Entanglement of Universe

Shintaro Sawayama[1]

[1] *Sawayama cram school of physics, Atsuhara 328,Fuji-shi,Shizuoka-ken, 419-0201, Japan*

Abstract

By the recent study of the quantum gravity, we can solved the inhomogeneous spacetime. And we quantized the enlarged Bianch I universe. As a result of quantization, we know the universe are entangled. The x,y, and z direction are entangled. So if we measure the x direction we know the information of the y and z directions.

1 Introduction

In the quantum gravity there are many difficulties i.e. problem of the time and problem of the norm and interpretation problem and problem of the inhomogeneous spacetime. One of the main difficulty comes form the Wheeler-DeWitt equation. The difficulty of the Wheeler-DeWitt still remain the Hamiltonian constraint of the loop quantum gravity. So we should remove the difficulty of the Wheeler-DeWitt equation.

So we construct the one method which called the up-to-down method. We explain what is the up-to-down method simply. Once we add an additional dimension to the usual 4-dimensional gravity. And we remove the additional dimension by using the problem of the time inversely. Then there appear the additional constraint equation. This is the up-to-down methodl.

The up-to-down method comes from the Isham's idea, that is quantum category. Imagine many world interpretation, and we assume may world is the infinitely many brane. We quantize the many brane at the same time.

In section II we write the result of the up-to-down method. In section III we quantize the enlarged Bianch I universe. And in section IV we discuss and conclude the presentation.

2 Up-to-down method

We skip the up-to-down method, because it is long. We cite the paper [5]. We start from 5-dimensional spacetime. There appear one additional constraint equation as,

$$\hat{H}_S \to -m\hat{H}_S := -\hat{K}^2 + \hat{K}^{ab}\hat{K}_{ab} - \frac{1}{2}\dot{P}. \tag{1}$$

Here, $\hat{H}_S$ is the Hamiltonian constraint of the 4+1 decomposition, and $\hat{K}_{ab}$ is the 4+1 extrinsic curvature and $\hat{K}$ is the it's trace and P is the 3+1 momentum. Now 4+1 decomposition is carried on the additional dimension s. This is the not usual decomposition. However, because we use the problem of the time inversely, the s direction vanished. We only consider the 4-dimensional quantum gravity. We obtain the next theorem.

Theorem 1. In this method, in the $\mathcal{H}_4$ additional constraint $m\hat{H}_S\Pi^3 = 0$ appears which we call static restriction, if there is no time evolution and the projection is defined by the difinition 3.

Here, Π^3 is the projection defined by $\Pi^3|g \quad\rangle = |g_s \quad = \text{const} g_{ij}\rangle$.

For example we first consider the Freedman universe as

$$g_{ab} := \begin{pmatrix} b & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix}. \tag{2}$$

[1] E-mail:sawayama0410@gmail.com

The modified Hamiltonian constraint becomes as

$$m\hat{H}_S = 6ab\frac{\partial}{\partial a}\frac{\partial}{\partial b} + a^2\frac{\partial^2}{\partial a^2} \tag{3}$$

$$= 6\frac{\partial}{\partial \eta}\frac{\partial}{\partial \eta_b} + \frac{\partial^2}{\partial \eta^2} = 0, \tag{4}$$

And the Hamiltonian constraint becomes as well known,

$$H_S = \frac{9}{2}a^2\frac{\partial}{\partial a^2} + \Lambda \tag{5}$$

$$= \frac{9}{2}\frac{\partial}{\partial \eta^2} + \Lambda = 0. \tag{6}$$

The 4-dimensional state becomes as

$$|\Psi^4(\eta)\rangle = \exp(i\frac{\sqrt{2}}{3}\Lambda^{1/2}\eta). \tag{7}$$

And it's enlargement of 4+1 is

$$|\Psi^{5(4)}(\eta_b, \eta)\rangle = \exp\bigg(-i\frac{\sqrt{2}}{3}\Lambda^{1/2}\eta_b\bigg)\exp(i\frac{\sqrt{2}}{3}\Lambda^{1/2}\eta). \tag{8}$$

We can proof the one of the universe are appropriated to the up-to-down method. There is one enlargement and measure of the projection is not zero.

Secondary, we think about the off-diagonal metric case.

$$g_{ab} = \begin{pmatrix} -c & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix}. \tag{9}$$

The modified Hamiltonian constraint become,

$$m\hat{H}_S = -6ac\frac{\partial}{\partial a}\frac{\partial}{\partial c} + (6a^2 - 2b^2)\frac{\partial^2}{\partial a^2} + 2(b^2 - a^2)\frac{\partial^2}{\partial b^2} = 0, \tag{10}$$

And the Hamiltonian constraint becomes as,

$$H_S = -(5a^2 - b^2)\frac{\partial}{\partial a^2} - (2b^2 - a^2)\frac{\partial^2}{\partial b^2} = 0. \tag{11}$$

This Hamiltonian constraint can not solved simply because this is the elliptic differential equation. However, if we use additional constraint as,

$$m\hat{H}_S\Pi^3 = (6a^2 - 2b^2)\frac{\partial^2}{\partial a^2} + 2(b^2 - a^2)\frac{\partial^2}{\partial b^2} = 0, \tag{12}$$

The Hamiltonian constraint can be solved.

$$|\Psi^{4(5)}(a, b)\rangle = E_1ab + E_2a + E_3b + E_4. \tag{13}$$

And the enlargement is

$$|\Psi^{5(4)}(a, b, c)\rangle = F_1ab + F_2a + F_3bc + F_4c + F_5. \tag{14}$$

This enlargement has also non zero measure of the projection.

Next we consider the Schwarzchild black holes as

$$g_{ab} = \begin{pmatrix} -f & 0 & 0 & 0 \\ 0 & f^{-1} & 0 & \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A\sin^2\theta \end{pmatrix}. \tag{15}$$

The Hamiltonian constraint becomes as

$$H_S = \frac{1}{2}\Big(- f^2 \frac{\partial^2}{\partial f^2} + Af \frac{\partial}{\partial f}\frac{\partial}{\partial A} - 2A^2 \frac{\partial^2}{\partial A^2}\Big) + \mathcal{R} = 0, \tag{16}$$

Usually this Hamiltonian constraint does not solved. However, if we think the additional constraint equation as,

$$m\hat{H}_S \Pi^3 = -2fA \frac{\partial}{\partial f}\frac{\partial}{\partial A} + A^2 \frac{\partial^2}{\partial A^2} \tag{17}$$

$$= A\frac{\partial}{\partial A}\Big(- 2f\frac{\partial}{\partial f} + A\frac{\partial}{\partial A}\Big) = 0, \tag{18}$$

we can solve the Hamiltonian constraint equation. Although the Hamiltonian and the additional constraint equation does not commute, if we make additional constraint parameter relations, we can solve the Hamiltonian constraint. The Hamiltonian constraint become simple second order ordinal functional equations.

$$H_S = -\frac{7}{8}A^2 \frac{\partial^2}{\partial A^2} + \frac{1 - c^2 A^2}{2A} = 0. \tag{19}$$

And this equation is solved easily by numerical method. The additional constraint or the enlargement is given by next formula.

$$m\hat{H}_S = -f^2 \frac{\partial^2}{\partial f^2} + \frac{4}{7}\frac{c - cf}{f^{1/2}} = 0. \tag{20}$$

In this case the enlargement be able and measure of the projection does not zero.

3 Enlarged Bianchi I universe

Now we consider the metric diagonal case. In this case the Hamiltonian constraint becomes as,

$$\mathcal{H}_S = \sum_{ij} \frac{\delta^2}{\delta\phi_i \delta\phi_j} + \sum_{i\neq j} (\phi_{i,jj} + \phi_{i,i}\phi_{j,i}) e^{\ i}. \tag{21}$$

Here, $g_{ii} = e^{\ i}$. And the commutation relation of the additional and the Hamiltonian constraint becomes as

$$[m\hat{H}_S, \mathcal{H}_4] = \delta^2 \sum_{i\neq j}(\phi_{i,ii} + \phi_{i,i}^2 + \frac{1}{2}\phi_{i,i}\phi_{j,i}) e^{\ i} - \frac{1}{2}\delta^2 \sum_{i\neq j} \phi_{i,0}(\phi_{j,ii} - 2\phi_{i,ij} - \phi_{i,j}\phi_{i,i}) e^{\ i}$$
$$-\frac{1}{2}\delta^2 \sum_{i\neq j}(\phi_{i,0jj} + \phi_{i,0j}\phi_{j,j} - \phi_{i,0ji} - \phi_{i,0i}\phi_{i,j}) e^{\ i}. \tag{22}$$

We know there are two case whose commutation relation is always zero. Such spacetime is following,

$$\begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & q_{11}(x_1) & 0 & 0 \\ 0 & 0 & q_{22}(x_2) & 0 \\ 0 & 0 & 0 & q_{33}(x_3) \end{pmatrix}. \tag{23}$$

and

$$\begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & q_{11}(x_2, x_3) & 0 & 0 \\ 0 & 0 & q_{22}(x_1, x_3) & 0 \\ 0 & 0 & 0 & q_{33}(x_1, x_2) \end{pmatrix}. \tag{24}$$

If we treat first metric, the Hamiltonian constraint become,

$$\mathcal{H}_S = \sum_i \frac{\delta^2}{\delta\phi_i^2} = 0. \tag{25}$$

And the additional constraint with cosmological constant becomes as

$$\mathcal{S} = \sum_{i \neq j} \frac{\delta^2}{\delta\phi_i \delta\phi_j} = \Lambda \tag{26}$$

The solution of the usual Hamilton constraint is,

$$|\Psi^4(\phi)\rangle = \prod \exp(a_i \Lambda^{1/2} \int \phi_i \delta\phi_i(x_i)). \tag{27}$$

In the a_i there are two equation as

$$a_1 a_2 + a_1 a_3 + a_2 a_3 = 1 \tag{28}$$
$$a_1^2 + a_2^2 + a_3^2 = 1. \tag{29}$$

Because there are two equation and three parameter, there are infinitely many basis. However, if we assume the one of the norm of a_i is one, we can obtain 12 basis as follows

$$a_1 = \pm\sqrt{3}, a_2 = \pm i, a_3 = \mp i \tag{30}$$
$$a_1 = \pm i, a_2 = \pm\sqrt{3}, a_3 = \mp i \tag{31}$$
$$a_1 = \pm i, a_2 = \mp i, a_3 = \pm\sqrt{3}. \tag{32}$$

Looking these basis we can easily know that the state is entangled.

4 Conclusion and Discussions

If we only solve the usual Bianch I universe, the spacetime are entangled. The difference of the enlarged Bianch I and usual Bianch I universe are eta square or only eta. Because three direction are entangled, by the measurement of the x direction, we know y direction and z direction. And we justify the up-to-down method. the measure of the all the projection treated here has non zero measure. In this case we only consider the cosmological constant. But we can consider the Schrodinger or the Krein-Gordon field instead of the cosmological constant. Then we can know the information of the spacetime by the measurement of the field. We can answer the philosophical problem " what is existence " by the physics.

References

[1] C.J.Isham and A.Ashtekar Class. Quant. Grav. **9** 1433 (1992)

[2] C.Rovelli Quantum Gravity; Cambridge monographs on mathematical physics (2004)

[3] T.Thiemann gr-qc/0110034 (2001)

[4] C.J.Isham Adv.Theor.Math.Phys. 7 331 (2003)

[5] S.Sawayama gr-qc/0604007 (2006)

[6] B.S.DeWitt Phys. Rev. **160** 1113 (1967)

[7] J.J.Halliwell and J.B.Hartle Phys. Rev. **D 43** 1170 (1991)

[8] Quantum computation and quantum information, M.A.Nielsen and I.L.Chang, Cambridge, (2000)

[9] A.Hosoya, privet discussions