Gravitational collapse in five dimensional space-time

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Abstract

We numerically investigate the gravitational collapse of collisionless particles in spheroidal configurations both in four and five-dimensional (5D) space-time. We repeat the simulation performed by Shapiro and Teukolsky (1991) that announced an appearance of a naked singularity, and also find that the similar results in 5D version. That is, in a collapse of a highly prolate spindle, the Kretschmann invariant blows up outside the matter and no apparent horizon forms. We also find that the collapses in 5D proceed rapidly than in 4D, and the critical prolateness for appearance of apparent horizon in 5D is loosened compared to 4D cases. We also show how collapses differ with spatial symmetries comparing 5D evolutions in single-axisymmetry, SO(3), and those in double-axisymmetry, U(1)×U(1).

1 Introduction

Motivated by the so-called "large extra-dimensional models", black-holes in higher dimensional space-time are extensively studied for a decade. Many interesting discoveries of new solutions have been reported, and their properties are also been revealing. However, fully relativistic dynamical features, such as the formation processes, stabilities and late-time fate, are still unknown and they are waiting to be studied.

In classical general relativity, there are two famous conjectures concerning the gravitational collapse. One is the *cosmic censorship conjecture* [1] which states that singularities are always clothed by event horizons. The other is the *hoop conjecture* [2] which states that black holes with horizons are formed when and only when a mass gets compacted into a small region. Shapiro and Teukolsky (ST91, hereafter) numerically showed that axisymmetric space-time with collisionless matter particles in spheroidal distribution will collapse to singularity, and there are no apparent horizon formed when the spheroids are highly prolate[3]. The behaviors supported the hoop conjecture.

Regarding to the 5D cases, the hoop conjecture is supposed to be replaced with the *hyper-hoop* version[4, 5], i.e. a criteria is not a hoop but a surface. In our previous work [6], we numerically constructed initial data sequences of non-rotating matter for 5D evolutions and examined the hyper-hoop conjecture using minimum *area* around the matter. The sequences suggest that a highly prolate spindle in 5D will form a naked singularity similar to the 4D cases.

In this note, we report our numerical simulations on gravitational collapse in axisymmetric space-time in (3+1)-dimensional space-time (4D, hereafter) and (4+1)-dimensional (5D) versions. We show that the naked singularity is formed for the gravitational collapse of spheroidal matter configuration in 5D. We also compare the dynamics between 4D and 5D. In 5D, two axes can be settled as rotational symmetric axes, so that we also compare gravitational collapses in axisymmetry with those in "doubly"-axisymmetric space-time.

2 Numerical code

We evolve five-dimensional axisymmetric [symmetric on z-axis, SO(3)] or doubly-axisymmetric [symmetric both on x and z-axes, U(1)×U(1)], asymptotically flat space-time (see Figure 1). For the comparison, we also performed four-dimensional axisymmetric space-time evolutions.

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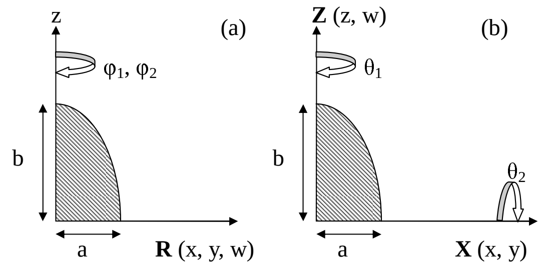

Figure 1: (above) we evolve five-dimensional (a) axisymmetric [SO(3)] or (b) double-axisymmetric [U(1)× U(1)], asymptotically flat space-time using the Cartesian grid. the initial matter configuration is expressed with parameters a and b.

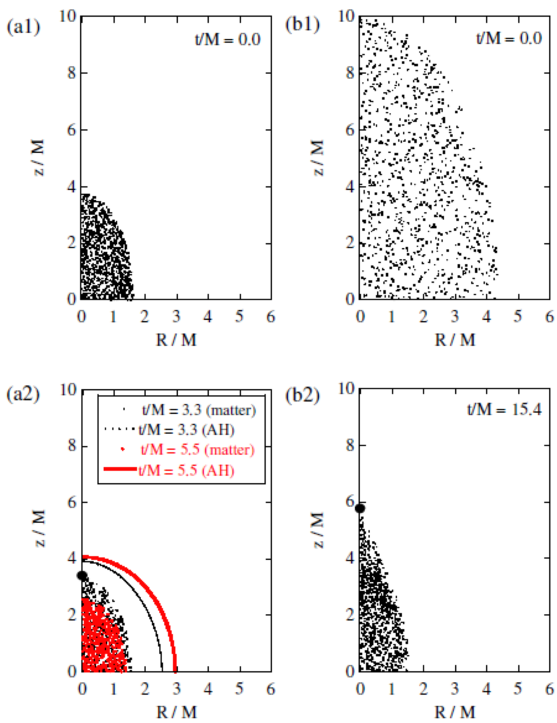

Figure 2: (right) Snapshots of 5D axisymmetric evolution with the initial matter distribution of $b/M = 4$ [Fig.(a1) and (a2); model 5DSβ in Table 1] and 10 [Fig.(b1) and (b2); model 5DSδ]. The big circle indicates the location of the maximum Kretschmann invariant $\mathcal{I}_{\max}$ at the final time at each evolution.

We start our simulation from time symmetric and conformally flat initial data, which are obtained by solving the Hamiltonian constraint equations [6]. The asymptotical flatness is imposed throughout the evolution, which settles the fall-off condition to the metric as $\sim 1/r$ for 4D cases and $\sim 1/r^2$ for 5D cases.

The matter is described with 5000 collisionless particles, which move along the geodesic equations. We smooth out the matter by expressing each particle with Gaussian density distribution function with its typical width is twice as much as the numerical grid. The particles are homogeneously distributed in a spheroidal shape, parametrized with a and b (Figure 1), or eccentricity $e = \sqrt{1 - a^2/b^2}$.

By imposing axisymmetry or double-axisymmetry, our model becomes practically a (2+1)-dimensional problem. We construct our numerical grids with the Cartesian coordinate (x, z), and apply the so-called Cartoon method [7] to recover the symmetry of space-time.

The space-time is evolved using the Arnowitt-Deser-Misner (ADM) evolution equations. It is known that the ADM evolution equations excite an unstable mode (constraint-violation mode) in long-term simulations [8, 9]. However, we are free from this problem since gravitational collapse occurs within quite short time. By monitoring the violation of constraint equations during evolutions, we confirm that our numerical code has second-order convergence, and also that the simulation continues in stable manner. The results shown in this report are obtained with numerical grids, 129×129×2×2. We confirmed that higher resolution runs do not change the physical results.

We use the maximal slicing condition for the lapse function α, and the minimal strain condition for the shift vectors β^i. Both conditions are proposed for avoiding the singularity in numerical evolutions [10], and the behavior of α and β^i roughly indicates the strength of gravity, conversely. The iterative Crank-Nicholson method is used for integrating ADM evolution equations, and the Runge-Kutta method is used for matter evolution equations.

For discussing physics, we search the location of apparent horizon (AH), calculate the Kretschmann invariant ($\mathcal{I} = R_{abcd}R^{abcd}$) on the spacial hypersurface.

b/M $(t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα AH-formed $e_{\rm AH}=0.90$ $e_{\rm f}=0.92$	4Dβ no $e_{\rm f}=0.89$	4Dγ no $e_{\rm f}=0.92$	4Dδ no $e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.82$	5DSβ AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.84$	5DSγ no $e_{\rm f}=0.88$	5DSδ no $e_{\rm f}=0.96$
5D double axisym. U(1)×U(1)	5DUα AH-formed $e_{\rm AH}=0.86$ $e_{\rm f}=0.79$	5DUβ AH-formed $e_{\rm AH}=0.87$ $e_{\rm f}=0.81$	5DUγ AH-formed $e_{\rm AH}=0.92$ $e_{\rm f}=0.90$	5DUδ no $e_{\rm f}=0.98$

Table 1: Model-names and the results of their evolutions whether we observed AH or not. The eccentricity e the collapsed matter configurations is also shown; $e_{\rm AH}$ and $e_{\rm f}$ are at the time of AH formed (if formed), and on the numerically obtained final hypersurface, respectively.

3 Results

We prepare several initial data keeping the total ADM mass and the eccentricity of distribution, $e=0.9$. By changing the initial matter distribution sizes, we observe the different final structures. Figure 2 shows snapshots of 5D axisymmetric evolutions of model $b/M=4$ and 10 (model 5DSβ and 5DSδ, respectively; see Table 1); the former collapses to a black hole while the latter collapses without AH formation.

All the models we tried result in forming a singularity (i.e., diverging $\mathcal{I}$). We stop our numerical evolutions when the shift vector is not obtained with sufficient accuracy due to the large curvature. For model 5DSδ, we integrated up to the coordinate time $t/M=15.4$ and the maximum of the Kretschmann invariant $\mathcal{I}_{\rm max}$ becomes $O(1000)$ on z-axis (see Figure 3), but AH is not formed.

When the initial matter is highly prolated, AH is not observed. This is consistent with 4D cases [3], and matches with the predictions from initial data analysis in 5D cases [5, 6]. The location of $\mathcal{I}_{\rm max}$ is on z-axis, and just outside of the matter. This is again the same with 4D cases [3]. The absence of AH with diverging $\mathcal{I}$ suggests a formation of naked singularity in 5D.

In order to compare the results with 4D and 5D, we reproduce the results of ST91. We then find that the $e=0.9$ initial data with $b/M=10$ (model 4Dδ) collapses without forming AH, and the code stops at the coordinate time $t=20.91$ with $\mathcal{I}_{\rm max}=84.3$ on the z-axis ($z/M=6.1$); all the numbers match quite well with ST91. (Note that our slicing conditions and coordinate structure is not the same with ST91.)

We also performed 5D collapses with doubly-axisymmetric [U(1)×U(1)] space-time. The matter and space-time evolve quite similar to 5D and 4D axisymmetric cases, but we find that the critical configurations for forming AH is different. Table 1 summarizes the main results of 4D and two 5D cases. We find that AH in 5D is formed in larger b initial data than 4D cases. This result is consistent with our prediction from the sequence of initial data [6]. AH criteria with initial b is loosened for 5D doubly-axisymmetric cases. We show the eccentricity, $e_{\rm AH}$ and $e_{\rm f}$, which tell us that the doubly-axisymmetric assumption makes collapse less sharp when it forms AH, and makes collapse similar to 4D cases when it does not form AH. Table 1 indicates that the eccentricity itself is not a guiding measure for AH formation.

In Figure 4, we plotted $\mathcal{I}$ at the point which gives $\mathcal{I}_{\rm max}$ on the final hypersurface as a function of proper time. We see that 5D-collapse is proceeding rapidly than 4D collapses. We also see that collapses in doubly-axisymmetric space-time is proceeding slowly than single axisymmetric cases.

4 Discussions

In this paper, we reported our numerical study of gravitational collapses in 5D space-time. The collapsing behaviors are quite similar to the cases in 4D, but we also found that (a) 5D-collapses proceed rapidly than 4D-collapses, (b) AH appears in highly prolate matter configurations than 4D cases, (c) doubly-axisymmetric [U(1)×U(1)] assumption makes collapse less sharp when it forms AH, and (d) the positive evidence for appearance of a naked singularity in 5D.

Up to this moment, we only checked the existence of apparent horizons, and not the event horizons. The system does not include any angular momentums. We are implementing our code to cover these studies.

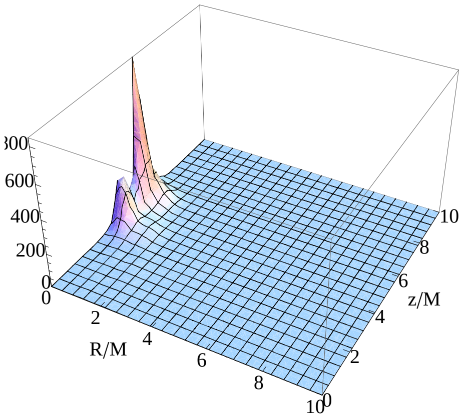

Figure 3: Kretschmann invariant $\mathcal{I}$ for model 5DSδ at $t/M = 15.4$. The maximum is $O(1000)$, and its location is on z-axis, just outside of the matter.

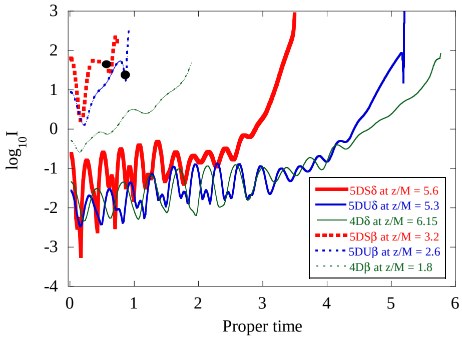

Figure 4: Kretschmann invariant $\mathcal{I}$ at the location of $\mathcal{I}_{\max}$ on the final hypersurface is plotted as a function of proper time at its location. labels indicate model-names in table 1. the time of ah formation (t=0.6 for model 5dsβ, t=0.9 for 5duβ) is shown by a dot.

We are now preparing our next detail report including the validity of hyper-hoop conjecture in 5D, and the cases of the ring objects.

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