VACUUM STABILITY AND SUPERSYMMETRY AT HIGH SCALES WITH TWO H DOUBLETS

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We consider two-H doublet models (THDMs) with a supersymmetric UV completion.[1] Contrary to the Standard Model, THDMs can be embedded in high-scale supersymmetry with a SUSY breaking scale as high as the scale of grand unification. The stability of the electroweak vacuum and experimental constraints point towards low values of $\tan\beta \lesssim 2$ and a pseudoscalar mass of at least about a TeV. If the higgsino superpartners of the H fields are also kept light, the conclusions are similar and essentially independent of the higgsino mass. However, if all gauginos are also given electroweak-scale masses (split supersymmetry with two H doublets), the predicted Standard Model-like H boson mass is always too large. Light neutral and charged higgsinos emerge as a promising signature of minimal theories with supersymmetric UV completions at high scales, and can be searched for at colliders.

1 Introduction

In the history of particle physics, we have found that the world tends to exhibit more and more symmetry when probing shorter and shorter distance scales. Therefore it seems not unreasonable to speculate that, at extremely high energies, the fundamental constituents of nature and their interactions will be governed by supersymmetry and possibly additional space-time dimensions. However, with our present knowledge we cannot predict at what scale these structures will appear.

Before the LHC era, a plausible and well-motivated possibility for the scale of supersymmetry was just above the electroweak scale. This has led to the expectation of a wealth of new particles in the mass range of a few hundred GeV waiting to be discovered at the LHC. But by now it seems that the SUSY scale is rather higher, and the main promise of sub-TeV supersymmetry — a simple and natural solution of the electroweak hierarchy problem — is no longer realistic; models compatible with the LHC's null findings so far tend to be either quite contrived or to be plagued by at least a "little hierarchy" problem.

In these proceedings we will adopt the point of view that, given that supersymmetry does not appear to completely resolve the fine-tuning problem of the electroweak scale, it may as well be completely unrelated to its eventual resolution (of which we are agnostic) and that the SUSY breaking scale may therefore be anywhere. In fact, the only scale of new physics we know about with certainty is the Planck scale of quantum gravity, $M_{\mathrm{Planck}} = 2.4 \cdot 10^{18}$ GeV, and from a top-down perspective of, for example, superstring theory, there is no a priori reason why the supersymmetry breaking scale should be parametrically lower than M_{Planck}. Thus, our working hypothesis will be that short-distance physics is described by the minimal supersymmetric Standard Model (for concreteness) which takes effect at a scale $M_S \sim 10^{14-17}$ GeV, corresponding to the largest superpartner masses in the theory. For an even higher M_S it would be hard to justify the use of the MSSM as an effective field theory to be UV-completed at M_{Planck}, since there is no large separation between M_S and M_{Planck}. Lower M_S are of course possible, but are not the subject of this study.

Our second hypothesis is that all particles in the spectrum are either extremely heavy, with masses of the order of M_S or at most one or two orders of magnitudes below, or have electroweak-scale masses at most of $\mathcal{O}$(TeV): There are no intermediate mass scales between the electroweak scale and M_S. This is motivated mostly by simplicity, and by the hope that some of the extra states may eventually be observable at colliders.

What can the low-energy field content be? It should include at least the Standard Model, but are just the Standard Model particles enough to match to high-scale SUSY, or do we need other states with electroweak-scale masses for consistency?

2 Supersymmetry at high scales

2.1 Just the Standard Model at low scales

The simplest scenario of this kind that one might imagine is usually called "high-scale supersymmetry": all superpartners and all additional Higgs bosons of the supersymmetric Standard Model have masses of the order of M_S. One should regard the Standard Model as an effective field theory which is matched, at the scale $M_S \sim 10^{14-17}$ GeV, to its supersymmetric extension.[2]

As is well known[3], this scenario is in some tension with experimental data due to the behaviour of the Higgs quartic coupling λ. For an electroweak-scale value of λ corresponding to the know value of the lightest Higgs boson mass $M_h = 125$ GeV, λ becomes negative in the ultraviolet during its evolution with the renormalization group. A $-|\lambda|\phi^4$ scalar potential is unbounded from below and therefore cannot be matched to global supersymmetry; in fact the tree-level matching condition for λ at the scale M_S reads

$$\lambda = \cos^2 2\beta \, \frac{g^2 + g'^2}{4} \tag{1}$$

and so $\lambda(M_S)$ is manifestly positive. The scale $M_{S,\,\max}$ where λ turns negative depends sensitively on the value of the top Yukawa coupling, and thus on m_t. For the current central value $m_t = 173$ GeV it is around $M_{S,\,\max} \approx 10^{10}$ GeV, which precludes in particular matching to the MSSM at scales close to M_{Planck}.[a]

2.2 Split SUSY

An alternative scenario which is theoretically appealing and has therefore been extensively studied is split supersymmetry[4,5]. Here the gaugino and higgsino superpartners of the Standard Model gauge and Higgs fields have masses close to the electroweak scale (with a somewhat heavier gluino to avoid LHC search bounds), whereas the scalar states of the MSSM are heavy with masses around M_S, except for the Standard Model Higgs field. Similarly as in the Standard Model, this model cannot be extrapolated and matched to the MSSM at very high scales because the Higgs quartic coupling becomes negative.[6,7] In fact, this behaviour is even more pronounced here than in the Standard Model, with $M_{S,\max} \approx 10^8$ GeV. The reason is that adding new fermions with Yukawa couplings to the Higgs field accelerates the running of the quartic coupling towards negative values in the UV.

2.3 Two-H doublet models

For the remainder of this proceedings contribution we will investigate models in which both of the MSSM Higgs doublets obtain electroweak-scale masses, whereas the other MSSM scalars

[a]It should be noted that $M_{S,\max} = M_{\text{Planck}}$ would still be possible if the top mass were about 2–3σ below its current central value.[3]

(and possibly the fermionic superpartners) are heavy with masses around M_S. This scenario has recently been studied in some detail by Lee and Wagner[8] who found that a two Higgs doublet model can indeed be matched to the MSSM at high scales while reproducing the known electroweak-scale observables, in particular the lightest Higgs mass of 125 GeV.

However, the renormalization group evolution of the scalar quartic potential[b]

$$V_4 = \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4|H_1^\dagger H_2|^2 \tag{2}$$

is still problematic in general. When imposing the matching conditions, the potential is positive by construction at the scale M_S, but the RG improved quartic potential at lower scales may still become formally unbounded from below, signalling the presence of unphysical vacua whose energy is in general much lower than the one of the electroweak vacuum. This implies that the electroweak vacuum would be unstable, and that the universe would eventually tunnel to a lower-energy configuration. The very minimum requirement one should impose in that case is that the lifetime of our universe should be larger than what we have observed, $\tau > 10^{10}$ yr. We follow the usual convention in calling such a configuration *metastable*. In a metastable model, cosmic history should still explain why our universe happened to end up in the false electroweak vacuum rather than the true one. One may avoid this by imposing the stricter requirement of *absolute stability*, i.e. the absence of any unphysical vacuum whatsoever.

We will show in the following that requiring absolute stability, or even just metastability, significantly reduces the viable parameter space of two Higgs doublet models with a SUSY UV completion at high scales.

3 Vacuum decay

The theory of vacuum tunnelling in quantum field theory was largely established in the seminal papers of Coleman[9] and Callan and Coleman[10], which showed that the decay rate times cosmic time τ (the "decay probability") is given by

$$p = \frac{\tau^4}{R^4} e^{-S_B} \,. \tag{3}$$

Here S_B is the euclidean action of a particular classical field configuration called the "bounce", and R is a scale which at one-loop order is essentially given by a certain functional determinant of fluctuations around this configuration. It can be identified with the characteristic scale of the bubble of true vacuum through which the tunnelling proceeeds. To be consistent with our survival until current cosmic times we should demand $p \ll 1$ for $\tau = 10^{10}$ yrs. For the case of a single real scalar field with a potential V, the bounce is an O(4) invariant field configuration whose radial part $\phi(r)$ solves the differential equation

$$\phi'' + \frac{3\,\phi'}{r} = \frac{dV}{d\phi} \tag{4}$$

subject to the boundary conditions

$$\phi'(0) = 0\,, \qquad\qquad \lim_{r\to\infty} \phi(r) = \langle\phi\rangle_{\text{false vacuum}} \,. \tag{5}$$

Such tunnelling solutions may exist even for potentials which also admit classical rolling solutions[11]. The case relevant for us is $-|\lambda|\phi^4$ theory, whose potential of course possesses neither a false nor a true vacuum, strictly speaking; nevertheless a field configuration at the origin of the potential at $\phi = 0$ is metastable, and can decay through a bounce of the form

$$\phi(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2\,R}{r^2 + R^2} \tag{6}$$

[b]Here we neglect all terms which are not generated by tree-level matching to SUSY.

whose euclidean action is

$$S_B = \frac{8\pi^2}{3|\lambda|}\,. \tag{7}$$

Owing to the classical scale invariance of the potential, R is undetermined in Eq. (6), and does not enter into Eq. (7). In quantum theory the potential is no longer scale invariant since λ evolves according to its RGE. The decay probability can then be calculated as[12]

$$p = \max_R \frac{\tau^4}{R^4} e^{-S_B(R)}\,, \qquad S_B(R) = \frac{8\pi^2}{3|\lambda(\frac{1}{R}|} + \Delta S\,, \tag{8}$$

where ΔS are numerically subdominant one-loop corrections. In this way one obtains a metastability criterion for the case of a single real scalar field, e.g. the Standard Model Higgs boson (whose potential is approximately $\lambda(\mu)\phi(\mu)^4$ at large RG scales $\mu \gg M_h$).

In the THDM case there are five real scalars and the four nonzero quartic couplings of Eq. (2). At the tree level, the conditions for absolute stability are well known[13]:

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 + (\lambda_1\lambda_2)^{1/2} > 0\,, \tag{9}$$

$$\lambda_3 + \lambda_4 + (\lambda_1\lambda_2)^{1/2} > 0\,. \tag{10}$$

Numerically, one finds that Eqns. (9) are always satisfied at all scales when matching to SUSY, but that Eq. (10) may be violated at intermediate scales below M_S. If this is the case, then the vacuum is not absolutely stable. The fact that at most one of the four conditions is violated allows us to derive a criterion for metastability analytically (except that we rely on a numerical solution for the RGEs): One identifies a particular direction in field space along which the quartic potential decreases most steeply. Along this direction ϕ the scalar potential turns out to be

$$V_{\text{eff}}(\phi) = \frac{\lambda}{4}\phi^4\,, \qquad \lambda = \frac{4\,(\lambda_1\lambda_2)^{1/2}\,\left(\lambda_3 + \lambda_4 + (\lambda_1\lambda_2)^{1/2}\right)}{\lambda_1 + \lambda_2 + 2\,(\lambda_1\lambda_2)^{1/2}}\,. \tag{11}$$

We can now apply the formalism for a single scalar field to calculate the decay probability of the electroweak vacuum along this direction (with respect to which all possible others are exponentially suppressed). This yields the metastability condition

$$\lambda(\mu) \gtrsim -\frac{2.82}{41.1 + \log_{10}\frac{\mu}{\text{GeV}}} \tag{12}$$

which must be satisfied at all RG scales μ for the decay probability to be < 1.

4 Numerical results and implications

We use two-loop RGEs generated by SARAH[14] and one-loop (partial two-loop) matching to the Standard Model observables with FlexibleSUSY[15]. We set all high-scale SUSY threshold corrections to zero, since the details of SUSY breaking are unknown, and assign a correspondingly large uncertainty of ± 3 GeV on the resulting low-scale prediction of the Higgs mass spectrum. (The effect of high-scale threshold corrections on the vacuum stability conditions is rather small.) There exist examples of GUT-scale models[16] where these thresholds are indeed suppressed with respect to the generic expectation, because of degeneracies in the leading-order soft term spectrum.

The results of our analysis for a particular choice of matching scale $M_S = 2 \cdot 10^{17}$ GeV are illustrated in Fig. 1. The remaining free parameters at low energies are the pseudoscalar Higgs mass m_A and the ratio of vacuum expectation values $\tan\beta$, before calculating the Higgs mass spectrum. Demanding $M_h = 125 \pm 3$ GeV effectively removes one more parameter. It is clear that most of the parameter space is ruled out by vacuum instability. In the right panel of Fig. 1 one sees that a small stable strip is remaining for very low $\tan\beta$ and large $m_A \gtrsim 1$ TeV.

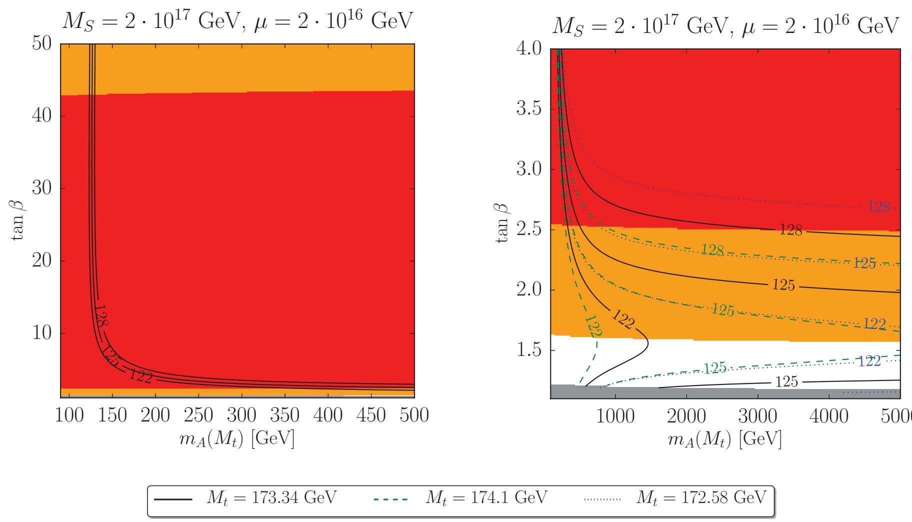

Figure 1 – Contours of the lightest Higgs mass M_h in the $m_A(M_t) - \tan\beta$ plane in the pure THDM for $M_S = 2{\cdot}10^{17}$ GeV. The Higgs mass prediction is computed for $M_t = 173.34 \pm 0.76$ GeV (solid black, dashed green and dotted blue). Left: full range of $\tan\beta$, low $m_A(M_t)$; right: region of low $\tan\beta$, large $m_A(M_t)$. Unshaded regions are allowed by vacuum stability. In the orange region, the electroweak vacuum is unstable but its lifetime is larger than the age of the universe. Red regions are excluded by vacuum stability.

Independently of vacuum stability, the metastable high $\tan\beta$, low m_A region in the left panel is ruled out by measurements such as BR($B \to s\gamma$) and $H, A \to \tau\tau$. This leaves the low $\tan\beta$, large m_A region as the only viable one. The constraints from vacuum stability become milder when lowering M_S; for example, one finds that for $M_S = 2 \cdot 10^{14}$ GeV, m_A is essentially unconstrained from absolute vacuum stability, and can be sub-TeV when allowing for a metastable electroweak vacuum.

Fig. 2 shows the RG evolution of the quartic couplings in a situation where the vacuum is not absolutely stable but still satisfies the metastability constraint.

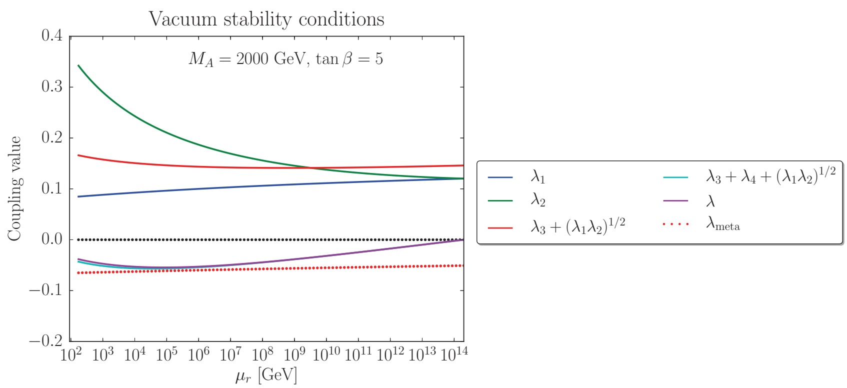

Figure 2 – RG evolution of selected couplings in the THDM. The coupling λ relevant for vacuum stability is defined in Eq. (11). The metastability bound of Eq. (12) is the dashed red line.

5 Variations of the model

Besides the two H doublets, it turns out that one may also keep the superpartners of the MSSM Higgs fields among the EW-scale degrees of freedom. The resulting "Higgsino-THDM" is even more constrained from vacuum stability, but may be experimentally more accessible. In particular, almost pure higgsino-like neutralino and chargino states may be probed by disappearing track searches at the LHC (as is already the case for the somewhat easier wino-like case[17,18]).

We have also studied a third option, namely, keeping all the MSSM gauginos and higgsinos light down to the electroweak scale. However, we find that in this case of a "split-THDM", the theory can no longer be matched to the MSSM at large scales when imposing $M_h = 125$ GeV. In fact, adding too many fermions with Yukawa couplings to the Higgs sector accelerates the running of the quartic couplings towards negative values at high energies, and thus tends to destabilize the electroweak vacuum.

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