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INCOMPATIBILITY OF THE QUARK-FIELD [U(6) $\otimes$ U(6)] COMMUTATION RELATIONS WITH VECTOR MESON DOMINANCE IN π° DECAY*

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ABSTRACT

Using the Bjorken limit theorem, we demonstrate that the commutation relations of the free-field quark model [or U(6) $\otimes$ U(6) commutation relations] are incompatible with the vector meson dominance of the $\pi^\circ \longrightarrow 2\gamma$ amplitude. The gauge field algebra does not suffer from this difficulty. The present status of vector meson dominance in π° decay is also reviewed.

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If the commutation relations of the vector and axial-vector currents are really part of the basic laws of physics, it is disturbing that as yet there has been no unanimity of opinions on the nature of the commutator between two space components of various current densities. For instance, the free-field quark model[1] [or U(6) × U(6)] leads to

$$\left[j_\mu^{em}(\vec{x}, 0), j_\nu^{em}(0)\right] = -2ie^2 \delta^{(3)}(\vec{x})\, \epsilon_{0\mu\nu\tau} \left[(2/3)^{3/2} j^0_{5\tau} + \frac{1}{3} j^3_{5\tau} + (1/3\sqrt{3})\, j^8_{5\tau}\right] + \text{S.T.} \tag{1}$$

whereas the same commutator is given in the gauge field algebra[2] by

$$\left[j_\mu^{em}(\vec{x}, 0), j_\nu^{em}(0)\right] = 0 + \text{S.T.}\,, \tag{2}$$

where S.T. stands for the Schwinger terms (which are irrelevant in the present discussion). We wish to point out that if the success of vector meson dominance in π° decay is not accidental, the commutation relations of the gauge field algebra must be favored.

According to Bjorken,[3] the high energy behavior of the matrix element for

$$\pi^\circ(q) \longrightarrow \gamma(k) + \gamma(k'),\ (q = k + k')$$

can be evaluated as follows:

$$M_{\mu\nu} = i\int d^4x\, e^{-ik.x} \left\langle 0 \left| T\left(j_\mu^{em}(x)\, j_\nu^{em}(0)\right)\right| \pi^\circ \right\rangle$$

$$\xrightarrow{|k_0| \to \infty} -\frac{1}{k_0}\int d^3x\, e^{-i\vec{k}.\vec{x}} \left\langle 0 \left| \left[j_\mu^{em}(\vec{x}, 0), j_\nu^{em}(0)\right] \right| \pi^\circ \right\rangle \tag{3}$$

where the limit is to be taken in such a way that $\vec{k}$, $\vec{k}'$, $\vec{q}$ and q_0 are kept finite as we let $|k_0| \approx |k'_0|$ go to infinity. The quark-field commutation relation (1) then gives

$$M_{\mu\nu} \longrightarrow \frac{2}{3}\left(e^2 c_\pi / k_0\right) \epsilon_{0\mu\nu\tau} q_\tau \tag{4}$$

(where c_π is the charged pion decay constant numerically equal to 94 MeV), a result already obtained by B.-L. Young.[4]

We now note that the behavior (4) is incompatible with the vector meson dominance model for $\pi°$ decay. If the $\pi°$ decay amplitude is dominated by ρ and ω (in the sense of Gell-Mann, Sharp and Wagner),[5,6] we expect

$$F(s,s') = \frac{g_{\rho\omega\pi}\ \lambda_\omega \lambda_\rho}{\left(m_\rho^2 - s\right)\left(m_\omega^2 - s'\right)} + (s \longleftrightarrow s') \tag{5}$$

where $F(s,s')$ is defined by

$$M_{\mu\nu} = \epsilon_{\mu\nu\lambda\tau}\, k_\lambda k'_\tau\ F(s,s') / m_\pi \tag{6}$$

with $s = -k^2$, $s' = -k'^2$ and[7]

$$\lambda_\rho = em_\rho^2 / f_\rho ,\ \lambda_\omega = -\left(em_\omega^2 / 2f_Y\right)\sin\theta_Y \quad . \tag{7}$$

As $|k_0| \approx |k'_0|$ goes to infinity with $\vec{k}$, $\vec{k}'$, $\vec{q}$ and q_0 finite, $M_{\mu\nu}$ based on (5) goes like $1/k_0^3$, in contradiction with the behavior (4). In contrast, the commutation relation of the gauge field algebra is consistent with the high-energy behavior of the vector meson dominance model since Eq. (2) requires the absence of the $1/k_0$ term in the Bjorken limit. Note also that any additional form factor effect one may care to consider in conjunction with the vector-meson dominance model tends to strengthen our argument against the quark-field commutation relations.

Next we comment on the paper of Young,[4] who claims to have shown that the observed pion lifetime is in good agreement with the quark-field commutation relation (1). Being aware of the incompatibility between the no-subtraction assumption for F(s, s') and the quark-field commutation relation, he proceeds to write down a once-subtracted dispersion relation in s and s':

$$F(s,s') - F(0,0) = \left[\frac{\lambda_\rho f_{\rho\gamma\pi} s}{m_\rho^2\left(m_\rho^2 - s\right)} + \frac{\lambda_\omega f_{\omega\gamma\pi} s}{m_\omega^2\left(m_\omega^2 - s\right)} + \frac{\lambda_\rho \lambda_\omega g_{\omega\rho\pi} s s'}{m_\rho^2 m_\omega^2\left(m_\rho^2 - s\right)\left(m_\omega^2 - s'\right)} + (s \leftrightarrow s')\right]$$

$$+ \text{ higher mass contributions} \tag{8}$$

where F(0, 0) is directly related to the π° lifetime via

$$\Gamma(\pi^\circ \rightarrow \gamma\gamma) = (1/64\,\pi)\left|F(0,0)\right|^2 m_\pi \ . \tag{9}$$

Comparing (8) with the Bjorken limit (3), he obtains

$$F(0,0) = \frac{2\lambda_\omega f_{\omega\lambda\pi}}{m_\rho^2} - \frac{e^2 c_\pi m_\pi}{3 m_\rho^2} \quad , \tag{10a}$$

$$g_{\rho\omega\pi} = \frac{f_\rho f_{\omega\gamma\pi}}{e} + \frac{f_\rho^2 c_\pi m_\pi}{6 m_\rho^2}\left(\frac{\lambda_\rho}{\lambda_\omega}\right) \tag{10b}$$

where $m_\omega = m_\rho$ and $\lambda_\rho f_{\rho\gamma\pi} = \lambda_\omega f_{\omega\gamma\pi}$ have been used[8] and the higher mass contributions have been neglected. At this point, however, Young uses an obsolete value for $\lambda_\omega/\lambda_\rho$ characteristic of unbroken SU(6), viz. $\lambda_\omega/\lambda_\rho = -1/3$; instead, one should take into account symmetry-breaking effects provided by spectral-function sum rules of the Weinberg type, which yield[9] $\lambda_\omega/\lambda_\rho = -0.81/3$. Thus we get in the free-field quark model[10]

$$|F(0,0)|_{\text{quark}} = \begin{cases} (3.9 \pm 0.4) \times 10^{-3} \\ (2.6 \pm 0.4) \times 10^{-3} \end{cases} , \tag{11a}$$

$$\frac{g^2_{\omega\rho\pi}}{4\pi} = \begin{cases} 0.58 \pm 0.11 \\ 0.25 \pm 0.06 \end{cases} \tag{11b}$$

if

$$\operatorname{sgn}\left(f_{\omega\gamma\pi} / f_\rho c_\pi\right) = \begin{cases} + \\ - \end{cases}$$

The predictions of the gauge field algebra can be obtained simply by setting $c_\pi = 0$ in Eqs. (10a) and (10b); we then recover the results of the vector meson dominance model (as we must). Numerically

$$|F(0,0)|_{\text{VMD}} = (3.3 \pm 0.4) \times 10^{-3} \tag{12a}$$

$$\frac{g^2_{\omega\rho\pi}}{4\pi} = 0.40 \pm 0.08 \ . \tag{12b}$$

Experimentally, the observed pion lifetime $\left[\tau_{\pi^\circ} = (0.89 \pm 0.18) \times 10^{-16} \text{ sec}\right]$ corresponds to

$$|F(0,0)|_{\text{exp}} = (3.3 \pm 0.4) \times 10^{-3} \tag{13}$$

which coincides exactly with (12a). Thus we see that the gauge field algebra (or the vector meson dominance model) is in even better agreement with the observed π° lifetime.[11]

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5. M. Gell-Mann, D. Sharp and W. G. Wagner, Phys. Rev. Letters 8, 216 (1962). See also M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).
6. We ignore the ϕ meson contribution because the $\rho\phi\pi$ coupling appears to be anomalously weak.
7. Our vector-meson coupling constants f_ρ and f_Y are normalized as in the papers of J. J. Sakurai [Annals of Phys. 11, 1 (1960)] and N. M. Kroll, T. D. Lee and B. Zumino [Phys. Rev. 157, 1376 (1967)]. The constants $g_{\rho\omega\pi}$, $f_{\rho\gamma\pi}$ and $f_{\omega\gamma\pi}$ are all dimensionless since we have taken out the factor $1/m_\pi$.
8. It may be mentioned that once we abandon vector meson dominance, the relation $\lambda_\rho f_{\rho\gamma\pi} = \lambda_\omega f_{\omega\gamma\pi}$ is more difficult to justify.
9. R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters 19, 1266 (1967); and references therein.
10. For numerical estimates we use $f_\rho^2/4\pi = 2.6$ and $f_{\omega\gamma\pi}^2/4\pi = 0.15\alpha \pm 20\%$. The major contribution to the errors in $F(0,0)$ and $g_{\rho\omega\pi}$ comes from $\Gamma(\omega \longrightarrow \pi^\circ\gamma)$.
11. Models based on integrally charged triplets [such as that of K. Johnson, F. E. Low and H. Suura, Phys. Rev. Letters 18, 1224 (1967)] are in worse agreement than the quark model.