Chapter 17
Spontaneous Symmetry Breaking in Particle Physics

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Abstract I will review the appearance of spontaneous symmetry breaking (SSB) in particle physics at the end of the fifties and beginning of the sixties of the XXth century. I will recall Heisenberg non-linear spinor theory and the genesis of the first model (NJL) of fermion mass generation developed in collaboration with Yoichiro Nambu, based on the idea of spontaneous symmetry breaking. Both the non-linear spinor theory and the NJL model are invariant under a chiral transformation (γ_5—invariance) which was introduced by Bruno Touschek in 1957 and named by Heisenberg the Touschek transformation. Then I will briefly describe the subsequent evolution where the NJL model became an effective theory for low energy QCD and SSB was the key for the electroweak unification. Finally I will consider SSB in non-equilibrium which may be of interest in cosmology.

17.1 Introduction

The phenomenon of Spontaneous Symmetry Breaking (SSB) has been known for a long time even if it did not have a name. In a remarkable paper of 1759 "*Sur la force des colonnes*" Euler [1] discussed the following problem: "*Il s'agit de determiner le poids qu'une colonne peut soutenir, sans etre sujette à se plier*" and obtained a formula for the critical force necessary for bending a thin bar. After bending, the equilibrium configurations of the bar are degenerate as they lie in a plane which can have any orientation breaking the original rotational symmetry. The degeneracy of equilibrium states is a main feature of the phenomenon.

The concept of spontaneous breakdown of a symmetry is applicable to systems with infinitely many degrees of freedom and permeated the physics of condensed matter for a long time, magnetism is a prominent example. However its formalization and the recognition of its importance has been an achievement of the second half

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of the XXth century and the name was adopted after the introduction in particle physics [2]. For the purpose of the present paper we can formulate this concept as follows:

SSB means that the lowest energy state of a system has a lower symmetry than the forces acting among its constituents or on the system as a whole.

The transfer of the idea of SSB from condensed matter to particle physics was an important case of *Cross Fertilization*. Heisenberg was probably the first to consider SSB as a possibly relevant concept in relativistic quantum field theory in the context of the comprehensive theory of elementary particles proposed by him and his collaborators [3, 4]. Mathematically the theory was based on a non-renormalizable non-linear spinor interaction. They tried to cope with the singularities of Quantum Field Theory by introducing an indefinite metric in the Hilbert space which made the approach very complicated, not transparent and comprehensible only to the intiated. Spontaneous symmetry breaking was really appreciated by the elementary particles community after Nambu and the present writer developed a specific model of relativistic field theory with a well defined physical interpretation [5, 6].

The NJL model was also non renormalizable and we simply introduced an invariant cut-off. Ideas like effective field theories [7] and asymptotic safety [8, 9], were not yet around. The model was formally close to Heisenberg theory in the sense that a non-linear spinor lagrangian was adopted and we considered the model with cut-off as a low energy theory of nucleons and mesons. I had been exposed more than once to the non-linear spinor theory, Heisenberg had visited Rome to explain it. Touschek was very interested in understanding the basic ideas and we had seminars on this subject.

We shared with Heisenberg symmetry properties because both approaches, considering the last version of his theory [4], were invariant under the following transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}, \tag{17.1}$$

$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}. \tag{17.2}$$

The second transformation was named after Touschek who introduced it [10] to insure that the neutrino mass be equal to 0 in the theory of weak interactions.

To appreciate the innovative character of SSB in particle physics one may recall that one of the axioms of quantum field theory was that the vacuum, i.e. the state of lowest energy, must be invariant under the symmetries of the theory implemented by unitary operators. Therefore SSB in relativistic field theories represented a real turning point.

To understand the path leading to the NJL model we must start from an observation of Nambu. When I arrived in Chicago in September 1959 he was writing a short paper on the axial vector current conservation in weak interactions: in nature, the axial current is only approximately conserved and Nambu's hypothesis was that a small violation of axial current conservation gives a mass to the massless boson, which is then identified with the π meson, and renormalizes the axial vector part of the β-decay constant. So there must be a relation between these quantities. Under strict invariance under the Touschek transformation, γ_5-invariance, the structure of the axial vector current is

$$\Gamma_\mu^A(p', p) = \left(i\gamma_5\gamma_\mu - \frac{2m\gamma_5 q_\mu}{q^2} \right) F(q^2) \qquad q = p' - p \tag{17.3}$$

We see that it is compatible with a non-vanishing fermion mass provided there exists a zero mass pseudoscalar particle. Under Nambu's hypothesis, one can write

$$\Gamma_\mu^A(p', p) \simeq \left(i\gamma_5\gamma_\mu - \frac{2m\gamma_5 q_\mu}{q^2 + m_\pi^2} \right) F(q^2) \qquad q = p' - p \tag{17.4}$$

This expression implies a relationship between the pion nucleon coupling constant G_π, the pion decay coupling g_π and the axial current β-decay constant g_A

$$2m g_A \simeq \sqrt{2} G_\pi g_\pi \tag{17.5}$$

This is the Goldberger–Treiman relation [11].

Nambu asked me to read a preliminary version of the paper. In order to support Nambu's idea one had to make some independent check. We did the following calculation: It was experimentally known that the ratio between the axial vector and vector β-decay constants $R = g_A/g_V$ was slightly greater than 1 and about 1.25. The following two hypotheses were then natural:

1. under strict axial current conservation there is no renormalization of g_A;
2. the violation of the conservation gives rise to the finite pion mass as well as to the ratio $R > 1$ so that there is some relation between these quantities.

Under these assumptions a perturbative calculation of the convergent difference of renormalization effects for $\mu_\pi \neq 0$ and $\mu_\pi = 0$ gives

$$R \simeq 1 + \Lambda(\mu_\pi) - \Lambda(0) \simeq 1 + \frac{G_\pi^2}{16\pi^2} \frac{\mu^2}{m^2} \ln \frac{m^2}{\mu^2} \simeq 1.24 \tag{17.6}$$

where Λ is the contribution of the diagrams shown in the figure. In the second approximate equality we have retained the dominant logarithmic term.

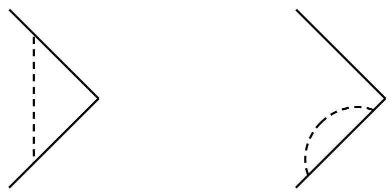

Fig. 17.1 Typical graphs considered in the evaluation of $R = g_A/g_V$

We did this calculation independently obtaining at first very different results. Mine supported Nambu's conjecture while his was definitely against. The question was not entirely trivial as the result was the difference between two divergent expressions. We discussed for several days and finally Nambu agreed that my result was correct. It was a perturbative calculation with a large coupling constant so the numerical result close to the experimental value could not be taken too seriously, it showed however that the renormalization effects due to the pion mass went in the right direction.

The interpretation that Nambu suggested in the paper [12] was

> This situation may be understood by making an analogy to the theory of superconductivity originated by Bardeen, Cooper and Schrieffer [13] and refined by Bogoliubov [15].

In the case of supercoductivity the symmetry spontaneously broken is gauge invariance which in the present case is replaced by γ_5 invariance. Encouraged by the above calculation the construction of a relativistic model was mandatory and this is what we did following the analogy with superconductivity.

17.2 Spontaneous Symmetry Breaking in Superconductivity

A turning point for understanding microscopically the phenomenon of superconductivity was the theory of Bardeen, Cooper and Schrieffer [13], known with the acronym BCS theory. A different approach arriving at similar conclusions was independently proposed by Bogoliubov [14]. Further developments and refinements were due to Anderson [18], Ricayzen [19], Nambu [16]. See also the monograph by Bogoliubov, Tolmachev and Shirkov [15]. In this section I will follow [16] where a formalism close to quantum field theory is used.

Electrons near the Fermi surface due to the attractive phonon interaction [17] are paired (Cooper pairs) and described by the following equation

$$E\psi_{p,+} = \epsilon_p \psi_{p,+} + \phi \psi^\dagger_{-p,-} \tag{17.7}$$

$$E\psi^\dagger_{-p,-} = -\epsilon_p \psi^\dagger_{-p,-} + \phi \psi_{p,+} \tag{17.8}$$

with eigenvalues

$$E = \pm\sqrt{\epsilon_p^2 + \phi^2} \tag{17.9}$$

Here, $\psi_{p,+}$ and $\psi^\dagger_{-p,-}$ are the wavefunctions for an electron and a hole of momentum p and spin $+$, ϕ is the energy necessary to break a pair. The corresponding eigenstates are called quasi-particles.

Formally this is very similar to the Dirac equation which in the Weyl representation reads

$$E\psi_1 = \boldsymbol{\sigma}\cdot\boldsymbol{p}\psi_1 + m\psi_2 \tag{17.10}$$

$$E\psi_2 = -\boldsymbol{\sigma}\cdot\boldsymbol{p}\psi_2 + m\psi_1 \tag{17.11}$$

with eigenvalues

$$E = \pm\sqrt{p^2 + m^2} \tag{17.12}$$

Here, ψ_1 and ψ_2 are the eigenstates of the chirality operator γ_5 and the mass corresponds to the gap ϕ. However the quasi-particles are not eigenstates of the charge therefore there must be a "*backflow*" to recover the conservation of current.

Approximate expressions for the charge density and the current associated to a quasi-particle in a BCS superconductor are given by

$$\rho(x,t) \simeq \rho_0 + \frac{1}{\alpha^2}\partial_t f$$

$$\boldsymbol{j}(x,t) \simeq \boldsymbol{j}_0 - \boldsymbol{\nabla} f$$

where $\rho_0 = e\Psi^\dagger\sigma_3 Z\Psi$ and $\boldsymbol{j}_0 = e\Psi^\dagger(\boldsymbol{p}/m)Y\Psi$ with Y, Z and α constants while f satisfies the wave equation

$$\left(\nabla^2 - \frac{1}{\alpha^2}\partial_t{}^2\right) f \simeq -2e\Psi^\dagger\sigma_2\phi\Psi$$

Here, $\Psi^\dagger = (\psi_1^\dagger, \psi_2)$

The Fourier transform of the wave equation for f gives

$$\tilde{f} \propto \frac{1}{q_0^2 - \alpha^2 q^2}$$

The pole at $q_0^2 = \alpha^2 q^2$ describes the excitation spectrum of a zero-mass boson. Due to the Coulomb force, the pole is cancelled [18] and the spectrum is shifted to the plasma frequency $e^2 n$, where n is the number of electrons per unit volume. In this way the electromagnetic field acquires a mass which lets the magnetic field penetrate only slightly in a superconductor (Meissner effect). This is the essence in a non-relativistic context of what will be known later as the Brout-Englert-Higgs-Guralnik-Hagen-Kibble mechanism. In the Landau-Ginzburg phenomenological theory of superconductivity [20] the vector potential $\mathbf{A}$ obeys the following equation

$$\nabla^2\mathbf{A} - e^2 n\mathbf{A} = 0 \tag{17.13}$$

that is the vector potential has a mass equal to the plasma frequency.

17.3 The NJL Model

The NJL model is a theory of nucleons and mesons based on a non-linear spinor lagrangian which however could be the limit of a theory with an intermediate particle of very high mass. The axial current is the analog of the electromagnetic current in BCS theory. In the hypothesis of exact conservation, as we already noted, the matrix elements of the axial current between nucleon states of four-momentum p and p' have the form of equation (17.3) and is compatible with a finite nucleon mass m provided there exists a massless pseudoscalar particle. Both superconductivity and the NJL model provide examples, of the following general statement, *Goldstone theorem* [36].

Whenever the original Lagrangian has a continuous symmetry group, which does not leave the ground state invariant, massless bosons appear in the spectrum of the theory.

The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu\partial_\mu\psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right] \tag{17.14}$$

It is invariant under the transformations (17.1) and (17.2)

By the Fierz transformation the non-linear term is equivalent to

$$-g\left[(\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2\right] \tag{17.15}$$

The simplest approximation we envisaged was a mean field approach.

$$m = -\frac{g_0 mi}{2\pi^4}\int \frac{d^4p}{p^2 - m^2 - i\varepsilon}\, F(p, \Lambda)$$

or

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2}\ln\left(1 + \frac{\Lambda^2}{m^2}\right)$$

where Λ is the invariant cut-off. The model exhibits an interesting spectrum of bound states as shown in the table.

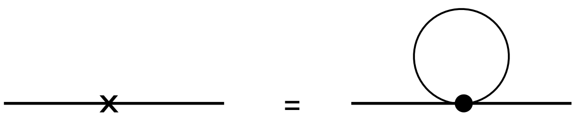

Fig. 17.2 The self-consistent equation for the mass of the fermion in diagrammatic form

The NJL model had a considerable follow up. Its structure was generalized in [21, 22] and shown to be equivalent, as far as the calculation of the S-matrix is concerned, to a more conventional renormalizable theory.

The NJL model has been mainly reinterpreted as an effective theory of low energy QCD where the nucleons of the original model are interpreted as quarks. The literature on the subject is rather extensive and we refer e.g. to the following reviews [23–25]. One is interested in the low energy degrees of freedom on a scale smaller than some cut-off $\Lambda \sim 1$ Gev. In [23] the short distance dynamics above Λ is dictated by perturbative QCD and is treated as a small perturbation. Confinement is also treated as a small perturbation. The total Lagrangian is then

Nucleon number	Mass μ	Spin-parity	Spectroscopic notation
0	0	0^-	${}^1S_0$
0	2 m	0^+	${}^3P_0$
0	$\mu^2 > \frac{8}{3}m^2$	1^-	${}^3P_1$
± 2	$\mu^2 > 2m^2$	0^+	${}^1S_0$

$$L_{\text{QCD}} \simeq L_{\text{NJL}} + L_{\text{KMT}} + \varepsilon\,(L_{\text{conf}} + L_{\text{OGE}})$$

where the Kobayashi–Maskawa–'t Hooft term

$$L_{\text{KMT}} = g_D \det_{i,j}\left[\bar{q}_i(1-\gamma_5)q_j + \text{h.c.}\right]$$

mimics the axial anomaly and L_{OGE} is the one gluon exchange potential. Applications in particle and nuclear physics of the NJL model are still quite frequent.

The argument showing that SSB actually takes place in the NJL model was based on a self-consistent field approximation and a formulation independent of any approximation was desirable. The similarities of the formalisms of quantum field theory and statistical mechanics is part of the common wisdom. This is emphasized for instance in the book of Bogoliubov and Shirkov [26]. In statistical mechanics both classical and quantum there are variational principles determining the equilibrium states of a system. In the quantum case variational principles have been introduced by Lee and Yang [27] followed by Balian, Bloch and De Dominicis [28] and generalized by De Dominicis and Martin [29]. The independent variables appearing in these principles are quantum averages of operators, that is c-numbers.

In a similar vein I found natural to characterize the vacuum and therefore SSB in terms of a variational principle for an effective action [30], a c-number action functional which turned out to be the generating functional of one-particle irreducible amplitudes. The independent functional variable is the vacuum expectation value of the field. It generalizes the effective-potential introduced by Goldstone whose theorem becomes very simple in this formalism. The effective action differs from a classical action as it is non-local in space and time and involves the whole history of the system.

Many years later I learnt that the effective action in a semi-classical context had appeared in a paper by Heisenberg and Euler in the thirties [31] where they calculated quantum corrections to Maxwell's equations. However the effective action was fully appreciated after its use to describe SSB. It became a standard approach to SSB in textbooks of quantum field theory [32, 33] to which the reader is referred. See also [34, 35].

17.4 SSB in Gauge Theories and the Electroweak Unification

We have seen in the case of superconductivity of charged fermions that the long range Coulomb interaction eliminates in the excitation spectrum the massless collective mode which becomes the plasmon and the electromagnetic vector potential acquires a mass. Several people observed that one can take advantage of this mechanism to eliminate unwanted zero mass Goldstone bosons and give a mass to vector mesons in gauge invariant theories. See the articles by Anderson [37], Brout and Englert [38], Higgs [39], Guralnik et al. [40, 41].

To illustrate the mechanism in relativistic field theories we consider the following simple example [38]. Consider a complex scalar field $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$ interacting with an abelian gauge field A_μ

$$H_{\text{int}} = ieA_\mu \varphi^\dagger \overleftrightarrow{\partial}_\mu \varphi - e^2 \varphi^\dagger \varphi A_\mu A_\mu$$

If the vacuum expectation value of φ is $\neq 0$, e.g. $\langle \varphi \rangle = \langle \varphi_1 \rangle / \sqrt{2}$, the polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory is

$$\Pi_{\mu\nu}(q) = (2\pi)^4 ie^2 \langle \varphi_1 \rangle^2 \left[g_{\mu\nu} - \left(q_\mu q_\nu / q^2 \right) \right]$$

Therefore the A_μ field acquires a mass $\mu^2 = e^2 \langle \varphi_1 \rangle^2$ and gauge invariance is preserved, $q_\mu \Pi_{\mu\nu} = 0$.

The discovery of how this could be used for the electroweak unification is due to Weinberg [42] and Salam [43] building on previous work by Glashow [44]. A very clear introduction to the path leading to the model presented in [42] is Weinberg's Nobel lecture [46]. We shall not describe in detail the electro-weak unification as there are comprehensive expositions in books, see e.g. [45]. The following quotation from [46] shows that the application of SSB to the electroweak unification was far from obvious.

> At some point in the fall of 1967, I think while driving to my office at MIT, it occurred to me that I had been applying the right ideas to the wrong problem [the strong interactions]. It is not the ρ meson that is massless: it is the photon. And its partner is not the A1, but the massive intermediate bosons, which since the time of Yukawa had been suspected to be the mediators of the weak interactions. The weak and electromagnetic interactions could then be described in a unified way in terms of an exact but spontaneously broken gauge symmetry.

17.5 SSB in Non-equilibrium

After my collaboration with Nambu I progressively shifted to many-body physics, still under the spell of BCS and Bogoliubov theories, and more generally to statistical mechanics where I continued to explore the analogies with field theory, in particular in critical phenomena and non-equilibrium states.

SSB has been studied so far mainly as an equilibrium phenomenon. It was discovered however that out of equilibrium SSB can take place through mechanisms not available in equilibrium: currents are flowing through the system and their dynamics is crucial.

> Stationary states are the obvious generalization of equilibrium states but the conditions under which SSB takes place in nonequilibrium are different from equilibrium. In stationary nonequilibrium states SSB may be possible even when it is not permitted in equilibrium.

To illustrate this statement let us consider the following toy model [47]: during a time interval dt three types of exchange events can take place between two adjacent sites, see Fig. 17.3

$$+0 \rightarrow 0+\,, \quad 0- \rightarrow -0\,, \quad +- \rightarrow -+\,, \tag{17.16}$$

with probability dt. The last one takes place only on the bridge. At the left of the access lane of plus particles we have

$$0 \rightarrow +\,, \tag{17.17}$$

with probability αdt. At the right end of the exit lane of plus particles

$$+ \rightarrow 0\,, \tag{17.18}$$

with probability βdt, and similarly for minus particles after reflection.

The model is clearly invariant under the discrete CP transformation. The authors have shown that that there is a phase transition spontaneously breaking CP invariance that manifests itself by blocking one of the access lanes so that in the stationary state charges of one sign prevail. On the other hand there is no phase transition in equilibrium. Unfortunately the study of SSB in non-equilibrium is considerably more difficult due to the lack of a general theory of non-equilibrium states.

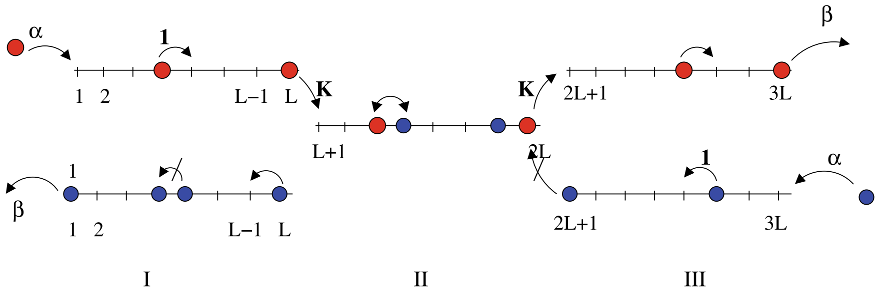

Fig. 17.3 The bridge model with two junctions from [47]. Positively (negatively) charged particles hop to the right (left). The model is invariant with respect to left-right reflection and charge inversion. Section 17.2 is the bridge. It contains positive and negative particles and holes. Sections 17.1 and 17.3 comprise parallel segments each containing pluses and holes or minuses and holes

The question naturally arises of how non-equilibrium SSB could be relevant in particle physics. At the cosmic scale matter is widespread and we do not see regions with antimatter. Explanations have been proposed invoking initial small asymmetries which are amplified over a long nonequilibrium evolution, See for example the following quotation [48].

> Baryogenesis gives a possible answer to the following question: Why there is no antimatter in the Universe? A (qualitative) solution to this problem is known already for quite some time: the Universe is charge asymmetric because it is expanding (the existence of arrow of time, in Sakharov's wording), baryon number is not conserved and the discrete CP-symmetry is broken. If all these three conditions are satisfied, it is guaranteed that some excess of baryons over anti-baryons will be generated in the course of the Universe evolution. However, to get the sign and the magnitude of the baryon asymmetry of the Universe (BAU) one has to understand the precise mechanism of baryon (B) and lepton (L) number non-conservation, to know exactly how the arrow of time is realized and what is the relevant source of CP-violation.

This type of explanation seems to shift the problem back in time. Non-equilibrium is considered after Sakharov [49] a precondition for explaining the matter-antimatter asymmetry in our universe. Phase transitions due to non-equilibrium spontaneous symmetry breaking may be relevant in cosmology. If the conditions are such that an approximately stationary state is an SSB phase, depending on the initial conditions a system will relax to one of the degenerate states avoiding the difficulties of reconstructing a history with many uncertainties. This may require a departure from the prevailing big-bang view of the origin of the universe. See, for an alternative, the recent theories of cyclic universes [50, 51].

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