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Wilson loop's phase transition probed by non-local observable

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Abstract

In order to give further insights into the holographic Van der Waals phase transition, it would be of great interest to investigate the behavior of Wilson loop across the holographic phase transition for a higher dimensional hairy black hole. We offer a possibility to proceed with a numerical calculation in order to discussion on the hairy black hole's phase transition, and show that Wilson loop can serve as a probe to detect a phase structure of the black hole. Furthermore, for a first order phase transition, we calculate numerically the Maxwell's equal area construction; and for a second order phase transition, we also study the critical exponent in order to characterize the Wilson loop's phase transition.

1. Introduction

According to AdS/CFT correspondence [1], an anti-de Sitter black hole in the bulk is dual to a strongly coupled large N gauge theory, and phase transitions in AdS backgrounds may provide an interpretation of holographic dual field theories. The best known Hawking–Page phase transition

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in Schwarzchild AdS spacetime [2], which presented the existence of a thermal radiation/large AdS black hole phase transition, is explained as the confinement/deconfinement phase transition in the dual gauge thermal field theory [3]. In addition, it has been shown that the scalar field condensation in four dimensional charged AdS black holes may be taken as holographically dual models of superconductors [4]. Therefore, it is interesting to extend the discussion of phase transitions to higher dimensional AdS spacetime to explore holographic properties.

Since a black hole possesses thermodynamical properties, it is natural to ask whether it can undergo a Van der Waals phase transition in the same manner as an ordinary liquid–gas thermodynamical system. In recent years, in the $P-V$ plane, the Van der Waals phase transition has been explored in various of AdS backgrounds [5–16], and most results indicated that, isotherms in the $P-V$ plane indeed present Van der Waals characterize: the system undergoes a second order phase transition at a second-order critical temperature, and undergoes a first order phase transition at sufficiently low temperature. In addition, in the $T-S$ plane, in the canonical ensemble, the isocharges also reveal Van der Waals behavior [17–21]. In [21], Caceres et al. have pointed out the connection between in the $P-V$ plane at constant T and in the $T-S$ plane at constant P, and found out dimensionless parameter that determines the transition.

Very recently, entanglement entropy and two point correlation function have been used to investigate Van der Waals phase transition [21,32–40]. In our previous paper [40], we have indicated the Van der Waals phase transition can be presented by employing black hole entropy, two point correction function and entanglement entropy. In this current paper, for a five-dimensional scalar hairy black hole, we will go on discussing whether Van der Waals phase transition can be presented by making use of Wilson loop.

Wilson loop is nonlocal probe, and it can also be used to investigate some properties of gauge field theories. In 1998, Maldacena proposed a method to calculate the expectation values of Wilson loop operators in the dual theory [22]. Following that, Wilson loop has attracted a lot of attentions for potential applications in AdS backgrounds. With Wilson loop/Wilson loop correlators in AdS/CFT by constructing space-like minimal surfaces, phase transitions in Wilson loop correlator were discussed in detail [23]. Minahan and Nedelin have shown how the phase transition affected the Wilson loop at strong coupling [24]. Furthermore, circular Wilson loop was studied as a probe, and the thermalization process of the dual boundary field theory was studied in GB-Vaidya model [25]. In addition, in order to give further insights into the holographic insulator/superconductor phase transition, Cai et al. presented the behavior of Wilson loop across the holographic phase transition [26], and showed that Wilson loop is a good probe to discuss the properties of the holographic superconductor phase transition.

Considering Wilson loop, in this paper, we will adapt the higher dimensional hairy black hole model to study Van der Waals-like phase transition. It is very interesting to investigate the conformal coupling of a scalar field in higher dimensional spacetime. Oliva and Ray first developed a novel construction of conformal couplings of a scalar field to arbitrary higher order Euler densities, which was done by constructing a four-rank tensor involving the curvature and derivatives of the field [27]. In higher dimensional AdS spacetime, Giribet et al. proved the analytic solutions to higher dimensional hairy black holes do exist and the scalar configuration is regular every where outside and on the horizon [28,29]. Further, Hennigar and Mann have first revealed a reentrant phase transition in a five dimensional gravitational system which does not include higher curvature corrections [15]. In addition, a tractable model to study the phase transition of hairy black holes in anti-de Sitter space was also discussed in detail [30]. Especially, in very recent years, Hennigar, Mann and Tjoa have found that, for a class of asymptotically AdS hairy black holes in Lovelock gravity where a real scalar field is conformally coupled to gravity, a novel form of phase transition akin to a superfluid phase transition can be observed [31]. Therefore, the phase transitions of higher dimensional hairy black holes have gained a lot of interest in AdS spacetime.

Motivated by the themes above, we attempt to discuss on the behavior of Wilson loop across the holographic Van der Waals-like phase transition in this work. The rest of this paper is organized as follows. In section 2, the Van der Waals phase transition of a five-dimensional hairy black hole in AdS background is investigated. The last section is devoted to the conclusions.

2. Van der Waals phase transition of Wilson loop

First, let us review a Van der Waals phase transition of the black hole entropy for a five-dimensional hairy black hole in AdS background. The corresponding action reads [28,29]

$$\mathrm{I} = \frac{1}{\kappa}\int d^5x\sqrt{-g}\,(R - 2\Lambda + \kappa L_m(\phi, \nabla\phi)), \tag{1}$$

where $\kappa = 16\pi G$, and the Lagrangian matter $L_m(\phi, \nabla\phi)$ takes the form

$$L_m(\phi, \nabla\phi) = \phi^{15}\left(b_0 S^{(0)} + b_1\phi^{-8}S^{(1)} + b_2\phi^{-16}S^{(2)}\right), \tag{2}$$

with

$$\begin{aligned} S^{(0)} &= 1, \\ S^{(1)} &= S \equiv g^{\mu\nu}S_{\mu\nu} \equiv g^{\mu\nu}\delta^{\rho}_{\sigma}S^{\sigma}_{\mu\rho\nu}, \\ S^{(2)} &= S_{\mu\nu\alpha\beta}S^{\mu\nu\alpha\beta} - 4S_{\mu\nu}S^{\mu\nu} + S^2, \end{aligned} \tag{3}$$

here b_0, b_1 and b_2 are the coupling constants of conformal field theory. The black hole's metric reads [15,30]

$$\mathrm{d}s^2 = -F(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + r^2\left[\mathrm{d}\varphi^2 + \sin^2\varphi(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\psi^2)\right] \tag{4}$$

with

$$F(r) = 1 - \frac{m}{r^2} - \frac{q}{r^3} + \frac{r^2}{l^2}, \tag{5}$$

where $l \equiv (-\Lambda/6)^{-1/2}$, m is mass parameter, and q stands for the black hole's charge under a scalar field. The scalar hair configuration of the theory is

$$\phi(r) = \frac{n}{r^{1/3}}, \tag{6}$$

where n in Eq. (6) and q in Eq. (5) are given by the following relationship,

$$q = \frac{64\pi G}{5}b_1 n^9, \qquad n = \varepsilon\left(-\frac{18}{5}\frac{b_1}{b_0}\right)^{1/6}, \tag{7}$$

with $\varepsilon = -1, 0, 1$. The scalar coupling constants satisfy the following constraint

$$10b_0b_2 = 9b_1^2. \tag{8}$$

The relationship between Hawking temperature T_h and the entropy S becomes [40]

$$T_h(S,q) = \frac{1}{2^{2/3} l^2 \pi^{5/3} (5\pi^2 q + 4S)^{4/3}} \Big[6l^2 \pi^{10/3} q + 4l^2 \pi^{4/3} S + 5\sqrt[3]{2}\pi^2 q (5\pi^2 q + 4S)^{2/3} + 4\sqrt[3]{2} S (5\pi^2 q + 4S)^{2/3} \Big]. \tag{9}$$

From $\left(\frac{\partial T_h}{\partial S}\right)_q = \left(\frac{\partial^2 T_h}{\partial^2 S}\right)_q = 0$, the critical temperature, critical scalar charge and critical entropy are given by

$$T_c = 5\sqrt{\frac{3}{10}} \frac{1}{2\pi l}, \tag{10}$$

$$S_c = \frac{9}{40}\sqrt{\frac{3}{10}} \pi^2 l^3, \tag{11}$$

$$q_c = -\frac{3}{50}\sqrt{\frac{3}{10}} l^3. \tag{12}$$

According to the temperature function $T_h(S, q)$ in Eq. (3), we can plot the isocharges in the T_h–S plane [40]. Here, we set $l = 1$ throughout. Based on the relation between $T_h - T_c$ and $S - S_c$, and heat capacity $C_q = T_h \frac{\partial S}{\partial T_h}|_q$, one has $C_q \sim (T_h - T_c)^{-2/3}$. That is to say, the second phase transition's critical exponent is $-2/3$. Taking logarithm form, we have

$$\log|T_h - T_c| = 3\log|S - S_c| + \text{constant}. \tag{13}$$

In the previous paper [40], the authors have concluded that the isocharges in the T_h–S plane present the Van der Waals phase transition, and the critical exponent has been calculated, which coincides with the one from the mean field theory. Now, we focus on exploring the phase transition of the hairy black hole by using Wilson hoop. Wilson loop corresponds to the minimal area surface by holography. In the form of the AdS/CFT correspondence, the expectation value of Wilson loop operator is approximated geometrically given by [22]

$$\langle W(c) \rangle \approx \exp\left(-\frac{A \sum}{2\pi\alpha'} \right), \tag{14}$$

in which α' is a Regge slope parameter, $\sum$ is the minimal bulk surface ending on C with its minimal area A, and C is the closed contour. Here, choosing the line with $\varphi = \pi/2$ and $\theta = \theta_0$ as loop, we are able to utilizing (φ, θ) to parameterize the minimal area surface. Consequently, the minimal area surface is

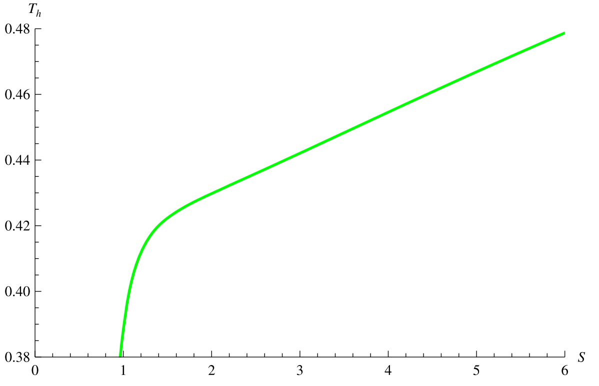
$$A = 2\pi \int_0^{\theta_0} r \sin\theta \sqrt{\frac{r'(\theta)^2}{F(r)} + r(\theta)^2} \mathrm{d}\theta, \tag{15}$$

where $r' \equiv \mathrm{d}r/\mathrm{d}A$. Through resolving the Euler–Langrange equation

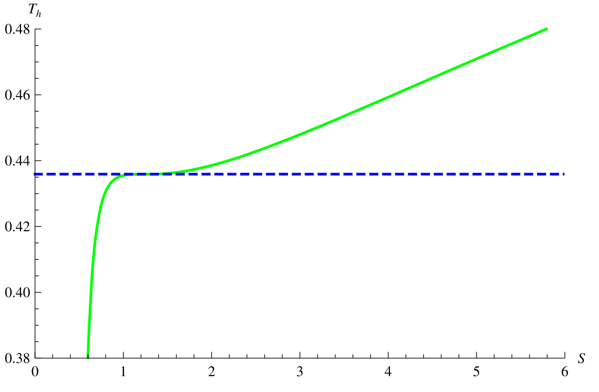
$$\frac{\partial L}{\partial r} = \frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\partial L}{\partial r'}\right), \tag{16}$$

one can obtain the motion equation of $r(\theta)$. Taking into account the boundary conditions

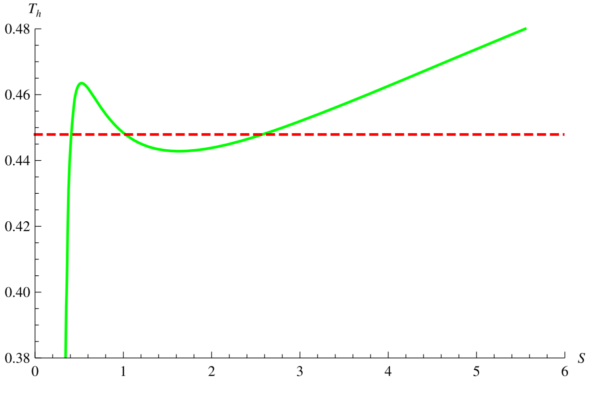
$$r'(0) = 0, \qquad r(0) = r_0, \tag{17}$$



(a) $|q = -0.05| > |q_c|$



(b) $|q| = |q_c|$



(c) $|q = -0.02| < |q_c|$

Fig. 1. Scalar isocharges in the T_h–S plane for different q with $l = 1$. The blue dashed line in panel (b) indicates the second phase transition's temperature $T_c = 0.4359$, and the red dashed line in panel (c) corresponds to the temperature of the first order phase transition $T^* = 0.4479$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

numerically, we obtain the result of $r(\theta)$. Notice that for a fixed θ_0, due to the minimal area surface is divergent, it needs to be regularized. We are able to accomplish it through subtracting the minimal area surface in pure AdS with the same boundary (which is denoted by A'). Namely, we first numerically get A by integrating the minimal area surface function in Eq. (9) to cut off $\theta_c \lesssim \theta_0$, then, repeating the same procedure in pure AdS, we can get A'. Thus, the regularized minimal area surface $\Delta A = A - A'$ is obtained. Here, we take $\theta_0 = 0.42$ and $\theta_0 = 0.50$ as example, and corresponding cutoffs are ordered to be $\theta_c = 0.419$ and 0.499. The scalar isocharges are presented in the T_h–ΔA plane in Fig. 2 and Fig. 3. Comparing with Fig. 1, we see that the Van der Waals phase transition can also be exhibited in the T_h–ΔA plane, like black hole entropy.

Next, for the first order phase transition, we check the Maxwell's equal area construction in the T_h–ΔA plane. We construct the similar equal area law of Wilson loop as

$$A_1 \equiv \int_{\Delta A_{min}}^{\Delta A_{max}} T_h\,(\Delta A, q)\,\mathrm{d}\Delta A = T^*\,(\Delta A_{max} - \Delta A_{min}) \equiv A_2, \tag{18}$$

where $T_h(\Delta A, q)$ is an interpolating function, which can be got from the numeric result, and ΔA_{min} and ΔA_{max} are the smallest and largest toots of the equation $T_h(\Delta A, q) = T^*$. Here, T^* is equal to the first order phase transition temperature of black hole entropy. For different θ_0, the values of ΔA_{min}, ΔA_{max}, A_1 and A_2 are tabulated in Table 1. As can be seen from this table, in our numeric accuracy, A_1 equals to A_2.

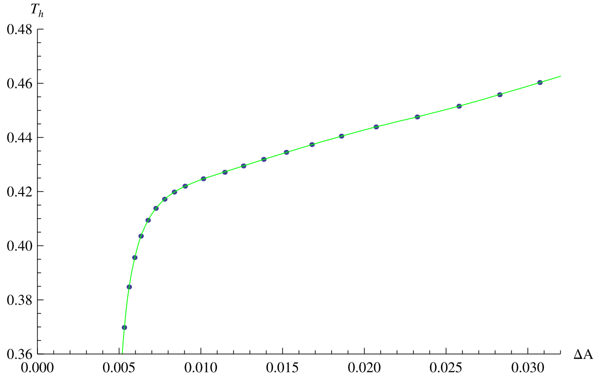
It is noticed that recent work [41] has called into question to Maxwell's equal area law in the holographic framework. Now, for the Wilson loop, we further check this problem with the different ratio $q/q_C = 0.9$ and 0.3. According to Ref. [41], we use following equal area law

$$\begin{aligned} A(I) &\equiv \int_{\triangle A_{min}}^{\triangle A_{int}} T_h(\triangle A, q)\,\mathrm{d}\triangle A - T^*(\triangle A_{int} - \triangle A_{min}) \\ &= T^*(\triangle A_{max} - \triangle A_{int}) - \int_{\triangle A_{int}}^{\triangle A_{max}} T_h(\triangle A, q)\,\mathrm{d}\triangle A \equiv A(II) \end{aligned} \tag{19}$$

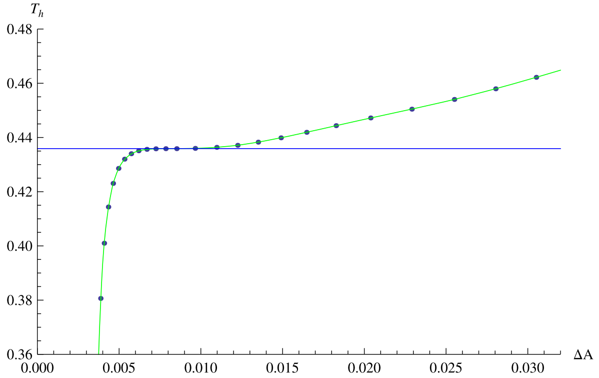
where $\triangle A_{min}$, $\triangle A_{int}$ and $\triangle A_{max}$ are the smallest, intermediate and largest root of the equation $T^* = T_h(\triangle A, q)$ with $A(I)$ and $A(II)$ the areas bounded above and below by $T_h(\triangle A, q)$ and T^*. Here, we take $\theta_0 = 0.42$ as example and perform numerical computations. The results are presented in Table 2 with the relative error defined as $\frac{A(I)-A(II)}{A(I)} \times 100\%$. From Table 2, it can be seen that, the equal area law holds just near criticality in the Wilson loop picture. However, as q/q_C decreases, the relative errors are significantly large and consequently Maxwell's equal area law does not hold on T–$\triangle A$ plane in regimes away from the critical point, which further strengthens the proposal of McCarthy et al. [41] who point out the breakdown of the equal area law in the holographic framework. In addition, we also find that, the relative errors become larger if we use the more precise areas given by Eq. (19) instead of that given by Eq. (18). With $A(I)$ and $A(II)$ used in Eq. (19), the relative errors observed are consistent with the result in Ref. [41].

For the second order phase transition, we go on verifying whether the minimal area surface has the same critical exponent with one obtained in the black hole entropy. In order to do so, firstly, we define a similar heat capacity of the Wilson loop.

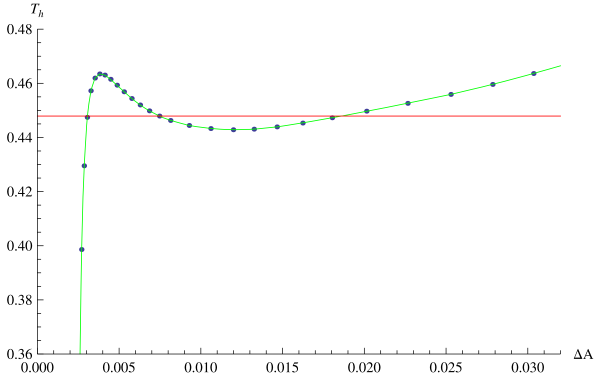
$$C_q = T_h \left.\frac{\partial \Delta L}{\partial T_h}\right|_q . \tag{20}$$



(a) $|q = -0.05| > |q_c|$

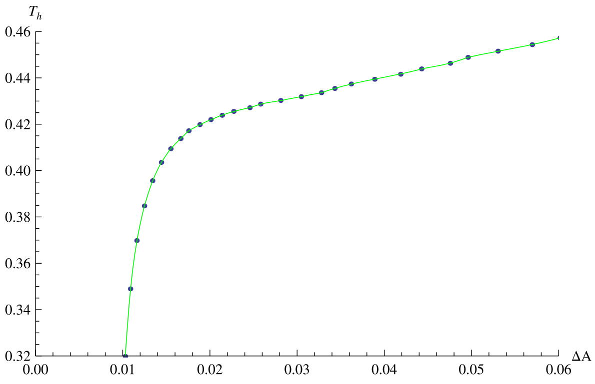


(b) $|q| = |q_c|$

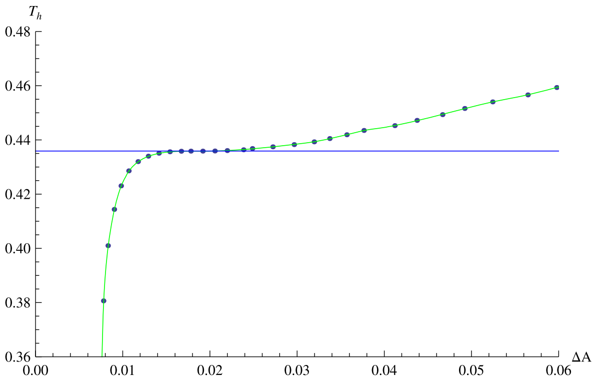


(c) $|q = -0.02| < |q_c|$

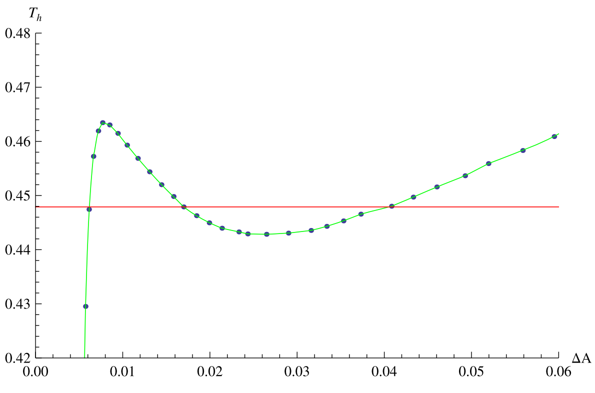
Fig. 2. Scalar isocharges in the T_h–ΔA plane for $\theta_0 = 0.42$. Panel (a): $|q = -0.05| > |q_c|$. Panel (b): $|q| = |q_c|$, the red isotherm $T_h = T_c$ corresponds the critical temperature for a second order phase transition. Panel (c): $|q = -0.02| < |q_c|$. The red isotherm $T_h = T^*$ indicates the temperature of a first order phase transition. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



(a) $|q = -0.05| > |q_c|$



(b) $|q| = |q_c|$



(c) $|q = -0.02| < |q_c|$

Fig. 3. Scalar isocharges in the T_h–ΔA plane for $\theta_0 = 0.05$. Panel (a): $|q = -0.05| > |q_c|$. Panel (b): $|q| = |q_c|$, the red isotherm $T = T_c$ corresponds the critical temperature for a second order phase transition. Panel (c): $|q = -0.02| < |q_c|$, the red isotherm $T_h = T^*$ indicates the temperature of a first order phase transition. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1
Check of the Maxwell's equal area construction with $T^* = 0.4479$ in the T_h–$\triangle S$ plane.

$\theta_0 = 0.42$	$\Delta A_{min} = 0.00305812$	$A_1 = 0.0069516$
	$\Delta A_{max} = 0.01857888$	$A_2 = 0.0069518$
$\theta_0 = 0.50$	$\Delta A_{min} = 0.00615366$	$A_1 = 0.015409$
	$\Delta A_{max} = 0.04052636$	$A_2 = 0.015396$

Table 2
Check of the equal area law in the T–$\triangle A$ plane for $\theta_0 = 0.42$.

T^*	$\lvert q\rvert$	q/q_c	A (I)	A (II)	Relative error
0.438406	0.03	0.9	1.54953×10^{-6}	1.53041×10^{-6}	0.0123
0.458343	0.01	0.3	1.57730×10^{-4}	1.36519×10^{-4}	0.1345

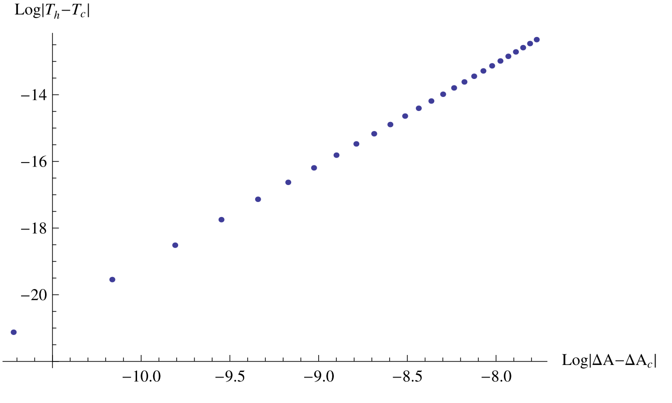


Fig. 4. $\log|T_h - T_c|$ versus $\log|\triangle A - \triangle A_c|$ for $\theta_0 = 0.42$.

Then, with the equation $T_h(\Delta A, q) = T_c$, the numeric result ΔA_c can be obtained. For different θ_0, we have plotted $\log(T - T_c)$ versus $\log|\Delta A - \Delta A_c|$ in Fig. 4 and Fig. 5, and the corresponding linear fit is

$$\log|T_h - T_c| = \begin{cases} 10.9198 + 2.99941 \log|\Delta A - \Delta A_c| & \text{for} \quad \theta_0 = 0.42 \\ 12.3146 + 2.94085 \log|\Delta A - \Delta A_c| & \text{for} \quad \theta_0 = 0.50 \end{cases} \tag{21}$$

Obviously, the slope of the each curve is around 3 for different θ_0. Compared with Eq. (7), the critical exponent of Wilson loop is consistent with that of the black hole entropy.

3. Conclusion

In this paper, making use of Wilson loop, we discuss on the Van der Waals phase transition for the five-dimensional scalar hairy black hole. The result shows that, like the black hole entropy, the Van der Waals phase transition can also be presented in the T–ΔA plane. To further characterize the phase transition, the equal area law is checked in the T–ΔA plane for a first order phase transition, and the critical exponent for the second order phase transition is also calculated numerically. Notice that recent work [41] has predicated that the breakdown of the equal area law

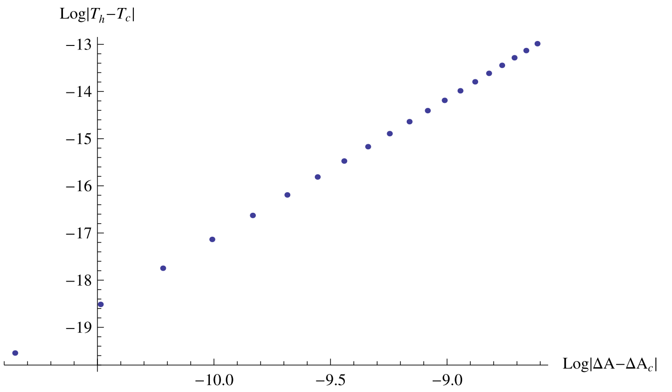


Fig. 5. $\log|T_h - T_c|$ versus $\log|\triangle A - \triangle A_c|$ for $\theta_0 = 0.50$.

for holographic entanglement entropy and two point correlation function. In this paper, for the Wilson loop, with different ratios q/q_C, we also perform numerically computation of the equal area law in the T–ΔA plane. As a result, we find out the equal area law is valid near criticality, however, it does not hold in regimes away from the critical point. To conclude, Wilson loop can serve as a probe to investigate the phase structure of a black hole, and a holographic equal area law of the Wilson loop is still an open question.

It is interesting to note that, for the charged hairy black hole, in [15], Hennigar and Mann have first revealed a reentrant phase transition, and have carefully studied the Van der Waals behavior in the P–V plane in both the charged/uncharged cases. Here, for uncharged case in the T–S plane and T–ΔA plane, by choosing some proper values of q, we also present the Van der Waals-like phase transition, thereby strengthening the conclusion of [15]. In fact, there is indeed a connection between (P, V, T) description and (T, S, Q) description [21]. Employing Wilson loop, we can explore reentrant phase transitions in the charged case. Wilson loop can serve as a probe of different phase transitions such as the Hawking–Page phase transition and the holographic superconductor phase transition. According to AdS/CFT correspondence, these phase transitions have been well understood in dual field theories. However, up to now the dual field theory interpretation of the Van der Waals-like phase transition is still, to a large extent, an open question, and it would be interesting to give a clear field theory interpretation in future works.

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