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The total cross section for the production of heavy quarks in hadronic collisions

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Abstract

We present the results of a full calculation of the QCD $O(\alpha_S^3)$ radiative corrections to the total cross section for the production of a heavy quark pair. We find large contributions for parton sub-energies near threshold and well above threshold. The implications for the production of top and bottom quarks at collider energies are discussed.

I. Introduction

The production of heavy quarks in hadronic collisions is a subject of great experimental and theoretical interest. The discovery of unknown heavy objects remains one of the main motivations for collider physics and heavy quark production provides an example of such a process. The comparison of calculated heavy quark cross-sections with experimental results tests the mechanism by which heavy objects are produced. Reliable predictions for heavy quark production rates are necessary in the search for the top quark and in the study of the properties of bottom and charmed quarks. In addition, heavy quarks are copious sources of leptons. Precise knowledge of the heavy quark production rates will permit the subtraction of leptons from heavy quark decays, perhaps revealing signals for new physics. For a sufficiently heavy quark the cross-section is calculable as a perturbation series in the QCD running coupling constant α_s , evaluated at the mass of the heavy quark. Thus the hadroproduction of heavy quarks can also be used as a testing ground for the underlying strong interaction dynamics.

The standard perturbative QCD formula for the inclusive production of a heavy quark Q of momentum p and energy E,

$$H_A(P_1) + H_B(P_2) \to Q(p) + X \tag{1}$$

determines the invariant cross-section as follows,

$$\frac{E \ d^3\sigma}{d^3p} = \sum_{i,j} \int dx_1 \ dx_2 \left[\frac{E \ d^3 \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, p, m, \mu)}{d^3p} \right] F_i^A(x_1, \mu) \ F_j^B(x_2, \mu).$$
(2)

The functions F_i are the number densities of light partons (gluons, light quarks and antiquarks) evaluated at a scale μ . The symbol $\hat{\sigma}$ denotes the short distance crosssection from which the mass singularities have been factored. Since the sensitivity to momentum scales below the heavy quark mass has been removed, $\hat{\sigma}$ is calculable as a perturbation series in $\alpha_s(\mu^2)$. The scale μ is a priori only determined to be of the order of the mass m of the produced heavy quark. The corrections to Eq. (2) are suppressed by powers of the heavy quark mass.

The first terms in the perturbation series which contribute are $O(\alpha_s^2)$. At this order there are contributions to $\hat{\sigma}$ due to gluon-gluon fusion and quark-antiquark

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annihilation,

$$q + \overline{q} \rightarrow Q + \overline{Q}$$

 $g + g \rightarrow Q + \overline{Q}.$ (3)

The diagrams contributing to the lowest order cross section are shown in Figs. 1,2. The invariant matrix elements squared and the cross-sections for these processes have been available in the literature for some time^[1-4]. Predictions for the hadronic cross-sections are obtained by inserting these parton cross-sections into Eq. (2). The theoretical justification for the use of Eq. (2) in heavy quark production has been discussed in refs. [5-7].

The phenomenological consequences of the lowest order formulae can be summarised as follows. The average transverse momentum of the heavy quark or antiquark is of the order of its mass and the p_T distribution falls rapidly to zero as p_T becomes larger than the heavy quark mass. The rapidity difference between the produced quark and antiquark is predicted to be of order one.

There is a widespread belief that charm production cannot be fully described by lowest order perturbative QCD. Because of the small value of the charmed quark mass, it may be that effects which are nominally suppressed by powers of the charmed quark mass are important for charm production. There is some experimental evidence to support this point of view^[8-10]. On the other hand, within the limited statistics available^[11,12], bottom production seems to be qualitatively in agreement with theoretical predictions^[13].

There are arguments^[14,15] which suggest that higher order corrections to heavy quark production could be large. These are mostly due to the observation that the fragmentation process,

$$\begin{array}{cccc} g+g & \to & g+g \\ & & & \downarrow & Q+\overline{Q} \end{array} \tag{4}$$

although formally of order α_S^3 , can be numerically as important as the lowest order $O(\alpha_S^2)$ cross section. This happens because the lowest order cross section for the process $gg \to q\bar{q}$ is about a hundred times smaller than the cross section for $gg \to gg$. A gluon jet will fragment into a pair of heavy quarks only a fraction $\alpha_S(m^2)/2\pi$ of the time. Because of the large cross-section for the production of gluons, the gluon

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fragmentation production process is still competitive with the production mechanisms of Eq. (3). The description of heavy quark production by the mechanism in Eq. (4) alone is appropriate only when the produced heavy quark is embedded in a high energy $jet^{[16]}$.

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The matrix elements squared for the production of a heavy quark pair plus a light parton have all been calculated^[14,17,18]. By themselves, they have physical significance only when the jet associated with the light parton has a large transverse momentum. When the produced light parton has small transverse momentum the matrix elements contain collinear and soft divergences, which cancel only when the virtual corrections to the diagrams of Figs. 1 and 2 are included, and the factorisation procedure is carried out.

A partial $O(\alpha_S^3)$ calculation involving the quark gluon fusion process which is free from soft gluon singularities, but contains collinear singularities has been presented in ref. [19]. This calculation provides a concrete example of the factorisation scheme and indicates the absence of large forward production of heavy quarks at ISR energies. However this calculation is valid only in the forward direction at energies where the stiffness of the valence quark distribution causes the quark gluon process to dominate over the competing processes. In the central region one cannot use any partial calculation of higher order effects; both real and virtual diagrams contribute. They separately contain divergences which cancel in a complete calculation.

We have performed a full calculation of the inclusive cross section for heavy quark production to order α_s^3 . We calculated the short distance cross sections $\hat{\sigma}$ for the inclusive production of a heavy quark of transverse momentum p_T and rapidity y. This requires the calculation of the cross-sections for the following parton inclusive processes,

$$g + g \to Q + X, \quad q + \overline{q} \to Q + X, \quad g + q \to Q + X, \quad g + \overline{q} \to Q + X$$
$$g + g \to \overline{Q} + X, \quad q + \overline{q} \to \overline{Q} + X, \quad g + \overline{q} \to \overline{Q} + X, \quad g + q \to \overline{Q} + X. \quad (5)$$

The inclusive cross-sections for the production of an anti-quark \overline{Q} differ from those for the production of a quark Q at a given y and p_T . This effect which first arises in $O(\alpha_S^3)$ is small^[20]. Using Eq. (2) we calculate the distributions in rapidity and transverse momentum of produced heavy quarks correct through $O(\alpha_S^3)$. The details of the above calculation will be presented elsewhere^[21]. At this point we list the

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parton sub-processes which contribute to the inclusive cross-sections.

$$q + \overline{q} \rightarrow Q + \overline{Q}, \quad \alpha_{S}^{2}, \alpha_{S}^{3}$$

$$g + g \rightarrow Q + \overline{Q}, \quad \alpha_{S}^{2}, \alpha_{S}^{3}$$

$$q + \overline{q} \rightarrow Q + \overline{Q} + g, \quad \alpha_{S}^{3}$$

$$g + g \rightarrow Q + \overline{Q} + g, \quad \alpha_{S}^{3}$$

$$g + q \rightarrow Q + \overline{Q} + q, \quad \alpha_{S}^{3}$$

$$g + q \rightarrow Q + \overline{Q} + q, \quad \alpha_{S}^{3}$$

$$(6)$$

Note the necessity of including both real and virtual gluon emission diagrams in order to calculate the full $O(\alpha_s^3)$ cross-section.

In this paper we concentrate on the calculation of the total cross section for the inclusive production of a heavy quark pair. Integrating Eq. (2) over the momentum p we obtain the total cross section for the production of a heavy quark pair,

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \, \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) \, F_i^A(x_1, \mu) F_j^B(x_2, \mu) \tag{7}$$

where S is the square of the centre of mass energy of the colliding hadrons A and B. The total short distance cross section $\hat{\sigma}$ for the inclusive production of a heavy quark from partons i, j can be written as,

$$\hat{\sigma}_{ij}(s,m^2,\mu^2) = \frac{\alpha_3^2(\mu^2)}{m^2} f_{ij}\left(\rho,\frac{\mu^2}{m^2}\right)$$
(8)

with $\rho = 4m^2/s$, and s the square of the partonic centre of mass energy. μ is the renormalisation and factorisation scale. We present a complete description of the functions f_{ij} including the first non-leading correction. These may be used by the reader to calculate heavy quark production at any energy and heavy quark mass.

The remainder of this paper is organised as follows: in section II we give all the relevant formulae and fits for the functions f_{ij} . In section III we present the application of our formulae to the production of heavy quarks in $p\overline{p}$ collisions at centre of mass energies of 0.63 and 1.8 TeV. In section IV we discuss our results, with particular emphasis on the following problems: the uncertainties in the knowledge of the gluon structure function and the significance of the large size of the corrections.

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II. The total parton cross-section

In this section we present our results which describe the total parton crosssection for the production of a heavy quark pair. Eq. (8) completely describes the short distance cross-section for the production of a heavy quark of mass m in terms of the functions f_{ij} , where the indices i and j specify the types of the annihilating partons. The dimensionless functions f_{ij} have the following perturbative expansion,

$$f_{ij}\left(\rho,\frac{\mu^2}{m^2}\right) = f_{ij}^{(0)}(\rho) + g^2(\mu^2) \left[f_{ij}^{(1)}(\rho) + \overline{f}_{ij}^{(1)}(\rho)\ln(\frac{\mu^2}{m^2})\right] + O(g^4)$$
(9)

In order to calculate the f_{ij} in perturbation theory we must perform both renormalisation and factorisation of mass singularities. The subtractions required for renormalisation and factorisation are done at mass scale μ . The dependence on μ is shown explicitly in Eq. (9). The energy dependence of the cross-section is given in terms of the ratio ρ ,

$$\rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1-\rho}. \tag{10}$$

The running of the coupling constant α_s is determined by the renormalisation group,

$$\frac{d\alpha_{S}(\mu^{2})}{d\ln\mu^{2}} = -b_{0}\alpha_{S}^{2} - b_{1}\alpha_{S}^{3} + O(\alpha_{S}^{4}), \quad \alpha_{S} = \frac{g^{2}}{4\pi}, \quad b_{0} = \frac{(33 - 2n_{if})}{12\pi}, \quad b_{1} = \frac{(153 - 19n_{if})}{24\pi^{2}}$$
(11)

where n_{lf} is the number of light flavours.

The quantities $f^{(1)}$ depend on the scheme used for renormalisation and factorisation. Therefore we must first specify the choices made in the definition of $f^{(1)}$. Our results are obtained in an extension of the \overline{MS} renormalisation and factorisation scheme^[22]. At one loop order, our renormalisation scheme is completely specified as follows. Graphs containing a light parton loop are renormalised using the normal \overline{MS} subtraction scheme. The following renormalisation conditions are chosen for $\Gamma^{(2)}(p,m)$, the two point function of the heavy quark field,

$$\Gamma^{(2)}(p,m)|_{p^2=m^2} = 0 \tag{12}$$

$$\frac{d}{d\hat{p}}\Gamma^{(2)}(p,m)\big|_{p^2=0}=1, \quad \hat{p}=\gamma^{\mu}p_{\mu}.$$
(13)

Eq. (12) implies that the mass m corresponds to a pole in the renormalised propagator. Eq. (13) fixes the wave function renormalisation for the heavy quark field.

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Eqs. (12,13) are sufficient to show that the anomalous dimensions associated with the mass renormalisation and the renormalisation of the heavy quark field are equal to zero. The renormalisation constant for the gluon- $Q-\overline{Q}$ vertex is then fixed by the Taylor-Slavnov identity. This completely specifies the treatment of primitively divergent graphs with heavy quarks on external lines. Lastly, graphs containing internal loops of heavy quarks are subtracted at zero momentum. In this scheme heavy quarks are decoupled at low energy^[23]. The light partons continue to obey the same renormalisation group equation as they would have done in the absence of the heavy quarks. Thus our results should be used in conjunction with the running coupling as defined in Eq.(11) and together with light parton densities evolved using the two loop \overline{MS} evolution equations^[24].

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We now present a complete description of the functions $f^{(0)}, \overline{f}^{(1)}$ and $f^{(1)}$ for all the contributing parton subprocesses. Thus all the information on the total heavy quark cross-section calculated through order α_S^3 is available to the reader. When combined with suitably evolved structure functions for the light quarks and gluons, our results can be used to calculate heavy quark production cross-sections at any energy and heavy quark mass.

The functions $f_{ij}^{(0)}$ defined in Eqs. (8,9) are,

$$f_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27} \left[2 + \rho \right]$$
(14)

$$f_{gg}^{(0)}(\rho) = \frac{\pi\beta\rho}{192} \left[\frac{1}{\beta} (\rho^2 + 16\rho + 16) \ln\left(\frac{1+\beta}{1-\beta}\right) - 28 - 31\rho \right]$$
(15)

$$f_{gq}^{(0)}(\rho) = f_{g\bar{q}}^{(0)}(\rho) = 0$$
(16)

We now turn to the higher order corrections in Eq.(9) which are separated into two terms. The $\overline{f}^{(1)}(\rho)$ terms are the coefficients of $\ln(\mu^2/m^2)$ and are determined by renormalisation group arguments from the lowest order cross-sections,

$$\overline{f}_{ij}^{(1)}(\rho) = \frac{1}{8\pi^2} \left[4\pi \ b_0 \ f_{ij}^{(0)}(\rho) - \int_{\rho}^{1} dz_1 f_{kj}^{(0)}(\frac{\rho}{z_1}) P_{ki}(z_1) - \int_{\rho}^{1} dz_2 f_{ik}^{(0)}(\frac{\rho}{z_2}) P_{kj}(z_2) \right].$$
(17)

Using the explicit forms^[24] for the lowest order Altarelli-Parisi kernels P_{ij} and

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Eqs. (14,15), we find the following analytic results for the $\overline{f}_{ij}^{(1)}(\rho)$ functions,

$$\begin{aligned} \overline{f}_{q\overline{q}}^{(1)} &= \frac{1}{8\pi^2} \left[\frac{16\pi}{81} \rho \ln \left(\frac{1+\beta}{1-\beta} \right) + \frac{1}{9} f_{q\overline{q}}^{(0)}(\rho) \left(127 - 6n_{lf} + 48 \ln \left(\frac{\rho}{4\beta^2} \right) \right) \right] \end{aligned} (18) \\ \overline{f}_{gg}^{(1)} &= \frac{1}{8\pi^2} \left[\frac{\pi}{192} \left\{ 2\rho (59\rho^2 + 198\rho - 288) \ln \left(\frac{1+\beta}{1-\beta} \right) \right. \\ &+ 12\rho (\rho^2 + 16\rho + 16) h_2(\beta) - 6\rho (\rho^2 - 16\rho + 32) h_1(\beta) \\ &- \frac{4}{15} \beta (7449\rho^2 - 3328\rho + 724) \right\} + 12 f_{gg}^{(0)}(\rho) \ln \left(\frac{\rho}{4\beta^2} \right) \right] \end{aligned} (19) \\ \overline{f}_{gq}^{(1)} &= \frac{1}{8\pi^2} \frac{\pi}{192} \left[\frac{4}{9} \rho (14\rho^2 + 27\rho - 136) \ln \left(\frac{1+\beta}{1-\beta} \right) - \frac{32}{3} \rho (2-\rho) h_1(\beta) \\ &- \frac{8}{135} \beta (1319\rho^2 - 3468\rho + 724) \right] \end{aligned} (20)$$

where the auxiliary functions h_1 and h_2 are given by,

$$h_{1}(\beta) = \ln^{2}\left(\frac{1+\beta}{2}\right) - \ln^{2}\left(\frac{1-\beta}{2}\right) + 2\operatorname{Li}_{2}\left(\frac{1+\beta}{2}\right) - 2\operatorname{Li}_{2}\left(\frac{1-\beta}{2}\right)$$
$$h_{2}(\beta) = \operatorname{Li}_{2}\left(\frac{2\beta}{1+\beta}\right) - \operatorname{Li}_{2}\left(\frac{-2\beta}{1-\beta}\right)$$
$$\operatorname{Li}_{2}(x) = -\int_{0}^{x}\frac{dz}{z}\ln(1-z).$$
(21)

The quantities $f^{(1)}$ in Eq.(9) can only be obtained by performing a complete $O(\alpha_S^3)$ calculation. We do not have exact analytical results for the quantities $f^{(1)}$. Instead we provide a physically motivated fit to the numerically integrated result. Near the endpoints we impose the correct asymptotic behaviour. Elsewhere our fit agrees with the numerically integrated result to better than 1%.

$$\begin{aligned} f_{q\bar{q}}^{(1)} &= \frac{\rho}{72\pi} \left[\frac{16}{3} \beta \ln^2(8\beta^2) - \frac{82}{3} \beta \ln(8\beta^2) - \frac{\pi^2}{6} \right] \\ &+ \beta \rho \left[a_0 + \beta^2 \left(a_1 \ln(8\beta^2) + a_2 \right) + \beta^4 \left(a_3 \ln(8\beta^2) + a_4 \right) + a_5 \beta^6 \ln(8\beta^2) + a_6 \ln \rho + a_7 \ln^2 \rho \right] \\ &+ \frac{1}{8\pi^2} (n_{lf} - 4) f_{q\bar{q}}^{(0)}(\rho) \left[\frac{2}{3} \ln \left(\frac{4}{\rho} \right) - \frac{10}{9} \right] \end{aligned}$$

$$(22)$$

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	$f^{(1)}_{qar q}$	$f^{(1)}_{gg}$	$f^{(1)}_{gq}$
<i>a</i> ₀	0.180899	0.108068	0.0110549
<i>a</i> 1	0.101949	-0.114997	-0.426773
a2	-0.234371	0.0428630	-0.00103876
a3	-0.0109950	0.131429	0.450626
a4	-0.0185575	0.0438768	-0.227229
a_5	0.00907527	-0.0760996	0.0472502
a ₆	0.0160367	-0.165878	-0.197611
a7	0.00786727	-0.158246	-0.0529130

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Table 1: Coefficients in the fits for $f_{ij}^{(1)}$.

$$\begin{aligned} f_{gg}^{(1)} &= \frac{7}{1536\pi} \left[12\beta \ln^2(8\beta^2) - \frac{366}{7}\beta \ln(8\beta^2) + \frac{11}{42}\pi^2 \right] \\ &+ \beta \left[a_0 + \beta^2 \left(a_1 \ln(8\beta^2) + a_2 \right) + a_3\beta^4 \ln(8\beta^2) + \rho^2 \left(a_4 \ln \rho + a_5 \ln^2 \rho \right) \right] \\ &+ \rho \left(a_6 \ln \rho + a_7 \ln^2 \rho \right) \right] + \left(n_{if} - 4 \right) \frac{\rho^2}{1024\pi} \left[\ln \left(\frac{1+\beta}{1-\beta} \right) - 2\beta \right] \end{aligned}$$
(23)
$$\begin{aligned} f_{gq}^{(1)} &= \beta \left[\beta^2 \left(a_0 \ln \beta + a_1 \right) + \beta^4 \left(a_2 \ln \beta + a_3 \right) + \rho^2 \left(a_4 \ln \rho + a_5 \ln^2 \rho \right) \right] \\ &+ \rho \left(a_6 \ln \rho + a_7 \ln^2 \rho \right) \right] \end{aligned}$$
(24)

The coefficients in the fit are given in Table (1).

The functions $f^{(0)}$, $f^{(1)}$ and $\overline{f}^{(1)}$ are shown plotted in Figs. 3, 4 and 5 for the cases of quark-antiquark, gluon-gluon and gluon-quark fusion respectively. Notice the strikingly different behaviour of the gluon-gluon and gluon-quark higher order terms in the high energy limit, $\rho \to 0$. These latter processes allow the exchange of a spin one gluon in the *t*-channel and are therefore dominant in the high energy limit. These cross sections tend to a constant at high energy. The lowest order terms involve fermion *t*-channel exchange and therefore fall off at large *s* as can be seen from Figs. 3 and 4. We find that,

$$f_{gg}^{(1)} \rightarrow 6k + O(\rho \ln^2 \rho)$$

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$$f_{gq}^{(1)} \rightarrow \frac{4}{3}k + O(\rho \ln^2 \rho) \tag{25}$$

where $k \approx 0.018$. At high energy the pair production of heavy quarks shares many features with the pair production of an electron positron pair in the field of a nucleus^[25]. The dominant diagrams are shown in Fig. 6. The asymptotic values of $f_{gg}^{(1)}$ and $f_{gq}^{(1)}$ are proportional to the colour charge of the line which provides the exchanged gluon, since in this limit the upper blob in Fig. 6 is the same for both diagrams and the lower vertex can be approximated by the eikonal form. In the gluon-gluon sub-process the exchanged spin one gluon can come from either incoming gluon, whereas in the gluon quark subprocess it can only come from the incoming quark line. This, together with the ratio of the gluon and quark charges, explains the relative factor of 9/2, shown in Eq.(25) and evident in Figs. 4 and 5. At high energy the functions $\overline{f}_{gg}^{(1)}$ and $\overline{f}_{gq}^{(1)}$ also tend to a constant. Again the ratio is 9/2 as can be seen from Eqs.(19,20).

A preliminary idea of the size of the corrections can be obtained from Figs. 3, 4 and 5 even before folding with the parton distribution functions. Taking a typical value for $g^2 \approx 2$, we see that the radiative corrections are large, particularly in the vicinity of the threshold. The significance of the constant cross-section region (gg, gq) at high energy will depend on the rate of fall-off of the structure functions with which the partonic cross-section must be convoluted.

Near threshold, $(\beta \rightarrow 0)$, we have,

$$\begin{aligned} f_{q\bar{q}}^{(1)} &\to \mathcal{N}_{q\bar{q}} \left[-\frac{\pi^2}{6} + \beta \left(\frac{16}{3} \ln^2 \left(8\beta^2 \right) - \frac{82}{3} \ln(8\beta^2) \right) + O(\beta) \right] \\ f_{gg}^{(1)} &\to \mathcal{N}_{gg} \left[\frac{11\pi^2}{42} + \beta \left(12 \ln^2 \left(8\beta^2 \right) - \frac{366}{7} \ln(8\beta^2) \right) + O(\beta) \right] \\ f_{gq}^{(1)} &\to O(\beta). \end{aligned}$$
(26)

The normalisation, \mathcal{N}_{ij} of the expressions in Eq.(26) is determined as follows,

$$\mathcal{N}_{ij} = \frac{1}{8\pi^2} \frac{f_{ij}^{(0)}(\rho)}{\beta} \bigg|_{\beta=0}, \quad \mathcal{N}_{q\bar{q}} = \frac{1}{72\pi}, \quad \mathcal{N}_{gg} = \frac{7}{1536\pi}.$$
 (27)

Notice that in this order in perturbation theory the cross-section is finite at threshold. This is due to the $1/\beta$ singularity which is responsible for the binding in a

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coulomb system. The coulomb attraction tends to increase the cross-section when the incoming partons are in a singlet state (gg), and decrease the cross-section when the incoming partons are in an octet state $(gg, q\bar{q})$. This results in a net positive term for the gg case.

In view of the numerical significance of the region near threshold we now examine it in more detail. The terms in Eq.(26) which are finite at threshold have already been explained. The $\ln^2(\beta^2)$ terms in Eq. (26) have a general origin. Consider the process depicted in Fig. 7. The gluon Bremsstrahlung leads to a term of the form,

$$\frac{\alpha_s}{2\pi} 2C_A \int dz \left[2 \frac{\ln(1-z)}{1-z} \right] \sigma_0(zs) \tag{28}$$

where σ_0 results from the upper blob in Fig. 7. The integral in Eq. (28) is divergent at z = 1. After inclusion of virtual diagrams the singularity is regulated as in Eq.(39). In the case of Bremsstrahlung from an incoming quark line, $C_A = 3$ is replaced by $C_F = 4/3$. The origin of the $2\ln(1-z)$ term is kinematic. In the appropriate axial gauge the amplitude squared for the process in Fig. 7 is

$$\frac{\alpha_s}{2\pi} C_A \int ds_0 \,\sigma_0(s_0) \int dx \, P_{gg}(x) \int \frac{dl_T^2}{l_T^2} \delta(s_0 - (q+p-l)^2) \tag{29}$$

where $x = (p-l) \cdot q/p \cdot q$, $q^2 = p^2 = 0$, $s = 2p \cdot q$. The l_T^2 integral, which contains the collinear divergence, has an effective upper bound,

$$l_T^2 < \frac{s}{4}(1-z)^2, \quad z = \frac{s_0}{s}$$
 (30)

which is derived from the condition that the argument of the delta function is satisfied for some value of x. Performing the collinear integral and retaining only the most singular terms in 1 - z, we obtain,

$$\frac{\alpha_S}{2\pi} 2C_A \int ds_0 \ \sigma_0(s_0) \int \frac{dx}{1-x} \delta(s_0 - (q+p-l)^2) \Big|_{l_T^2=0} \ln(1-z)^2$$

$$\to \frac{\alpha_S}{2\pi} 2C_A \int dz \ \sigma_0(zs) \frac{2\ln(1-z)}{1-z}.$$
 (31)

In deep inelastic scattering one has a similar contribution of the form,

$$F(x_{BJ},Q^{2}) = \frac{\alpha_{S}}{2\pi} 2C_{F} \int dz \ F(\frac{x_{BJ}}{z},Q^{2}) \frac{\ln(1-z)}{1-z}$$
(32)

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Note the factor of two difference between Eqs.(31,32). This is due to the different kinematics for Deep Inelastic Scattering, (DIS). In DIS the upper bound on the l_T^2 integration is,

$$l_T^2 < \frac{\nu}{2}(1-x_{BJ}), \quad x_{BJ} = \frac{Q^2}{2\nu}.$$
 (33)

This mismatch is one of the sources of the large radiative corrections to Drell-Yan pair production as already pointed out in ref. [26].

Up to now all results have been presented in the modified \overline{MS} subtraction scheme described above. We now consider a more physical factorisation scheme which can be defined for the quark and antiquark distribution functions^[26]. In this scheme the quark distributions are defined directly in terms of the DIS structure function F_2 . The $O(\alpha_S)$ corrections are completely absorbed into the definitions of the distribution functions. The 'physical' $f_{q\bar{q}}^{(1p)}$ and $f_{gq}^{(1p)}$ are defined as follows,

$$f_{q\bar{q}}^{(1p)}(\rho) = f_{q\bar{q}}^{(1)}(\rho) - \int_{\rho}^{1} dz_{1} f_{q\bar{q}}^{(0)}(\frac{\rho}{z_{1}}) c_{q}(z_{1}) - \int_{\rho}^{1} dz_{2} f_{q\bar{q}}^{(0)}(\frac{\rho}{z_{2}}) c_{q}(z_{2})$$
(34)

$$f_{gq}^{(1p)}(\rho) = f_{g\bar{q}}^{(1p)}(\rho) = f_{g\bar{q}}^{(1)}(\rho) - \int_{\rho}^{1} dz_{1} f_{q\bar{q}}^{(0)}(\frac{\rho}{z_{1}}) c_{g}(z_{1})$$
(35)

where $c_q(z)$ and $c_g(z)$ are given by^[26],

$$c_{q}(z) = \frac{4}{3} \frac{1}{8\pi^{2}} \left\{ (1+z^{2}) \left[\frac{\ln(1-z)}{1-z} \right]_{+} - \frac{3}{2} \left[\frac{1}{1-z} \right]_{+} - (1+z^{2}) \frac{\ln z}{1-z} + 3 + 2z - \left(\frac{9}{2} + \frac{\pi^{2}}{3} \right) \delta(1-z) \right\}$$
(36)

$$c_g(z) = \frac{1}{2} \frac{1}{8\pi^2} \left\{ \left(z^2 + (1-z)^2 \right) \ln \left(\frac{1-z}{z} \right) + 6z(1-z) \right\}$$
(37)

and the plus distributions are given by,

$$\int_{0}^{1} dz f(z) \left[\frac{1}{1-z} \right]_{+} = \int_{0}^{1} dz \frac{f(z) - f(1)}{(1-z)}$$
(38)

$$\int_{0}^{1} dz f(z) \left[\frac{\ln (1-z)}{1-z} \right]_{+} = \int_{0}^{1} dz \left(f(z) - f(1) \right) \frac{\ln (1-z)}{(1-z)}$$
(39)

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Performing the integrations in Eqs.(34,35) we obtain the analytic results,

$$\begin{aligned} f_{q\bar{q}}^{(1p)}(\rho) &= f_{q\bar{q}}^{(1)}(\rho) + \frac{1}{8\pi^2} \Biggl\{ \frac{8}{3} f_{q\bar{q}}^{(0)}(\rho) \Biggl[\frac{13}{18} + \frac{2\pi^2}{3} - \ln^2 \left(\frac{\rho}{4\beta^2} \right) - \frac{25}{6} \ln \left(\frac{\rho}{4\beta^2} \right) \Biggr] \\ &+ \frac{8}{3} \frac{\pi\rho}{27} \Biggl[\frac{7}{3}\beta - 2 \Biggl(4 + \ln \left(\frac{\rho}{4\beta^2} \right) \Biggr) \ln \left(\frac{1+\beta}{1-\beta} \right) - 2h_2(\beta) \Biggr] \Biggr\} \end{aligned}$$
(40)
$$f_{gq}^{(1p)}(\rho) &= f_{gq}^{(1)}(\rho) + \frac{1}{8\pi^2} \frac{\pi\rho}{27} \Biggl\{ (1 - \frac{3}{4}\rho^2) \Biggl[h_2(\beta) + \ln \left(\frac{1+\beta}{1-\beta} \right) \ln \left(\frac{\rho}{4\beta^2} \right) \Biggr] \\ &+ \beta \Bigl(\frac{13}{6}\rho - \frac{8}{3} \Bigr) \ln \left(\frac{\rho}{4\beta^2} \right) - \Bigl(1 + \frac{9}{4}\rho^2 \Bigr) \ln \Bigl(\frac{1+\beta}{1-\beta} \Bigr) + \beta \Bigl(\frac{191}{18}\rho - \frac{46}{9} \Bigr) \Biggr\}. \end{aligned}$$
(41)

The function $h_2(\beta)$ is as given in Eq.(21).

Because of the importance of gluons in the hadroproduction of heavy quarks it would be desirable to have a physical definition of the gluon distribution function beyond the leading order. This is not provided by DIS because the gluons only enter as a higher order correction in that process. In principle, such a definition is provided by any process in which the gluons enter at the leading order, for which the relevant higher order corrections have been calculated and for which sufficiently accurate data is available. For example, direct photon production could be used to define the gluon distribution^[27].

III. Phenomenological Applications

In this section we examine the effect of the radiative corrections on the production of heavy quarks at the energies of current $p\bar{p}$ colliders. The total cross section is obtained by integrating the product of the short distance cross sections and the parton fluxes. We use the EHLQ^[28] structure functions set 1, which have $\Lambda_{LO} =$ 0.2 GeV. Using the one loop expression for α_s , this gives $\alpha_s(\mu = 20 \text{ GeV})=0.158$. The EHLQ structure functions include only the lowest order contributions. To be consistent, when calculating a next to leading order correction one should use structure functions evolved according to the next to leading order evolution equations. However, next to leading order evolution usually gives small corrections and we will ignore it in most of what follows. In order to determine the sensitivity of our predictions to the choice of the structure functions, we also present results with the DFLM structure functions of ref. [29], which include next to leading order evolution.

To assess the significance of the radiative corrections and the relative importance of the various kinematic regions we must know the flux of incoming partons. We define the parton flux function Φ ,

$$\Phi_{ij}(\tau,\mu) = \tau \int_0^1 dx_1 \int_0^1 dx_2 F_i^A(x_1,\mu) F_j^B(x_2,\mu) \delta(x_1x_2-\tau) \quad . \tag{42}$$

In terms of these parton fluxes the hadronic cross-section is given by,

$$\sigma(S,m^2) = \frac{\alpha_S^2(\mu^2)}{m^2} \sum_{i,j} \int_{\rho^H}^1 \frac{d\tau}{\tau} \Phi_{ij}(\tau,\mu) f_{ij}(\frac{\rho^H}{\tau},\frac{\mu^2}{m^2}), \quad \rho^H = \frac{4m^2}{S} \quad .$$
(43)

The function Φ is shown in Figs. 8 and 9 for two different choices of μ , the scale at which the parton distributions are evaluated. In Fig. 8 we also show the thresholds for the production of a bottom quark with a mass of 5 GeV at centre of mass energies of 0.63 and 1.8 TeV and in Fig. 9 we show the thresholds for the production of a heavy quark with a mass of 40 GeV at the same centre of mass energies. Note that bottom quark production at collider energies is predominantly due to gluons with quite small values of x. In these small x regions the form of the gluon distribution functions is unmeasured. In fact theoretical arguments^[30] suggest that the gluon distribution behaves at low x as $1/x^{1+\delta}$ with $\delta \approx 0.5$, rather than the 1/x behaviour assumed in the EHLQ parametrisation. Thus any conclusions which depend on the behaviour of the gluon distribution for $x < 10^{-2}$ should be considered suspect.

In Figs. 10-13 we show the product of the parton fluxes with the total partonic cross sections for centre of mass energies of 0.63 and 1.8 TeV and for top and bottom production. We have arbitrarily set the top quark mass equal to 40 GeV. The renormalisation scale, μ , is chosen to be equal to the heavy quark mass, m. In Figs. 10-13 the function $f^{(1)}$ has been multiplied by $g^2(\mu^2)$ so that it is included with the correct weight, (see Eq. (9)). The vertical scales are in arbitrary units of $\alpha_S^2(\mu^2)/m^2$; the horizontal scales are logarithmic, so that the areas in the figures are proportional to the size of a given contribution to the total cross section, (see Eq.(43)).

Fig. 10 shows the rate for bottom production at the $Sp\overline{p}S$ collider. The dominant contribution is clearly the gluon-gluon subprocess. The contributions from $q\overline{q}$ and $(q + \overline{q})g$ initial states are small. From Fig. 10, we see that there are large $O(\alpha_S^3)$

positive corrections to the gluon gluon subprocess both from the threshold region and from the region of high s, where the radiative corrections dominate over the lowest order contribution, for the reasons discussed in section II. The contributions to top production at the $Sp\overline{p}S$ are shown in Fig. 11. For a top quark of 40 GeV, the lowest order contributions coming from the gluon-gluon and the quark-antiquark sectors are roughly equal; for a heavier top quark, the $q\overline{q}$ annihilation mechanism becomes dominant. The $O(\alpha_S^3)$ corrections to top production are larger for ggscattering than for $q\overline{q}$ scattering. The $(q + \overline{q})g$ initial state contributes little to the top quark total cross section.

The various terms in bottom and top production at Tevatron energies are shown in Figs. 12 and 13, respectively. We see that the $O(\alpha_s^3)$ corrections to the gluon gluon fusion process are large both at threshold and in the region of high s for bottom production. The high s region is more important than it was at $Sp\bar{p}S$ collider energies. From Fig. 13 we see that for $m_t = 40$ GeV, top production is still dominated by the gg subprocess with large positive corrections near threshold.

In Figs. 14 and 15, we show the ratio of the total hadronic cross section calculated to $O(\alpha_S^3)$ divided by the cross section predicted by lowest order QCD evaluated at the scale $\mu = m$. We have included the corrections specified by Eqs. (34,35). The gluons are treated in the \overline{MS} factorisation scheme. The numerator of this ratio is calculated at three different values of the subtraction scale; $\mu = m/2, m$, and 2m. (Note that the scale at which the lowest order prediction is evaluated is kept fixed at $\mu = m$). The corrections tend to decrease as the mass of the heavy quark increases; this is because the quark antiquark annihilation subprocess, which receives smaller corrections, becomes dominant at high values of the mass. The sensitivity of the $O(\alpha_S^3)$ total cross-section to a change in the subtraction scale is moderate. For a top mass of 40 GeV and a centre of mass energy of 0.63 TeV, the ratio of the radiatively corrected cross section to the lowest order prediction changes by about 30% as μ is varied from m/2 to 2m.

The ratio of the lowest order QCD prediction evaluated at $\mu = \sqrt{2m}$ to the lowest order result with $\mu = m$ is shown by the dotted lines in Figs. 14 and 15. The value $\mu = \sqrt{2m}$ was chosen to facilitate comparison with curves plotted with a choice of $\mu = m_T = \sqrt{p_T^2 + m^2}$, since, on the average $p_T^2 \sim m^2$. The dotted curves are near one at values of the mass near 5 GeV. In general, the μ dependence of the lowest order result can be understood as follows. As μ increases, the gluon structure function decreases at large x, but increases at small x. The coupling constant, on the other hand, increases as μ decreases. These two effects tend to compensate at the values of x corresponding to bottom production at collider energies. Therefore, at these energies the sensitivity of the lowest order result for bottom production to changes in the scale μ is deceptively small.

In Figs. 16 and 17 we show plots similar to Figs. 14 and 15, obtained using structure functions of ref. [29] which include the next to leading log evolution. These parton distributions require $\Lambda_{\overline{MS}} = 0.3$ GeV. In Figs. 16 and 17 we have used the two loop expression for the running coupling constant, normalised by the inclusion of a non-leading term so that $\alpha_S(\mu = 20 \text{ GeV})=0.158$. Note that Figs. 16 and 17 are still normalized to the lowest order EHLQ set 1 result with $\mu = m$. At high values of the mass the curves lie lower than the curves in Figs. 14 and 15. Relative to the DFLM lowest order result (not shown) the radiative corrections are large and positive. The DFLM structure functions predict that the gluon content of the proton falls faster at large values of x than it does for the EHLQ structure functions. Thus the quark-antiquark annihilation mechanism becomes dominant at lower values of the quark mass. Comparing Figs. 14 and 15 with Figs. 16 and 17 we see that at large values of the quark mass there are significant uncertainties due to the ignorance of the gluon structure function.

The results presented so far have all been for the total cross-section. However, at collider energies it is experimentally much easier to detect bottom quarks which lie above a certain p_T . We therefore present in Figs. 18-20 the results for the shortdistance cross-sections when the cut $p_T > 2m$ is made. It is clear from Figs. 19 and 20 that the significance of the region of constant cross-section is greatly enhanced by performing this cut. With this p_T cut, the $O(\alpha_S^3)$ contributions to the bottom cross sections are considerably larger than the lowest order predictions. Furthermore, as the subtraction scale μ is varied from 2m to m/2, the bottom cross section with this p_T cut changes by about a factor of two.

Experimental considerations make it extremely attractive to know the ratio of the production rate of bottom quarks above a certain p_T and the production rate of a top quark of a given mass. Many systematic errors will cancel from such a ratio. The theoretical problems with such a prediction should be apparent from the

above discussion. The mechanisms for bottom production with a p_T cut and top production are quite different. Because bottom quark production with this cut is largely an $O(\alpha_S^3)$ process due to the high s region it is very sensitive to the choice of scale. We therefore conclude that this ratio is subject to a large theoretical error.

IV. Conclusions.

Our calculation of $O(\alpha_S^3)$ effects has clarified the role of higher order corrections in the hadroproduction of heavy flavours. We have found significant corrections near to the threshold of the parton sub-process. In addition in the case of processes which allow gluon exchange in the *t*-channel there is a large correction in the region well above the partonic threshold.

The large correction coming from the high s region in the gg and gq subprocesses is due to the onset of a new phenomenon at $O(\alpha_s^3)$, namely the exchange of vector gluons in the *t*-channel. The consequent constant behaviour of the short distance cross-section may also lead to a different rapidity distribution for the produced pair of heavy quarks. Let us assume that the number density of gluons is given by,

$$F_g^A(x) \sim \frac{1}{x}.$$
 (44)

and denote by $\sigma^{(0)}$ the part of the cross-section which dies at large s and by $\sigma^{(1)}$ the part of the cross-section which persists at high s, first present in $O(\alpha_S^3)$. With the assumption of Eq. (44) it follows that the rapidity distribution of a pair of heavy quarks due to the lowest order is uniform in rapidity,

$$\frac{d\sigma^{(0)}}{dy} \sim \text{constant}$$
 (45)

whereas $\sigma^{(1)}$ leads to a rapidity distribution growing with y,

$$\frac{d\sigma^{(1)}}{dy} \sim y. \tag{46}$$

Consequently the lowest order and corrected formulae for the total hadronic crosssections are of the form,

$$\sigma^{(0)} \sim rac{lpha_S^2}{m^2} Y$$

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$$\sigma^{(1)} \sim \frac{\alpha_s^3}{m^2} \frac{Y^2}{2}, \quad Y \sim \ln\left(\frac{S}{4m^2}\right) \tag{47}$$

When $\alpha_S Y$ is of order one, the perturbation series in the running coupling alone will break down and one must sum all powers of $(\alpha_S Y)^n$. There are techniques^[31] to sum all terms of the form $(\alpha_S Y)^n$. After resummation these terms give rise to hadronic cross-sections which grow approximately like S^{δ} where $\delta \sim 0.5$. An explanation of this behaviour can be found in ref. [32] where a similar problem involving minijets is treated.

At present energies the form of the structure functions in the relevant regions differs substantially from Eq.(44). The decrease of the structure function as x grows tends to reduce the importance of above effects. We find them of minor importance for bottom production at the $Sp\overline{p}S$ collider. At very high energies these effects must appear. However uncertainties about the form of the gluon distribution function cloud the issue for both the Tevatron and the LHC/SSC. Because of the large value of the coupling these effects may appear in charm production at relatively low energy. The uncertainties alluded to above make a more precise statement difficult.

Reliable predictions of heavy flavour production at collider energies are plagued by the uncertainty in the gluon distribution function. This is a problem both at the theoretical level and at a practical level. Theoretically, it is uncertain how much of the positive correction near threshold should be considered physical. Our result is the correct one in the \overline{MS} factorisation scheme. But because of the kinematic nature of some of the large terms near threshold, explained in section II, it is possible that a considerable part of the correction is common to all processes with initial state gluons and should be absorbed into the structure function. At a practical level, the form of the gluon structure function to be used for bottom production at the highest collider energies is unknown. These questions can be answered definitively when reliable structure functions are forthcoming from experiments in which incoming gluons contribute in an essential way.

At present collider energies the production of very heavy quarks, m > 40 GeV is less subject to the above uncertainties, since a greater fraction of the production is due to quarks and antiquarks, which are well defined from DIS. The prediction of the top quark rate should be considered quite reliable. For example, at m = 60 GeV and $\sqrt{S} = 0.63$ TeV the uncertainty in the gluon contribution should be numerically irrelevant. Note however that the prediction of the absolute rate is subject to the overall uncertainty in the value of the coupling constant α_s .

Although we have not undertaken a systematic study of all the theoretical uncertainties, Figs. 14-17 suggest that the lowest order result, EHLQ set 1, $\Lambda = 0.2$ GeV and scale choice $\mu = \sqrt{2m}$, can be considered as a lower bound on the heavy quark total cross-section.

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References

- 1. M. Gluck, J. F. Owens and E. Reya, Phys. Rev. <u>D17</u>, (1978) 2324.
- 2. B. L. Combridge, Nucl. Phys. <u>B151</u>, (1979) 429.
- 3. J. Babcock, D. Sivers and S. Wolfram, Phys. Rev <u>D18</u>, (1978) 162.
- 4. K. Hagiwara and T. Yoshino, Phys. Lett. <u>80B</u>, (1979) 282;
 L. M. Jones, and H. Wyld, Phys. Rev. <u>D17</u>, (1978) 782;
 H. Georgi et al., Ann. Phys. <u>114</u>, (1978) 273.
- 5. P. Mazzanti and S. Wada, Phys. Rev. <u>D26</u>, (1982), 602.
- S. J. Brodsky, J. C. Collins, S.D. Ellis, J. F. Gunion and A. H. Mueller, Proc. 1984 Summer Study on the Design and Utilization of the SSC, R. Donaldson and J. Morfin, (eds.), APS (1984).
- 7. J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. <u>B263</u>, (1986) 37.
- 8. A. Kernan and G. VanDalen, Phys. Rep. 106, (1984) 298.
- 9. J. L. Ritchie, Proc. 1984 Summer Study on the Design and Utilization of the SSC, R. Donaldson and J. Morfin, (eds.), APS (1984) and references therein.
- M. Aguilar-Benitez et al., Phys. Lett. <u>161B</u>, (1985) 400;
 M. E. Duffy et al., Phys. Rev. Lett. <u>55</u>, (1985) 1816.
- M. G. Catanesi *et al.*, Phys. Lett. <u>187B</u>, (1987) 431;
 J. P. Albanese *et al.*, Phys. Lett. <u>158B</u>, (1987) 186.
- C. J. Albajar et al., (UA1 collaboration) CERN-EP/87-189, (1987);
 Phys. Lett. <u>186B</u>, (1987) 237.
- 13. For a review see, E. L. Berger, Argonne preprint ANL-HEP-PR-87-90, to be published in Nucl. Phys. B (Proceedings supplements), Proceedings of the topical seminar on heavy flavours, San Miniato, Italy, May 1987 and references therein.
- 14. Z. Kunszt and E. Pietarinen, Nucl. Phys. <u>B164</u>,(1980) 45.

-20- FERMILAB-Pub-87/222-T

- F. Halzen and G. Herzog, Phys. Rev. <u>D30</u>, (1984) 2326;
 F. Halzen and P. Hoyer, Phys. Lett. <u>154B</u>, (1985) 324;
 V. Barger and R. J. N. Phillips, Phys. Rev. <u>D31</u>, (1985) 215;
 A. Ali and G. Ingelman, Phys. Lett. <u>156B</u>, (1985) 111;
 G. Kopp, J. H. Kuhn and P. Zerwas, Phys. Lett. <u>153B</u>, (1985) 315.
- A. H. Mueller and P. Nason, Phys. Lett. <u>156B</u>, (1985) 226; Nucl. Phys. <u>B266</u>, (1986) 265.
- 17. R. K. Ellis and J.C. Sexton, Nucl. Phys. <u>B269</u>, (1986) 445.
- 18. Z. Kunszt and J. F. Gunion, Phys. Lett. <u>178B</u>, (1986) 296.
- 19. R. K. Ellis, in Proceedings of the XXI Rencontre de Moriond: Strong Interactions and gauge theories, Les Arcs, France (1986).
- 20. F. Halzen, P. Hoyer and C.S.Kim, Phys. Lett. <u>195B</u>, (1987) 74.
- 21. P. Nason, S. Dawson and R. K. Ellis, in preparation.
- W.A. Bardeen, A. Buras, D. W. Duke and T. Muta, Phys. Rev. <u>D18</u>, (1978) 3998.
- J. Collins, F. Wilczek and A. Zee, Phys. Rev. <u>D18</u>, (1978) 242;
 W. J. Marciano, Phys. Rev. <u>D29</u>, (1984) 580.
- 24. G. Altarelli, Phys. Rep. <u>81</u>, (1982) 1.
- 25. H. Bethe and W. Heitler, Proc. Roy. Soc. (London) <u>146</u>, (1934) 83.
- 26. G. Altarelli, R. K. Ellis and G. Martinelli, Nucl. Phys. <u>B157</u>, (1979) 461.
- 27. P. Aurenche et al., Orsay preprint, LPTHE-87/30, May 1987 and references therein.
- E. J. Eichten, I. Hinchliffe, C. Quigg and K. D. Lane, Rev. Mod. Phys. <u>56</u>, (1984) 579, (E:<u>58</u>,(1986) 1065).
- M. Diemoz, F. Ferroni, E. Longo, G. Martinelli, CERN-TH-4751/87 (1987);
 J. V. Allaby et al., CERN-EP-87-145 (1987).

- 30. J. Collins, Proceedings of the UCLA workshop on Observable standard model physics at the SSC, Los Angeles, CA (1986), (eds. H. Bengtsson *et al.*).
- 31. L. V. Gribov, E. M. Levin and M. G. Ryskin Phys. Rep. 100, (1983) 1.
- 32. A. H. Mueller and H. Navelet, Nucl. Phys. <u>B282</u>, (1987) 727.

Figure Captions

- Fig. 1: Lowest order Feynman diagram for heavy quark production $q + \overline{q} \rightarrow Q + \overline{Q}$.
- Fig. 2: Lowest order Feynman diagrams for heavy quark production $g + g \rightarrow Q + \overline{Q}$.
- Fig. 3: The quark-antiquark contributions to the parton cross section plotted vs. $1/\rho$. The functions $f^{(0)}, f^{(1)}$ and $\overline{f}^{(1)}$ are defined in Eq.(9). A solid line, $f(\rho) = 0$, is shown for comparison.
- Fig. 4: The gluon-gluon contributions to the parton cross section plotted vs. $1/\rho$. The functions $f^{(0)}, f^{(1)}$ and $\overline{f}^{(1)}$ are defined in Eq.(9). A solid line, $f(\rho) = 0$, is shown for comparison.
- Fig. 5: The gluon-quark contributions to the parton cross section as a plotted vs. $1/\rho$. The functions $f^{(0)}, f^{(1)}$ and $\overline{f}^{(1)}$ are defined in Eq.(9). A solid line, $f(\rho) = 0$, is shown for comparison.
- Fig. 6: The diagrams responsible for the constant behaviour of the cross-section.
- Fig. 7: Bremsstrahlung of an initial state gluon.
- Fig. 8: The parton flux $\Phi(r,\mu)$ versus r, for $\mu=5$ GeV. The thresholds for production of a bottom quark at centre of mass energies of 0.63 and 1.8 TeV are shown. Φ is defined in Eq. (42).
- Fig. 9: The parton flux $\Phi(\tau, \mu)$ versus τ , for $\mu=40$ GeV. The thresholds for production of a top quark with mass 40 GeV at centre of mass energies of 0.63 and 1.8 TeV are shown. Φ is defined in Eq. (42).

- Fig. 10: The product of short distance cross-sections and parton fluxes showing the relative importance of the various contributions for bottom quark production at $\sqrt{S}=0.63$ TeV. The upper (lower) solid line is the lowest order contribution from the $gg~(q\bar{q})$ initial state. The upper dotted line is the $O(\alpha_S^3)$ contribution from gg, the dotted line which falls below zero for small r is the $(q + \bar{q})g$ initial state, and the remaining dotted line is the contribution from $q\bar{q}$.
- Fig. 11: The product of short distance cross-sections and parton fluxes showing the relative importance of the various contributions for the production of a quark of mass m=40 GeV at $\sqrt{S}=0.63$ TeV. The 2 solid lines are the $O(\alpha_S^2)$ contributions from gg and $q\bar{q}$ initial states. The upper dotted line is the $O(\alpha_S^3)$ contribution from gg. The dotted line which has a positive (negative) slope at the threshold value of τ is the $q\bar{q}$ $((q+\bar{q})g)$ contribution.
- Fig. 12: The product of short distance cross-sections and parton fluxes showing the relative importance of the various contributions for bottom quark production at $\sqrt{S}=1.8$ TeV. The upper (lower) solid line is the lowest order contribution from the $gg(q\bar{q})$ initial state. The upper dotted line is the $O(\alpha_s^3)$ contribution from gg interactions, the dotted line which falls below zero for small τ is the $(q + \bar{q})g$ initial state, and the remaining dotted line is the $q\bar{q}$ contribution.
- Fig. 13: The product of short distance cross-sections and parton fluxes showing the relative importance of the various contributions for the production of a quark of mass m=40 GeV at $\sqrt{S}=1.8$ TeV. The upper (lower) solid line is the lowest order contribution from the $gg~(q\bar{q})$ initial state. The upper dotted line is the $O(\alpha_S^3)$ contribution from gg interactions, the dotted line which falls below zero for small τ is the $(q + \bar{q})g$ initial state, and the remaining dotted line is the $q\bar{q}$ contribution.
- Fig. 14: The ratio of the $O(\alpha_S^3)$ heavy quark cross-section evaluated at various subtraction scales μ to the $O(\alpha_S^2)$ cross-section as a function of the heavy quark mass m at $\sqrt{S}=0.63$ TeV. The lowest order prediction is evaluated at a fixed value of $\mu = m$. The parton distributions of EHLQ are used.

- Fig. 15: The ratio of the $O(\alpha_S^3)$ heavy quark cross-section to the $O(\alpha_S^2)$ crosssection as a function of the heavy quark mass m at $\sqrt{S}=1.8$ TeV. The parton distributions of EHLQ are used.
- Fig. 16: The ratio of the $O(\alpha_S^3)$ heavy quark cross-section to the $O(\alpha_S^2)$ crosssection as a function of the heavy quark mass m at $\sqrt{S}=0.63$ TeV. The parton distributions of DFLM are used.
- Fig. 17: The ratio of the $O(\alpha_S^3)$ heavy quark cross-section to the $O(\alpha_S^2)$ crosssection as a function of the heavy quark mass m at $\sqrt{S}=1.8$ TeV. The parton distributions of DFLM are used.
- Fig. 18: The quark-antiquark contributions to the parton cross section plotted vs. $1/\rho$, with the p_T cut, $p_T > 2m$.
- Fig. 19: The gluon-gluon contributions to the parton cross section plotted vs. $1/\rho$, with the p_T cut, $p_T > 2m$.
- Fig. 20: The gluon-quark contributions to the parton cross section plotted vs. $1/\rho$, with the p_T cut, $p_T > 2m$.











Figure 3



Figure 4



Figure 5















Figure 8



Figure 9



Figure 10



Figure 11



Figure 12



Figure 13



Figure 14



Figure 15



Figure 16

Ratio of cross-sections



Figure 17



Figure 18



Figure 19



Figure 20