

Orbits and probabilities of primordial black hole–star mergers

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Abstract: Primordial black holes (PBHs) are classical candidates for dark matter and may be captured by stars when passing through galaxies at suitable velocities. If the PBH follows a highly eccentric orbit with a periastron inside the stellar radius, it can penetrate the stellar interior, experiencing dynamical friction and accreting matter. Taking an asteroid-mass PBH orbiting the Sun as an example, we find that the conditions for eventual capture are extremely stringent, requiring eccentricities near unity and approximating radial infall. The effects of friction and accretion on orbital energy are minimal. Depending on the PBH–star configuration, capture timescales range from millions of years to the age of the Universe. Through physically motivated estimates, we find that the cosmological probability of such events is exceedingly low.

1. Introduction

Primordial black holes (PBHs) were first postulated by Hawking and Carr in the 1970s as potential products of high-density fluctuations in the early universe. Unlike black holes of stellar origin, PBHs could have formed during radiation domination through various mechanisms such as inflationary perturbations, phase transitions, or bubble collisions [1–7]. Given their formation before baryogenesis, PBHs are naturally non-luminous, electrically neutral, and nearly collisionless—characteristics that render them viable dark matter candidates [8–9].

Numerous observational strategies have been employed to constrain the abundance of PBHs across mass ranges, including microlensing surveys, cosmic microwave background (CMB) analyses, and gravitational wave detections [10–12]. While substantial parameter space has been ruled out, certain mass windows remain unconstrained, especially in the sub-lunar and asteroid-mass ranges [13–15].

If PBHs are indeed constituents of the dark matter halo, their occasional passage through stars could lead to energy loss via accretion and dynamical friction, potentially resulting in gravitational capture. In such scenarios, the PBH transitions from an unbound hyperbolic orbit to a bound elliptical one, and may ultimately spiral into the stellar center [16–18]. The outcome—a PBH–stellar merger—could give rise to novel astrophysical signatures or serve as a precursor to exotic compact objects, such as Hawking stars [19–21]. While previous studies have analyzed drag and radiation during single transits, few have systematically modeled the multi-pass, secular orbital decay, nor have they computed the statistical likelihood of eventual merger.

This paper addresses this gap by constructing a physically consistent, semi-analytical framework for modeling PBH capture and inspiral in stellar environments. We incorporate energy loss from dynamical friction, consider realistic stellar structure, and evaluate the evolution of orbital elements through successive transits. We then perform Monte Carlo integrations over initial conditions to derive expected merger probabilities and event rates. These calculations are based on physically motivated but idealized assumptions—chosen specifically to maximize the merger probability; the resulting estimates should thus be interpreted as optimistic upper bounds, reinforcing the robustness of our conclusions.



2. Method

2.1. Keplerian Orbital Equation

We begin with the two-body problem under Newtonian gravity. In this scenario, assuming only gravitational interaction and setting the orbital plane of the primordial black hole (PBH) as the coordinate plane, the total energy and angular momentum of the system are given by:

$$E = \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}, \quad L = \sqrt{a\mu(1-e^2)} = rv_\theta \quad (1)$$

Here, $\mu = G(M_\star + m_{pbh})$, where G is the gravitational constant, M_\star is the stellar mass, m_{pbh} is the PBH mass, a is the semi-major axis, and e is the orbital eccentricity.

2.2. Energy Dissipation Mechanisms

When a primordial black hole passes through the interior of a star (e.g., the Sun), energy loss mechanisms must be considered, including dynamical friction, accretion, and radiation. The total dissipative force is:

$$f = f_{df} + f_{ac} + f_\gamma \quad (2)$$

2.2.1 Dynamical Friction. Dynamical friction depends on the density and temperature distribution of the stellar interior as well as the PBH velocity. We adopt the formalism from Thun et al. (2016) [23], in which the acceleration due to gravitational drag in a bound gaseous medium is given by:

$$f_{df} = \frac{4\pi G^2 \rho m_{pbh}^2}{v^2} I(v, \Lambda) \quad (3)$$

Here, ρ is the local stellar density around the PBH. Since the PBH considered is of asteroid mass and atomic-scale size, the density gradient in its vicinity is negligible, and we directly adopt the solar density profile. The function $I(v, \Lambda)$ is a dimensionless quantity dependent on temperature, encapsulating the hydrodynamic details of dynamical friction.

For the Sun, which can be approximated as a monatomic ideal gas, the adiabatic index is $\gamma = 5/3$. According to [23], we have:

$$I_T(v, \Lambda) = \ln \left[2\Lambda \left(1 - \frac{1}{\mathcal{M}^2} \right) \right] \quad (4)$$

The Coulomb logarithm $\Lambda = R_{max} / R_{min}$ describes the effective range of hydrodynamic drag. For stellar interiors, we take $R_{max} = R_\odot$, and for PBHs, $R_{min} = (2Gm_{pbh})/v^2$. Particles with distances smaller than R_{min} move faster than the PBH and thus do not contribute to drag. Hydrodynamic friction is also dependent on the sound speed of the medium, and the above formula applies for Mach number $\mathcal{M} = v/v_{sound_speed} > 1$. In the case of PBHs transiting through the Sun, the escape velocity near the solar surface is approximately $v \sim 615 \text{ km/s}$, and the local sound speed is $v_{sound_speed} \sim 350 \text{ km/s}$, satisfying $\mathcal{M} > 1$ [24]. Once the stellar model is specified, the accretion by PBHs can be evaluated using:

$$p = \frac{k_B}{\mu m_H} \rho T, \quad v_{sound_speed} = \sqrt{\gamma \frac{p}{\rho}}, \quad \mathcal{M} = \frac{v}{\sqrt{\frac{5k_B T}{3\mu m_H}}} \quad (5)$$

2.2.2 Accretion and Radiation. Solving the equations of motion under general relativity for moving PBHs is notoriously difficult [22]. The stellar density is orders of magnitude lower than that of neutron stars, and the PBH is of atomic scale. The accretion region is negligible compared to the stellar radius. Prior works [16, 19] have ignored the effects of accretion and radiation. However, given that the capture timescale could be comparable to the age of the solar system, we adopt a conservative upper bound using the Eddington limit to estimate the maximum mass increase of a PBH during stellar passage:

$$L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T}, \quad L_{Edd} = \eta \dot{M} c^2 \quad (6)$$

Here, m_p is the proton mass, σ_T is the Thomson scattering cross-section, and η is the accretion efficiency factor, typically $\eta \sim 0.1$ for black holes. The Eddington limit assumes complete conversion of gravitational potential energy into radiation, and we apply this only when the PBH is inside the star. Numerical simulations (see Section 4) show that for asteroid-mass PBHs, the total mass gain during the

process is only on the order of 1000 tons, and thus negligible. Therefore, radiation from accretion is also negligible and is not considered further in this work.

2.3. Merger Timescale and Probability

2.3.1. Initial Condition Constraints. It is impractical to compute the full trajectory of a primordial black hole (PBH) from an initial position to merger with a star, especially when considering all stars in the universe. When the PBH is initially very far from the star, the orbital eccentricity must be sufficiently large for the PBH to penetrate the stellar interior. Furthermore, if the initial distance from the PBH to the Sun is sufficiently large, then even for a parabolic orbit, the initial tangential-to-radial velocity ratio $|v_\theta/v_r|$ is very small. Therefore, only PBHs whose initial velocities result in nearly radial (free-fall-like) trajectories can penetrate the stellar interior, provided that their periastron distances lie within the stellar radius.

2.3.2. Physical Model. Performing full MESA evolution simulations for every star is unfeasible. Instead, we assume a uniform stellar density profile, such that:

$$\rho_{\text{eff}} = \begin{cases} \frac{M_*}{V_*} = \frac{3M_*}{4\pi R_*^3} & r \leq R_* \\ 0 & r > R_* \end{cases} \quad (7)$$

Here, R_* and M_* are the stellar radius and mass, respectively. Under this approximation, the PBH motion inside the star reduces to a simple harmonic oscillator:

$$F = -\frac{4}{3}\pi G\rho_{\text{eff}}r \Rightarrow \ddot{r} = -\omega^2 r \quad (8)$$

with $\omega^2 = \frac{4}{3}\pi G\rho_{\text{eff}}$, yielding a period: $T = 2\pi\sqrt{\frac{3}{4\pi G\rho_{\text{eff}}}}$

We define the energy dissipation during a single periastron passage (fall from distance r_0 , then rise back to the distance where the speed is 0) as:

$$\delta E = |E_{\text{final}} - E_{\text{ini}}| = \left| -\frac{GM_*m_{\text{pbh}}}{r_0} + \frac{GM_*m_{\text{pbh}}(r_0)}{r_1} \right| = \delta E_{\text{acc}} + \delta E_{\text{dif}} \quad (9)$$

where m_{pbh} is the initial PBH mass and $m(r_0)$ is the PBH mass after accretion. δE_{acc} is the contribution from mass accretion, and δE_{dif} is due to dynamical friction. We analyze these two components separately:

- Energy Loss from Accretion δE_{acc}

Assuming Eddington-limited accretion and $m(r_0) \approx m_0$, the accretion rate can be approximated as constant: $\dot{m}_{\text{Edd}} = \frac{4\pi G m_{\text{pbh}}(r_0) m_p}{\sigma_T c \eta} \approx \frac{4\pi G m_{\text{pbh}} m_p}{\sigma_T c \eta}$. The PBH traverses the stellar interior in approximately

half an oscillation period: $t_{\text{in}} \approx \pi\sqrt{\frac{3}{4\pi G\rho_{\text{eff}}}}$. Assuming accretion occurs only within the stellar interior, the mass gain is: $\Delta m_{\text{pbh}}(r_0) \approx \dot{m}_{\text{Edd}} \times t_{\text{in}}$

- Energy Dissipation Due to Dynamical Friction δE_{dif}

The rate of energy dissipation due to dynamical friction can be expressed as:

$$\frac{dE}{dt} = -F_{\text{dif}}v \approx -\frac{4\pi G^2 \rho(r) m_{\text{pbh}}^2 I_0}{v(r)} \quad (10)$$

Inside the stellar interior, as the PBH travels from the stellar surface ($r = R_*$) to the core and then back outward, the total energy loss becomes:

$$\delta E_{\text{dif}} = \int F_{\text{dif}}v dt = \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{4\pi G^2 \rho(r) m_{\text{pbh}}^2 I_0}{v(r)} dr \quad (11)$$

Here, we approximate the stellar density as a uniform effective value ρ_{eff} , and adopt a typical PBH velocity $\bar{v} \approx \sqrt{2GM_*/R_*}$. The path length is approximated as $l = 2R_*$.

- Combined Energy Loss

Combining the contributions from both accretion and dynamical friction, the total energy loss per stellar passage is approximated as:

$$\delta E(r_0) = \delta E_{\text{acc}}(r_0) + \delta E_{\text{dif}} \approx \frac{C_1}{r_0} + C_2 \quad (12)$$

$$\text{where: } C_1 = \frac{4\pi^2 G^2 m_{\text{pbh}} m_p}{\sigma_T c \eta} \sqrt{M_* R_*^3}, \quad C_2 = \frac{3Gm_{\text{pbh}}^2 I_0}{R_*}.$$

2.3.3. *Number of Passages and Capture Timescale.* Provided that the energy loss per passage $\delta E \ll E_{\text{ini}}$ is much smaller than the total orbital energy E , the number of stellar passages required for merger can be approximated as:

$$N \sim \left| \frac{E_{\text{ini}}}{\delta E} \right| \quad (13)$$

The mechanical energy of the system is conserved and given by: $E = \frac{1}{2}v^2 - \frac{GM_*}{r} = -\frac{GM_*}{r_0}$. Then, the time differential is: $dt = \frac{dr}{v(r)} = \frac{dr}{\sqrt{2GM_*\left(\frac{1}{r} - \frac{1}{r_0}\right)}}$.

$$\text{Integrating from } r_0 \text{ to } 0 \text{ gives the free-fall timescale: } t(r_0) = \pi \sqrt{\frac{r_0^3}{2GM_*}}.$$

The total PBH capture time is then: $T_{\text{capture}} = N \times t(r_0)$.

In our calculation of the capture timescale, this simplification neglects the gradual circularization of the orbit and feedback between orbital parameters and dissipation efficiency; it intentionally maximizes the cumulative energy loss rate and thus shortens the predicted capture time. In more realistic scenarios, orbital precession and angular momentum buildup would typically lengthen the timescale or suppress capture altogether. Therefore, our approximation yields a conservative upper limit on the merger probability and serves to demonstrate that, even under assumptions favorable to capture, the expected event rate remains negligible.

2.3.4. *Cosmological Assumptions and Statistical Distributions.* We assume that the mass m of primordial black holes (PBHs) is uniformly distributed within a narrow mass range. PBHs are treated as a subcomponent of dark matter, comprising no more than 1% of the total dark matter abundance, and are locally uniformly distributed around stars in space. The stellar mass M_* follows the Salpeter Initial Mass Function (IMF). Under the gravitational influence of an individual star, PBHs fall in from isotropically distributed initial positions, with the radial distance x satisfying: $x \in [R_*, 10^4 R_*]$, where R_* is the stellar radius.

Finally, we consider the effect of directional selection. The periastron distance r_p of a PBH determines whether it will merge with a star or bypass it. If $r_p > R_*$, the PBH will not enter the stellar interior and will therefore bypass the star without undergoing a merger. To evaluate this probability, we assume that PBHs are isotropically distributed in incident direction over a spherical surface. Under this assumption, the polar angle θ must satisfy the condition that the tangential velocity component remains below a critical threshold: $v_\theta = v \sin \theta = \sqrt{\frac{2GM_*}{r}} \sin \theta$. To ensure the PBH passes within the stellar radius R_* , the condition is: $\theta \leq \theta_0 = \arcsin \sqrt{\frac{R_*}{r}}$. The corresponding probability that the PBH trajectory is sufficiently radial to intersect the stellar interior is then:

$$P_{\text{dir}}(r) = \int_0^{\theta_0} \frac{\sin \theta}{2} d\theta \times 2 = 1 - \sqrt{1 - \frac{R_*}{r}} \quad (14)$$

Considering all the above factors—namely the spatial distribution of PBHs, the probability of directional incidence, the merger energy loss mechanism, and capture timescale—the average PBH–star merger rate per star is denoted by:

$$R_{\text{per star}} = \rho_{\text{PBH}} \int_{m_{\text{min}}}^{m_{\text{max}}} \frac{dm}{m_{\text{max}} - m_{\text{min}}} \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{M^{-2.35}}{\Phi_{\text{norm}}} dM \int_{R_*(M)}^{10^4 R_*(M)} \frac{3x^2}{(10^4 R_*)^3 - R_*^3} \frac{1}{T_{\text{capture}}(m, x, M)} \left[1 - \sqrt{1 - \frac{R_*(M)}{x}} \right] \quad (15)$$

where Φ_{norm} is the normalization constant, and the total mass density of primordial black holes (PBHs) is given by: $\rho_{\text{PBH}} = f_{\text{PBH}} \cdot \rho_{\text{DM}}$, $f_{\text{PBH}} = 0.01$.

The assumption of uniform PBH mass and spatial distributions in our Monte Carlo sampling is deliberately chosen to maximize the local merger probability. Realistic PBH populations are expected to be more sparsely distributed and may exhibit clustering or exclusion effects at sub-galactic scales. Similarly, the adopted mass range and uniform probability density neglect theoretical suppression mechanisms and observational constraints that would further reduce the number of PBHs capable of producing stellar-scale mergers. Thus, our calculated rates should be understood as upper bounds derived under optimistic premises. The fact that even under these conditions, the predicted merger probability remains orders of magnitude below detectability strengthens the robustness of our conclusion.

3. Conclusion

3.1. Orbital Analysis

Taking the Sun as a representative case for orbital analysis, we employ a solar model computed by the *Modules for Experiments in Stellar Astrophysics* (MESA) framework, which describes a star of mass $1 M_{\odot}$ and metallicity $Z = 0.02$ at an age of 4.5 billion years [25–29]. The mass of the primordial black hole (PBH) is chosen to be $m_{pbh} \sim 10^{-12} M_{\odot} \sim 2 \times 10^{21} g$. To generalize the effect of dynamical friction, we introduce a dimensionless damping coefficient \mathcal{K} , and rewrite the frictional force as:

$$f_{df} = \mathcal{K} \times \frac{4\pi G^2 \rho m_{pbh}^2}{v^2} I(v, \Lambda) \quad (16)$$

By varying the value of \mathcal{K} , we can not only explore the orbital behavior more conveniently but also quantify the relative strength of dynamical friction concerning gravitational attraction. For instance, considering an orbit with semi-major axis $a = 10 R_{\odot}$ and eccentricity $e = 0.95$, the trajectory exhibits noticeable precession. As \mathcal{K} increases, keeping all other parameters constant, the orbit spirals inward. As shown in Figure 1, When $\mathcal{K} \sim 10^8$, the trajectory terminates near the stellar core as modeled by MESA, forming a spiral orbit with decreasing semi-major axis and periastron distance. These results demonstrate that the total energy dissipation due to dynamical friction is small throughout the inspiral process, thereby supporting the physical approximations adopted in the capture timescale and merger probability estimates.

3.2. Merger Timescale

We compute the merger (capture) timescale $T_{capture}$ for all combinations of stellar mass M_* in $(0.08 M_{\odot}, 100 M_{\odot})$ and PBH mass m_{pbh} in $(10^{18}, 10^{24}) g$. For each mass pair, the initial orbital radius is taken from: r_0 in $(R_*, 10000 R_*)$. The stellar radius is modeled using a simplified scaling relation:

$$R_* = R_{\odot} \times \begin{cases} \left(\frac{M_*}{M_{\odot}}\right)^{0.8} & M_* \leq M_{\odot} \\ \left(\frac{M_*}{M_{\odot}}\right)^{0.5} & M_* > M_{\odot} \end{cases} \quad (17)$$

The results yield the characteristic time required for the PBH to spiral in from r_0 and merge with the star. When both m_{pbh} and M_* are small (e.g. $m_{pbh} = 10^2 g$, $M_* = 0.08 M_{\odot}$, green dashed line in Figure 2), the capture time is typically long. For $r_0 \gtrsim 10^2 R_{\odot}$, $T_{capture}$ exceeds the age of the Universe. Only for very close-in systems with $r_0 \lesssim 10 R_{\odot}$ does the merger time fall below 10^9 year. Furthermore, as the PBH mass increases to $10^{24} g$ (blue dotted line in Figure 2), the capture process accelerates significantly: even for loose orbits with $r_0 \sim 10^3 R_{\odot}$, mergers can complete within 10^6 – 10^7 year.

Stellar mass also significantly affects the timescale. For example, with $m_{pbh} = 10^{21} g$ and $r_0 \lesssim 10^2 R_{\odot}$, merger time is less than 10^8 year if $r_0 \lesssim 10^2 R_{\odot}$. In the case of a massive star with $M_* = 100 M_{\odot}$ (green dash-dotted line in Figure 2), stronger gravitational focusing further reduces the timescale by an order of magnitude. The purple dashed line compares mergers involving $1 M_{\odot}$ star with

PBHs of mass $10^{18}g$ and $10^{24}g$, respectively—showing that a six-order-of-magnitude increase in PBH mass reduces the merger time by roughly 10^6 times.

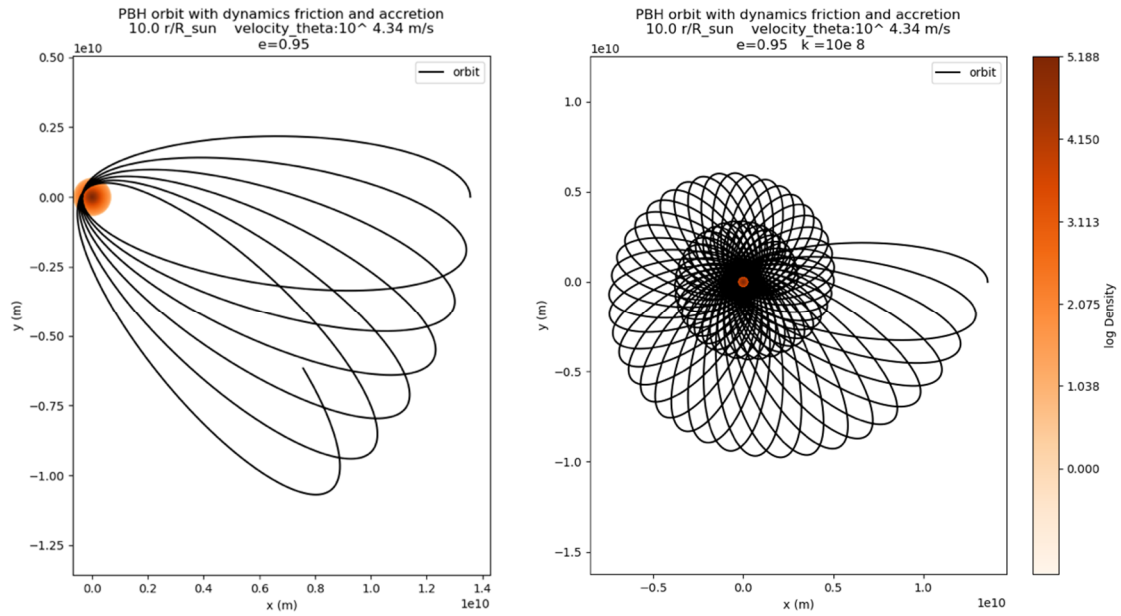


Figure 1. Orbital trajectories of primordial black holes with identical semi-major axis and eccentricity, evolved over the same timescale, for $\mathcal{K} = 1$ and $\mathcal{K} = 10^8$.

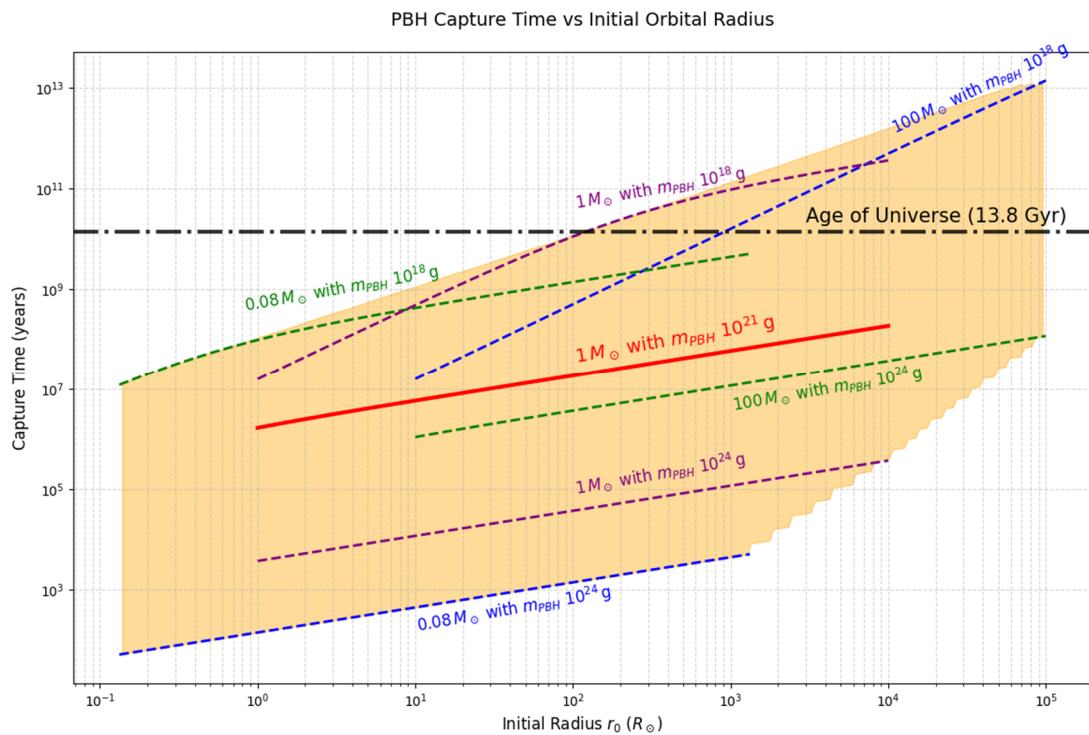


Figure 2. Merger timescales of primordial black holes and stars for different mass combinations.

In general, the merger timescale is highly sensitive to PBH mass, with heavier PBHs ($\gtrsim 10^{22}$ g) leading to rapid inspirals. More massive stars, due to stronger gravity, further accelerate mergers—especially for intermediate and large orbital separations ($r_0 \sim 10^2\text{--}10^4 R_\odot$).

3.3. Merger Probability Estimation

Assuming the dark matter density is approximately $\rho_{DM} \sim 10^{-27} \text{ kg/m}^3$, and that PBHs constitute a fraction $f_{PBH} = 0.01$, the mean PBH–star merger rate per star is estimated as: $R_{\text{per star}} \sim 10^{-38}$ [1/star/year]

Given an estimated total of $\sim 10^{23}$ stars in the Universe, the global PBH–star merger rate is: $R_{\text{per year}} \sim 10^{-15}$ [1/year]

Assuming the age of the Universe is $\sim 10^{10}$ year, the total number of PBH–star merger events up to the present is: $n \sim 10^{-5}$

These results demonstrate that even under assumptions that maximize the likelihood of PBH–star mergers, the expected event rate remains vanishingly small.

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