode of the TC-fermion propagators. However, in Sec. II D we argued vanishing $m_{\mathcal{F}}$ and $m_{\mathcal{S}}$ in order to preserve the global flavor symmetries. Additionally, TC masses will trigger the decoupling of the TC fields from the quantum corrections to the TC gauge coupling and hence invalidate the possibility of a walking regime. For this reason, the fundamental masses are taken to be $m_{\mathcal{F}}, m_{\mathcal{S}} \leq \Lambda_c$. With $m_{\mathcal{F}} = 0$, each diagram vanishes, and we are led to

$$\gamma_{f\mathcal{B}} = 0. \quad (52)$$

It follows from this result that the linear mixing operators in (51) are not generated by quantum fluctuations from the fundamental theory in the absence of a fundamental TC-fermion mass. However, we have discussed around (32) that the fundamental Yukawa coupling y_{TC}^f in the absence of the composites are only related but not equal to $y_{c,\text{TC}}^f$ in the presence of composites. The former is known to show

nonvanishing quantum contributions $\partial_t y_{\text{TC}}^f \neq 0$ in the fundamental theory, see, e.g., [18,21]. These contributions are not reproduced by γ_{fB} in the current approximation but accounted in $\partial_t y_{c,\text{TC}}^f$. On the other hand, contributions of $\partial_t y_{\text{TC}}^f$ in the chirally broken phase are reproduced in γ_{fB} as reflected by its proportionality to $m_{\mathcal{F}}$.

Finally, the leading contribution in (48) remains to be determined. The momentum scale variation of the wavefunction renormalization can be obtained from the composite's two-point function

$$\Gamma_{B\bar{B},k}^{(2)} = \frac{\delta\Gamma_k}{\delta B(p)\delta\bar{B}(-p)} = iZ_B\sigma_\mu p_\mu. \quad (53)$$

Choosing the adequate projection onto the kinetic part of the two-point function and evaluating at vanishing external momenta, the anomalous dimension reads

$$\gamma_B = \frac{\partial_t Z_B}{Z_B} = \frac{-i\partial_{p^2}(\sigma_\mu p_\mu \partial_t \Gamma_{B\bar{B},k}^{(2)})}{Z_B \text{tr}[\sigma_\mu \sigma_\nu]} \Big|_{p=0}. \quad (54)$$

The diagrammatic form of $\partial_t \Gamma_{B\bar{B},k}^{(2)}$ is depicted in the second row of Fig. 3. The explicit expressions for the flow are provided in the Appendix. The diagram with the fermionic flow, Fig. 3(a), vanishes in the $p = 0$ limit. To scrutinize this further we reevaluated the diagram employing numerical regulators which implement a richer momentum shell integration. In fact, regulators featuring a more elaborated momentum shell structure always show Fig. 3(a) to have a negligible contribution. Moreover, the analytic result for the flat or Litim regulator is properly reproduced from our numerical regulators, which supports the neglect of the contribution of Fig. 3(a).

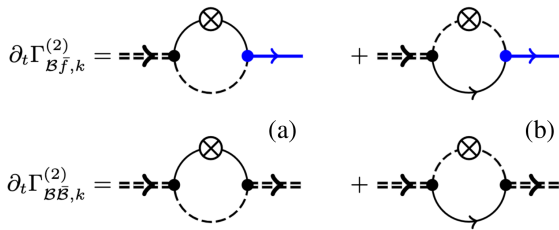


FIG. 3. Diagrammatic flow of the composite-SM-fermion two-point function (first line) and the composite's two-point function (second line), necessary for the computation of the vertex flow (51) and the composite's anomalous dimension (54). All vertices (black circles) and regulated propagators are full correlation functions. Double dashed lines with an arrow correspond to the composite regulated propagator, plain lines with an arrow correspond to fundamental TC-fermion regulated propagators, dashed lines to fundamental TC-scalar regulated propagators and blue lines with an arrow to the SM fermion regulated propagators. Crossed circles depict the insertion of $\partial_t R_k^{(\phi)}$.

Consequently, the anomalous dimension at vanishing external momentum is given by Fig. 3(b) and reads (see the Appendix for details)

$$\gamma_B = -\frac{h_B^2}{16\pi^2} \frac{N_{\text{TC}}}{2} \text{tr}[T^2] \left(1 + \frac{\gamma_S}{5}\right), \quad (55)$$

where N_{TC} is the dimension of the fundamental representation of the $\text{SU}(N_{\text{TC}})$ -gauge group and γ_S is the anomalous dimension of the fundamental scalar TC field coming from the cutoff derivative $\partial_t R_k$. The presence of the scalar anomalous dimension reflects the higher loop character of the fRG.

Moreover, all parameters and couplings in (55) are positive constants, leading to

$$\gamma_B < 0 \quad (56)$$

as a natural limit. Equation (56) reproduces the unitarity bound on fermionic fields [2] purely derived from the effective action. As discussed in the Appendix, the choice of the regulator does not alter the sign of the anomalous dimension as this can be written as a total scale derivative of the one-loop diagram. See (A10) for more details. However, γ_B could turn positive if the scalar anomalous dimension is largely negative, $\gamma_S < -5$. This would require an extremely strong coupling leading to contributions from higher n -point functions becoming relevant. As this scenario is outside the scope of this computation, we consider the one-loop expression and neglect the suppressed scalar anomalous dimension.

Collecting (49), (52), and (55), we find that the scaling of the linear mixing couplings is mainly determined by the anomalous dimension of the composite field

$$\gamma_\lambda \approx -1 - \gamma_B/2, \quad (57)$$

leading to the relation

$$m^f = \frac{\mathcal{Y}_f v}{\sqrt{2}} \left(\frac{\Lambda_c}{\Lambda_{\text{UV}}}\right)^{2\gamma_\lambda} \sim \frac{\mathcal{Y}_f v}{\sqrt{2}} \left(\frac{\Lambda_c}{\Lambda_{\text{UV}}}\right)^{-2-\gamma_B}, \quad (58)$$

for the SM fermion masses. In Fig. 4 we have used (58) to illustrate the SM fermion masses as a function of the anomalous dimension γ_λ . Note that the parameter scan presented here is independent of the derivation of γ_λ , but subject to the phenomenological characteristics of the theory such as the size of the walking regime $\Delta\Lambda = \log(\Lambda_c/\Lambda_{\text{UV}})$ and the value of fundamental couplings at the onset scale (Λ_{UV}). We will provide the connection to our results for the anomalous dimension further below. Nine different walking regimes of sizes $\Delta\Lambda \subset [-2, -10]$ are shown as clouds of points of different colors whose steepness increases with the regime's size. We consider the couplings $\bar{\lambda}_f$ and g_B to have a natural value at the UV scale.

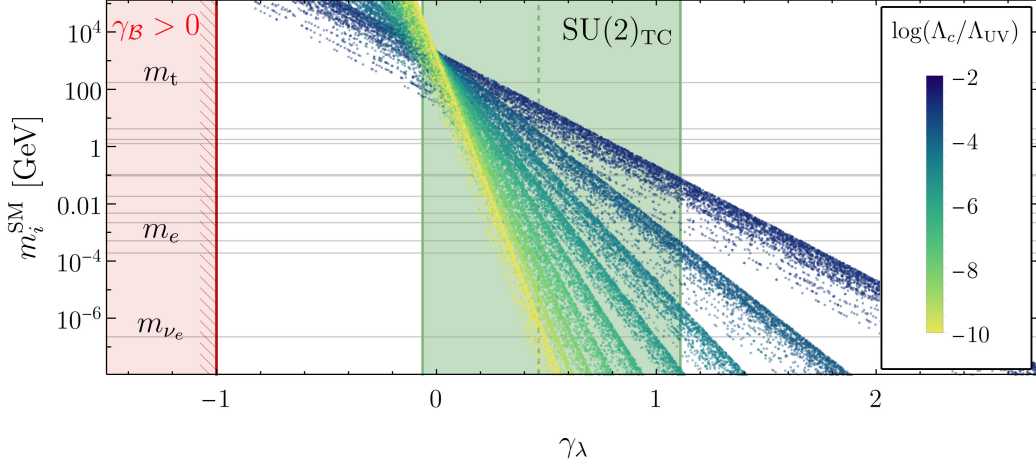


FIG. 4. SM fermion masses as a function of the anomalous scaling γ_λ defined in (48). The colorful clouds of points show a parameter investigation in which $\mathcal{V}_f \in [0.1, 4\pi]$ and $\gamma_\lambda \in [-1, 3]$. From blue to yellow, the size of the walking regimes is discretely increased by varying $\Delta\Lambda = \log(\Lambda_c/\Lambda_{UV})$ from $\Delta\Lambda = -2$ to $\Delta\Lambda = -10$. The red-shaded region depicts the prohibited area inaccessible due to the found negativity of the composite's anomalous dimension and coincidentally to the unitarity bound of the coupling λ_f . The green dashed line indicates the estimated ballpark of γ_λ in the MFPC scenario with two additional Dirac fermions in (59). The green-shaded region shows a 20% variation in the estimate of h_B in (43) and serves only as an indicative measure of the dependence of γ_λ on h_B .

Therefore, varying $\mathcal{V}_f \in [0.1, 4\pi]$, as defined in (42), covers the expected region. We note that the wider the range of $\mathcal{V}_f$, the broader the spread of points and that $\mathcal{V}_f \rightarrow 0$ for $y_{TC}^f \rightarrow 0$, see (32).

For reproducing the SM flavor hierarchies within PC, different composites and scalings are necessary, as already reviewed in Sec. I. As the top Yukawa coupling is approximately unity, $\gamma_\lambda \sim 0$ is required, independently of the walking-regime size. This coupling could be predicted from the existence of the BZ fixed point, namely $\gamma_\lambda = 0$. On the other hand, the lightest fermions require $\gamma_\lambda \sim 0.5\text{--}2.0$, which is subject to the size of the walking regime. As is apparent in (55), considering different excited radial, angular and spin states (encoded in $\mathcal{T}$) could facilitate generating the nondegeneracy. In this work, we do not discuss further the impact of these parameters and consider the particular case $\text{tr}[\mathcal{T}^2] = 1$.

In order to estimate the properties of the composite sector along the walking regime, the magnitude of the TC coupling, given by the fixed point solution of the beta function, is key. The MFPC matter content consists of four chiral fermions and 12 complex scalars transforming in the fundamental representation of $SU(2)_{TC}$ [18,19,21]. Altogether, the TC-matter content affects the gauge beta function equivalently to the presence of five Dirac fermions. For such field content and representation, the $SU(2)_{TC}$ gauge coupling is known to not show a BZ-like IR fixed point [51,52]. four-loop $\overline{MS}$ results show the BZ window to be present from 8 to 11 Dirac fermions. For fewer number of fermions (six to eight), the IR FP exists but is not reached due to chiral symmetry breaking triggering before reaching the conformal regime [54,55,65,72]. As

both properties, conformal scaling and chiral symmetry breaking, are necessary for the current setup, we expect the desired dynamics in the boundary of the BZ window. Therefore we assume additional matter content charged under the TC-gauge group which decouples by the exit of the walking regime. Then, we consider the beta functions with six, seven, and eight Dirac fermions and employ the four-loop $\overline{MS}$ [51–53,73,74] results to estimate h_B . For it, we recall the discussion in Sec. II E and the estimate in (43). Altogether, the estimated anomalous scaling with one, two, and three additional Dirac fermions reads

$$\gamma_\lambda \sim -1 - \frac{1}{2} \left[-\frac{N_{TC}}{32\pi^2} \left(\frac{(1 + \bar{m}_B^2) r_g (g_{TC}^*)^4}{\bar{m}_B} \right) \right], \quad (59)$$

$$\sim \{4.74, 0.47, -0.42\},$$

where we have assumed the scenario in which the gauge corrections are dominant over the Yukawa corrections, $g_{TC}^* \gg y_{TC}^*$ and considered $\bar{m}_B = 1$. Furthermore, we made the simplification $r_g \approx 1$. This rough estimate lacks a full determination of the flows of the couplings and combinatorial factors hence, must not be taken as conclusive for the validity of the models. Nevertheless, it serves at a conceptual level.

In Fig. 4, we depict the estimate for MFPC with two additional Dirac fermions with a green vertical dashed line. The green shaded region indicates a 20% variation in the estimate of h_B which serves only as a visual guide of the dependence of γ_λ on h_B .

IV. SUMMARY AND CONCLUSIONS

In this work, we have explored the generation of the SM fermion mass hierarchies in the framework of FPC. Aside from a viable model building, two key features need to be satisfied for a realization of the SM Yukawa couplings: the existence of a walking regime and an adequate scaling of the linear mixing couplings λ_f between SM fermions and composites of the new sector. While we assume the underlying FPC model to provide a walking regime, we have employed the nonperturbative fRG to address the latter point and determine whether the adequate scaling is realizable in different scenarios.

In contrast to the original idea of PC, the bound states considered here are built out of a TC-fermion and a TC-scalar field, $\mathcal{B} \sim \mathcal{SF}$. In Secs. II A and II B, we introduced the effective action formalism particularized to FPC and derived in detail the hadronization procedure for the composites of interest. This latter transformation consists in rewriting the two-TC-fermion two-TC-scalar four-point function in a specific momentum channel as the exchange of the composite. To tackle the nonperturbative phenomenon of composite emergence and perform this field redefinition in a mathematically well-defined manner, we employ the fRG. This dynamical treatment of the emergence of resonances allows us to investigate the theory in a global and complete manner, taking into account fundamental and composite degrees of freedom simultaneously as well as their interplay. Accordingly, this formalism allows us to access the properties of the composites (e.g., couplings, masses, ...) from the parameters of the fundamental effective action as shown in Sec. II E.

In Sec. II D we derived the anomalous scaling of the linear mixing couplings from the momentum scaling of the two-point functions in the effective action and resolved it in Sec. III B with the fRG. While the scaling of the vertex is found to vanish in the chiral limit, the composite anomalous dimension naturally reproduces the conformal bound. In the walking regime, we are able to reformulate the couplings of the composite action in terms of couplings of the fundamental FPC effective action. With this, we present an estimate for the MFPC scenario with two additional Dirac fermions coupled to the TC-gauge group. Lacking a full computation, this estimate does not allow us to conclusively judge the viability of the different models but furnishes a conceptual understanding of the underlying dynamics.

In general, we provide a novel application of functional methods to new physics scenarios involving strong dynamics and the emergence of composites. Using the fRG allows us to account for all quantum fluctuations and to investigate theories in a complete manner, taking into account fundamental and composite degrees of freedom simultaneously as well as their interplay. Importantly, the framework presented here provides the versatility needed for large theory space investigations required in model building.

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APPENDIX: ANOMALOUS DIMENSIONS

In this appendix, we provide the explicit computation of the composite two-point function flow equation, necessary for the determination of the anomalous dimension. In this computation we consider the TC fields to be massive to exemplify the mass-dependent nature of the fRG.

The necessary ingredients for the resolution of the diagrammatic flows in the second line of Fig. 3 are the Hessians or two-point point functions, and the full three-point functions. The former allows us access to the propagators of the fields in the theory. The regularized propagators for the dispersive modes of the different fields read

$$G_{\psi\bar{\psi},k}(p) = \left[\Gamma_{\psi\bar{\psi},k}^{(2)}(p) + R_k^\psi(p) \right]^{-1},$$

$$= \frac{-i\sigma_\mu p_\mu \left(1 + r^{(\psi)}\left(\frac{p^2}{k^2}\right) \right)}{Z_\psi \left(p^2 \left(1 + r^{(\psi)}\left(\frac{p^2}{k^2}\right) \right) + m_\psi^2 \right)}, \quad (\text{A1})$$

and

$$G_{\varphi\varphi^\dagger,k}(p) = \left[\Gamma_{\varphi\varphi^\dagger,k}^{(2)} + R_k^\varphi \right]^{-1},$$

$$= \left[Z_\varphi \left(p^2 \left(1 + r^{(\varphi)}\left(\frac{p^2}{k^2}\right) \right) + m_\varphi^2 \right) \right]^{-1}. \quad (\text{A2})$$

where φ stands for bosonic fields and ψ for fermionic fields in the super field ϕ . For the regulator functions R_k we employ the flat or Litim regulator [75]

$$R_k^\varphi(p^2) = Z_\varphi p^2 r^{(\varphi)}(p^2/k^2),$$

$$R_k^\psi(p^2) = iZ_\psi \sigma_\mu p_\mu r^{(\psi)}(p^2/k^2), \quad (\text{A3})$$

with

$$r^{(\varphi)}(x) = (-1 + 1/x)\theta(1-x),$$

$$r^{(\psi)}(x) = (-1 + 1/\sqrt{x})\theta(1-x). \quad (\text{A4})$$

This regulator choice allows for analytical access to the flow of n -point functions at vanishing momentum. For fully quantitative studies smooth regulators (in momentum space) are better suited.

The full effective vertex can be read off from the effective action (20) by performing three functional derivatives,

$$\Gamma_{S\mathcal{F}\bar{B},k}^{(3)} = \Gamma_{S^\dagger B\mathcal{F},k}^{(3)} = h_B \sqrt{Z_B Z_S Z_{\mathcal{F}}}. \quad (\text{A5})$$

The anomalous dimension can be obtained from the composite two-point function

$$\partial_t \Gamma_{B\bar{B},k}^{(2)}(p) = \partial_t [Z_B (i\sigma_\mu p_\mu + m_B)], \quad (\text{A6})$$

leading to

$$\gamma_B = \frac{\partial_t Z_B}{Z_B} = \frac{-i\partial_{p^2} \left(\text{tr} [\sigma_\mu p_\mu \partial_t \Gamma_{B\bar{B},k}^{(2)}(p)] \right)}{Z_B \text{tr} [\sigma_\mu \sigma_\mu]} \Big|_{p=0}. \quad (\text{A7})$$

Collecting all pieces and integrating the loop momenta

$$\begin{aligned} \gamma_B = & -\frac{N_{\text{TC}}}{2} h_B^2 \text{tr}[T^2] \left\{ +\partial_{p^2} \left[\int \frac{d^4 q}{(2\pi)^4} \frac{q^2 (\gamma_{\mathcal{F}}(k/q+1) - k/q) \theta(1 - q^2/k^2)}{((p+q)^2 (1 + r^{(\phi)}((p+q)^2/k^2)) + m_S^2)} \frac{(p \cdot q)(1 + r^{(\psi)}(q^2/k^2))^2}{(q^2 (1 + r^{(\psi)}(q^2/k^2)) + m_{\mathcal{F}}^2)^2} \right]_{p=0} \right. \\ & + \partial_{p^2} \left[\int \frac{d^4 q}{(2\pi)^4} \frac{q^2 (\gamma_S(k^2/q^2 - 1) + 2k^2/q^2) \theta(1 - q^2/k^2)}{(q^2 (1 + r^{(\phi)}(q^2/k^2)) + m_S^2)} \frac{(p \cdot (p+q)(1 + r^{(\psi)}((p+q)^2/k^2))^2}{((p+q)^2 (1 + r^{(\psi)}((p+q)^2/k^2)) + m_{\mathcal{F}}^2)^2} \right]_{p=0} \left. \right\}, \\ = & -\frac{N_{\text{TC}}}{2} h_B^2 \text{tr}[T^2] \left\{ + \int \frac{d^4 q}{(2\pi)^4} \frac{k^2 q^2 x_1^2 (\gamma_{\mathcal{F}}(k/q+1) - k/q) \theta(1 - q^2/k^2)}{(q^2 + (k-q)(k+q) + m_{\mathcal{F}}^2)^2 (q^2 + (k-q)(k+q) + m_S^2)^2} \right. \\ & \times [(1 - q^2/k^2) \delta(-1 + q^2/k^2) + (-1 + \theta(1 - q^2/k^2))] \\ & + \int \frac{d^4 q}{(2\pi)^4} \frac{kq(-1 + x_1^2)(k^2 + m_{\mathcal{F}}^2)(\gamma_S(k^2/q^2 - 1) + 2k^2/q^2) \theta(1 - q^2/k^2)}{(q^2 + (k-q)(k+q) + m_{\mathcal{F}}^2)^2 (q^2 + (k-q)(k+q) + m_S^2)^2} \left. \right\}, \\ = & -\frac{h_B^2}{16\pi^2} \frac{N_{\text{TC}}}{2} \text{tr}[T^2] \frac{(1 + \gamma_S/5)}{(1 + \bar{m}_{\mathcal{F}}^2)(1 + \bar{m}_S^2)^2}, \end{aligned} \quad (\text{A8})$$

where the first term of each derivation step corresponds to Fig. 3(a) and the second to Fig. 3(b). γ_S and $\gamma_{\mathcal{F}}$ are the anomalous dimensions of the TC-scalar and TC-fermion fields, respectively. In contrast with (55), the explicit contribution of the TC-field masses is here shown to illustrate the mass-dependent nature of the fRG. The integral $d^4 q$ has been performed in spherical coordinates

$$d^4 q = dq dx_1 dx_2 d\phi \sqrt{1 + x_1 q^3}, \quad (\text{A9})$$

with $x_i = \cos \theta_i$ integrated from $x_i \in [-1, 1]$. θ_1 is the angle between the external p and internal q momenta in spherical coordinates and in the two-point function, there is no dependence on a second external momenta x_2 . Last ϕ is the azimuthal angle $\phi \in [0, 2\pi]$.

Figure 3(a), in the first line of (A8), is always vanishing due to the momentum structure of the diagram. This can be read off from the first term in the second step of the derivation. This result is in agreement with previous computations of fermion-scalar systems [66,76].

One may employ other types of regulators that better implement the momentum shell integration. This computation was repeated for smooth- and exponential-type regulators. For all types of numerical regulators, we found the analytical result with (A3) in the respective limit. In the cases in which Fig. 3(a) did not vanish, its contribution was negligible in comparison to Fig. 3(b).

Aside from the regulator discussion, the result (56) can be shown to remain unaffected by regulator choices. The two-point function flow (second line in Fig. 3) can be rewritten as a total scale derivative of the uncut one-loop diagram. Consequently, the sign of the anomalous dimension is given by this diagram's contribution

$$\text{sign}[\gamma_B] = \text{sign} \left[\partial_t \left(\text{diagram} \right) \right] < 0 \quad (\text{A10})$$

and is unchanged by a cutoff insertion.

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PHASE STRUCTURE OF EXTRA-DIMENSIONAL GAUGE THEORIES WITH FERMIONS

This Appendix contains [4], one of the core papers of this thesis, contextualized in Section 4.3.

Phase structure of extra-dimensional gauge theories with fermions

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We study the phase structure of five-dimensional Yang-Mills theories coupled to Dirac fermions. In order to tackle their nonperturbative character, we derive the flow equations for the gauge coupling and the effective potential for the Aharonov-Bohm phases employing the functional renormalization group. We analyze the infrared and ultraviolet fixed-point solutions in the flow of the gauge coupling as a function of the compactification radius of the fifth dimension. We discuss various types of trajectories which smoothly connect both dimensional limits. Last, we investigate the phase diagram and vacuum structure of the gauge potential for different fermion content.

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I. INTRODUCTION

Many quantum field theories (QFTs), like the Standard Model (SM) of particle physics, are formulated as non-Abelian gauge theories. Understanding the phase structure and dynamics of this family of theories is one of the central tasks carried on by theoretical and phenomenological efforts. Whereas our Universe is well-described by a four-dimensional spacetime system, the possible emergence of extra spacetime dimensions at higher-energy scales is not yet excluded. These could be used to solve longstanding open questions in fundamental physics such as the nature of the electroweak scale [1,2]. For example, extra-dimensional gauge theories have been studied as models for gauge-Higgs unification [3–12] where one of the components in the gauge field is identified with the Higgs field. This idea has been extended to more realistic setups beyond the SM (BSM). See e.g., Refs. [13–28].

Four-dimensional ($D = 4$) non-Abelian gauge theories such as quantum chromodynamics (QCD) have been thoroughly studied with various QFT methods. One of their noteworthy features is asymptotic freedom [29,30] in the ultraviolet (UV) limit which allows us to define them as continuum QFTs. In the infrared (IR) limit, the theories evolve into a strongly interacting sector with a rich phase structure and nontrivial phenomena such as dynamical chiral symmetry breaking and color confinement. In the many-flavor limit, additional nontrivial IR fixed-point

solutions can appear. In $D = 4$, these are known as known as Caswell-Banks-Zaks (CBZ) fixed points [31,32].

Conversely, extra-dimensional gauge theories have not been as studied due to their perturbatively nonrenormalizable character. Nevertheless, they show several attractive features for example evidence for the existence of a nontrivial UV fixed-point solution providing the theory with UV completeness [33–39] and hence, asymptotic safety. Moreover, in this scenario, the theory although perturbatively not renormalizable is nonperturbatively renormalizable.

An important object to properly understanding the phase structure of gauge theories is the gauge potential. A key phenomenon in gauge theories, especially on compactified spacetimes e.g., $\mathbb{R}^4 \times S^1$, is the Hosotani mechanism [6–8] which is analogous to the Aharonov-Bohm (AB) effect in quantum mechanics. Here, the extra-dimensional component of the five-dimensional gauge field shows a nontrivial background which, depending on its configuration, can lead to the breaking of the gauge symmetry into one of its subgroups. This dynamical process can be used to formulate the SM electroweak and Higgs sectors in a more minimal manner where all degrees of freedom are embedded into a five-dimensional gauge theory.

In this work, we study the phase structure of five-dimensional Yang-Mills theories coupled with Dirac fermions transforming in different representations of the gauge group. To continuously connect with the well-known four-dimensional limit we formulate the theory in a compactified manner on $\mathbb{R}^4 \times S^1$ spacetime. To tackle the nonperturbative character of these theories, we employ the functional renormalization group (FRG) [40–55] which has proven its applicability in similar frameworks. We employ the background field approximation [56–60] and derive the flow equations employing heat-kernel techniques. For the fixed-point analysis, we help ourselves

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by employing perturbative results for the anomalous dimensions which include all fermionic contributions up to three loops. While the truncation employed in this work grants access to the qualitative features, it fails to quantitatively address the strong dynamics and the associated phenomena such as color confinement and chiral symmetry breaking. Therefore, we study the behavior of the gauge coupling and configurations of the background gauge fields depending on fermion numbers.

This work is organized as follows. After introducing the theoretical framework and the effective action formalism in Sec. II, in Sec. III we derive the flow equation for the gauge coupling and study the fixed-point solutions as a function of the compactification radius and fermion content. In Sec. IV we derive the flow of the effective potential and study the AB phase diagram as a function of the compactification radius and fermion masses for different number of fundamental and adjoint fermions. Section V is devoted to summarizing the results in this work and their relevance.

II. THEORETICAL FRAMEWORK

In this work, we employ the FRG to tackle the non-perturbative character of five-dimensional gauge theories. In QFT, the FRG is formulated as a functional partial differential equation describing the change of the effective average action (one-particle irreducible generating functional) Γ_k for the varying energy scale k . The scale dependence of Γ_k obeys the Wetterich equation [40] whose form is given by

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right], \quad (1)$$

where $\partial_t = k \partial_k$ is the dimensionless scale and Tr is the functional trace acting on all spaces in which the fields are defined. Here, $\Gamma_k^{(2)}$ is the full two-point correlation function obtained by performing the second-order functional derivative for Γ_k and $\mathcal{R}_k$ is the regulator function realizing the coarse graining process within the path integral.

We are especially interested in the impact of fermionic fluctuations on the gauge coupling and the nontrivial background field of the gauge field in five-dimensional (Euclidean) spacetime. For this purpose, we make the following ansatz for the effective action Γ_k :

$$\Gamma_k = \Gamma_{\text{gauge}} + \Gamma_{\text{fermion}} + S_{\text{gf}} + S_{\text{gh}} + \Gamma_{\text{AB}}. \quad (2)$$

Here, Γ_{gauge} is the effective action for the $SU(N_c)$ Yang-Mills gauge field,

$$\Gamma_{\text{gauge}} = \frac{Z_k}{4g^2} \int d^5x F_{MN}^a F^{aMN}, \quad (3)$$

which in the current approximation has been truncated at lowest order of gauge-invariant operators. Here, $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a - f^{abc} A_M^b A_N^c$ is the field strength of A_M^a with

f^{abc} the structure constants of $SU(N_c)$ and Z_k is the field renormalization factor for the gauge field. Note that the squared gauge coupling has a canonical mass dimension of

$$[g^2] = -1, \quad (4)$$

making apparent the power counting nonrenormalizable character of the theory. Here and hereafter, capital Latin characters $M, N, \dots (=0, \dots, 3, 5)$ stand for Lorentz indices in five-dimensional spacetime, while four-dimensional Lorentz indices are denoted by Greek letters $\mu, \nu, \dots (=0, \dots, 3)$ and indices for the fundamental and adjoint representations of $SU(N_c)$ are denoted by small Latin characters $i, j, \dots (=N_c)$ and $a, b, \dots (=N_c^2 - 1)$, respectively.

We consider fermion fields in both fundamental and adjoint representations of $SU(N_c)$, which are denoted by $\psi = (\psi)_i$ and $\chi = (\chi_a \tau^a)_{ij}$, respectively, where τ^a are generators of $SU(N_c)$ in the fundamental representation. Note that the normalization for these generators is chosen to be $\text{tr}(\tau^a \tau^b) = \frac{1}{2} \delta^{ab}$. The truncated effective action reads,

$$\begin{aligned} \Gamma_{\text{fermion}} = & \int d^5x \bar{\psi} (i Z_\psi \Gamma^M \nabla_M + m_f) \psi \\ & + \int d^5x \bar{\chi}_a (i Z_\chi \Gamma^M D_M + m_{\text{ad}})^{ab} \chi_b, \end{aligned} \quad (5)$$

where Γ_M are the Dirac matrices in five-dimensional spacetime and are defined by those in four dimensions,

$$\Gamma_M = \gamma_\mu \quad (\text{for } M = \mu = 0, \dots, 3), \quad (6)$$

$$\Gamma_5 = \gamma_5 \quad (\text{for } M = 5). \quad (7)$$

satisfying the Clifford algebra $\{\Gamma_M, \Gamma_N\} = 2\delta_{MN}$. The covariant derivatives acting on spinor fields are given respectively by

$$\nabla_M \psi = \partial_M \psi - i A_M \psi, \quad (8)$$

$$D_M \chi = \partial_M \chi - i [A_M, \chi], \quad (9)$$

where the fields in adjoint representation are $\chi = \chi_a \tau^a$ and $A_M = A_M^a \tau^a$.

Additionally in the gauge sector, the gauge fixing and ghost actions associated to the gauge symmetry read

$$S_{\text{gf}} = \frac{Z_k}{\xi} \int d^5x \text{tr}[\mathcal{F}]^2, \quad (10)$$

$$S_{\text{gh}} = Z_{\text{gh}} \int d^5x \text{tr}[\bar{c} \mathcal{M} c], \quad (11)$$

with ξ being the gauge-fixing parameter and Z_{gh} the ghost-field renormalization factor. Here, $c = \tau^a c^a$ and $\bar{c} = \tau^a \bar{c}^a$ denote the ghost and antighost fields, respectively and tr

acts on $SU(N_c)$ spaces. In this work, we intend to investigate the nontrivial background generation of A_M^a , so that it is convenient to introduce the gauge-fixing function $\mathcal{F}$ such that some interactions involving both the background and fluctuation fields in Eq. (3) are appropriately canceled. To this end, assuming the linear splitting $A_M^a = \bar{A}_M^a + a_M^a$, we give a type of the R_ξ -gauge fixing function for $\mathcal{F}$, namely

$$\mathcal{F} = \delta^{\mu\nu} \bar{D}_\mu a_\nu + \xi \bar{D}_5 a_5, \quad (12)$$

where bar on the field stands for the background gauge field and then $\bar{D}_\mu a_\nu = \partial_\mu a_\nu - i[\bar{A}_\mu, a_\nu]$ is the covariant derivative with the background gauge field $\bar{A}_M^a$ and the fluctuation a_M . Once $\mathcal{F}$ is fixed, the derivative operator $\mathcal{M}$ acting on the ghost fields is automatically determined by the BRST quantization procedure [61] to be

$$\mathcal{M} = \delta^{\mu\nu} \bar{D}_\mu D_\nu + \xi \bar{D}_5 D_5. \quad (13)$$

In this study, we choose the Feynman gauge $\xi = 1$ to appropriately cancel the $\bar{A}_5$ contributions appearing in the gauge and ghosts modes. Hereafter, we provide all formulas in this gauge.

When a five-dimensional spacial direction is compactified such that $\mathbb{R}^4 \times S^1$ in which the circle radius is denoted by $R \geq |x_5|$, its Fourier modes are discretized and known as the Kaluza-Klein (KK) modes. More specifically, the fluctuation fields for gauge and ghost fields ($\varphi \equiv a_M, c, \bar{c}$) in position space are given by

$$\varphi(x, x_5) = \sum_{n=-\infty}^{\infty} \varphi^{(n)}(x) \frac{e^{inx_5/R}}{\sqrt{2\pi R}}, \quad (14)$$

while the fermionic fields, ψ, χ , are expanded as

$$\psi(x, x_5) = \sum_{n=-\infty}^{\infty} \psi^{(n)}(x) \frac{e^{i(n+\beta_f/2\pi)(x_5/R)}}{\sqrt{2\pi R}}, \quad (15)$$

$$\chi(x, x_5) = \sum_{n=-\infty}^{\infty} \chi^{(n)}(x) \frac{e^{i(n+\beta_{ad}/2\pi)(x_5/R)}}{\sqrt{2\pi R}}. \quad (16)$$

These KK expansions are associated to the boundary conditions,

$$\varphi(x, x_5 + 2\pi R) = \varphi(x, x_5), \quad (17)$$

$$\psi(x, x_5 + 2\pi R) = e^{i\beta_f} \psi(x, x_5), \quad (18)$$

$$\chi(x, x_5 + 2\pi R) = e^{i\beta_{ad}} \chi(x, x_5), \quad (19)$$

where $\beta_f \in [0, 2\pi)$ and $\beta_{ad} \in [0, 2\pi)$.

For the compactification $\mathbb{R}^4 \times S^1$ we consider the form,

$$\bar{A}_\mu^a = 0, \quad \bar{A}_5^a = \frac{\vartheta^a}{2\pi R}, \quad (20)$$

for the background gauge field, which is written in a unified manner as $\bar{A}_M^a = \vartheta^a \delta_M^5 / (2\pi R)$. The dimensionless parameters ϑ^a are the so-called AB phases. The use of the $SU(N_c)$ generators in the fundamental representation gives the other parametrization in a diagonal form

$$(\bar{A}_5)_{ij} = \frac{1}{2\pi R} \begin{pmatrix} \theta_1 & & \\ & \ddots & \\ & & \theta_{N_c} \end{pmatrix} = \frac{\theta_i}{2\pi R} \delta_{ij}, \quad (21)$$

whose AB phases θ_i satisfy $\sum_{i=1}^{N_c} \theta_i = 0 \pmod{2\pi}$ due to the traceless property for the generators of $SU(N_c)$. This parametrization can be obtained by an $SU(N_c)$ transformation $\mathcal{U}$ such that $\mathcal{U}^\dagger \vartheta^a \tau^a \mathcal{U} = \text{diag}(\theta_1, \dots, \theta_{N_c})$. Inserting the KK expansions exhibited in Eq. (14), the covariant derivative $\bar{D}_5^2$ acting on φ and the background field (20) gives rise to the effective masses of $\varphi_{ij}^{(n)} = (\varphi_a^{(n)} \tau^a)_{ij}$,

$$M_{ij,n}^2 = \frac{1}{R^2} \left(n - \frac{\theta_i - \theta_j}{2\pi} \right)^2. \quad (22)$$

We note here that the difference $\theta_i - \theta_j$ originates from the commutator involved in the covariant derivative acting on the fields, $(\bar{D}_M \varphi)_{ij} = (\bar{D}_M \varphi^a \tau^a)_{ij} = \partial_M \varphi_{ij} - i[\bar{A}_5, \varphi]_{ij} = \partial_M \varphi_{ij} - \frac{i}{2\pi R} (\theta_i - \theta_j) \varphi_{ij}$. It is clear now that even for the zero KK mode $n = 0$, the gauge field acquires a finite mass from the AB phases for finite values of R . Furthermore, the phenomenon that the gauge symmetry is spontaneously broken by finite AB phase is known as the Hosotani mechanism [6–8]. See also Refs. [9,10]. In the five-dimensional spacetime limit $R \rightarrow \infty$, all KK modes become massless (for vanishing AB phases) and thus are dynamical degrees of freedom. In the four-dimensional limit ($R \rightarrow 0$), all KK modes are infinitely massive and only the massless zero KK mode ($n = 0$) $\varphi^{(0)}$ contributes to the quantum dynamics in the system. The covariant derivatives acting on the fermionic fields given in Eqs. (8) and (9) now read,

$$\nabla_M \psi = \partial_M \psi - ia_M \psi - \bar{A}_5 \psi, \quad (23)$$

$$D_M \chi = \partial_M \chi - i[a_M, \chi] - i[\bar{A}_5, \chi]. \quad (24)$$

Thus, together with the Dirac masses in Eq. (5), the effective squared masses for the fermionic fields $\psi_i^{(n)}$ and $\chi_{ij}^{(n)} = (\chi_a^{(n)} \tau^a)_{ij}$ are found to be

$$M_{f,i,n}^2 = m_f^2 + \frac{1}{R^2} \left(n + \frac{\beta_f - \theta_i}{2\pi} \right)^2, \quad (25)$$

$$M_{ad,ij,n}^2 = m_{ad}^2 + \frac{1}{R^2} \left(n + \frac{\beta_{ad} - (\theta_i - \theta_j)}{2\pi} \right)^2. \quad (26)$$

Finally, Γ_{AB} contains the effective action for AB phases,

$$\Gamma_{AB} = \int d^5x V_{5D}(\theta_H) = \int d^4x (2\pi R) V(\theta_H), \quad (27)$$

where $\theta_H = \{\theta_1, \dots, \theta_{N_c}\}$ is a set of AB phases and we have performed the integral for the fifth direction of spacetime $\int_0^{2\pi R} dx_5 = 2\pi R$.

For the total effective action (2), the Wetterich equation (1) is expressed diagrammatically as

$$\begin{aligned} \partial_t \Gamma_k = & \frac{1}{2} \text{(curly loop)} + \frac{1}{2} \text{(dashed loop)} - \text{(dotted loop)} \\ & - N_f \text{(solid loop)} - N_{ad} \text{(double solid loop)}, \end{aligned} \quad (28)$$

where curly, dashed, and dotted lines represent the propagators of the four-dimensional gauge ($M = 0, \dots, 3$), the extra-dimensional gauge field component ($M = 5$) and ghost fields, respectively. Single and double solid lines stand for those of fundamental and adjoint fermion fields, respectively, and the crossed circles denote the insertion of $\partial_t \mathcal{R}_k$.

A crucial object for deriving the beta functions from the Wetterich equation is the regulated propagators (inverse two-point functions). For the gauge and ghost fields these have been computed in Ref. [39]. The second-order functional derivatives for the fermion fields in the effective action (5) on the background field (20) with respect to the pairs of $(\psi_i^{(n)}, \bar{\psi}_i^{(n)})$ and $(\chi_{ij}^{(n)}, \bar{\chi}_{ij}^{(n)})$ yield,

$$\Gamma_{\bar{\psi}\psi}^{(2)}|_{a_M=0} = i\hat{p} + \frac{i}{R} \left(n + \frac{\beta_f - \theta_i}{2\pi} \right) \gamma^5 + m_f, \quad (29)$$

$$\Gamma_{\bar{\chi}\chi}^{(2)}|_{a_M=0} = i\hat{p} + \frac{i}{R} \left(n + \frac{\beta_{ad} - (\theta_i - \theta_j)}{2\pi} \right) \gamma^5 + m_{ad}, \quad (30)$$

where $\hat{p} = \gamma^\mu p_\mu$ with p_μ being the four-dimensional momenta.

The cutoff regulator function is introduced as $\mathcal{R}_k(p) = i\hat{p}r_k(p^2/k^2)$ which replaces $i\hat{p}$ in the inverse two-point functions to $i\hat{p}(1 + r_k^F(p^2/k^2))$. In this work, we use the Litim-type cutoff function [62], i.e., $r_k^F(x) = (1/\sqrt{x} - 1)\theta(1 - x)$ where $\theta(1 - x)$ is the step function. The denominators of the regulated propagators for $\psi_i^{(n)}$ and $\chi_{ij}^{(n)}$ take the forms of $P_k^F(p) + M_{f,i,n}^2$ and $P_k^F(p) + M_{ad,ij,n}^2$, respectively, in which we defined $P_k^F(p) = p^2(1 + r_k^F(p^2/k^2))^2$. Note that for the bosonic fields

$\varphi_{ij}^{(n)}$, we employ the cutoff function $\mathcal{R}_k(p)$ such that the four-dimensional squared momentum p^2 is replaced to $P_k^B(p) = p^2(1 + r_k^B(p^2/k^2))$ with $r_k^B(x) = (1/x - 1)\theta(1 - x)$. For the momentum integral which is a part of the functional trace in the Wetterich equation, both P_k^F and P_k^B turn to k^2 because of $\theta(1 - p^2/k^2)$.

In the following sections, we derive the flow equations for $V(\theta_H)$ and the gauge coupling g^2 . To this end, we use the heat-kernel method as previously developed in [39]. For obtaining the potential of AB phases, we project out $\bar{A}_5^a$ (or equivalently ϑ^a) from loop effects. On the other hand, the beta function for the gauge coupling is extracted by projecting on $\bar{F}_{\mu\nu} \bar{F}^{a\mu\nu}$ which is the field strength for $\bar{A}_\mu^a$. Hence, to derive the flow equation for the gauge coupling, we assume the background field of the gauge field such that $\bar{F}_{MN}^a \bar{F}^{aMN} = \bar{F}_{\mu\nu}^a \bar{F}^{a\mu\nu}$ for which the effective action (3) reads,

$$\frac{Z_k}{4g^2} \int d^5x \bar{F}_{MN}^a \bar{F}^{aMN} = \frac{2\pi R Z_k}{4g^2} \int d^4x \bar{F}_{\mu\nu}^a \bar{F}^{a\mu\nu}. \quad (31)$$

The scale derivative on the left-hand side of the Wetterich equation acts on Z_k , while we extract the operator $\int d^4x \bar{F}_{\mu\nu}^a \bar{F}^{a\mu\nu}$ with loop coefficients and the threshold functions from the right-hand side. Note that $g_{4D}^2 = g^2/(2\pi R)$ is identified with the dimensionless gauge coupling in four-dimensional spacetime.

III. FLOW OF THE GAUGE COUPLING AND FIXED POINTS

In this section, we derive the flow of the gauge coupling in the background field approximation and analyze the fixed-point solutions as a function of the gauge group, fermion content, and the effective dimension. Additionally, we discuss interdimensional trajectories which smoothly connect all UV and IR solutions found.

A. Flow equation for the gauge coupling

We first deal with the gauge coupling flow whose detailed derivation except for fermionic loop effects has been discussed in Ref. [39]. Here, we exhibit the final form of the flow equation. The running of the gauge coupling arises from that of Z_k for which we define the anomalous dimension of the gauge field as

$$\begin{aligned} \eta_g = & -\frac{\partial_t Z_k}{Z_k} \\ = & -\frac{1}{(4\pi)^2} \frac{Z_k^{-1} g^2}{2\pi R} \left\{ +7N_c \left(1 - \frac{19\eta_g}{42} + \frac{\eta_{gh}}{21} \right) J(\bar{R}; 0) \right. \\ & - \frac{4}{3} N_f (1 - \eta_f) J(\bar{R}; \bar{m}_f) \\ & \left. - \frac{8}{3} N_c N_{ad} (1 - \eta_{ad}) J(\bar{R}; \bar{m}_{ad}) \right\}, \end{aligned} \quad (32)$$

where the anomalous dimensions for each of the remaining fields read $\eta_{\text{gh}} = -\partial_t Z_{\text{gh}}/Z_{\text{gh}}$, $\eta_f = -\partial_t Z_f/Z_f$ and $\eta_{\text{ad}} = -\partial_t Z_{\text{ad}}/Z_{\text{ad}}$. The threshold functions for massive fields read,

$$J(\bar{R}; \bar{m}) = \sum_{n=-\infty}^{\infty} \frac{1}{1 + (\frac{n}{\bar{R}})^2 + \bar{m}^2} = \frac{\pi \bar{R} \coth(\sqrt{1 + \bar{m}^2} \pi \bar{R})}{\sqrt{1 + \bar{m}^2}}, \quad (33)$$

with dimensionless quantities $\bar{R} = Rk$ and $\bar{m}^2 = m^2/k^2$. The first term in the curly brackets on the right-hand side of Eq. (32) corresponds to the contributions from gauge and ghost fields, while the second and third, to quantum corrections from N_f fundamental fermions and N_{ad} adjoint ones, respectively.

Let us now define the dimensionless renormalized gauge coupling [39] as

$$\tilde{g}^2 = \frac{Z_k^{-1} g^2}{2\pi R} J(\bar{R}; 0) = Z_k^{-1} g_{4D}^2 J(\bar{R}; 0). \quad (34)$$

Acting ∂_t on this yields the flow equation for $\tilde{g}$ as

$$\partial_t \tilde{g}^2 = \beta_{\tilde{g}^2} = (d_{\text{eff}}(\bar{R}) - 4 + \eta_g) \tilde{g}^2. \quad (35)$$

Here, we have defined the effective dimension

$$d_{\text{eff}}(\bar{R}) = 4 + \frac{d \log J(\bar{R}; 0)}{d \log \bar{R}}, \quad (36)$$

as a function of $\bar{R}$ [39,63]. As depicted in Fig. 1, the limit $\bar{R} \rightarrow \infty$ corresponds to $D = 5$ and gives $d_{\text{eff}}(\infty) = 5$, while in the $D = 4$ limit ($\bar{R} \rightarrow 0$), we have $d_{\text{eff}}(0) = 4$. Hence, Eq. (36) allows us to smoothly and continuously interpolate the spacetime dimensionality of the system depending on the compactification radius.

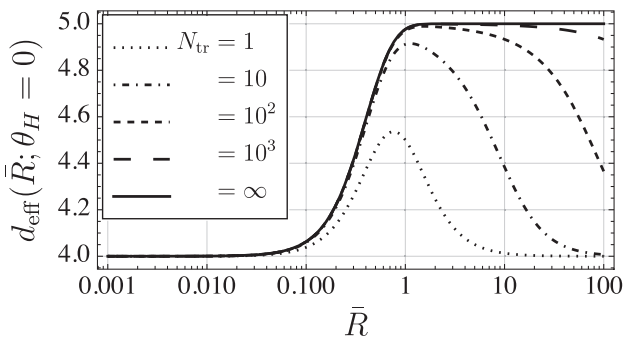


FIG. 1. Effective dimension for a trivial gauge background and as a function of the dimensionless compactification radius for different truncations of the KK-mode tower (N_{tr}). Figure adapted from Ref. [39].

The anomalous dimension η_g appears on both sides of Eq. (32) and hence can easily be solved so as to be

$$\eta_g = \frac{-\frac{\tilde{g}^2}{(4\pi)^2} \left(\frac{N_c}{3} (22 - 1) - \frac{4}{3} N_f - \frac{8}{3} N_{\text{ad}} N_c \right)}{1 - \frac{\tilde{g}^2}{(4\pi)^2} \frac{19}{6} N_c}, \quad (37)$$

where for simplicity we have considered massless fermions ($\bar{m}_f = \bar{m}_{\text{ad}} = 0$) and have approximated $\eta_{\text{ad}} = \eta_f = \eta_{\text{gh}} \approx 0$.

By performing a series expansion in the gauge coupling, we have

$$\eta_g = -\frac{\tilde{g}^2}{(4\pi)^2} \left(\frac{N_c}{3} (22 - 1) - \frac{4}{3} N_f - \frac{8}{3} N_c N_{\text{ad}} \right) + \mathcal{O}(\tilde{g}^4), \quad (38)$$

where now the various matter contributions can easily be identified and compared to the well-known four-dimensional structure of non-Abelian gauge theories. The shown anomalous dimension at the one-loop level in Eq. (38) or the numerator of Eq. (37) coincides with the beta function for a $SU(N_c)$ gauge group in the presence of N_c real scalar fields transforming in the adjoint representation and N_f fundamental and N_{ad} adjoint fermions. The scalar-mode contribution appears from the emergence of an additional mode in the fifth-dimensional gauge field A_5 . Note that fermionic fields do not induce additional modes as five-dimensional Dirac fields are considered. While the anomalous dimension η_g has a correct one-loop structure, higher-order contributions disagree due to lack of inclusion of some diagrams in the background approximation. In particular, higher-order fermionic effects are missing as can be seen from the fact that the denominator of Eq. (37) depends neither on N_f nor on N_{ad} . These contributions can partially be included when feeding back the anomalous dimensions in Eq. (32) arising from the cut lines in Eq. (28). In fact, these terms are relevant in the $g \gg 0$ limit where the contributions from the anomalous dimensions are not negligible and cannot be dropped.

Furthermore, we comment on the $\bar{R}$ dependence in the resummed anomalous dimension (37). No compactification radius dependence appears explicitly after the redefinition of the gauge coupling. However, this is a special case in which the anomalous dimensions contain only one specific class of loop diagrams which yield the same form of the threshold function $J(\bar{R}, 0)$ as in Eq. (33). See the left-hand side diagram in Fig. 2. Consequently, the flow equation for the gauge coupling takes a form as

$$\partial_t \left(\frac{Z_k^{-1} g^2}{2\pi R} \right) = \left(c_1 J(\bar{R}, 0) \left(\frac{Z_k^{-1} g^2}{2\pi R} \right) + c_2 [J(\bar{R}, 0)]^2 \left(\frac{Z_k^{-1} g^2}{2\pi R} \right)^2 + \dots \right) \frac{Z_k^{-1} g^2}{2\pi R}, \quad (39)$$

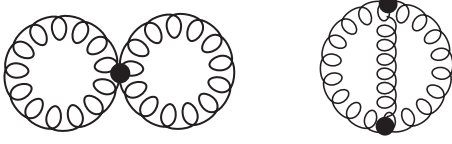


FIG. 2. Examples of two-loop diagrams. The left-hand side diagram produces the two-loop effect proportional to $[J(\bar{R}, 0)]^2$, where $J(\bar{R}, 0)$ is defined in Eq. (33), while the right-hand side one gives a new type of threshold function $I(\bar{R}, 0)$ different from $J(\bar{R}, 0)$.

where c_i depend only on N_f and N_c . The redefinition of the gauge coupling (34) leads to

$$\partial_t \tilde{g}^2 = [(d_{\text{eff}}(\bar{R}) - 4) + (c_1 \tilde{g}^2 + c_2 \tilde{g}^4 + \dots)] \tilde{g}^2. \quad (40)$$

Here, the term in the second parentheses in square brackets corresponds to the anomalous dimension (37) free from the compactification radius. However, in more general case beyond the one-loop resummation such as the right-hand side diagram in Fig. 2, explicit $\bar{R}$ dependencies appear that are not absorbed by the definition of $\tilde{g}$. These cannot be appreciated in the background anomalous dimension given that only accounts for one class of diagrams with the same $\bar{R}$ dependence properly absorbed in the definition of $\tilde{g}$. More specifically, the flow equation reads,

$$\begin{aligned} \partial_t \left(\frac{Z_k^{-1} g^2}{2\pi R} \right) &= \left[c_1 J(\bar{R}, 0) \left(\frac{Z_k^{-1} g^2}{2\pi R} \right) \right. \\ &\quad + (c_2 [J(\bar{R}, 0)]^2 + d_2 I(\bar{R}, 0) + \dots) \\ &\quad \left. \times \left(\frac{Z_k^{-1} g^2}{2\pi R} \right)^2 + \dots \right] \left(\frac{Z_k^{-1} g^2}{2\pi R} \right), \end{aligned} \quad (41)$$

for which after the redefinition of the gauge coupling (34), we have

$$\begin{aligned} \partial_t \tilde{g}^2 &= \left[(d_{\text{eff}}(\bar{R}) - 4) \right. \\ &\quad \left. + \left\{ c_1 \tilde{g}^2 + \left(c_2 + d_2 \frac{I(\bar{R}, 0)}{J(\bar{R}, 0)} + \dots \right) \tilde{g}^4 + \dots \right\} \right] \tilde{g}^2, \end{aligned} \quad (42)$$

where a new threshold function $I(\bar{R}, 0)$ with a factor d_2 has been introduced. $\bar{R}$ dependences such as $I(\bar{R}, 0)/J(\bar{R}, 0)$ remain. In the following section, we discuss how the resumed computation can be improved by adopting $\overline{\text{MS}}$ results.

B. UV and IR fixed points

Let us study the fixed-point structure of the gauge coupling using the flow equation (35) and as a function of the compactification radius. Yang-Mills theories coupled

to fermions are known to feature a very rich phase structure in the four-dimensional limit ($\bar{R} \rightarrow 0$). Depending on the number of fermions and the representation under which they transform, we can find different fixed-point solutions. For example, in the QCD-like scenario with small number of fundamental fermions, the theory has a trivial UV fixed point ($\tilde{g}_* = 0$) associated to asymptotic freedom [29,30] and in the IR regime the system turns strongly coupled. Increasing the number of fermions from the QCD scenario, nontrivial IR solutions appear in the beta function. These are known as CBZ fixed points and are roots of the beta function caused by the cancellations of different loop orders. Therefore, contributions beyond one loop are crucial in this limit. Furthermore, in the many fermion limit, asymptotic freedom is lost leading to a UV Landau pole and the nontrivial IR fixed point evolves into a trivial (Gaussian) solution.

Here, we analyze the fixed-point solutions as a function of the effective dimensionality parametrized by the dimensionless compactification radius $\bar{R}$. For simplicity, we consider the massless fermion limit and with the periodic boundary conditions $\beta_f = \beta_{\text{ad}} = 0$. The presence of masses simply triggers the decoupling of fermionic degrees of freedom and hence the UV and IR fixed points are not significantly affected by such parameters. However, masses, boundary conditions or a nontrivial vacuum structure will leave imprints in the integrated flows at finite $\bar{R}$ and k , see Ref. [39]. We denote $g_*^2(\bar{R}, N_f, N_{\text{ad}}, N_c)$ as the nontrivial fixed-point solutions for arbitrary compactification radius, number of fermionic fields and colors. These solutions are found in Eq. (35) by solving $d_{\text{eff}}(\bar{R}) - 4 + \eta_g = 0$,

$$\tilde{g}_*^2(\bar{R}, N_f, N_{\text{ad}}, N_c) = \frac{(d_{\text{eff}}(\bar{R}) - 4)(4\pi)^2}{b_0 + c(d_{\text{eff}}(\bar{R}) - 4)}, \quad (43)$$

where $b_0 = 7N_c - \frac{4}{3}N_f - \frac{8}{3}N_{\text{ad}}N_c$ and $c = \frac{19}{6}N_c$. For $\bar{R} = 0$ ($D = 4$), the fixed-point solution (43) vanishes leading to the trivial UV fixed point characterizing asymptotic freedom. As the compactification radius increases, g_*^2 takes a finite value. This fact provides evidence for the nonperturbatively renormalizability, in sense of asymptotic safety [64,65], of the theory. See the discussion at the end of this section.

From Eq. (35) we find that in the absence of threshold effects, the dependence on the compactification radius only enters via the effective dimensionality $d_{\text{eff}}(\bar{R})$. As stated in the end of the previous subsection, this follows from the background approximation only accounting for one specific family of diagrams producing $J(\bar{R}, 0)$ whose $\bar{R}$ dependence is fully absorbed in the definition of $\tilde{g}$. Moreover, neglecting these dependencies permits us to conveniently improve Eq. (43) by implementing η_g 's from computations accounting for the complete higher-order fermionic contributions lacking in the background

approximation. Furthermore, we will employ three-loop $\overline{\text{MS}}$ results [66–68] for η_g instead of Eq. (37) for the following study. This approximation implicitly assumes the ratio $I(\bar{R}, 0)/J(\bar{R}, 0) \approx 1$ and hence the $\bar{R}$ dependence of the diagrams not accounted in the background approximation to be small.

Before proceeding with the analysis, let us first investigate the appearance of fixed point solutions entertaining a perturbative expansion of the anomalous dimension in Eq. (35),

$$4 - d_{\text{eff}}(\bar{R}) = \eta_g = \tilde{g}_*^2(\bar{R})[f^{(1)} + \tilde{g}_*^2(\bar{R})f^{(2)}(\bar{R}) + \dots]. \quad (44)$$

Here, $f^{(i)}$ are the i th-order loop contributions dependent on the gauge group and fermion content. While due to the redefinition of the gauge coupling (34), the full R -dependence of the one-loop coefficient $f^{(1)}$ is translated into the effective dimension (36), for higher-loop terms a residual contribution remains in the $f^{(i>1)}$ functions given the nontrivial powers of the threshold function (33). In $D = 4$ ($R \rightarrow 0$), we find two solutions vanishing the right-hand side of Eq. (44),

$$\begin{cases} \tilde{g}_*^2(0) = 0 & \text{(Gaussian),} \\ f^{(1)} + \tilde{g}_*^2(0)f^{(2)}(0) + \dots = 0 & \text{(CBZ).} \end{cases} \quad (45)$$

While the Gaussian fixed point is a UV fixed point and characterises asymptotic freedom, the nontrivial CBZ fixed point is an IR solution given as a root of the several loop contributions in polynomial of g^2 .

On the other hand, in the $D = 5$ ($R \rightarrow \infty$) limit, the nontrivial fixed-point solution is obtained when

$$\tilde{g}_*^2(\infty)f^{(1)} + \tilde{g}_*^4(\infty)f^{(2)}(\infty) + \dots = 4 - d_{\text{eff}}(\infty) = -1, \quad (46)$$

is satisfied. In this limit, the roots of the beta function cannot be only linked to the roots of the anomalous dimension and hence the new fixed points cannot be related to the four-dimensional CBZ solution.

In Fig. 3 we display the regions on the $N_c - N_f/N_{\text{ad}}$ planes where the beta function for the gauge coupling (35) shows together both nontrivial UV and IR fixed points. In the $\bar{R} \rightarrow 0$ limit, we find the nontrivial IR fixed-point solutions for N_f and N_{ad} which satisfy $\eta_g = 0$. This region is known as the CBZ window and their properties such as boundaries and magnitude of the fixed point have been widely studied with perturbative [32,68–75], and nonperturbative

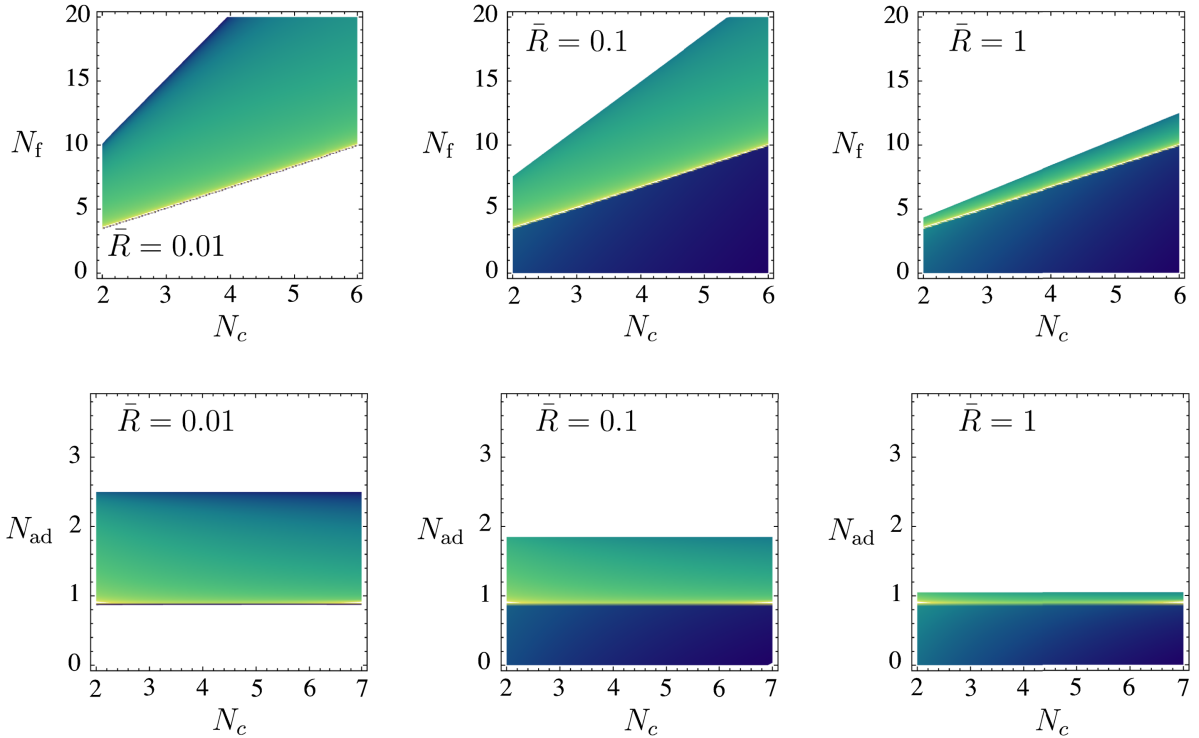


FIG. 3. In the top (bottom) row, the nontrivial UV and IR fixed-point solutions for an $SU(N_c)$ gauge coupling in the presence of N_f (N_{ad}) fermion fields transforming in the fundamental (adjoint) representations and at different compactification radii; $\bar{R} = 0.01$ (left column), $\bar{R} = 0.1$ (center column), and $\bar{R} = 1$ (right column). Brighter colors indicate a larger magnitude of the fixed point. In order to account for beyond one-loop fermionic contributions, we have employed three-loop $\overline{\text{MS}}$ results in the anomalous dimension η_g in Eq. (35).

methods [76–78] as well as lattice simulations [79–85]. For a larger number of fermions, the nontrivial IR fixed point turns into a Gaussian fixed point, losing asymptotic freedom in the UV limit. It is important to recall a difference with the standard gauge-fermion CBZ window picture; in the current $D = 5$ scenario there are additional N_c scalar degrees of freedom in the $\bar{R} \rightarrow 0$ limit associated to the five-dimensional modes of the gauge field. Their contribution reflects in a subtle fewer number of fermionic degrees of freedom necessary to reach the conformal window.

As the compactification radius is increased, the growth of dimensionality causes the canonical dimension of the gauge coupling to increment turning the marginal gauge coupling into a relevant one. Moreover, as was shown in Ref. [38,39], pure Yang-Mills theories displaying asymptotic freedom in the UV limit evolve to asymptotic safety. This is evidently seen in the appearance of a nontrivial fixed point in the lower boundary of the CBZ window and below. The increment of the canonical dimension of the gauge coupling contributes positively to the flow alike the addition of fermionic matter. Hence, increasing the dimensionality will require a lower number of fermions to trigger a singular solution. This causes the CBZ window to shrink with the increment of $\bar{R}$. However, we note a remanent of the CBZ fixed point at $\bar{R} \rightarrow \infty$ in the lower boundary of the window (bright yellow band), leading to the simultaneous presence of IR and UV fixed-point solutions. This will be discussed in detail below.

The dependence of the beta function on the change of dimension is best studied as a function of the gauge coupling. This is shown in Fig. 4. Here, both of the approximations employed for η_g , the resummed shown in Eq. (37) (black lines)

and the three-loop perturbative (red-orange lines), are shown for different $\bar{R}$. While both computations agree well at small $\tilde{g}^2$, the resummed anomalous dimension lacks to reproduce the nontrivial CBZ fixed point as does not account for the complete two-loop and higher structure.

As shown in Ref. [39], in the pure Yang-Mills or QCD-like limit, the beta function experiences a relevant transformation as the gauge coupling's dimension changes with the compactification radius. In the left most panels of Fig. 4, it can be seen how the beta function with a single Gaussian UV fixed point develops an additional nontrivial solution. In other words, the change in dimensionality turns the Gaussian UV fixed point into an IR fixed-point solution and generates a nontrivial UV fixed point.

Furthermore, configurations which in $D = 4$ show a CBZ fixed point can evolve to two different scenarios. In the upper boundary of the CBZ window, where $g_*^2(\bar{R} = 0, N_f, N_{ad}, N_c)$, the nontrivial IR fixed point is lost leading to a five-dimensional theory with a trivial IR and a UV Landau pole (third plot in each row in Fig. 4). In the second case, we find that the IR CBZ fixed point does not disappear in the $\bar{R} \rightarrow \infty$ limit and as the new nontrivial UV fixed point emerges. This leads to three solutions being present and to UV and IR finite trajectories to be realizable (second plot in each row of Fig. 4).

The many flavor limit is unaffected by the change of dimensionality. This can be seen in the last panels of each row in which the five-dimensional beta function does not present any new solutions.

In Fig. 5, we depict different trajectories of the integrated gauge coupling flow as a function of the cutoff scale and considering various onsets of the extra dimension. For

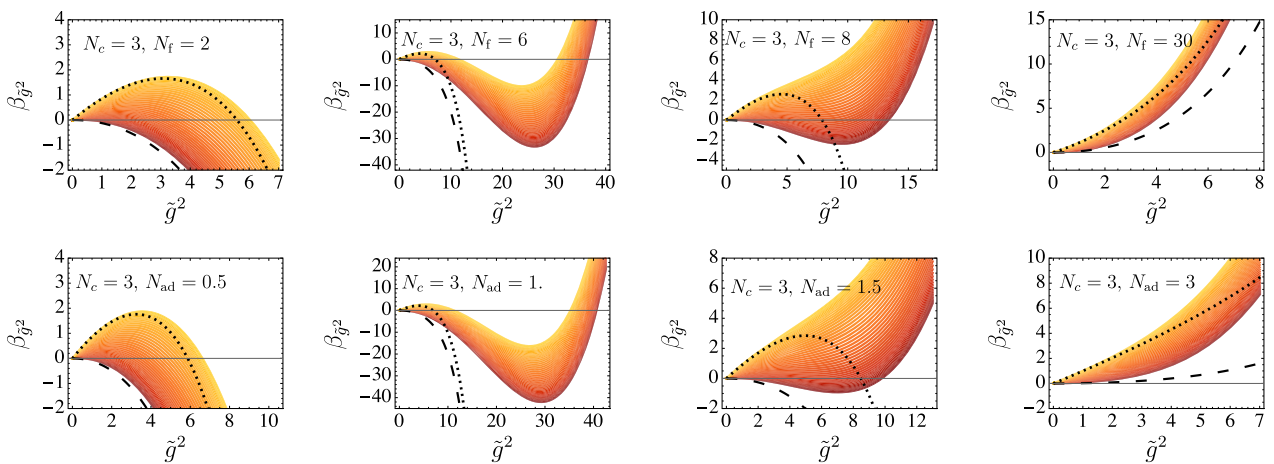


FIG. 4. $SU(3)$ gauge coupling beta functions as a function of the coupling for different fermion content in the fundamental (top row) and adjoint (bottom row) representations. As plain lines from red (dark) to yellow (bright), we show the evolution from $\bar{R} = 0$ to $\bar{R} \rightarrow \infty$ employing the three-loop $\overline{MS}$ results. In black we show the resummed beta functions at $\bar{R} = 0$ (black dashed) and $\bar{R} \rightarrow \infty$ (black dotted). The resummed anomalous dimension does not show the CBZ fixed point in the $\bar{R} = 0$ as lacks the complete two-loop contributions. Nevertheless, in the large and few N_f, N_{ad} limits, both beta functions qualitatively agree. Noninteger number of fermions are employed in some instances for best display of qualitative features.

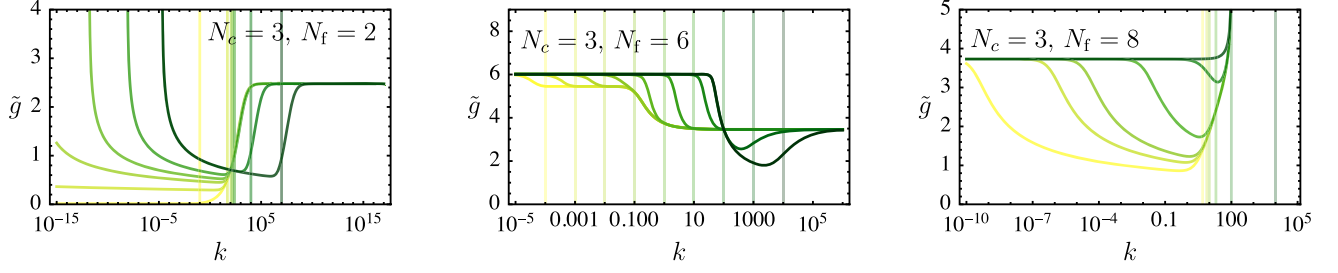


FIG. 5. Integrated renormalization group flows of the $SU(3)$ coupling for different fermion content. From left to right, we increase the number of fermions $N_f = 2, 6, 8$ transforming in the fundamental representation, while $N_{ad} = 0$. In each plot we show different curves for which the onset of the extra-dimension triggers at different scales. The vertical lines denote the onset of the extra dimension where $R/k = 1$ is satisfied.

a fixed number of colors $N_c = 3$ and $N_f = 2$, the inter-dimensional flows connect a four-dimensional strong IR QCD-like scenario with a UV finite five-dimensional setting as shown in the left-most panel of Fig. 5. Here, from bright yellow to dark green the onset scale of the extra dimension is delayed with respect to the cutoff scales k . Depicted by vertical lines with the respective color of the trajectory, we show the scale at which $R/k = 1$ and hence, the onset of the extra dimension.

For $N_f = 6$ we find a particularly interesting scenario (middle panel in Fig. 5). From the UV, we commence the flow at approximately the nontrivial fixed point in the $D = 5$ limit. If the system is kept in the $\bar{R} \rightarrow \infty$ limit, trajectories evolve towards either a Gaussian or interacting IR fixed point. The former can be appreciated in the darkest trajectories and the latter in the brightest. However, as soon as the system senses the $D = 4$ limit, both IR solutions are attracted towards the CBZ fixed point.

The rightmost plot in Fig. 5 shows trajectories for $N_f = 8$ in which the CBZ solution exists in the $D = 4$ limit but no fixed point is present in $D = 5$.

Finally, we discuss the critical exponent of the gauge coupling denoted here by $\nu_{\tilde{g}^2}$. At a fixed point $\tilde{g}_*^2$, the gauge coupling behaves as a power law,

$$\tilde{g}^2 \sim k^{-\nu_{\tilde{g}^2}^{-1}}. \quad (47)$$

From the beta function of the gauge coupling (35), the critical exponent is given by

$$\begin{aligned} \nu_{\tilde{g}^2}^{-1} &= -\left. \frac{\partial \beta_{\tilde{g}^2}}{\partial \tilde{g}^2} \right|_{\tilde{g}^2 = \tilde{g}_*^2} \\ &= -(d_{\text{eff}}(\bar{R}) - 4 + \eta_g)|_{\tilde{g}^2 = \tilde{g}_*^2} - \left. \frac{\partial \eta_g}{\partial \tilde{g}^2} \right|_{\tilde{g}^2 = \tilde{g}_*^2} \tilde{g}_*^2. \end{aligned} \quad (48)$$

At the Gaussian fixed point $\tilde{g}_*^2 = 0$ for which $\eta_g = 0$, one obtains $\nu_{\tilde{g}^2}^{-1} = 4 - d_{\text{eff}}(\bar{R})$. Since $d_{\text{eff}}(\bar{R}) > 4$ for finite $\bar{R}$ as shown in Fig. 1, we observe $\nu_{\tilde{g}^2}^{-1} < 0$ and therefore the gauge coupling at the Gaussian fixed point is an irrelevant

coupling in extra-dimensional spacetimes. In other words, the Gaussian fixed point is an IR attractive fixed point. The gauge coupling becomes marginal only when $d_{\text{eff}}(\bar{R} = 0) = 4$. As is well-known, Yang-Mills theories in $D = 4$ exhibit the marginally relevant behavior of the gauge coupling if the fermion numbers do not exceed a certain threshold. This fact can be seen from $\eta_g < 0$ for a small perturbation from the Gaussian fixed point as $0 < g_*^2 \ll 1$. In such a case, one has $\nu_{\tilde{g}^2}^{-1} \ll 1$ so that,

$$\tilde{g}^2 \sim e^{-\log k / \nu_{\tilde{g}^2}^{-1}} \sim \nu_{\tilde{g}^2}^{-1} \log k. \quad (49)$$

This logarithmic running is a natural result inferred from the perturbative computation.

On the other hand, at the nontrivial fixed points $\tilde{g}_*^2 \neq 0$, the first term on the right-hand side in Eq. (48) vanishes, so that the critical exponent reads,

$$\nu_{\tilde{g}^2}^{-1} = -\left. \frac{\partial \eta_g}{\partial \tilde{g}^2} \right|_{\tilde{g}^2 = \tilde{g}_*^2} \tilde{g}_*^2. \quad (50)$$

For the anomalous dimension (37), there is the nontrivial UV fixed point (43) at which the critical exponent is found to be

$$\nu_{\tilde{g}^2}^{-1} = \frac{(b_0 + c(d_{\text{eff}}(\bar{R}) - 4))(d_{\text{eff}}(\bar{R}) - 4)}{b_0}. \quad (51)$$

For $\bar{R} > 0$, we see that $\nu_{\tilde{g}^2}^{-1} > 0$ for $b_0 + c(d_{\text{eff}}(\bar{R}) - 4) > 0$, namely the gauge coupling is relevant. For large fermion numbers, $b_0 + c(d_{\text{eff}}(\bar{R}) - 4) < 0$, leading to the fixed point (43) turning positive. Hence, for positive values of the nontrivial UV fixed point, the corresponding critical exponent always takes positive values, implying a relevant direction.

The existence of such a nontrivial UV fixed point provides evidence for extra-dimensional Yang-Mills theories to be nonperturbatively renormalizable in terms of asymptotic safety. However, the current setup for the truncated effective action cannot determine the number

of relevant couplings and hence, the nonperturbative renormalizability of the theory cannot be concluded. In order to determine the dimensionality of the UV critical surface, higher-dimensional operators such as $(F_{\mu\nu}F^{\mu\nu})^n$ have to be taken into account. This project will be reported elsewhere.

IV. GAUGE POTENTIAL AND AHARONOV-BOHM PHASES

In this section, we study the phase structure of the four- and five-dimensional gauge effective. To this end, we consider the effective potential for the AB phases and the vacuum structure in the presence of fermions. The relevant flow equation for the effective potential can be obtained employing heat-kernel techniques and reads

$$\begin{aligned} \partial_i V(\theta_H) = & + \frac{5k^4}{2(4\pi)^2} \left(1 - \frac{\eta_g}{6}\right) \sum_{i,j=1}^{N_c} h(\bar{R}; \theta_i - \theta_j; 0) \\ & - \frac{k^4}{(4\pi)^2} \left(1 - \frac{\eta_{gh}}{6}\right) \sum_{i,j=1}^{N_c} h(\bar{R}; \theta_i - \theta_j; 0) \\ & - \frac{2N_{ad}k^4}{(4\pi)^2} \left(1 - \frac{\eta_{ad}}{5}\right) \sum_{i,j=1}^{N_c} h(\bar{R}; \theta_i - \theta_j - \beta_{ad}; \bar{m}_{ad}) \\ & - \frac{2N_fk^4}{(4\pi)^2} \left(1 - \frac{\eta_f}{5}\right) \sum_{i=1}^{N_c} h(\bar{R}; \theta_i - \beta_f; \bar{m}_f). \end{aligned} \quad (52)$$

Here, the threshold function with a mass and AB phases is given by

$$h(\bar{R}; \theta_H; \bar{m}) = \frac{i\pi\bar{R}}{2\sqrt{1+\bar{m}^2}} \left[\cot\left(\frac{1}{2}\bar{R}(\theta_H + 2i\pi\sqrt{1+\bar{m}^2})\right) + \cot\left(-\frac{1}{2}\bar{R}(\theta_H - 2i\pi\sqrt{1+\bar{m}^2})\right) \right]. \quad (53)$$

Note that setting $\theta_H = 0$ in Eq. (53) reduces to $J(\bar{R}; \bar{m})$ defined in Eq. (33), i.e., $h(\bar{R}; 0; \bar{m}) = J(\bar{R}; \bar{m})$. For a detailed derivation we refer to Appendixes of Ref. [39].

A key quantity for understanding the gauge phase structure is the Polyakov loop which is defined as $P = \text{tr}W/N_c$. Here, W denotes the Wilson line along S^1 compactified direction (see e.g., Ref. [39] for its explicit definition). The Polyakov loop is the gauge-independent order parameter for distinguishing the confined ($P = 0$) and deconfined ($P \neq 0$) phases. Especially $P \neq 0$ implies that the global Z_{N_c} center symmetry of $SU(N_c)$ is spontaneously broken. While in $SU(N_c)$ gauge theories with adjoint matter the Z_{N_c} center symmetry is preserved, the introduction of fundamental fermions leads to the explicit Z_{N_c} center symmetry breaking. As a result, if no other matter fields are included, the minimum is always in the deconfined phase [39].

As a specific example, we consider $N_c = 3$ for which there are three AB phases ($\theta_1, \theta_2, \theta_3$), but only two phases among them are independent parameters thanks to the traceless condition $\theta_1 + \theta_2 + \theta_3 = 0$. Depending on the configurations of AB phases, $SU(3)$ gauge symmetry is broken into different subgroups $\mathcal{H}_{\text{sym}}$. The breaking patterns for possible configurations of AB phases are listed in Fig. 6. Moreover, for these AB phases, we show the values of the Polyakov loop defined as

$\mathcal{H}_{\text{sym}}/\text{Phase}$	config.	θ_1	θ_2	θ_3
$SU(3)/\text{confined}$	X	—	—	—
$SU(3)/\text{deconfined}$	A_1	0	0	0
	A_2	$\frac{2}{3}\pi$	$\frac{2}{3}\pi$	$\frac{2}{3}\pi$
	A_3	$-\frac{2}{3}\pi$	$-\frac{2}{3}\pi$	$-\frac{2}{3}\pi$
$SU(2) \times U(1)/\text{split}$	B_1	α	α	-2α
	B_2	α	-2α	α
	B_3	-2α	α	α
$U(1) \times U(1)/\text{reconfined}$	C_1	β_1	β_2	$-\beta_1 - \beta_2$
	C_2	β_1	$-\beta_1 - \beta_2$	β_2
	C_3	$-\beta_1 - \beta_2$	β_1	β_2

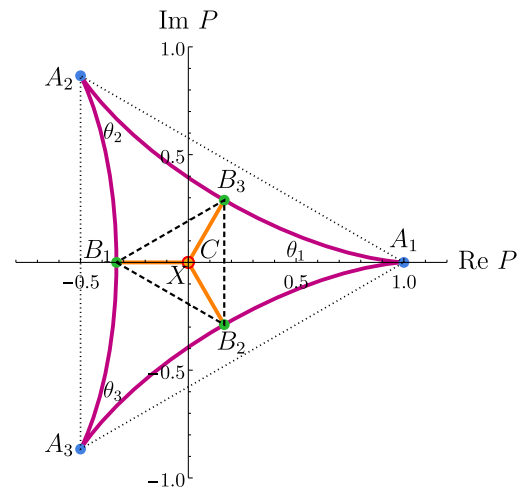


FIG. 6. Configurations of Aharonov-Bohm phases and breaking patterns of $SU(3)$ gauge symmetry, namely $SU(3) \rightarrow \mathcal{H}_{\text{sym}}$ via the Hosotani mechanism [86,87]. Here, $\alpha \neq 0$, $\beta_1 \neq \beta_2$, $-\frac{1}{2}\beta_2, -2\beta_2$. The confined configuration X arises from the Haar-measure distribution which is uniformly and randomly distributed so as to be $P = 0$ [87]. On the right-hand side, the Polyakov loop configurations [defined in Eq. (54)] for the phases summarized on the left-hand side Table are displayed.

$$P = \frac{1}{3}(e^{i\theta_1} + e^{i\theta_2} + e^{i\theta_3}), \quad (54)$$

on the right-hand side panel of Fig. 6.

As a first approximation, we neglect the effect of the anomalous dimensions in Eq. (52) leading to an effective potential independent of the gauge coupling. Moreover, this implies that the confined phase (denoted by X) cannot be accessed in the current simplified approximation. Additionally, we assume a cutoff-independent fermion mass and hence its quantum corrections do not feed back to the potential. This approximation neglects higher-order loop effects which become sizeable in the strong coupling limit where rich dynamics are expected.

In Fig. 7, we investigate the gauge vacuum phase diagram as a function of the compactification radius and the mass of one adjoint fermion. As previously known from Monte Carlo simulations based on lattice gauge field theory [86,87] and effective model studies [88,89], three phases are present in both dimensional limits. While for large masses the fermion contributions decouple and hence the deconfined phase is present, for small masses the reconfined phase is always found. On the right-hand side of Fig. 7, the leftmost set of plots shows the relevant slice of the effective potential ($\theta_1 = 0$) at a fixed adjoint fermion mass of $\bar{m}_{\text{ad}} = 0.4$ and for different compactification radius. On the top plot, the compactification radius is slightly decreased from the deconfined phase (blue line) to the split phase (green lines). In the bottom plot, we show the evolution from the split to the reconfined phase (yellow lines).

In comparison to previous studies [88,89], in the present work we can continuously connect from deeply in both

$D = 4$ and $D = 5$ limits. The fRG accounts for the threshold effects associated to inherent scales or field masses. As shown in Eq. (53), the propagators contain threshold functions which are sensitive to the compactification radius and the explicit fermionic mass. We find that the split phase only appears in the interdimensional limit (finite $\bar{R}$) while in the $D = 4, 5$ limits vanishes as is squeezed by both deconfined and reconfined phases. In the rightmost panel of Fig. 7, we show the gauge potential evolution as the adjoint fermion mass is decreased in both of the dimensional limits. We see how the split phase [minimum at $(\theta_1, \theta_2) = (0, \pi)$] is not realized in any of the limits. While in the $D = 5$ limit the potential flattens along this transition, in the $D = 4$ limit a barrier separates all three phases is present. Additionally, we note the clear relation between the profile of the phase boundaries with the dependence of the effective dimensionality on the compactification radius.

We also investigated the AB phase diagram for larger numbers of adjoint fermions. While the dependence on the compactification radius was found to be exactly the same, larger adjoint fermion masses are necessary in both dimensional limits for the transition between the deconfined and reconfined phases to occur. In other words, the profile shown in Fig. 7 shifts to larger masses.

On the other hand, the inclusion of fundamental fermions is known to explicitly break the center symmetry of the gauge group; see e.g., Refs. [90,91]. As a result, the gauge potential is always found in the deconfined phase ($P \neq 0$). However, including additional adjoint matter leads to a very rich phase structure. In the left plot of Fig. 8, we show the gauge potential phase diagram for $(N_f, N_{\text{ad}}) = (1, 1)$ with periodic boundary conditions ($\beta_f = \beta_{\text{ad}} = 0$) and $\bar{m}_f = 0$, as a

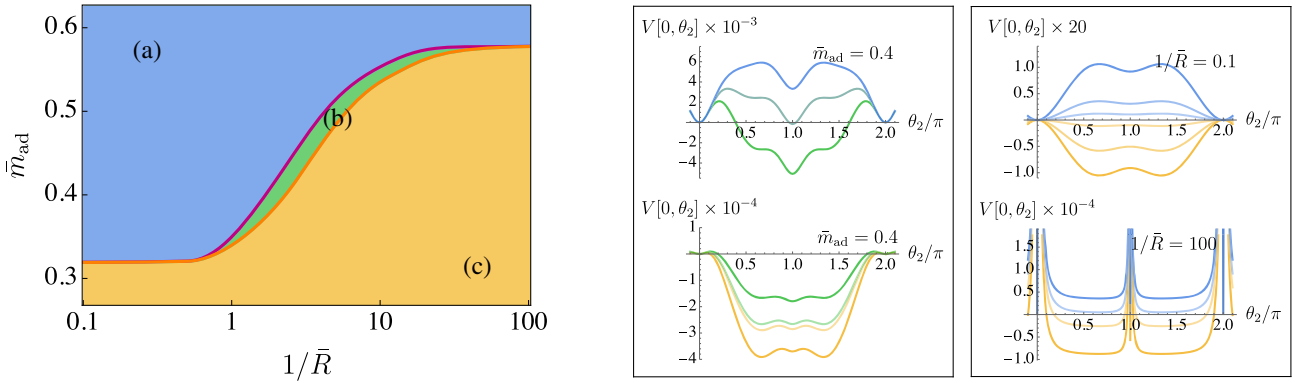


FIG. 7. On the left-hand side plot, the AB phase diagram in the presence of one adjoint fermion $(N_f, N_{\text{ad}}) = (0, 1)$ as a function of the compactification radius and the adjoint fermion mass. Three phases are found to appear: deconfined [(a), in blue], split [(b), in green], and reconfined [(c), in orange]. On the first set of plots on the right-hand side panel, we show the evolution of a slice of the effective potential along a fixed $\bar{m}_{\text{ad}} = 0.4$ for different finite compactification radii. In the top plot, we show the evolution from the deconfined (blue lines) to the split phase (green lines) and in the bottom plot, from the split (green lines) to the reconfined phase (yellow lines). We slightly vary the compactification radius in the interval $1/\bar{R} = [0.42, 0.66]$. In the rightmost set of plots, we show the evolution of the gauge potential at two fixed radii $1/\bar{R} = \{0.1, 10^2\}$. The adjoint fermion mass is slightly decreased showing the evolution of the potential from the deconfined (dark blue lines) to the split phase (dark yellow lines) in both dimensional limits. While in the $D = 5$ limit (top plot), this phase transition displays a separating barrier, in the $D = 4$ limit (bottom plot) the potential undergoes a flat shape.

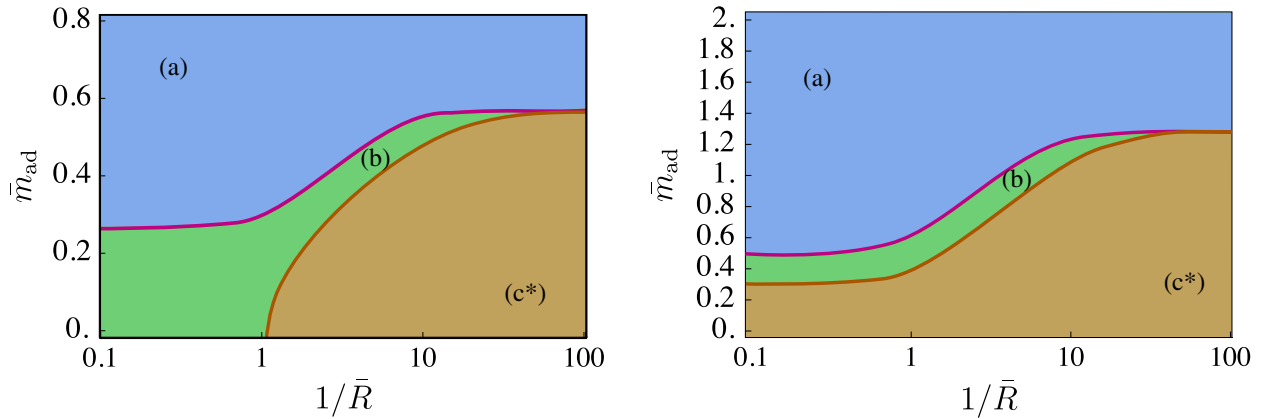


FIG. 8. AB phase diagram as a function of the compactification radius and $\bar{m}_{\text{ad}}$. On the left-(right-)hand side plot, we show the phase diagram for $(N_f, N_{\text{ad}}) = (1, 1)$ [$(N_f, N_{\text{ad}}) = (2, 2)$] with periodic boundary conditions and $m_f = 0$. In both cases, three phases are found; deconfined [(a), in blue], split [(b), in green] and pseudoreconfined [(c*), in brown].

function of the compactification radius and for different values of the adjoint fermion mass. The plots here shown are the interpolated version of the numerical results obtained. In the $D = 5$ limit ($1/\bar{R} \lesssim 1$), we identify predominantly two phases; the deconfined and split. In between both ($\bar{m}_{\text{ad}} \sim 0.25$), we find a region in which the potential shows a flat shape in the $(\theta_1 = \theta_2, \theta_2)$ plane in which both minima coexist alike in the top plot in the rightmost panel of Fig. 8. This evolution along decreasing m_{ad} resembles a second-order phase transition. For small masses in the $D = 4$ limit, we find the so-called “pseudoreconfined” phase (C^*) [88]. In this phase (not present in Fig. 6), the Polyakov loop is slightly shifted from the central values $\text{Im}P = 0$ and $\text{Re}P = 0$ but the minimum of the potential still satisfies those conditions of the reconfined phase.

We have also investigated the phase diagram for $(N_f, N_{\text{ad}}) = (2, 2)$, this is shown in Fig. 8. While the $D = 4$ limit is qualitatively similar to the $(N_f, N_{\text{ad}}) = (1, 1)$, the $D = 5$ limit shows the presence of the pseudoreconfined phase at small masses. Moreover, increasing the fermion number lifts the split phase to larger values of $\bar{m}_{\text{ad}}$ and triggers the appearance of the pseudoreconfined phase at smaller ones. This observation was already pointed out in Ref. [88]. In other words, increasing fermion numbers are equivalent to considering parity-pair [92] e.g., on Z_2 orbifolding (S^2/Z_2).

V. SUMMARY AND CONCLUSIONS

In this work we have studied the properties of five-dimensional Yang-Mills theories coupled to Dirac fermions. To tackle the nonperturbative character of these theories, we have employed the functional renormalization group to derive the flow equations for the gauge coupling and the effective potential.

First, we have investigated the structure of the flow equations and the fixed-point solutions of the four- and

five-dimensional setups with different fermionic content. Considering the extra dimension to be compactified and employing a mass-dependent renormalization scheme allows to continuously connect the phase diagram in both dimensional limits. In Fig. 3, we summarize the IR and UV fixed-point solutions found at different compactification radii. Particularly, we find theories in which both IR and UV fixed points are simultaneously present. We display the trajectories between fixed points considering different onsets of the extra dimension.

We also studied the gauge potential and AB phase diagram as a function of the compactification radius. We particularized on $N_c = 3$ theories with both adjoint and fundamental fermions. The findings discussed are in agreement with previous results from Monte Carlo simulations based on lattice gauge field theory [86,87] and effective-model studies [88,89]. However, we find qualitative advances given we employ a mass-dependent renormalization scheme with sensitivity to threshold effects.

Compactifying the extra dimension and defining an effective dimensionality has allowed us to continuously interpolate between the four- and five-dimensional limits of the theory. This is particularly useful for the implementation of the present theoretical results along functional constructions of the SM [93,94] to describe and test phenomenological models of gauge-Higgs unification, see e.g., Ref. [95].

Given the intricacy of Yang-Mills theories, in the present work we have focused on the qualitative features. In addition to the dimensionality of the gauge coupling, five-dimensional gauge theories also require a nonperturbative treatment as the gauge interactions turn strong. In this limit important phenomena such as dynamical chiral symmetry breaking or color confinement take place. Here, we have worked in the background field approximation and used heat-kernel techniques to derive the flows. This approximation does not provide quantitative access to the strong

sector and hence we focused in the discussion of the qualitative aspects. However, to improve the current approximation, we have made use of three-loop $\overline{\text{MS}}$ results accounting for high order fermionic contributions. This allowed to track the nontrivial CBZ fixed-point solution in the five-dimensional limit.

Several improvements can be made for conclusive statements and quantitative precision. First, the flow equation for the gauge coupling can be derived from a vertex expansion of the effective action and from the scaling of 3- and 4-point functions containing gauge fields. This method facilitates the study considering higher-dimensional operators. This step is crucial in order to determine the dimensionality of the critical surface and hence the predictivity of the theory. Both analyses, the fixed point landscape and the phase diagram, can be improved by feeding back the anomalous dimensions of the gauge, ghost, and fermionic fields. In the fixed-point study performed, this will lead to the inclusion of more contributions at two and higher loops which at the current point have been included by employing perturbative results. In the AB phase analysis, feeding the anomalous dimension will make the gauge potential dependent on the gauge coupling. This will lead to qualitative access to the strong coupling confinement phase.

Finally, let us briefly mention some prospects of the study of extra-dimensional Yang-Mills theories. From the theoretical point of view, the nonperturbative renormalizability, i.e., the asymptotic safety scenario for extra-dimensional Yang-Mills theories, is still an open question. Within the truncation employed in this work, the $\text{tr}[F_{\mu\nu}F^{\mu\nu}]$ operator is

found to be relevant. Nevertheless, one cannot exclude the possibility of higher-dimensional operators additionally being relevant and hence theory being less predictive. To address this issue, we need to determine the dimensionality of the UV critical surface in theory space spanned by an infinite number of effective operators such as $(\text{tr}[F_{\mu\nu}F^{\mu\nu}])^n$ and $\text{tr}[F_{\mu\nu}F^{\nu\sigma}F_{\sigma}{}^{\mu}]$. This is under investigation as an ongoing project.

On the phenomenological side, UV complete extra-dimensional Yang-Mills theories are necessary to construct predictive models for gauge-Higgs unification. The findings here discussed are relevant for phenomenological implementations of the Hosotani mechanism into viable models. However, defining chiral fermions is crucial for realistic setups of gauge-Higgs unification models compatible with the SM. Although one cannot define chiral fermions on $\mathbb{R}^4 \times S^1$ spacetimes, implementations are possible on the orbifold [13], e.g., $\mathbb{R}^4 \times S^1/Z_2$. Additionally, realistic gauge-Higgs unification models are usually defined as a $SU(3)_c \times SO(5) \times U(1)_X$ gauge theory [15–19]. Furthermore, gauge-Higgs grand unification theories have been studied as BSM models [20–28]. For these reasons, it may be worthy to study the non-perturbative features of extra-dimensional gauge theories with other gauge groups and spacetime structures.

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ACRONYMS

$\overline{\text{MS}}$ Minimal Sustraction scheme	fRG Functional Renormalisation Group
χSB Spontaneous symmetry breaking	GUT Grand Unification Theory
$\text{d}\chi\text{SB}$ Dynamical chiral symmetry breaking	IR Infrared
k_{SSB} Spontaneous symmetry breaking scale	KK Kaluza-Klein
$k_{\chi\text{SB}}$ $\text{d}\chi\text{SB}$ scale	LEFT Low energy effective theory
k_{conf} Confinement scale	LHC Large Hadron Collider
AB Aharonov-Bohm	mSTI Modified Slavnov-Taylor identities
ASQG Asymptotically safe quantum gravity	NGB Nambu-Goldstone boson
ASSM Asymptotically safe Standard Model	PC Partial Compositeness
BG Barbieri-Giudice	pNGB Pseudo Nambu-Goldstone boson
BSM Beyond the Standard Model	QCD Quantum Chromodynamics
CBZ Caswell-Banks-Zaks	QFT Quantum Field Theory
CH Composite Higgs	RG Renormalisation Group
CKM Cabibbo-Kobayashi-Maskawa matrix	SM Standard Model
CP Charge-Parity	STI Slavnov-Taylor identities
DM Dark Matter	SUSY Supersymmetry
DSE Dyson-Schwinger equation	TC Technicolor
EoM Equations of motion	UV Ultraviolet
EW Electroweak	
EWSB Electroweak symmetry breaking	
FPC Fundamental Partial Compositeness	

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