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Interplay between the pairing and quadrupole interactions in the algebraic realization of the microscopic shell model

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Abstract. We explore the algebraic realization of the Pairing-Plus-Quadrupole Model /PQM/ in the framework of the Elliott's SU(3) Model with the aim to obtain the complementary and competing features of the two interactions through the relation between the pairing and the SU(3) bases. First, we establish a correspondence between the SO(8) pairing basis and the Elliott's SU(3) basis. It is derived from their complementarity to the same LST coupling chain of the shell-model number-conserving algebra. The probability distribution of the SU(3) basis states within the SO(8) pairing states is also obtained and allows the investigation of the interplay between the pairing and quadrupole interactions in the Hamiltonian of the PQM, containing both of them as limiting cases. The description of some realistic N∼Z nuclear systems is investigated in a SU(3)-symmetry-adapted basis within a model space of one and two oscillator shells.

keywords: algebraic models, pairing-plus-quadrupole model, microscopic shell model

1. Introduction

Since the early years of the development of the nuclear structure physics, it has been understood that the pairing [1] and the quadrupole-quadrupole interactions [2] are the most important short- and long-range interactions that have to be taken into account in the shell-model description of the nuclear systems [3]. Being with different range of action on the nucleons in the valence shells it is quite clear that these interactions actually influence the behavior of the systems in different parts of the shells. Hence, in some nuclei each of these interactions could reproduce relatively well the observed behavior of the nuclear system, but in most of the cases the study of the relationship between them is of great importance. This is the main motivation for the development of the Pairing-plus-Quadrupole Model /PQM/ [4] for the description of the nuclear excitation spectra. It is most successfully done in the framework of the basic shell model representation of the employed interactions, but the applications to real nuclear systems are rather complicated and cumbersome, due to the enormous dimensionality of the model space in particular for the heavy nuclei. Such a problem is easily avoidable by employing a group-theoretical approach [5], which introduces symmetry principles useful in particular for reducing the basis spaces and in the calculation of matrix elements of transitional operators. In this work we present such an approach by considering the basic interactions of the PQM as invariants of two respective algebras, which reduce the general symmetry of the shell model in a dynamical way. At the same time the two so defined dynamical symmetry chains are both complementary to the Wigner's spin-isospin

$SU_{ST}(4)$ symmetry, which establishes the direct connection between these two limiting cases. The latter allows for the investigation of the competing and complementarity features of the pairing and quadrupole interactions in the description of the realistic nuclear systems in the lower shells up to mass numbers $A \sim 100$.

2. Dynamical Symmetries in the Many-Particle Shell-Model Scheme

In this work we explore the reduction of the algebraic realization of the shell model algebra $U(4\Omega) \supset U_{ST}(4) \otimes U(\Omega))$[5] into the spatial $U(\Omega)$ and spin-isospin $U_{ST}(4)$ branches which are complementary. The chain $SU_{ST}(4) \supset SU_S(2) \otimes SU_T(2)$ of the Wigner's supermultiplet model [6] gives the spin S and isospin T of the basis states of the shell model. Further we consider the two possible reductions of the spatial part $U(\Omega)$ to the SO(3) algebra of the angular momentum. The chain $U(\Omega) \supset SU(3) \supset SO_L(3)$ [2] gives the rotational limit of the model with only quadrupole-quadrupole interaction taken into account. The other one through $SO(\Omega)$ which is isomorphic to SO(8), the algebra of the isoscalar and isovector pairing interaction, represents the pairing limit of the shell model algebra. Both these chains are complementary to the spin-isospin $U_{ST}(4)$ algebra. The above chains are unified into the following generalized reduction scheme:

$$
\begin{array}{ccccccc}
\{1^m\} & & & & U(4\Omega) & & \\
 & & & & \downarrow & & \\
(\tilde{f}\} & [U(\Omega) & & & \otimes & U_{ST}(4)] & \{f\} \\
 & \downarrow & \searrow & \alpha & & \downarrow & \\
[\tilde{\mu}] & [SO(\Omega) & [SU(3) & & \otimes & SU_{ST}(4)] & \{f'\} \\
(\nu[p]) & \Leftrightarrow SO(8) & (\lambda,\mu) & & & \sim SO(6) & [P] \\
\beta & \downarrow & \swarrow & K & & \downarrow & \\
L & [SO_L(3) & & & \otimes & SU_S(2)] \otimes SU_T(2) & S,T \\
 & & & & \downarrow & \downarrow & \\
J & & & & SU_J(2) & \otimes SU_T(2) & T
\end{array}
\qquad (1)
$$

Now we obtain the important result that the spatial subalgebra $U(\Omega)$ of the shell-model algebra $U(4\Omega)$ contains two distinct dynamical symmetries defined by the reduction chains: left branch through $SO(\Omega)$ and middle branch through SU(3). Consequently, both chains determine full-basis sets and could be expressed through each other. The basis states labeled by the quantum numbers of the representations of the algebras in the SU(3) chain $|\Psi_R\rangle \equiv |\{f\}\alpha(\lambda,\mu)KL,S;JM\rangle$ are eigenstates of the rotational limit of the model with quadrupole-quadrupole interaction. Correspondingly, the basis states in which the pairing interaction is diagonal [7] are labeled as $|\Psi_P\rangle \equiv |\{f\}\nu[p_1,p_2,p_3]\beta L,S;JM\rangle$. In both types of states α, β and K give the multiplicity labels of the corresponding reductions. Since the microscopic SU(3) model based on the three-dimensional harmonic oscillator has a well-developed theory, including the Wigner-Racah algebra for the calculation of matrix elements [8] in the SU(3) basis and various successful applications in real nuclei, we choose to expand the states of the pairing basis $|\Psi_P\rangle$ in the set of basis states $|\Psi_R\rangle$, i.e. $|\Psi_P\rangle_i = \sum_j C_{ij} |\Psi_R\rangle_j$. Using the above expansion and the diagonalization procedure for the pairing interaction in the SU(3) basis:

$$
{}_i\langle\Psi_P|H_{Pair}|\Psi_P\rangle_i = E_{Pair}(m,i,[P],(ST)) = \sum_{jk} C^*_{ki} C_{ij} . \delta_{kj} . {}_k\langle\Psi_R|H_{Pair}|\Psi_R\rangle_j \qquad (2)
$$

we obtain numerically the probability $|C_{ij}|^2$ with which the states of the SU(3) basis enter into the expansion of the pairing basis. In this way we actually calculate the transformation brackets between the two chains [9], which is of great use when calculating the matrix elements

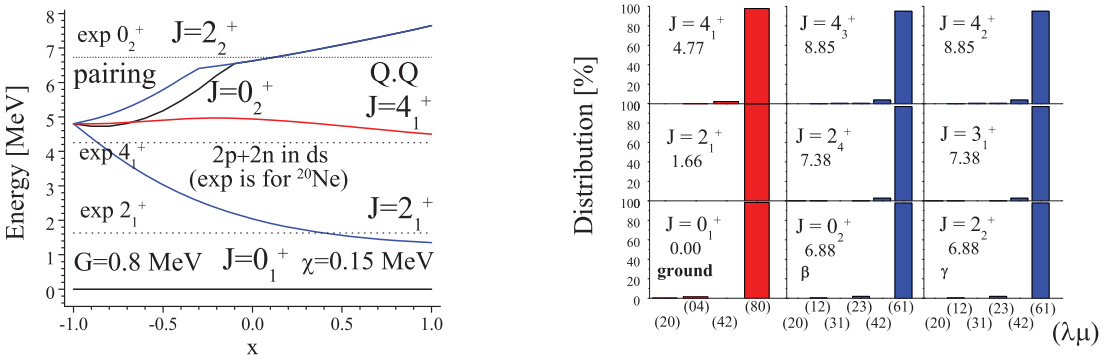

Figure 1. Excitation energies (left) and the SU(3) contents of the low-lying states (right) for the system of 2 protons and 2 neutrons (the nucleus ^{20}Ne) in the *ds* shell.

of different operators in each of the chains. This is important for example for the calculation of transition probabilities. Also, this expansion could help evaluate the importance (weight) of the different SU(3) states, when we need to impose restrictions on the basis because of computational difficulties. The known relations of the SU(3) labels (λ, μ) and the β, γ shape variables of the geometrical model can be used for the analysis of the deformations of the pairing states, expressed through the respective SU(3) ones.

For the purpose of our investigation we use the Hamiltonian $H = H_0 + V_{res}$ of the PQM [10], where H_0 is the harmonic oscillator term and the single-particle interactions and

$$V_{res} = \frac{1}{2}(1-x)\mathbf{G}(S^{\dagger}_{\mu}.S_{\mu} + P^{\dagger}_{\mu}.P_{\mu}) - \frac{1}{2}(1+x)\chi Q.Q, \tag{3}$$

At $x = -1$ we have pure pairing interaction with equal strengths of the isoscalar and isovector terms, and at $x = 1$ the limiting case of pure quadrupole interaction is realized. At $x = 0$ we have both interactions mixed with their respective strengths. This allows us to investigate the influence of these residual interactions on the spectra in real nuclear systems.

3. Results and discussion

After presenting in short the algebraic realization of the dynamical symmetries that appear in the microscopic shell model we would now like to exploit their applications in realistic nuclear systems. We start with a real test case for the applications of the theory - the *ds* shell, which is the first one, where both deformation and pairing phenomena play an important role [11], [12]. Our proof-of-case example presents the simple but complete system of 4 particles in the *ds* shell which allows us to study the PQM without any truncation of the model space.

First, we demonstrate the complementarity of the two competing modes by showing the energy spectrum (the left panel of Fig. 1) and the SU(3) contents (the right panel of the same figure) of the first few low-lying energy eigenstates for the system of 2 protons and 2 neutrons in the ds shell (^{20}Ne). Different spin-value dominance in the bands is represented by the respective colors - red for $S = 0$ and blue - for $S = 1$. Below the angular momentum labels on the right, the excitation energy in MeV is given. Both β and γ bands have equal excitation energies and are dominated by the same SU(3) irrep (61).

Next, for the same system, we extend our investigation beyond one oscillator shell and demonstrate to what degree the two-shell eigenfunction can be described by its shell-plus-a-single-orbital and one-shell restrictions. For this purpose, we add the single-particle term to

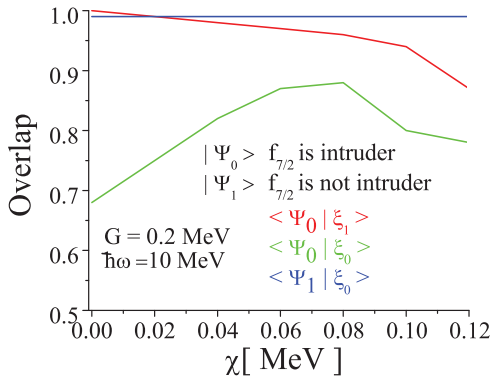

Figure 2. The overlap of the eigenstate, obtained in the two-shell model space $(|\Psi_0\rangle, |\Psi_1\rangle)$, with the result in the ds shell plus the $f_{7/2}$ orbital as an intruder $|\xi_1\rangle$ and with the single ds-shell result $|\xi_0\rangle$ for the system of 2 protons and 2 neutrons in the $dsfp$ shell at $G = 0.2$ MeV and varying values of the parameter χ. In the $|\Psi_0\rangle$ eigenfunction, the $f_{7/2}$ orbit is considered energetically to belong to the ds shell. In the $|\Psi_1\rangle$ case, $f_{7/2}$ belongs to the fp shell.

the Hamiltonian which accounts for the separation of the orbits in the two adjacent shells. So, on Fig. 2 we demonstrate that the two-shell results $|\Psi_0\rangle, |\Psi_1\rangle$ can be quite well represented at certain interaction strengths by the eigenfunctions in only one shell $|\xi_0\rangle$ or the shell plus a single orbital configuration $|\xi_1\rangle$. The strongest overlap, almost independent of the interaction strengths, is obtained for the case $\langle\Psi_1|\xi_0\rangle$, followed by the $\langle\Psi_0|\xi_1\rangle$, which starts to diminish at higher values of χ.

In conclusion, the algebraic structure of the shell-model algebra $U(4\Omega)$ is investigated to obtain its reductions through the microscopic pairing algebra SO(8), containing both isoscalar $(T = 0, S = 1)$ and isovector $(T = 1, S = 0)$ pairing operators and Elliott's SU(3) algebra. A relation between these chains is established on the basis of the complementarity to the Wigner's spin-isospin $U_{ST}(4)$ symmetry. This elucidates the algebraic structure of an extended Pairing-plus-Quadrupole Model, realized in the framework of the Elliott's SU(3) scheme [5]. This approach is used to study the combined effects of the quadrupole-quadrupole and pairing interactions on the energy spectra of the nuclear systems in a single shell, a shell plus an orbital and two shells.

Acknowledgments

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References

[1] Flowers B H 1952 *Proc. Roy. Soc. London*, Ser. A **212** 248
[2] Elliott J P 1958 *Proc. Roy. Soc. London*, Ser. A **245** 128; **245** 562
[3] Bohr A, Mottelson B R and Pines D 1958 *Phys. Rev.* **110** 936
[4] Belyaev S T 1959 *Mat. Fys. Medd. Dan. Vid. Selsk.* **31** No. 11; Kisslinger L S and Sorensen R A 1963 *Rev. Mod. Phys.* **35** 853; Baranger M and Kumar K 1965 Nucl. Phys. **62** 113
[5] Draayer J P 1993 in *Algebraic Approaches to Nuclear Structure: Interacting Boson and Fermion Models*, Contemporary Concepts in Physics VI, edited by R.F. Casten (Harwood Academic, Chur, Switzerland) 423
[6] Wigner E P 1937 *Phys. Rev.* **51** 106
[7] Kota V K B, Castilho Alcaras J A 2006 *Nucl. Phys.* A **764** 181
[8] Akiyama Y and Draayer J P 1973 *Comput. Phys. Commun.* **5** (1973) 405; Bahri C and Draayer J P 1994 *Comput. Phys. Commun.* **83** 59; Draayer J P, Leschber Y, Park S C, and Lopez R 1989 *Comput. Phys. Commun.* **56** 279
[9] Santopinto E, Bijker R and Iachello F 1996 *J. Math. Phys.* **37** 2674
[10] Bahri C, Escher J, Draayer J P 1995 *Nucl. Phys.* A **592** 171
[11] Elliott J P and Harvey M 1963 *Proc. Roy. Soc. London*, Ser. A **272** 557; Elliott J P and Wilsdon C E 1968 *Proc. Roy. Soc. London*, Ser. A **302** 509
[12] Vargas C E, Hirsch J G, Draayer J P 2001 *Nucl. Phys.* A **690** 409