

# PARTICLE-IN-CELL SIMULATIONS OF A PLASMA LENS AT DARESBUURY LABORATORY

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## Abstract

Feasibility of a focusing element using the transverse fields provided by a plasma cell was studied numerically. In this paper, an experimental set up is proposed for various beam parameters available from the VELA and CLARA beam lines at Daresbury Laboratory. 2D simulation results from VSim [1], and expected results from planned measurement stations are presented. Field properties and the advantages and disadvantages of such an instrument compared to conventional focusing elements are discussed.

## INTRODUCTION

Focusing of a particle beam is conventionally carried out using quadrupole magnets, either normal or superconducting, or solenoids. For final focus elements in particle colliders, the smallest possible beam size at the interaction region is desirable to achieve the highest possible luminosity. A plasma lens provides a possible means of achieving extremely large focusing gradients in both transverse directions simultaneously, but also provides a new set of challenges to overcome [2]. Study of plasma lenses using the electron beam from VELA and CLARA will provide results for comparison with previous experiments performed elsewhere and will provide a groundwork for further experiments using present and proposed accelerator test facilities at Daresbury Laboratory.

## EXPECTED FOCUSING GRADIENTS

When a negatively charged particle bunch enters a plasma, the space charge force expels electrons from the path of the bunch. This leaves the bunch propagating in a region of net positive charge. If the background plasma density  $n_p$  is larger than the density of the bunch  $n_b$  (the overdense regime), the plasma left behind will completely neutralize the bunch. If  $n_p$  is lower than  $n_b$  (the underdense regime), the plasma electrons will be completely expelled leaving behind an ion channel. Applying Gauss's and Ampère's laws, the forces due to the electric and magnetic fields on a particle of charge  $q$  at a radius  $r$  within a homogenous, cylindrical bunch of charge density  $\rho_b$  and current density  $j$ , travelling at velocity  $v_b$ , can be calculated:

$$F = q \left( \frac{\rho_b r}{2\epsilon_0} + v_b \frac{\mu_0 r j}{2} \right) \quad (1)$$

In the overdense regime the net force on the particle is due to the magnetic field and is proportional to  $r$ . For comparison with magnetic quadrupoles, this can be quoted as a magnetic field gradient  $G$ :

$$G = \frac{B}{r} = \frac{\mu_0 j}{2} \quad (2)$$

Reasonable parameters for a future linear collider before final focus, with a peak beam current of 3.2 kA [3] and transverse bunch size of 100  $\mu\text{m}$ , give a magnetic field gradient of 100 kT m<sup>-1</sup>. In the underdense regime, more realistic for very high density beams, the focusing gradient will be smaller by, to a first approximation, a factor of  $n_p/n_b$ . This can still lead to very large focusing gradients compared to conventional magnets e.g. 200 T m<sup>-1</sup> for quadrupoles for an LHC upgrade [4].

## EFFECT OF ABERRATIONS

Compared to conventional magnets, plasma lenses have a significant disadvantage of increasing the emittance of the beam during focusing. This is caused by two main factors: the deviation of the focusing force from linearity in  $r$ , the spherical aberration, and the variation in focusing force with longitudinal position in the bunch, the longitudinal aberration. Scattering from plasma particles is not expected to be a significant source of emittance growth over the length scale of a plasma lens [5]. The emittance growth due to the spherical aberration can be calculated by considering the force on a particle at a distance from the axis of  $r = 1\sigma_r$ , being focused to a focal length  $f$ , in a beam of initial beta function  $\beta_0$  and emittance  $\epsilon_0$  [6]. The fractional deviation in focusing strength from linearity,  $\frac{\Delta K}{K}$  leads to a deviation from expected angular deflection of  $\delta\theta$  given by:

$$\delta\theta = \frac{\sqrt{\beta_0\epsilon_0}}{f} \frac{\Delta K}{K} \quad (3)$$

The effective emittance  $\epsilon_{\text{eff}}$  due to the spherical aberrations is given by:

$$\epsilon_{\text{eff}} = \sqrt{\epsilon_0^2 + \beta_0\epsilon_0\delta\theta^2} \quad (4)$$

The longitudinal aberration requires consideration of the phase space of the bunch as it passes through the lens. The head of the bunch enters an unperturbed plasma, and as such its space charge is not neutralized and it sees no net focusing force. Moving back along the bunch, the focusing force seen by slices of the beam will be proportional to the perturbation in plasma density. Focusing corresponds to a rotation of the phase space of the bunch, and non-uniform focusing will lead to smearing out of the phase space.

## PROPOSED EXPERIMENTAL SETUP

VELA (Versatile Electron Linear Accelerator) is an S-band RF photoinjector at Daresbury Laboratory. It is well suited for initial studies of the plasma lens in the overdense regime. The low beam density allows a low plasma density to be used, and its low beam energy, while still being relativistic, allows the lens length to be short making for a compact experiment. Parameters for the VELA beam at Beam Area 1 multiuser station are shown in Table 1.

Table 1: VELA Beam Parameters

Charge $Q$ (pC)	250
Energy $E$ (MeV)	4.8
Population $N$ ( $10^6$ )	1563
RMS bunch length $\sigma_z$ ( $\mu\text{m}$ )	3300
RMS bunch size $\sigma_x$ ( $\mu\text{m}$ )	800
RMS bunch size $\sigma_y$ ( $\mu\text{m}$ )	250
Norm. emittance $x/y$ ( $\mu\text{m}$ )	6 / 7
Bunch density $n_b$ ( $\text{m}^{-3}$ )	$1.5 \times 10^{17}$
Peak current (A)	22.7

Due to the short plasma lengths that are appropriate for plasma lens experiments with VELA, the most suitable plasma source is a laser-ionized gas jet as this source can produce a variable length plasma with a minimum length shorter than 1 mm. The use of an axicon lens enables the laser to be focused to uniform intensity over the gas jet, achieving a constant plasma density [7]. Figure 1 shows a conceptual experimental layout using such a plasma source.

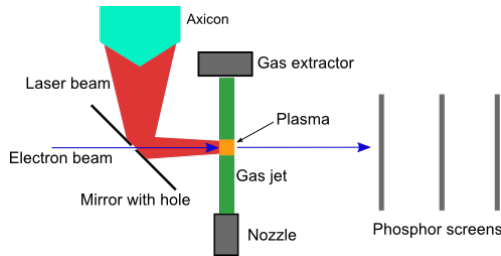


Figure 1: Conceptual layout of a plasma lens experiment using a laser-ionized gas jet plasma source.

The expected focusing strength that can be achieved using VELA can be calculated using Eq. 2. The peak current density is  $113.5 \text{ MA m}^{-2}$  giving a maximum focusing gradient of  $71 \text{ T m}^{-1}$ , comparable to the typical gradient of a magnetic quadrupole.

## PARTICLE-IN-CELL SIMULATION RESULTS

In order to validate the use of the VELA beam and the chosen experimental concept, 2D particle-in-cell simulations were carried out using VSim [1]. Studies were carried out of the effect of varying the plasma density and the length of the plasma lens. In order to model the correct bunch density in 2D, the average of the transverse bunch parameters were

used. The bunch size at a longitudinal distance  $z = 20 \text{ cm}$  was recorded, as was final emittance.

Figure 2 shows the evolution of the electron bunch from its initial state (a), through its minimum rms size (b), and beyond its focal point (c). The plasma length in this case is 9 mm. It can be seen in fig 2(b) that the maximum focusing strength occurs near the rear of the bunch, while the head of the bunch has not been focused. Beyond its focal point, the most strongly focused part of the bunch begins to diverge, while a point nearer to the head of the bunch reaches its focus. For all results in this paper,  $z = 0$  is the initial centre of the bunch, with the plasma lens starting at  $z = 26.4 \text{ mm}$ . As the bunch projection deviates significantly from a Gaussian profile, different fitting schemes were considered. Taking the rms bunch size was found to give the most consistent results, and ensures results are not affected unduly by particles far from the axis.

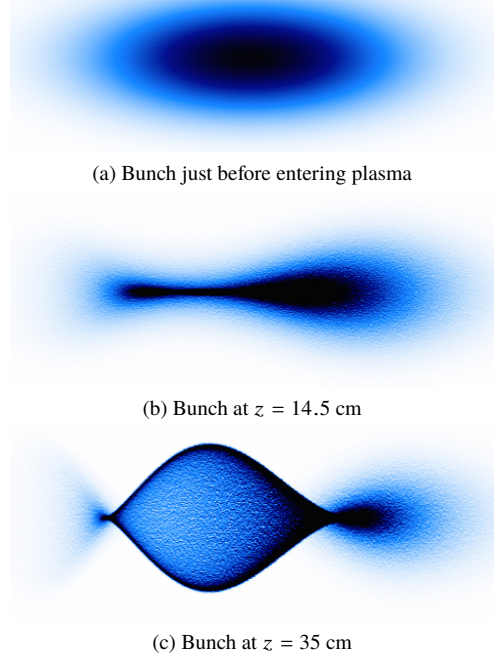


Figure 2: The bunch charge distribution at various points in the simulation for a plasma lens of length 9 mm and density  $9 \times 10^{18} \text{ m}^{-3}$ . The direction of motion of the bunch is to the right. All plots are to the same scale and use the same colour scale.

The change in rms bunch size with position is shown in Fig. 3 for a plasma length of 9 mm and a plasma density of  $9 \times 10^{18} \text{ m}^{-3}$ . These results show a demagnification factor of approximately 2. The focal length of 20 cm gives a focusing strength  $K = 1/fl = 556 \text{ m}^{-2}$ . This corresponds to an average magnetic field gradient of  $9.4 \text{ T m}^{-1}$ .

Figure 4 shows the results of the plasma density scan. As expected for the overdense regime, the focusing strength of the lens is approximately constant for plasma densities much

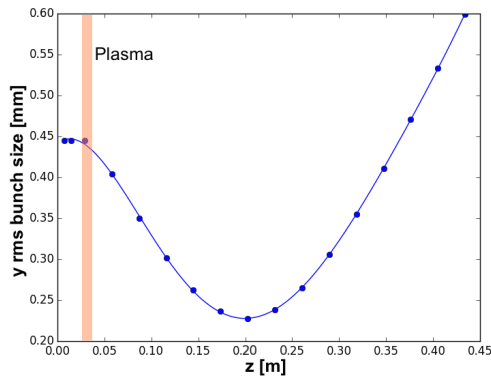


Figure 3: Variation of bunch RMS size with  $z$  for a plasma length of 9 mm and a plasma density of  $9 \times 10^{18} \text{ m}^{-3}$ , with 5th-order spline fitting.

greater than the bunch density of  $1.5 \times 10^{17} \text{ m}^{-3}$ , and the focusing force depends only on the bunch density.

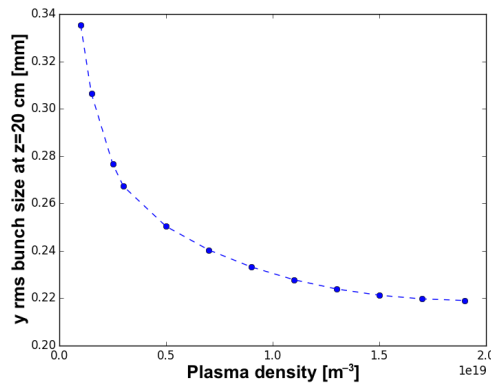


Figure 4: Variation with plasma density of bunch RMS size at  $z=20$  cm, with linear interpolation.

Figure 5 shows the results of the plasma length scan. A linear fit was performed, neglecting the last six points as the thin lens approximation is less valid for  $l_{\text{lens}} \geq 0.1f$ . The results show that the focusing strength of the lens is proportional to the length of the plasma, as long as the plasma is much shorter than the focal length.

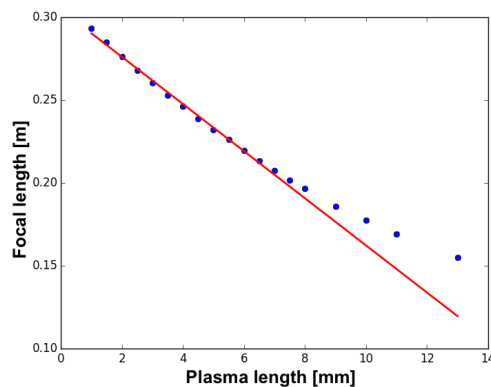


Figure 5: Variation of focal length with plasma length, with a linear fit calculated using a least squares method.

Figure 6 shows the increase in emittance for the plasma density scan. The emittance growth is smaller at low plasma densities as the focusing strength is less. At higher plasma densities the growth in emittance begins to fall as the plasma wavelength approaches the size of the bunch, and thus the focusing force reaches a maximum within the bunch length. Estimation of the spherical aberration and application of equation 3 gives a maximum emittance growth due to the spherical aberration of approximately 2 mm mrad. The majority of the emittance growth is attributable to the longitudinal aberration.

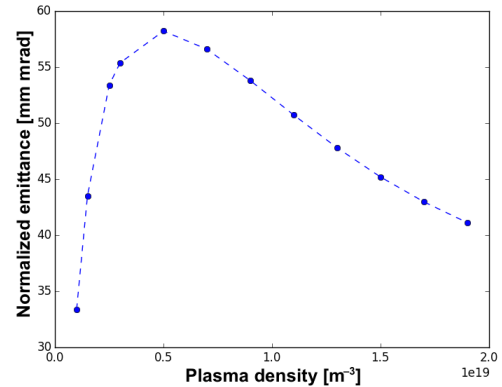


Figure 6: Variation of final normalized RMS emittance with plasma density, with linear interpolation.

## CONCLUSION

Simulations show that a plasma lens using the VELA beam would be capable of demonstrating significant demagnification and would allow the investigation of the effects of the longitudinal aberration. Further studies using higher density beams (e.g. the CLARA beam) will allow investigation of the underdense regime and will give more freedom to choose parameters to minimize emittance growth and maximize focusing strength.

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