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3 + 1 active–sterile neutrino mixing in $B - L$ model with $S_3 \times Z_4 \times Z_2$ symmetry for normal neutrino mass ordering

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Abstract We propose a non-renormalizable $B - L$ model with $S_3 \times Z_4 \times Z_2$ symmetry which successfully accommodates the current active–sterile neutrino mixing in $3 + 1$ scheme. The S_3 flavor symmetry is supplemented by $Z_4 \otimes Z_2$ symmetry to consolidate the Yukawa interaction of the model. The presence of $S_3 \otimes Z_4 \otimes Z_2$ flavour symmetry plays an important role in generating the desired structure of the neutrino mass matrix. The model can reproduce the recent observed active-neutrino neutrino oscillation data for normal ordering in which two sterile–active mixing angles $\theta_{14,24}$ get the best-fit values and the obtained values of θ_{34}, δ_{14}, δ_{14}, the sum of neutrino mass and the effective neutrino masses are within their currently allowed ranges.

1 Introduction

The neutrino mass and mixing is one of the most exciting issues of modern physics. In recent years, the leptonic mixing angles and neutrino mass squared differences are measured with a high precision [1]. Apart from the currently unknown parameters in the neutrino sector, such as mass hierarchy, leptonic Dirac CP violating phase [1], there are some experimental observations that cannot be accommodated in the three neutrino framework, see for instance LSND [2], MiniBooNE [3] and the other experiments [4–15]. These observations could be explained by supplementing at least an additional fourth neutrino having non-trivial mixing with active neutrinos, called sterile neutrinos, with mass in the eV range, i.e., $\Delta m^2_{41} \gg |\Delta m^2_{31}|$. For the neutrino mass spectrum, the global analysis shows a hint in favor of the normal ordering (NO) over the inverted ordering (IO) at more than 3σ [16] and a preference for NO at about 2σ [17,18], i.e., NO seems to be favored than IO and we thus only consider the NO.

The best-fit values and 3σ ranges of neutrino mass squared differences and the mixing angles in the three neutrino framework for NO [1] and $(3 + 1)$ neutrino framework [10] are given in Table 1. In order to explain the observed lepton flavor mixing pattern, flavour symmetries have proven many advantages which have been widely used in different works, see for instance, S_3 [19–25], S_4 [26–35], T' [36–38], D_4 [39–46], $\Delta(27)$ [47–56]. In recent years, there have been various proposals to generate the active–sterile neutrino mass matrix within different seesaw frameworks [15,57–115]. The combination of $B - L$ model and discrete symmetries S_3, Q_6, D_4, $\Sigma(18)$, $\Delta(27)$ has been done in Refs. [24,116–120] in which the observed structure of lepton and quark masses and mixing angles in the three neutrino scheme has been studied. In $(3 + 1)$ neutrino scheme, the $B - L$ model with S_3 symmetry was first presented in Ref. [64], however, the active–sterile neutrino mass and mixing has not been addressed.[1] In other recent works, active–sterile neutrino mass and mixing have been considered with A_4 [66,68,90,104,105], D_4 [77] and texture zeros with Z_N [87,92,95]. To our best knowledge S_3 symmetry has not been considered before in the $3 + 1$ neutrino scheme with $B - L$ model. We thus suggest a $B - L$ extension based on $S_3 \otimes Z_4 \otimes Z_2$ symmetry which successfully accommodates the observed sterile–active neutrino mixing patterns within the framework of the $3 + 1$ scheme.

The remaining of this work is organised as follows. A brief description of the model is presented in Sect. 2. Section 3 is devoted to the active−sterile neutrino mixing. The numerical

[1] In the extension with $U(1)_{B-L}$ gauge symmetry, the anomalies can be cancelled in different ways [121–136] with various charge assignments of $B - L$. We develop the model proposed in Refs. [122,132] whereby all of three right-handed neutrinos owns $B - L = -1$ which is different from the previous work [64] in which two of right-handed neutrinos owns $B - L = -4$ and the other one has $B - L = 5$.

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Table 1 The global analysis of neutrino oscillation data with three light active neutrinos at 3σ range and the best fit results taken from [1], and one extra sterile neutrino constraints taken from Ref. [10]

Parameters	The best-fit values	3σ range
$\Delta m^2_{21}(10^{-5}\,\mathrm{eV}^2)$	7.42	$6.82 \to 8.04$
$\Delta m^2_{31}(10^{-3}\,\mathrm{eV}^2)$	2.517	$2.435 \to 2.598$
$\sin^2\theta_{12}$	0.304	$0.269 \to 0.343$
$\sin^2\theta_{23}$	0.573	$0.415 \to 0.616$
$\sin^2\theta_{13}$	0.02219	$0.02032 \to 0.02410$
$\delta(^\circ)$	197	$120 \to 369$
$\|U_{14}\|^2$	0.020	0.0098–0.031
$\|U_{24}\|^2$	0.015	0.006–0.026
$\|U_{34}\|^2$	✓	0–0.039
δ_{14}	✓	0–2π
δ_{24}	✓	0–2π

analysis is given in Sect. 4. Finally, some conclusions are drawn in Sect. 5.

2 The model

Apart from the SM symmetry, one $U(1)_{B-L}$ gauge symmetry and one non-Abelian discrete symmetry S_3 together with two Abelian symmetries Z_4, Z_2 are supplemented. The symmetry of the model is $\mathrm{G}\times U(1)_{B-L}\otimes S_3\otimes Z_4\otimes Z_2 \equiv \Gamma$ where G is the gauge symmetry of the SM. Regarding the particle content, three right-handed neutrinos $(\nu_{1R}, \nu_{\alpha R})$, one sterile neutrino (S) and five $SU(2)_L$ singlet scalars are introduced in comparison with the SM. The assignments of lepton and scalar fields in the considered model are summarized in Table 2.

The Yukawa couplings, up to five-dimension, which are invariant under Γ symmetry, are:

$$\begin{aligned}
-\mathcal{L}^{lep}_Y =\; & \frac{h_1}{\Lambda}(\bar{\psi}_{1L}l_{1R})_{\underline{1}}(H\phi)_{\underline{1}} + \frac{h_2}{\Lambda}(\bar{\psi}_{\alpha L}l_{\alpha R})_{\underline{1}}(H\phi)_{\underline{1}} \\
& + \frac{h_3}{\Lambda}(\bar{\psi}_{\alpha L}l_{\alpha R})_{\underline{1}'}(H\phi')_{\underline{1}'} \\
& + \frac{x_1}{\Lambda}(\bar{\psi}_{1L}\nu_{1R})_{\underline{1}}(\widetilde{H}\widetilde{\phi})_{\underline{1}} + \frac{x_2}{\Lambda}(\bar{\psi}_{\alpha L}\nu_{\alpha R})_{\underline{1}}(\widetilde{H}\widetilde{\phi})_{\underline{1}} \\
& + \frac{x_3}{\Lambda}(\bar{\psi}_{\alpha L}\nu_{\alpha R})_{\underline{1}'}(\widetilde{H}\widetilde{\phi}')_{\underline{1}'} \\
& + \frac{y_1}{2\Lambda}(\overline{\nu^c_{\alpha R}}\nu_{\alpha R})_{\underline{2}}(\chi\varphi)_{\underline{2}} \\
& + \frac{y_2}{2\Lambda}\left[(\overline{\nu^c_{1R}}\nu_{\alpha R})_{\underline{2}} + (\overline{\nu^c_{\alpha R}}\nu_{1R})_{\underline{2}}\right](\chi\varphi)_{\underline{2}} \\
& + \frac{z_1}{\Lambda}(\bar{S}^c\nu_{1R})_{\underline{1}'}(\varphi\xi)_{\underline{1}'} \\
& + \frac{z_2}{\Lambda^2}(\bar{S}^c\nu_{\alpha R})_{\underline{2}}(\phi\varphi\xi)_{\underline{2}} + \mathrm{H.c.} \qquad (1)
\end{aligned}$$

It should be noted that the following terms $(\overline{\psi}_{1L}l_{\alpha R})_{\underline{2}}(H\chi)_{\underline{2}}$, $(\overline{\psi}_{\alpha L}l_{1R})_{\underline{2}}(H\chi)_{\underline{2}}$, $(\overline{\psi}_{\alpha L}l_{\alpha R})_{\underline{2}}(H\chi)_{\underline{2}}$, $(\overline{\psi}_{1L}\nu_{\alpha R})_{\underline{2}}(\widetilde{H}\widetilde{\chi})_{\underline{2}}$, $(\overline{\psi}_{\alpha L}\nu_{1R})_{\underline{2}}(\widetilde{H}\widetilde{\chi})_{\underline{2}}$, $(\overline{\psi}_{\alpha L}\nu_{\alpha R})_{\underline{2}}(\widetilde{H}\widetilde{\chi})_{\underline{2}}$ and $(\overline{\nu}^c_{1R}\nu_{1R})_{\underline{1}}(\xi\varphi)_{\underline{1}}$ are forbidden by Z_2 symmetry while $(\overline{\nu}^c_{\alpha R}\nu_{\alpha R})_{\underline{1}'}(\xi\varphi)_{\underline{1}'}$ is forbidden by Z_4 symmetry.

We will show that the following VEV alignments

$$\begin{aligned}
&\langle H\rangle = (0\quad v_H)^T,\quad \langle\phi\rangle = v_\phi,\quad \langle\phi'\rangle = v_{\phi'},\quad \langle\varphi\rangle = v_\varphi, \\
&\langle\chi\rangle = (\langle\chi_1\rangle, \langle\chi_2\rangle),\quad \langle\chi_2\rangle = \langle\chi_1\rangle = v_\chi,\quad \langle\xi\rangle = v_\xi, \qquad (2)
\end{aligned}$$

satisfy the minimum condition of $\mathrm{V}_{\mathrm{scalar}}$ in Appendix A. Namely, in the minimum minimization condition of $\mathrm{V}_{\mathrm{scalar}}$, let us put $v_{\chi_2} = v_{\chi_1} = v_\chi$ and $v^*_H = v_H$, $v^*_\phi = v_\phi$, $v^*_{\phi'} = v_{\phi'}$, $v^*_\varphi = v_\varphi$, $v^*_\chi = v_\chi$ and $v^*_\xi = v_\xi$ which leads to

$$\begin{aligned}
\frac{\partial \mathrm{V}_{\mathrm{scalar}}}{\partial v^*_i} &= \frac{\partial \mathrm{V}_{\mathrm{scalar}}}{\partial v_i},\quad \frac{\partial^2 \mathrm{V}_{\mathrm{scalar}}}{\partial v^{*2}_i} \\
&= \frac{\partial^2 \mathrm{V}_{\mathrm{scalar}}}{\partial v^2_i}\quad (v_i = v_H, v_\phi, v_{\phi'}, v_\varphi, v_\chi, v_\xi), \qquad (3)
\end{aligned}$$

and the minimum condition becomes

$$\begin{aligned}
&\mu^2_H + 2\lambda_H v^2_H + \lambda_{H\phi}v^2_\phi + \lambda_{H\phi'}v^2_{\phi'} \\
&\quad + \lambda_{H\varphi}v^2_\varphi + 2\lambda_{H\chi}v^2_\chi + \lambda_{H\xi}v^2_\xi = 0, \qquad (4)\\
&\mu^2_\phi + \lambda_{H\phi}v^2_H + 2\lambda_\phi v^2_\phi + \lambda_{\phi\phi'}v^2_{\phi'} \\
&\quad + \lambda_{\phi\varphi}v^2_\varphi + 2\lambda_{\phi\chi}v^2_\chi + \lambda_{\phi\xi}v^2_\xi = 0, \qquad (5)\\
&\mu^2_{\phi'} + \lambda_{H\phi'}v^2_H + \lambda_{\phi\phi'}v^2_\phi + 2\lambda_{\phi'}v^2_{\phi'} \\
&\quad + \lambda_{\phi'\varphi}v^2_\varphi + 2\lambda_{\phi'\chi}v^2_\chi + \lambda_{\phi'\xi}v^2_\xi = 0, \qquad (6)\\
&\mu^2_\varphi + \lambda_{H\varphi}v^2_H + \lambda_{\phi\varphi}v^2_\phi + \lambda_{\phi'\varphi}v^2_{\phi'} \\
&\quad + 2\lambda_\varphi v^2_\varphi + 2\lambda_{\varphi\chi}v^2_\chi + \lambda_{\varphi\xi}v^2_\xi = 0, \qquad (7)\\
&\mu^2_\chi + \lambda_{H\chi}v^2_H + \lambda_{\phi\chi}v^2_\phi + \lambda_{\phi'\chi}v^2_{\phi'} \\
&\quad + \lambda_{\varphi\chi}v^2_\varphi + 2\lambda_\chi v^2_\chi + \lambda_{\chi\xi}v^2_\xi = 0, \qquad (8)\\
&\mu^2_\xi + \lambda_{H\xi}v^2_H + \lambda_{\phi\xi}v^2_\phi + \lambda_{\phi'\xi}v^2_{\phi'} \\
&\quad + \lambda_{\varphi\xi}v^2_\varphi + 2\lambda_{\chi\xi}v^2_\chi + 2\lambda_\xi v^2_\xi = 0, \qquad (9)\\
&\lambda_H v_H > 0,\quad \lambda_\phi v_\phi > 0,\quad \lambda_{\phi'}v_{\phi'} > 0, \\
&\quad \lambda_\varphi v_\varphi > 0,\quad \lambda_\chi v_\chi > 0,\quad \lambda_\xi v_\xi > 0, \qquad (10)
\end{aligned}$$

where, for simplicity, the following notations have been used:

$$\begin{aligned}
&\lambda_\chi = 2\lambda_{1\chi} + \lambda_{3\chi}, \\
&\lambda_{H\phi} = \lambda_{1H\phi} + \lambda_{2H\phi},\quad \lambda_{H\phi'} = \lambda_{1H\phi'} + \lambda_{2H\phi'}, \\
&\quad \lambda_{H\varphi} = \lambda_{1H\varphi} + \lambda_{2H\varphi}, \\
&\lambda_{H\chi} = \lambda_{1H\chi} + \lambda_{2H\chi},\quad \lambda_{H\xi} = \lambda_{1H\xi} + \lambda_{2H\xi}, \\
&\quad \lambda_{\phi\phi'} = \lambda_{1\phi\phi'} + \lambda_{2\phi\phi'}, \\
&\lambda_{\phi\varphi} = \lambda_{1\phi\varphi} + \lambda_{2\phi\varphi},\quad \lambda_{\phi\chi} = \lambda_{1\phi\chi} + \lambda_{2\phi\chi}, \\
&\quad \lambda_{\phi\xi} = \lambda_{1\phi\xi} + \lambda_{2\phi\xi},
\end{aligned}$$

Table 2 The particle contents of the model and their transformations under the chosen symmetries

	ψ_{1L}	$\psi_{\alpha L}$	l_{1R}	$l_{\alpha R}$	H	ϕ	ϕ'	φ	χ	ξ	ν_{1R}	$\nu_{\alpha R}$	S
$SU(2)_L$	2	2	1	1	2	1	1	1	1	1	1	1	1
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	$\frac{1}{2}$	0	0	0	0	0	0	0	0
$U(1)_{B-L}$	-1	-1	-1	-1	0	0	0	2	0	0	-1	-1	-1
S_3	$\underline{1}$	$\underline{2}$	$\underline{1}$	$\underline{2}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{2}$	$1'$	1	2	$1'$
Z_4	1	1	$-i$	$-i$	1	i	i	i	i	-1	i	i	1
Z_2	+	+	+	+	+	+	+	−	−	−	+	+	+

$$\lambda_{\phi'\varphi} = \lambda_{1\phi'\varphi} + \lambda_{2\phi'\varphi}, \quad \lambda_{\phi'\chi} = \lambda_{1\phi'\chi} - \lambda_{2\phi'\chi},$$
$$\lambda_{\phi'\xi} = \lambda_{1\phi'\xi} + \lambda_{2\phi'\xi},$$
$$\lambda_{\varphi\chi} = \lambda_{1\varphi\chi} + \lambda_{2\varphi\chi}, \quad \lambda_{\varphi\xi} = \lambda_{1\varphi\xi} + \lambda_{2\varphi\xi},$$
$$\lambda_{\chi\xi} = \lambda_{1\chi\xi} - \lambda_{2\chi\xi}. \tag{11}$$

The system of Eqs. (4)–(9) always own the solution as given in Appendix B.

We will show that the inequalities in Eq. (10) are always satisfied by the solutions of Eqs. (4)–(9) as shown in Appendix B. For instance, for the following benchmark point

$$\lambda_\phi = \lambda_{\phi'} = \lambda_\varphi = \lambda_\chi = \lambda_\xi = \lambda_x = 10^{-3}, \tag{12}$$
$$\lambda_{H\varphi} = \lambda_{H\chi} = \lambda_{H\phi} = \lambda_{H\phi'} = \lambda_{H\xi} = -\lambda_y = -10^{-4}, \tag{13}$$
$$\mu_H = \mu_\phi = \mu_{\phi'} = \mu_\varphi = \mu_\chi = \mu_\xi = 10^8\,\mathrm{eV}, \tag{14}$$
$$v_H = v_{\phi'} = 10^{11}\,\mathrm{eV}, \quad v_\varphi = v_\chi = v_\phi = v_\xi = 10^{13}\,\mathrm{eV}, \tag{15}$$

the inequalities in Eq. (10) are always satisfied. Therefore, the VEV alignments in Eq. (2) is a solution of the potential minimum condition.

3 3 + 1 active–sterile neutrino mixing

Using the Clebsch–Gordan coefficients of S_3 group presented in Ref. [137], from Eqs. (1) and (2), we find the charged lepton masses and the charged-lepton diagonalization matrices as follows

$$m_e = a_{1l}, \quad m_\mu = a_{2l} - a_{3l}, \quad m_\tau = a_{2l} + a_{3l}, \tag{16}$$
$$U_{lL} = U_{lR} = 1, \tag{17}$$

with

$$a_{1l} = h_1 v_\phi \lambda_H, \quad a_{2l} = h_2 v_\phi \lambda_H,$$
$$a_{3l} = h_3 v_{\phi'} \lambda_H \quad (\lambda_H = v_H/\Lambda). \tag{18}$$

Because the charged-lepton diagonalization matrices U_{lL}, U_{lR} take the diagonal form, the lepton mixing matrix is that of the neutrinos. Furthermore, Eq. (16) tells us that m_μ and m_τ are distinguished from each other by ϕ', thus, ϕ' is introduced in accompany with ϕ.

Next, comparing the obtained result in Eq. (16) with the experimental values of $m_{e,\mu,\tau}$ taken from Ref. [138] $m_e = 0.51099\,\mathrm{MeV}$, $m_\mu = 105.65837\,\mathrm{MeV}$, $m_\tau = 1776.86\,\mathrm{MeV}$ yields

$$a_{1l} = 5.11 \times 10^5\,\mathrm{eV}, \quad a_{2l} = 9.41 \times 10^8\,\mathrm{eV},$$
$$a_{3l} = 8.36 \times 10^8\,\mathrm{eV}. \tag{19}$$

Let us now consider the neutrino sector. From Eq. (1), after symmetry breaking, i.e., when the scalar fields get the VEVs, we find the 7×7 neutrino mass matrix in the basis (ν_L, ν_R^c, S^c) as follows

$$M_\nu^{7\times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}, \tag{20}$$

where M_D, M_R and M_S are, respectively, the Dirac, Majorana and sterile neutrino mass matrices that take the following forms:

$$M_D = \begin{pmatrix} a_{1D} & 0 & 0 \\ 0 & a_{2D} - a_{3D} & 0 \\ 0 & 0 & a_{2D} + a_{3D} \end{pmatrix},$$
$$M_R = \begin{pmatrix} 0 & a_{2R} & a_{2R} \\ a_{2R} & a_{1R} & 0 \\ a_{2R} & 0 & a_{1R} \end{pmatrix}, \tag{21}$$
$$M_S = (a_{1S} \quad a_{2S} \quad a_{2S}), \tag{22}$$

where

$$a_{1D} = \frac{x_1 v_H^* v_\phi^*}{\Lambda}, \quad a_{2D} = \frac{x_2 v_H^* v_\phi^*}{\Lambda}, \quad a_{3D} = \frac{x_3 v_H^* v_{\phi'}^*}{\Lambda},$$
$$a_{1R} = y_1 v_\chi v_\varphi, \quad a_{2R} = y_2 v_\chi v_\varphi,$$
$$a_{1S} = \frac{z_1 v_\xi v_\varphi}{\Lambda}, \quad a_{2S} = \frac{z_2 v_\chi v_\phi v_\varphi}{\Lambda^2}. \tag{23}$$

In the case where $M_R \gg M_S > M_D$, similar to the typical type-I seesaw model, one can block-diagonalize the full mass matrix 7×7 by using the seesaw formula, the effective 4×4 light neutrino mass matrix in the basis (ν_L, S^c) can be written as [68]

$$M_\nu = -\begin{pmatrix} M_D M_R^{-1} M_D^T & M_D M_R^{-1} M_S^T \\ M_S (M_R^{-1})^T M_D^T & M_S M_R^{-1} M_S^T \end{pmatrix}. \tag{24}$$

Combining Eqs. (21), (22) and (24) yields the 4×4 active–sterile mass matrix in the explicit form as follows

$$M_\nu^{4\times 4} = M_0 + \delta M_1 + \delta M_2, \tag{25}$$

where

$$M_0 = \begin{pmatrix} a_1 & -b_1 & -b_1 & b_S \\ -b_1 & -a_2 & a_2 & -c_S \\ -b_1 & a_2 & -a_2 & -c_S \\ b_S & -c_S & -c_S & a_S \end{pmatrix}, \tag{26}$$

$$\delta M_1 = \begin{pmatrix} 0 & u_1 & -u_1 & 0 \\ u_1 & u_2 & 0 & u_S \\ -u_1 & 0 & -u_2 & -u_S \\ 0 & u_S & -u_S & 0 \end{pmatrix},$$

$$\delta M_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\varepsilon & -\varepsilon & 0 \\ 0 & -\varepsilon & -\varepsilon & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{27}$$

with

$$a_1 = \frac{a_{1D}^2 a_{1R}}{2a_{2R}^2}, \quad a_2 = \frac{a_{2D}^2}{2a_{1R}},$$
$$b_1 = \frac{a_{1D}a_{2D}}{2a_{2R}}, \quad c_S = \frac{a_{1S}a_{2D}}{2a_{2R}},$$
$$a_S = \frac{a_{1S}(a_{1R}a_{1S} - 4a_{2R}a_{2S})}{2a_{2R}^2},$$
$$b_S = \frac{a_{1D}(a_{1R}a_{1S} - 2a_{2R}a_{2S})}{2a_{2R}^2}, \tag{28}$$
$$u_1 = \frac{a_{1D}a_{3D}}{2a_{2R}}, \quad u_2 = \frac{a_{2D}a_{3D}}{a_{1R}},$$
$$u_S = \frac{a_{1S}a_{3D}}{2a_{2R}}, \quad \varepsilon = \frac{a_{3D}^2}{2a_{1R}}. \tag{29}$$

Provided that $x_1 \simeq x_2 \simeq x_3 \simeq y_1 \simeq y_2 = \lambda_{xy}$ and $z_1 \simeq z_2 = 10^{-2}\lambda_{xy}$ and using the benchmark point in Eq. (15), from Eqs. (23), (28) and (29), we can evaluate

$$\left(\frac{v_{\phi'}}{\Lambda}\right)^2 \sim \varepsilon \ll u_1 \approx u_2 \approx u_S \sim \frac{v_{\phi'}}{\Lambda} \ll \frac{v_x}{\Lambda} \sim a_1 \approx a_2 \approx b_1 \approx a_S \approx b_S, \tag{30}$$

with $v_x = \{v_\phi, v_\varphi, v_\chi, v_\xi\}$.

On the other hand, it is noted that, in Eq. (25), H, ϕ, φ, χ and ξ are responsible for the first matrix M_0 providing the $\mu-\tau$ symmetry while ϕ' contributes to the second and third terms $(\delta M_1, \delta M_2)$. We will therefore consider the contribution of ϕ' as a small perturbation with the perturbation parameter $\frac{v_{\phi'}}{\Lambda}$. Furthermore, $\varepsilon \sim \left(\frac{v_{\phi'}}{\Lambda}\right)^2$ is a second-order parameter thus δM_2 will be ignored in calculation process, i.e.,

$$\delta M = \delta M_1 + \delta M_2 \simeq \delta M_1. \tag{31}$$

The matrix M_0 in Eq. (26) owns four eigenvalues and the corresponding mixing matrix as follows

$$m_1 = 0, \quad m_2 = \Gamma_3 - \Gamma_4, \quad m_3 = \Gamma_2, \quad m_4 = \Gamma_3 + \Gamma_4, \tag{32}$$

$$U_0 = \begin{pmatrix} g_1 & n_1 & 0 & r_1 \\ g_2 & n_2 & -\frac{1}{\sqrt{2}} & r_2 \\ g_2 & n_2 & \frac{1}{\sqrt{2}} & r_2 \\ g_0 & n_0 & 0 & r_0 \end{pmatrix}, \tag{33}$$

where

$$\Gamma_2 = -\frac{a_{2D}^2}{a_{1R}}, \quad \Gamma_3 = \frac{a_{1R}(a_{1D}^2 + a_{1S}^2) - 4a_{1S}a_{2S}a_{2R}}{4a_{2R}^2},$$
$$\Gamma_4 = \frac{\sqrt{(a_{1D}^2 + a_{1S}^2)\left[a_{1D}^2 a_{1R}^2 + 8a_{2D}^2 a_{2R}^2 + (a_{1R}a_{1S} - 4a_{2R}a_{2S})^2\right]}}{4a_{2R}^2}, \tag{34}$$

and

$$g_0 = \frac{1}{\sqrt{K_1^2 + 2K_2^2 + 1}}, \quad n_0 = \frac{1}{\sqrt{N_1^2 + 2N_2^2 + 1}},$$
$$r_0 = \frac{1}{\sqrt{R_1^2 + 2R_2^2 + 1}}, \tag{35}$$
$$g_{1,2} = \frac{K_{1,2}}{\sqrt{K_1^2 + 2K_2^2 + 1}}, \quad n_{1,2} = \frac{N_{1,2}}{\sqrt{N_1^2 + 2N_2^2 + 1}},$$
$$r_{1,2} = \frac{R_{1,2}}{\sqrt{R_1^2 + 2R_2^2 + 1}}, \tag{36}$$

with $K_{1,2}$, $N_{1,2}$ and $R_{1,2}$ are defined in Appendix C and satisfy the following relations

$$1 + K_1N_1 + 2K_2N_2 = 0, \quad 1 + K_1R_1 + 2K_2R_2 = 0, \quad 1 + N_1R_1 + 2N_2R_2 = 0. \tag{37}$$

At the first order of perturbation theory, the matrix δM_1 in Eq. (27) does not effect on neutrino mass eigenvalues but it changes the corresponding eigenvectors. The perturbed leptonic mixing matrix are obtained as:

$$U = U_0 + \Delta U = \begin{pmatrix} g_1 & n_1 & -\sqrt{2}(g_1g_u - n_1n_u + r_1r_u) & r_1 \\ g_2 - g_u & n_2 + n_u & -\frac{1}{\sqrt{2}} - \sqrt{2}(g_2g_u - n_2n_u + r_2r_u) & r_2 - r_u \\ g_2 + g_u & n_2 - n_u & \frac{1}{\sqrt{2}} - \sqrt{2}(g_2g_u - n_2n_u + r_2r_u) & r_2 + r_u \\ g_0 & n_0 & -\sqrt{2}(g_0g_u - n_0n_u + r_0r_u) & r_0 \end{pmatrix}, \tag{38}$$

where

$$g_u = \frac{g_1u_1 + g_2u_2 + g_0u_S}{m_3}, \quad n_u = \frac{n_1u_1 + n_2u_2 + n_0u_S}{m_2 - m_3},$$

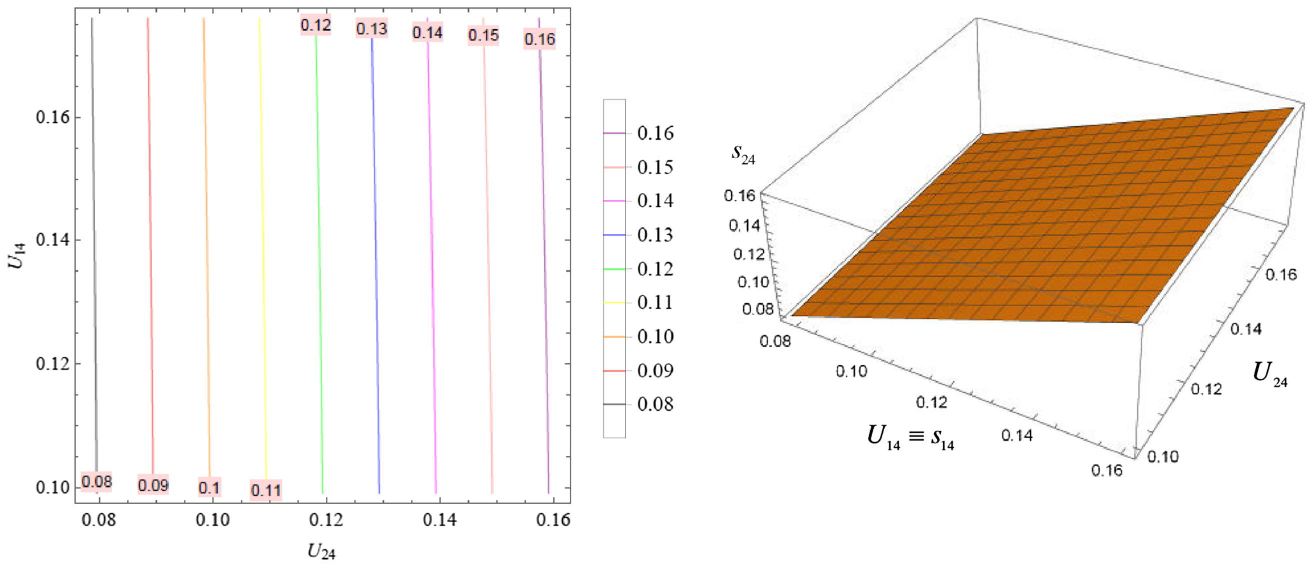

Fig. 1 $U_{14} - U_{24}$ lines corresponding to the specific values $s_{24} = 0.08, 0.09, 0.10, 0.11, 0.12, 0.13, 0.14, 0.15$ and 0.16 from left to right (left panel), and the $3D$ plot of s_{24} versus U_{14} and U_{24} with $U_{14} \in (0.099, 0.176)$ and $U_{24} \in (0.0775, 0.161)$ (right panel)

$$r_u = \frac{r_1 u_1 + r_2 u_2 + r_0 u_S}{m_3 - m_4}. \tag{39}$$

In $3+1$ scenario, the four favor eigenstates $(\nu_e, \nu_\mu, \nu_\tau, \nu_s)$ are related to the mass eigenstates $(\nu_1, \nu_2, \nu_3, \nu_4)$ via a 4×4 unitary lepton mixing matrix possessing the following form

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{31} & U_{32} & U_{33} & U_{34} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{pmatrix}, \tag{40}$$

which can be parameterized by six neutrino mixing angles $\theta_{12}, \theta_{13}, \theta_{23}, \theta_{14}, \theta_{24}, \theta_{34}$, three Dirac phases $\delta_{13} \equiv \delta_{CP}$, δ_{14}, δ_{24} and three Majorana phases α, β, γ [66]:

$$\begin{aligned} U &= R_{34}\widetilde{R}_{24}\widetilde{R}_{14}R_{23}\widetilde{R}_{13}R_{12}P \\ &= \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12}e^{i\frac{\alpha}{2}} & c_{14}s_{13}e^{i\frac{\beta}{2}} & s_{14}e^{i\frac{\gamma}{2}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & c_{14}s_{24}.\mathrm{e}^{i(\delta_{14}-\delta_{24}+\frac{\gamma}{2})} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & c_{14}c_{24}s_{34}.\mathrm{e}^{i(\delta_{14}+\frac{\gamma}{2})} \\ U_{s1} & U_{s2} & U_{s3} & c_{14}c_{24}c_{34}.\mathrm{e}^{i(\delta_{14}+\frac{\gamma}{2})} \end{pmatrix}, \end{aligned} \tag{41}$$

where $U_{\mu j}, U_{\tau j}, U_{sj}$ $(j = 1, 2, 3)$ are defined in Appendix D. The analysis is approximately independent of the angle θ_{14} and $\delta_{13}, \delta_{14}, \delta_{24}$ [11,12], so in this work, $\delta_{14}, \delta_{24}, \alpha, \beta, \gamma$ are set to zero and $\delta_{13} = \delta_{CP}$. From Eqs. (38), (40) and (41) we obtain:

$$\begin{aligned} g_0 &= U_{41}, \quad n_0 = U_{42}, \quad r_0 = U_{44}, \\ g_1 &= U_{11} = c_{12}c_{13}c_{14}, \quad n_1 = U_{12} = c_{12}c_{13}c_{14}t_{12}, \\ r_1 &= U_{14} = s_{14}, \\ r_2 &= \frac{1}{2}\left(r_0 t_{34} + c_{14}s_{24}\right), \quad r_u = \frac{1}{2}\left(r_0 t_{34} - c_{14}s_{24}\right), \\ g_u &= g_2 + c_{23}c_{24}s_{12} + c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} \\ &\quad + c_{12}c_{13}s_{14}s_{24}, \\ n_u &= \frac{c_{12}c_{13}c_{14}g_u + c_{14}s_{13}/\sqrt{2} + s_{14}r_u}{c_{12}c_{13}c_{14}t_{12}}. \end{aligned} \tag{42}$$

Furthermore, Eqs. (40) and (41) yield the relations between neutrino mixing angles $\theta_{14}, \theta_{24}, \theta_{34}$ and $|U_{14}|, |U_{24}|, |U_{34}|$:

$$\begin{aligned} &\sin\theta_{14} = |U_{14}|, \quad \sin\theta_{24} = \frac{|U_{24}|}{\sqrt{1-|U_{14}|^2}}, \\ &\sin\theta_{34} = \frac{|U_{34}|}{\sqrt{1-|U_{14}|^2-|U_{24}|^2}}. \end{aligned} \tag{43}$$

In the next section, we will show that our model is in good agreement with the recent global analysis for NO which seems favored by the global analysis given in Refs. [16–18] as given in Table 1.

4 Numerical analysis

In this work, we use the active–sterile neutrino mixing constraints taken from Ref. [10] as shown in Table 1 for which at 3σ range,

$$\begin{aligned} &0.0098 \le |U_{14}|^2 \le 0.031, \\ &0.006 \le |U_{24}|^2 \le 0.026, \quad 0 \le |U_{34}|^2 \le 0.039, \end{aligned} \tag{44}$$

thus we get $0.099 \le s_{14} \le 0.176$ and $0.0782 \le s_{24} \le 0.163$, $0.025 \le s_{34} \le 0.178$ which are, respectively, plotted in Figs. 1 and 2.

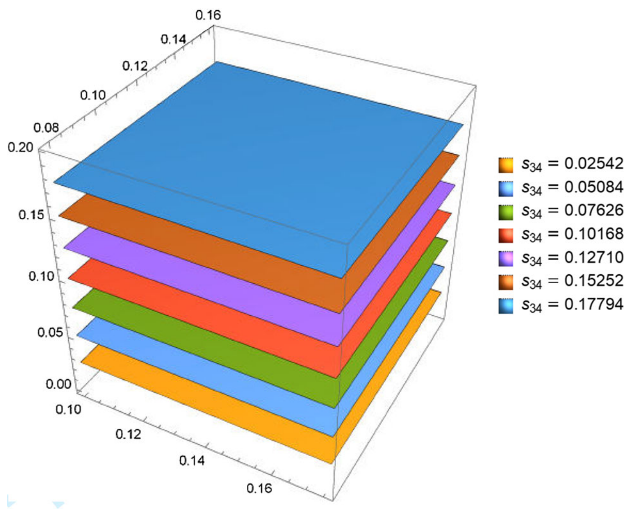

Fig. 2 s_{34} versus U_{14}, U_{24} and U_{34} with $U_{14} \in (0.099, 0.176)$, $U_{24} \in (0.0775, 0.161)$ and $U_{34} \in (0, 0.197)$

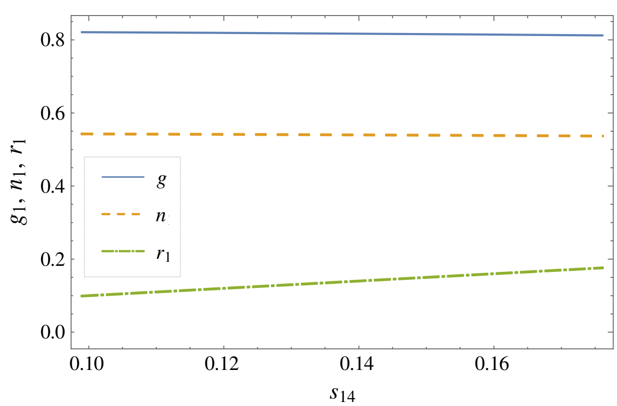

Fig. 3 g_1, n_1 and r_1 versus s_{14} with $s_{14} \in (0.099, 0.176)$

In the case where U_{14} and U_{24} take their best fit values as given in Table 1 and $U_{34} = 0.150$, we get

$$s_{14} = 0.141\,(\theta_{14} = 8.13^\circ), \quad s_{24} = 0.124\,(\theta_{24} = 7.11^\circ),$$
$$s_{34} = 0.153\,(\theta_{34} = 8.78^\circ), \quad U_{44} = 0.971. \tag{45}$$

If three neutrino mixing angles $s_{12,23,13}$ and leptonic Dirac CP violation phase δ_{CP} take their best-fit values as given in Table 1, three parameters g_1, n_1 ad r_1 depend only on θ_{14} which is plotted in Fig. 3.

In the three neutrino scheme, tribimaximal mixing form [139–142] in which $U^{\mathrm{TBM}}_{21} = U^{\mathrm{TBM}}_{31} = -\frac{1}{\sqrt{6}}$ and $U^{\mathrm{TBM}}_{22} = U^{\mathrm{TBM}}_{32} = \frac{1}{\sqrt{3}}$, can be considered as a good approximation for the neutrino mixing form. Furthermore, Eq. (38) implies $U_{21} = g_2 - g_u$, $U_{31} = g_2 + g_u$, $U_{22} = n_2 + n_u$, $U_{32} = n_2 - n_u$ where g_u and n_u comes from the constribution of ϕ', i.e, $|g_u|, |n_u| \ll |g_2|, |n_2|$. Thus, we can set $g_2 = -\frac{1}{\sqrt{6}}$ and $n_2 = \frac{1}{\sqrt{3}}$, g_u then depends on s_{14} and s_{24} which is plotted in Fig. 4.

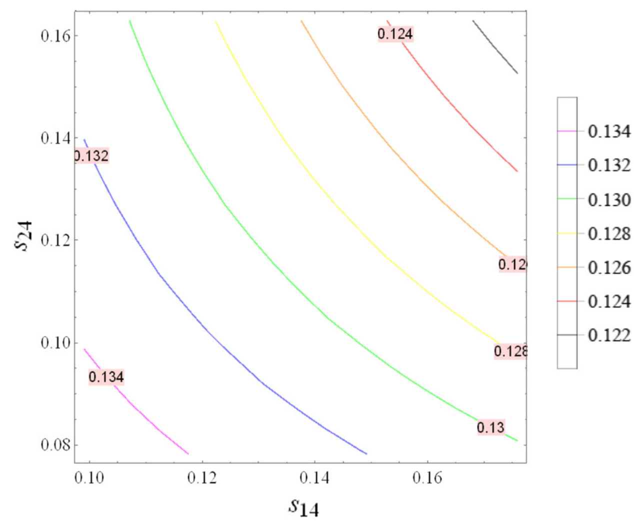

Fig. 4 $|g_u|$ versus s_{14} and s_{24} with $s_{14} \in (0.099, 0.176)$ and $s_{24} \in (0.0782, 0.163)$

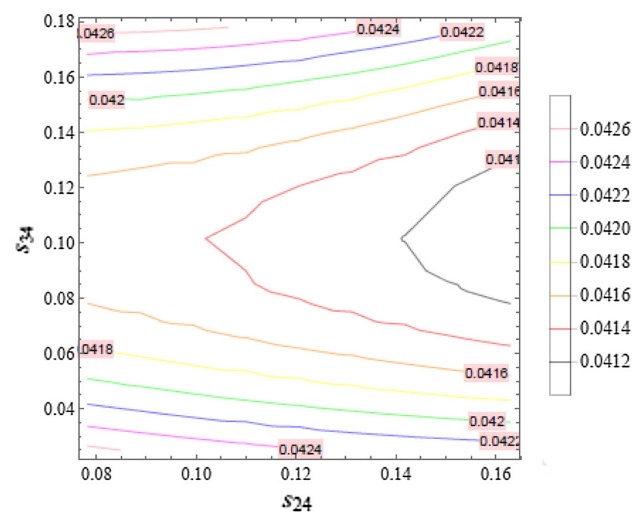

Fig. 5 $|n_u|$ versus s_{24} and s_{34} with $s_{24} \in (0.0782, 0.163)$ and $s_{34} \in (0.025, 0.178)$

In the case $s_{14} = 0.141$, we get

$$g_1 = 0.817, \quad n_1 = 0.54, \quad r_1 = 0.141, \tag{46}$$

and n_u, r_u depend on s_{23} and s_{34} which are plotted in Figs. 5 and 6, respectively.

For NH, $0 = m_1 < m_2 < m_3 < m_4$, the neutrino mass eigenvalues can be written in terms of three mass squared differences as

$$m_2 = \sqrt{\Delta m^2_{21}}, \quad m_3 = \sqrt{\Delta m^2_{31}}, \quad m_4 = \sqrt{\Delta m^2_{41}}. \tag{47}$$

In the case two squared neutrino mass differences in three neutrino scheme Δm^2_{21} and Δm^2_{31} take the best fit values as given in Table 1 and $s_{24} = 0.124$, the effective neutrino masses [143–147] $m_\beta = \left(\sum_{i=1}^4 |U_{ei}|^2 m_i^2\right)^{1/2}$ and $\langle m_{ee}\rangle =$

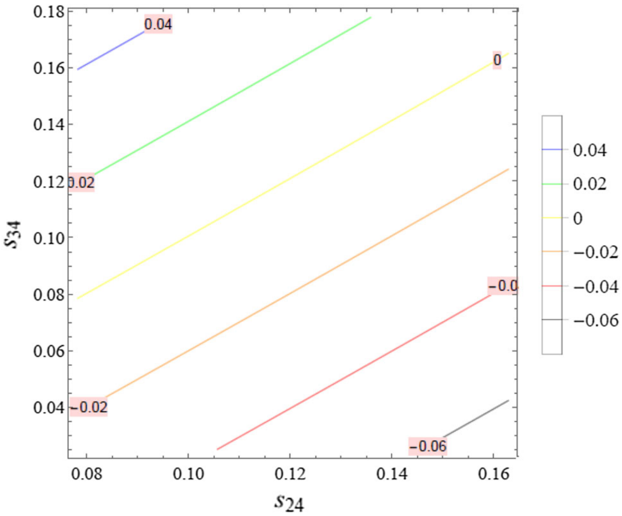

Fig. 6 r_u versus s_{24} and s_{34} with $s_{24} \in (0.0782, 0.163)$ and $s_{34} \in (0.025, 0.178)$

$\left|\sum_{i=1}^{4} U_{ei}^2 m_i\right|$ depend on s_{34} and Δm_{41}^2 which are plotted in Fig. 7. If s_{34} takes its value as given in Eq. (45), $s_{34} = 0.153$, we get:

$$g_2 = -0.408, \quad n_2 = 0.577, \quad r_2 = 0.137,$$
$$g_u = -0.126 - 0.0273i, \quad n_u = 0.00685 - 0.0413i,$$
$$r_u = 0.0138, \tag{48}$$

and the 4×4 lepton mixing matrix gets the explicit form:

$$U^{4\times4} = \begin{pmatrix} 0.817 & 0.54 & 0.147 & 0.141 \\ -0.283 + 0.0273i & 0.584 - 0.0413i & -0.777 - 0.0495i & 0.123 \\ -0.534 - 0.0273i & 0.571 + 0.0413i & 0.638 - 0.0495i & 0.150 \\ \checkmark & \checkmark & \checkmark & 0.971 \end{pmatrix}. \tag{49}$$

At present, there are various experimental constraints on Δm_{41}^2 [2–15], for instance, $\Delta m_{41}^2 \in (0.2, 10.0)\,\mathrm{eV}^2$ [2], $\Delta m_{41}^2 \in (0.01, 1.0)\,\mathrm{eV}^2$ [3], $\Delta m_{41}^2 \geq 0.1\,\mathrm{eV}^2$ [4], $\Delta m_{41}^2 \in (0.1, 1.0)\,\mathrm{eV}^2$ [5], $\Delta m_{41}^2 > 1.5\,\mathrm{eV}^2$ [6], $\Delta m_{41}^2 \in (10^{-3}, 10^{-1})\,\mathrm{eV}^2$ [7], $\Delta m_{41}^2 > 0.1\,\mathrm{eV}^2$ [8], $\Delta m_{41}^2 = 0.041\,\mathrm{eV}^2$ [9], $\Delta m_{41}^2 = 1.7\,\mathrm{eV}^2$ [10], $\Delta m_{41}^2 > 10^{-2}\,\mathrm{eV}^2$ [11], $\Delta m_{41}^2 < 10.0\,\mathrm{eV}^2$ [12] $\Delta m_{41}^2 = 1.45\,\mathrm{eV}^2$ [13], $\Delta m_{41}^2 = 1.0\,\mathrm{eV}^2$ [14,15]. In the considered model, the obtained parameters corresponding to some values of Δm_{41}^2 as listed in Table 3.

5 Conclusion

We propose a non-renormalizable $B - L$ model with $S_3 \times Z_4 \times Z_2$ symmetry which successfully accommodates the current active–sterile neutrino mixing in 3 + 1 scheme. The S_3 flavor symmetry is supplemented by $Z_4 \otimes Z_2$ symmetry to consolidate the Yukawa interaction of the model. The presence of $S_3 \otimes Z_4 \otimes Z_2$ flavour symmetry plays an important role in generating the desired structure of the neutrino mass matrix.

The light neutrino mass matrix from type I seesaw mechanism is obtained in Eq. (25). The obtained leptonic mixing matrix is given in Eq. (38) and it can also be parametrized by the matrix U in Eq. (41). On the other hand, the neutrino masses of the model are obtained in Eq. (32) and neutrino mass eigenvalues for the NO are also defined by Eq. (47). After fixing $\Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2, U_{14}, U_{24}$ at their best-fit values and $U_{34} = 0.150$, we find the sterile–active neutrino mixing angles $\theta_{14}, \theta_{24}, \theta_{34}$ and the effective neutrino masses $\langle m_{ee}\rangle, m_\beta$ within their currently allowed ranges.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The experimental data used in our work are the ones quoted from Refs. [1,10] reported in Table 1, and the experimental values of the charged lepton masses taken from Ref. [138] that we use to compare the predictions of our model with the experimental data. Thus, it was not needed to be deposited.].

Funded by SCOAP[3].

Appendix A: Renormalizable Higgs potential invariant under Γ symmetry

The total scalar potential invariant under Γ symmetry, up to five-dimension, is given by[2]:

$$\begin{aligned} \mathrm{V}_{\text{scalar}} = {} & \mathrm{V}(H) + \mathrm{V}(\phi) + \mathrm{V}(\phi') + \mathrm{V}(\varphi) + \mathrm{V}(\chi) \\ & + \mathrm{V}(\xi) + \mathrm{V}(H, \phi) + \mathrm{V}(H, \phi') \\ & + \mathrm{V}(H, \varphi) + \mathrm{V}(H, \chi) + V(H, \xi) \\ & + \mathrm{V}(\phi, \phi') + \mathrm{V}(\phi, \varphi) + \mathrm{V}(\phi, \chi) + \mathrm{V}(\phi, \xi) \\ & + \mathrm{V}(\phi', \varphi) + \mathrm{V}(\phi', \chi) + \mathrm{V}(\phi', \xi) \\ & + \mathrm{V}(\varphi, \chi) + \mathrm{V}(\varphi, \xi) + \mathrm{V}(\chi, \xi), \end{aligned} \tag{A1}$$

where

$$\mathrm{V}(H) = \mu_H^2 (H^\dagger H)_{\underline{1}} + \lambda_H (H^\dagger H)_{\underline{1}} (H^\dagger H)_{\underline{1}}, \tag{A2}$$
$$\mathrm{V}(\phi) = \mathrm{V}(H \to \phi), \quad \mathrm{V}(\phi') = \mathrm{V}(H \to \phi'),$$

[2] Here we have used the notation $\mathrm{V}(a \to a_1, b \to b_1, \ldots) \equiv \mathrm{V}(a, b, \cdots)|_{\{a=a_1, b=b_1, \ldots\}}$.

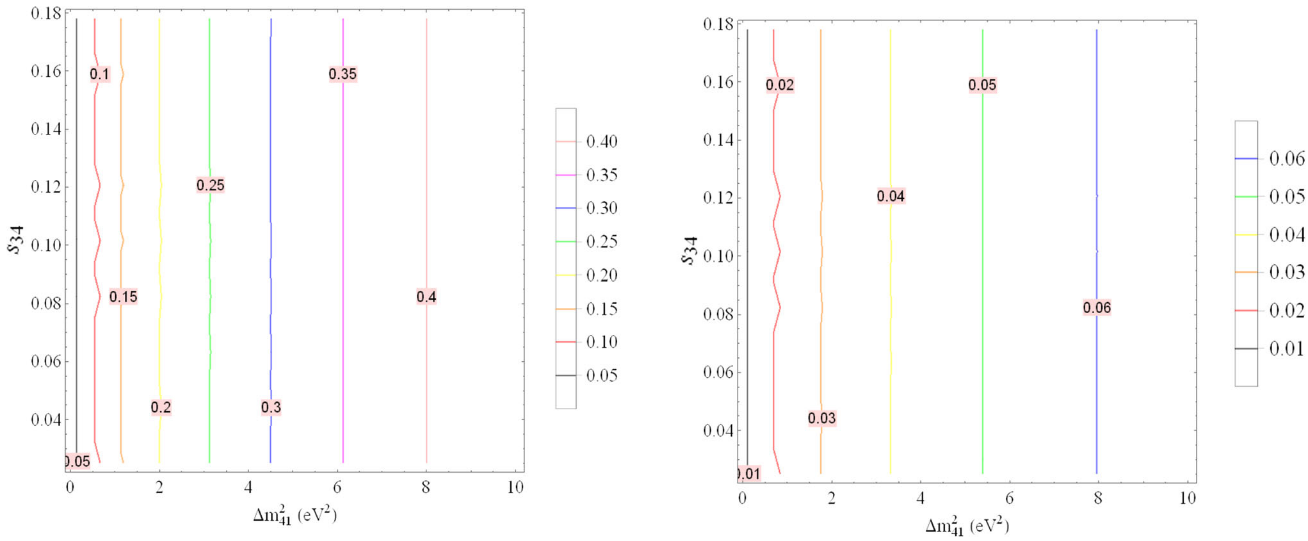

Fig. 7 m_β (left panel) and $\langle m_{ee}\rangle$ (right panel) versus s_{34} and Δm^2_{41} with $s_{34} \in (0.025, 0.178)$ and $\Delta m^2_{41} \in (0.1, 10.0)$ eV^2

Table 3 Obtained parameters corresponding to some values of Δm^2_{41}

Δm^2_{41} (eV^2)	m_2 (eV)	m_3 (eV)	m_4 (eV)	m_β (eV)	$\langle m_{ee}\rangle$ (eV)	$\sum_{i=1}^4 m_{\nu_i}$ (eV)
10^{-2}	0.00861	0.0502	0.1	0.0166	0.0056	0.159
0.1	0.00861	0.0502	0.316	0.0456	0.00992	0.375
1.0	0.00861	0.0502	1.0	0.142	0.0236	1.06
10	0.00861	0.0502	3.16	0.447	0.0668	3.22

$$\mathrm{V}(\varphi) = \mathrm{V}(H \to \varphi), \quad \mathrm{V}(\xi) = \mathrm{V}(H \to \xi), \tag{A3}$$

$$\begin{aligned} V(\chi) &= \mu_\chi^2 (\chi^\dagger\chi)_{\underline{1}} + \lambda_{1\chi}(\chi^\dagger\chi)_{\underline{1}}(\chi^\dagger\chi)_{\underline{1}} \\ &\quad + \lambda_{2\chi}(\chi^\dagger\chi)_{\underline{1}'}(\chi^\dagger\chi)_{\underline{1}'} + \lambda_{3\chi}(\chi^\dagger\chi)_{\underline{2}}(\chi^\dagger\chi)_{\underline{2}}, \end{aligned} \tag{A4}$$

$$\begin{aligned} V(H,\phi) &= \lambda_{1H\phi}(H^\dagger H)_{\underline{1}}(\phi^\dagger\phi)_{\underline{1}} \\ &\quad + \lambda_{2H\phi}(H^\dagger\phi)_{\underline{1}}(\phi^\dagger H)_{\underline{1}}, \end{aligned} \tag{A5}$$

$$V(H,\phi') = V(H,\phi \to \phi'), \quad V(H,\varphi) = V(H,\phi \to \varphi), \tag{A6}$$

$$\begin{aligned} V(H,\chi) &= \lambda_{1H\chi}(H^\dagger H)_{\underline{1}}(\chi^\dagger\chi)_{\underline{1}} \\ &\quad + \lambda_{2H\chi}(H^\dagger\chi)_{\underline{2}}(\chi^\dagger H)_{\underline{2}}, \end{aligned} \tag{A7}$$

$$\begin{aligned} V(H,\xi) &= \lambda_{1H\xi}(H^\dagger H)_{\underline{1}}(\xi^\dagger\xi)_{\underline{1}} \\ &\quad + \lambda_{2H\xi}(H^\dagger\xi)_{\underline{1}'}(\xi^\dagger H)_{\underline{1}'}, \end{aligned} \tag{A8}$$

$$V(\phi,\phi') = V(H \to \phi, \xi \to \phi'), \quad V(\phi,\varphi) = V(\phi,\phi' \to \varphi), \tag{A9}$$

$$V(\phi,\chi) = V(H \to \phi,\chi), \quad V(\phi,\xi) = V(H \to \phi,\xi), \tag{A10}$$

$$\begin{aligned} V(\phi',\varphi) &= V(H \to \phi', \phi \to \varphi), \quad V(\phi',\xi) \\ &= V(\phi \to \phi', \varphi \to \xi), \end{aligned} \tag{A11}$$

$$\begin{aligned} V(\varphi,\chi) &= V(\phi \to \varphi,\chi), \quad V(\varphi,\xi) = V(\phi \to \varphi,\xi), \quad V(\chi,\xi) \\ &= V(\varphi \to \chi,\xi). \end{aligned} \tag{A12}$$

It is noted that all the other triple, quartic and quintic interaction terms, up to five-dimension, of three or four or five differential scalar fields such as $H^\dagger H\phi^\dagger\phi'$, $\chi^\dagger\chi\phi\phi'\xi$, $H\phi\phi'\chi\xi$, e.c.t are forbidden by one (or some) of the model symmetries.

Appendix B: The solution of Eqs. (4)–(9)

$$\begin{aligned} \lambda^H &= -\Big[\mu_H^2 + \lambda_{H\varphi}v_\varphi^2 + 2\lambda_{H\chi}v_\chi^2 + \lambda_{H\phi}v_\phi^2 \\ &\quad + \lambda_{H\phi'}v_{\phi'}^2 + \lambda_{H\xi}v_\xi^2\Big] \Big/ \left(2v_H^2\right), \end{aligned} \tag{B1}$$

$$\begin{aligned} \lambda_{\phi\varphi} &= -\frac{1}{2v_\varphi^2 v_\phi^2}\Big[\mu_\varphi^2 v_\varphi^2 + 2\lambda_\varphi v_\varphi^4 - 2\mu_\chi^2 v_\chi^2 \\ &\quad - 4\lambda_\chi v_\chi^4 + \lambda_{H\varphi}v_\varphi^2 v_H^2 - 2\lambda_{H\chi}v_\chi^2 v_H^2 \\ &\quad + \mu_\phi^2 v_\phi^2 + \lambda_{H\phi}v_H^2 v_\phi^2 + 2\lambda_\phi v_\phi^4 - \mu_{\phi'}^2 v_{\phi'}^2 \\ &\quad - 4\lambda_{\phi'\chi}v_\chi^2 v_{\phi'}^2 - \lambda_{H\phi'}v_H^2 v_{\phi'}^2 \\ &\quad - 2\lambda_{\phi'}v_{\phi'}^4 + (\mu_\xi^2 + 2\lambda_{\varphi\xi}v_\varphi^2 + \lambda_{H\xi}v_H^2 \\ &\quad + 2\lambda_{\phi\xi}v_\phi^2)v_\xi^2 + 2\lambda_\xi v_\xi^4\Big], \end{aligned} \tag{B2}$$

$$\begin{aligned} \lambda_{\phi'\varphi} &= -\frac{\mu_{\phi'}^2 + 2\lambda_{\phi'\chi}v_\chi^2 + \lambda_{H\phi'}v_H^2 + \lambda_{\phi\phi'}v_\phi^2 + 2\lambda_{\phi'}v_{\phi'}^2}{v_\varphi^2} \\ &\quad + \frac{v_\xi^2\left(\mu_\xi^2 + \lambda_{\varphi\xi}v_\varphi^2 + 2\lambda_{\chi\xi}v_\chi^2 + \lambda_{H\xi}v_H^2 + \lambda_{\phi\xi}v_\phi^2 + 2\lambda_\xi v_\xi^2\right)}{v_\varphi^2 v_{\phi'}^2}, \end{aligned} \tag{B3}$$

$$\lambda_{\varphi\chi} = \frac{1}{4v_\varphi^2 v_\chi^2}\Big[-\mu_\varphi^2 v_\varphi^2 - 2\lambda_\varphi v_\varphi^4 - 2\mu_\chi^2 v_\chi^2 -4\lambda_\chi v_\chi^4 - \lambda_{H\varphi} v_\varphi^2 v_H^2 - 2\lambda_{H\chi} v_\chi^2 v_H^2 +\mu_\phi^2 v_\phi^2 + \lambda_{H\phi} v_H^2 v_\phi^2 + 2\lambda_\phi v_\phi^4 + \mu_{\phi'}^2 v_{\phi'}^2 +\lambda_{H\phi'} v_H^2 v_{\phi'}^2 + 2\lambda_{\phi\phi'} v_\phi^2 v_{\phi'}^2 +2\lambda_{\phi'} v_{\phi'}^4 - (\mu_\xi^2 + 2\lambda_{\varphi\xi} v_\varphi^2 +4\lambda_{\chi\xi} v_\chi^2 + \lambda_{H\xi} v_H^2) v_\xi^2 - 2\lambda_\xi v_\xi^4\Big], \tag{B4}$$

$$\lambda_{\phi\chi} = -\frac{1}{4v_\phi^2 v_\chi^2}\Big[-\mu_\varphi^2 v_\varphi^2 - 2\lambda_\varphi v_\varphi^4 + 2\mu_\chi^2 v_\chi^2 + 4\lambda_\chi v_\chi^4 -\lambda_{H\varphi} v_\varphi^2 v_H^2 + 2\lambda_{H\chi} v_\chi^2 v_H^2 +\mu_\phi^2 v_\phi^2 + \lambda_{H\phi} v_H^2 v_\phi^2 + 2\lambda_\phi v_\phi^4 + \mu_{\phi'}^2 v_{\phi'}^2 +4\lambda_{\phi'\chi} v_\chi^2 v_{\phi'}^2 + \lambda_{H\phi'} v_H^2 v_{\phi'}^2 + 2\lambda_{\phi\phi'} v_\phi^2 v_{\phi'}^2 +2\lambda_{\phi'} v_{\phi'}^4 - v_\xi^2(\mu_\xi^2 +2\lambda_{\varphi\xi} v_\varphi^2 + \lambda_{H\xi} v_H^2 + 2\lambda_\xi v_\xi^2)\Big], \tag{B5}$$

$$\lambda_{\phi'\xi} = -\Big[\mu_\xi^2 + \lambda_{\varphi\xi} v_\varphi^2 + 2\lambda_{\chi\xi} v_\chi^2 + \lambda_{H\xi} v_H^2 +\lambda_{\phi\xi} v_\phi^2 + 2\lambda_\xi v_\xi^2\Big]/v_{\phi'}^2. \tag{B6}$$

Appendix C: The explicit expressions of $K_{1,2}$, $N_{1,2}$ and $R_{1,2}$

$$K_1 = -\frac{a_{1S}}{a_{1D}}, \quad K_2 = -\frac{a_{2S}}{a_{2D}}, \tag{C1}$$

$$N_1 = \frac{a_{1D}\left\{\left[(a_{1D}^2 - a_{1S}^2)a_{1R} - \kappa_{11}\kappa_{12}\right]a_{2S} + 2(a_{2D}^2 + 2a_{2S}^2)a_{1S}a_{2R}\right\}}{2\left[(a_{1S}^2 a_{2D}^2 - 2a_{1D}^2 a_{2S}^2)a_{2R} + a_{1D}^2 a_{1R} a_{1S} a_{2S}\right]}, \tag{C2}$$

$$N_2 = \frac{2a_{2D}a_{2R}\kappa_{12}}{(4a_{2R}a_{2S} - a_{1S}a_{1R})\kappa_{12} + a_{1S}\kappa_{11}}, \tag{C3}$$

$$R_1 = \frac{a_{1D}\left\{\left[(a_{1D}^2 - a_{1S}^2)a_{1R} + \kappa_{11}\kappa_{12}\right]a_{2S} + 2(a_{2D}^2 + 2a_{2S}^2)a_{1S}a_{2R}\right\}}{2\left[(a_{1S}^2 a_{2D}^2 - 2a_{1D}^2 a_{2S}^2)a_{2R} + a_{1D}^2 a_{1R} a_{1S} a_{2S}\right]},$$

$$R_2 = \frac{2a_{2D}a_{2R}\kappa_{12}}{(4a_{2R}a_{2S} - a_{1S}a_{1R})\kappa_{12} - a_{1S}\kappa_{11}}, \tag{C4}$$

where

$$\kappa_{11} = \sqrt{a_{1D}^2 a_{1R}^2 + 8a_{2D}^2 a_{2R}^2 + (a_{1R}a_{1S} - 4a_{2R}a_{2S})^2}, \quad \kappa_{12} = \sqrt{a_{1D}^2 + a_{1S}^2}, \tag{C5}$$

with $a_{1,2,3D}$, $a_{1,2R}$, $a_{1,2S}$ are defined in Eq. (23).

Appendix D: The explicit expressions of $U_{i,j}$ ($i = \mu, \tau, s$; $j = 1, 2, 3$)

$$\begin{aligned}
U_{\mu1} &= -c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})},\\
U_{\mu2} &= c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})},\\
U_{\mu3} &= c_{13}c_{24}s_{23}e^{i\delta_{13}} - s_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})},\\
U_{\tau1} &= -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23} -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} +s_{24}s_{34}(c_{23}s_{12} + c_{12}e^{i\delta_{13}}s_{13}s_{23})e^{i\delta_{24}},\\
U_{\tau2} &= -c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} - c_{12}c_{34}s_{23} -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} - s_{24}s_{34}(c_{12}c_{23} - e^{i\delta_{13}}s_{12}s_{13}s_{23})e^{i\delta_{24}},\\
U_{\tau3} &= -c_{24}s_{13}s_{14}s_{34}e^{i\delta_{14}} + c_{13}(c_{23}c_{34} -s_{23}s_{24}s_{34}e^{i\delta_{24}})e^{i\delta_{13}},\\
U_{s1} &= -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} + c_{34}s_{24}(c_{23}s_{12} +c_{12}s_{13}s_{23}e^{i\delta_{13}})e^{i\delta_{24}} +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} - s_{12}s_{23}s_{34},\\
U_{s2} &= -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} - c_{34}s_{24}(c_{12}c_{23} -e^{i\delta_{13}}s_{12}s_{13}s_{23})e^{i\delta_{24}} +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} + c_{12}s_{23}s_{34},\\
U_{s3} &= -c_{24}c_{34}s_{13}s_{14}e^{i\delta_{14}} - c_{13}(c_{34}e^{i\delta_{24}}s_{23}s_{24} +c_{23}s_{34})e^{i\delta_{13}}.
\end{aligned} \tag{D1}$$

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