A TEST OF NEWTON'S GRAVITATIONAL LAW IN THE RANGE OF 0.6 M TO 3.6 M

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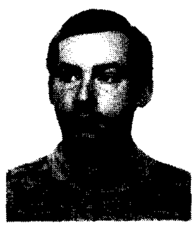

ABSTRACT

We have performed an experiment to test the $1/r^2$ dependence of Newton's law of gravitation and to determine the gravitational constant G, using a microwave resonator. The gravitational force of a laboratory test mass acts on this resonator and the resulting change of the resonator frequency is used to determine the gravitational force as a function of distance. In a first series of experiments we have investigated the inverse square law in the range of distances of 0.6 m to 3.6 m and determined the gravitational constant with a relative error of $1.6 \cdot 10^{-3}$. No deviations from Newton's inverse square law or the CODATA value of G were found.

I. INTRODUCTION

Many aspects of Newton's law of gravitation have been investigated experimentally during the last three centuries and the precision of these experiments has increased continuously. Until today the most precise determination of the gravitational constant G is possible by means of the famous Cavendish torsion balance. It has been highly developed in the last century and G can now be determined with a relative error of about 10^{-4}. It is however remarkable, that none of the experiments which have been performed to obtain a precision value for G have at the same time tested the inverse square law. Only a few experiments in the range of a few millimeter up to 10 m have been performed to test the inverse square law using a laboratory test mass. But none of them result in values for the gravitational constant. This is astonishing, because a lot of experiments are limited by systematic influences and the gravitational constant cannot be investigated independent of the $1/r^2$ law of gravitation [1].

The reports of possible intermediate-range deviations from Newton's gravitational law have stimulated a fifth-force discussion during the last five years [2]. This discussion has motivated us to develop a novel gravimeter. The gravimeter is based on a Fabry-Perot microwave resonator. External forces acting on the resonator can be detected due to the shift of its resonant frequency which can be measured with very high accuracy. The gravimeter was designed to measure the gravitational field of a test mass as a function of distance and from this to determine the gravitational constant G. A possible intermediate-range force can be investigated without assuming an explicit dependence on the material in use. Systematic errors which could influence the precision of the gravitational constant will in general result in deviations from the inverse square law. Their dependence on the distance of the interacting masses is a useful information to identify and to eliminate the source of these effects. The very sensitive check of systematic influences allows to enhance the precision of the measurement of the gravitational force and this again allows to enhance the precision of the gravitational constant.

II. THE GRAVIMETER

We have started our experiments to test Newton's law with measurements of the gravitational force of a test mass in a distance of about 10 cm to the resonator [3]. This prototype set-up was used to get an idea of the achievable resolution and accuracy and to develop solutions for various problems.

Based on this experience an improved set-up was designed to measure the gravitational force in the range of 0.6 to 3.6 m [4]. The main part of the gravimeter consists of two Fabry-Perot mirrors suspended as a pair of pendula (Fig. 1). The pendula have a length l of about 2.6 m and the distance between the two mirrors is about 0.24 m. The gravitational force of a test mass M displaces both pendula and results in a change of the mirror separation b. Therefore, the length of the Gaussian beam inside the resonator changes and results in a change of the resonator frequency f. That is, the gravitational force of the test mass is measured by means of the frequency shift of the Fabry-Perot resonator.

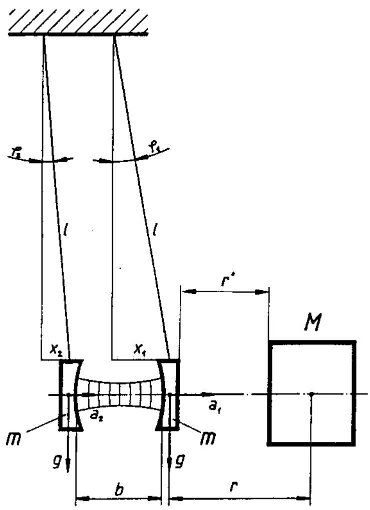

Fig. 1: The principle of the Fabry-Perot gravimeter.

A resonator frequency in the microwave range of approximately 22 GHz is chosen. A high quality factor of about $2 \cdot 10^5$ is obtained experimentally which is limited only by reflection losses of the copper mirrors. Furthermore, in the microwave region it is possible to meet all requirements to transmit the power of a source with appropriate stability to the resonator, to couple them into the resonator in order to excite the desired Fabry-Perot mode, and then to transmit them to a detector system. Furthermore, the limitation of the quality factor due to reflection losses of the copper mirrors can be overcome by using superconducting mirrors. We have demonstrated that at a temperature of 4.2 K a quality factor of $2 \cdot 10^7$ can be obtained with niobium mirrors [5]. Recent results of High-T_c-Superconductors seem to open the possibility for an improved gravimeter at 77 K [6].

Fig. 2 shows the schematic arrangement of the experimental set-up. Its main part, the two copper Fabry-Perot mirrors, are suspended as pendula inside of a vacuum tank which is mounted into a supporting steel construction.

The test mass is a cylinder of 576 kg positioned outside of the vacuum tank. Their dimensions were chosen in a way that the gravitational force between test mass and resonator is nearly the same as the gravitational force of point masses positioned at the centers of gravity. The test mass rests on a special guide rail and glides on rollers which are rotating on ball bearings fixed to this rail. The test mass can be positioned precisely by means of a motor driven spindle. All movable parts of this rail perform a strictly rotational motion and do not contribute to the mass distribution of the movable cylinder. The guide

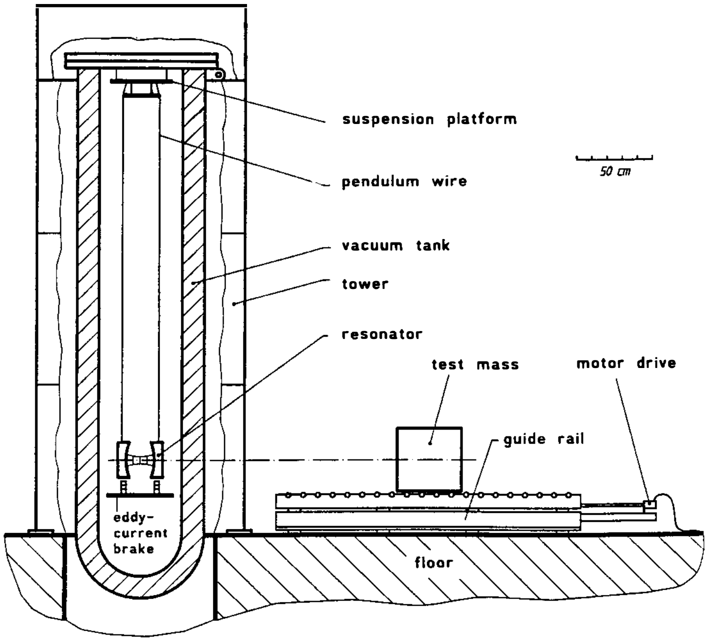

Fig. 2: Schematic arrangement of the experimental set-up.

rail is aligned such that the test mass has the same height as the resonator and moves along a line extending the resonator axis, i.e. the centers of gravity of the pendula.

III. THE EXPERIMENTAL PROCEDURE

The test mass alternates between a position with a distance r to the resonator and a reference position r_{Ref}. It rests in each position for about 15 minutes, a time which is much larger than the time constant of the pendula. The motion of the pendula is damped by eddy-current brakes and they reach their new equilibrium position in a few seconds.

The gravitational force from the moving test mass results in a change of the mirror distance Δb of maximal 12 nm and changes the resonant frequency Δf by approximately 1 kHz. The relation between Δb and Δf is determined from the measured frequency spectrum of the Fabry-Perot resonator with appropriate accuracy. An external force acting on the pendula is related to the change Δb in mirror separation via the eigenfrequency of the pendula which is determined from an additional measurement. This procedure results in a calibration between the measured quantity, the change of the resonator frequency, and the quantity of interest, the gravitational force.

To obtain a single measurement of the resonant frequency of the chosen Fabry-Perot mode, the frequency of the microwave generator is swept across the resonance curve and the resonant frequency is calculated by a least-squares fit of the transmitted power to the Lorentzian shaped resonance curve. This procedure is repeated in equidistant time steps and results in a time series of resonant frequencies which is further analysed and processed.

The periodic motion of the test mass results in a modulation of the resonator frequency with nearly rectangular shape. The modulation amplitude is determined by means of a demodulation technique which allows a strong suppression of random noise and thermal drift of the mirror separation. So far we have achieved a resolution of the change of the mirror separation of 1 to 4 pm, depending on the integration time. Therefore, the change in mirror separation Δb can be measured with high resolution and with high precision.

IV. RESULTS

The modulation procedure described above has been repeated with the test mass in different positions (but with the same reference position), and the shift of the resonant frequency has been measured as a function of distance. The distances r' between test mass and resonator are measured from the front side of the test mass to the back of the mirrors (Fig. 1). The results of the shift of the mirror distance Δb versus the distance r' are plotted in Fig. 3.

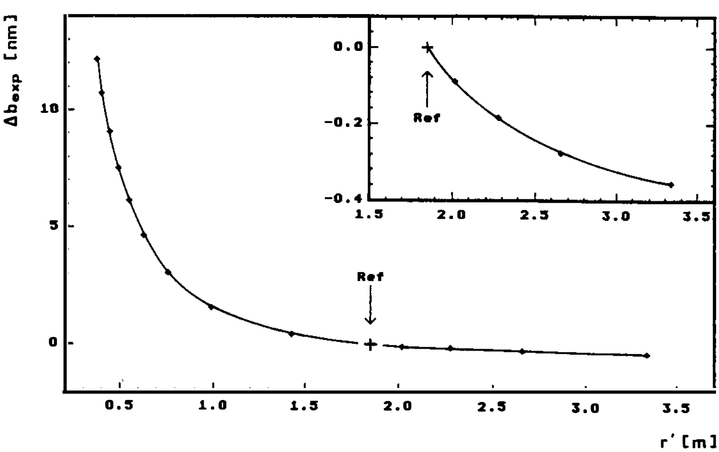

Fig. 3: The measured values of the change Δb in mirror separation due to the gravitational force of the test mass versus the distance r' of the testmass.

The corresponding distances r from the center of the test mass to the center of the pendulum next to the test mass are within the range of 0.6 m to 3.6 m. The residuals of Δb are shown in Fig. 4 in some more detail.

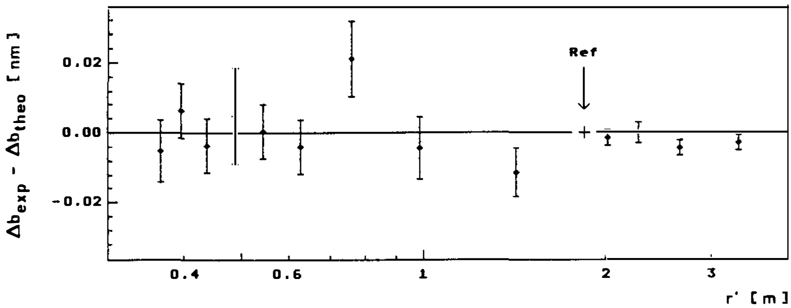

Fig. 4: The difference between measured and calculated values.

The full line in Fig. 3 is a least-squares fit of the theoretical values which are computed using Newton's inverse square law. The data points are normally distributed and no significant deviations from the inverse-square law are observed. From this fit the gravitational constant is determined to be

$$G = (6.6613 \pm 0.0011 \pm 0.0093) \cdot 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

and

$$\frac{G - G_c}{G_c} = (-1.69 \pm 0.17 \pm 1.39) \cdot 10^{-3} .$$

The first error quoted is the statistical error as determined from the least-squares fit. The second error is the systematic error which has to be taken into account additionally [4]. The value of the gravitational constant G as determined in this experiment agrees with the CODATA-value G_c very well.

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