Renormalization of the baryon axial vector current in large-N_c chiral perturbation theory

Rubén Flores-Mendieta[1], María A. Hernández Ruíz[2], and Christoph P. Hofmann[3]

[1] Instituto de Física, Universidad Autónoma de San Luis Potosí,
Álvaro Obregón 64, Zona Centro, San Luis Potosí, San Luis Potosi 78000, México.
[2]Facultad de Ciencias Químicas, Universidad Autónoma de Zacatecas,
Apartado Postal 585, Zacatecas, Zacatecas 98060, México.
[3] Facultad de Ciencias, Universidad de Colima,
Bernal Díaz del Castillo 340, Colima, Colima 28045, México.

E-mail: mahernan@uaz.edu.mx

Abstract. The baryon axial vector current is considered within the combined framework of large-N_c baryon chiral perturbation theory (where N_c is the number of colors) and the baryon axial vector couplings are extracted. Loop graphs with octet and decuplet intermediate states are systematically incorporated into the analysis.

1. Introduction

The theory of the strong interactions is quantum chromodynamics (QCD). Different methods have been used to extract low-energy consequences of QCD. In this work, we use a combined expansion in m_q (where m_q is the quark mass) and $1/N_c$ [1]. The $1/N_c$ chiral effective Lagrangian for the lowest-lying baryons was constructed in Ref. [2].

On the one hand, chiral perturbation theory exploits the symmetry of the QCD Lagrangian under $SU(3)_L \times SU(3)_R \times U(1)_V$ transformations on the three flavors of light quarks u, d and s in the limit that the quark masses m_u, m_d and m_s vanish. Chiral symmetry is spontaneously broken to the vector subgroup $SU(3) \times U(1)_V$ by the QCD vacuum, giving rise to the octet of pseudoscalar Goldstone bosons (π, K and η). When chiral perturbation theory is extended to include baryons, it is convenient to introduce velocity-dependent baryon fields [3], so that the expansion of the baryon chiral Lagrangian in powers of m_q and $1/M_B$ (where M_B is the baryon mass) is manifest. This is the so-called heavy baryon chiral perturbation theory [3]. The inclusion of decuplet baryon intermediate states yields sizable cancellations between one loop corrections [4]. This phenomenological observation can be explained in the context of the $1/N_c$ expansion.

On the other hand, large-N_c QCD is the $SU(N_c)$ gauge theory of quarks and gluons where the number of colors, N_c, is a parameter of the theory [2]. Large-N_c is the generalization of QCD from $N_c = 3$ to $N_c \gg 3$ colors. A spin-flavor symmetry emerges for baryons in the large-N_c limit and can be used to classify large-N_c baryon states and matrix elements [2], which has led to remarkable insights into the understanding of the nonperturbative QCD dynamics of hadrons.

In particular, in this work we will describe the baryon axial-vector couplings, and as a result we obtain corrections at relative orders $1/N_c$ and $1/N_c^2$.

2. The chiral Lagrangian for baryons in the $1/N_c$ expansion

The $1/N_c$ chiral Lagrangian for baryons reads [2]

$$\mathcal{L}_{\text{baryon}} = i\mathcal{D}^0 - \mathcal{M}_h + Tr(\mathcal{A}^k\lambda^c)A^{kc}\frac{1}{N_c}Tr\left(\mathcal{A}^k\frac{2I}{\sqrt{6}}\right)A^k + \ldots \tag{1}$$

where

$$\mathcal{D}^0 = \partial^0 1 + Tr(\mathcal{V}^0\lambda^c)T^c. \tag{2}$$

Each term in Eq. (1) involves a baryon operator which can be expressed as a polynomial in the SU(6) spin-flavor generators [2]

$$J^k = q^\dagger\frac{\sigma^k}{2}q, \qquad T^c = q^\dagger\frac{\lambda^c}{2}q, \qquad G^{kc} = q^\dagger\frac{\sigma^i}{2}\frac{\lambda^a}{2}q, \tag{3}$$

where $q^\dagger$ and q are $SU(6)$ operators that create and annihilate states in the fundamental representation of $SU(6)$, and σ^k and λ^c are the Pauli spin and Gell-Mann flavor matrices, respectively.

The baryon operator $\mathcal{M}_h$ denotes the spin splittings of the tower of baryon states with spins $1/2, \ldots, N_c/2$ in the flavor representations. Furthermore, the vector and axial vector combinations of the meson fields,

$$\mathcal{V}^0 = \frac{1}{2}(\xi\partial^0\xi^\dagger + \xi^\dagger\partial^0\xi), \qquad \mathcal{A}^k = \frac{i}{2}(\xi\nabla^k\xi^\dagger - \xi^\dagger\nabla^k, \xi),$$

couple to baryon vector and axial vector currents, respectively. Here $\xi = \exp[i\Pi(x)/f]$, where $\Pi(x)$ stands for the nonet of Goldstone boson fields and $f \approx 93$ MeV is the meson decay constant.

The QCD operators involved in $\mathcal{L}_{baryon}$ in Eq. (1) have well-defined $1/N_c$ expansions. Specifically, the baryon axial vector current A^{kc} is a spin-1 object, an octet under $SU(3)$, and odd under time reversal. Its $1/N_c$ expansion reads

$$A^{kc} = a_1 G^{kc} + \sum_{n=2,3}^{N_c} b_n\frac{1}{N_c^{n-1}}\mathcal{D}_n^{kc} + \sum_{n=3,5}^{N_c} c_n\frac{1}{N_c^{n-1}}\mathcal{O}_n^{kc}, \tag{4}$$

where the unknown coefficients a_1, b_n, and c_n have expansions in powers of $1/N_c$ and are order unity at leading order in the $1/N_c$ expansion. The first few operators in expansion (4) are

$$\mathcal{D}_2^{kc} = J^k T^c, \tag{5}$$

$$\mathcal{D}_3^{kc} = \{J^k, \{J^r, G^{rc}\}\}, \tag{6}$$

$$\mathcal{O}_3^{kc} = \{J^2, G^{kc}\} - \frac{1}{2}\{J^k, \{J^r, G^{rc}\}\}, \tag{7}$$

while higher order terms can be obtained as $\mathcal{D}_n^{kc} = \{J^2, \mathcal{D}_{n-2}^{kc}\}$ and $\mathcal{O}_n^{kc} = \{J^2, \mathcal{O}_{n-2}^{kc}\}$ for $n \geq 4$. Notice that $\mathcal{D}_n^{kc}$ are diagonal operators with non-zero matrix elements only between states with the same spin, and the $\mathcal{O}_n^{kc}$ are purely off-diagonal operators with non-zero matrix elements only between states with different spin. At $N_c = 3$ the series (4) can be truncated as

$$A^{kc} = a_1 G^{kc} + b_2\frac{1}{N_c}\mathcal{D}_2^{kc} + b_3\frac{1}{N_c^2}\mathcal{D}_3^{kc} + c_3\frac{1}{N_c^2}\mathcal{O}_3^{kc}. \tag{8}$$

The matrix elements of the space components of A^{kc} between SU(6) symmetric states yield the values of the axial vector couplings. For the octet baryons, the axial vector couplings are g_A, as defined in experiments in baryon semileptonic decays.

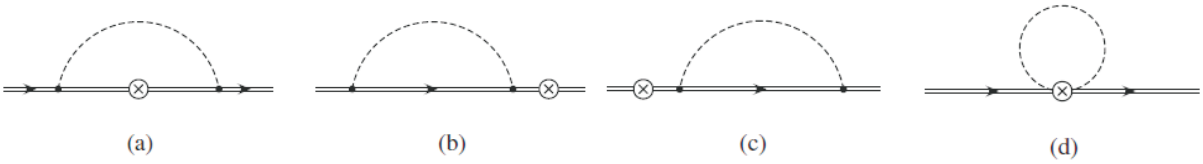

Figure 1. One-loop corrections to the baryon axial vector current.

3. Renormalization of the baryon axial vector current

The baryon axial vector current A^{kc} is renormalized by the one-loop diagrams displayed in Fig. 1. These loop graphs have a calculable dependence on the ratio Δ/m_Π, where $\Delta \equiv M_\Delta - M_N$ is the decuplet-octet mass difference and m_Π is the meson mass.

The correction arising from the sum of the diagrams of Figs. 1(a)-1(c), containing the full dependence on the ratio Δ/m_Π, reads [4]

$$\begin{aligned}\delta A^{kc} &= \frac{1}{2}\left[A^{ja},\left[A^{jb},A^{kc}\right]\right]\Pi^{ab}_{(1)} - -\frac{1}{2}\{A^{ja},\left[A^{kc},\left[\mathcal{M},A^{jb}\right]\right]\}\Pi^{ab}_{(2)} \\ &+ \frac{1}{6}\left(\left[A^{ja},\left[\left[\mathcal{M},\left[\mathcal{M},A^{jb}\right]\right],A^{kc}\right]\right] - \frac{1}{2}\left[\left[\mathcal{M},A^{ja}\right],\left[\left[\mathcal{M},A^{jb}\right],A^{kc}\right]\right]\right)\Pi^{ab}_{(3)} + \ldots\end{aligned}$$

Here $\Pi^{ab}_{(n)}$ is a symmetric tensor which contains meson-loop integrals with the exchange of a single meson: A meson of flavor a is emitted and a meson of flavor b is reabsorbed. $\Pi^{ab}_{(n)}$ descomposes into flavor singlet, flavor **8** and flavor **27** representations

$$\Pi^{ab}_{(n)} = F^{(n)}_{\mathbf{1}}\delta^{ab} + F^{(n)}_{\mathbf{8}}d^{ab8} + F^{(n)}_{\mathbf{27}}\left[\delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888}\right], \tag{9}$$

where

$$\begin{aligned}F^{(n)}_{\mathbf{1}} &= \frac{1}{8}\left[3F^{(n)}(m_\pi,0,\mu) + 4F^{(n)}(m_K,0,\mu) + F^{(n)}(m_\eta,0,\mu)\right], \\ F^{(n)}_{\mathbf{8}} &= \frac{2\sqrt{3}}{5}\left[\frac{3}{2}F^{(n)}(m_\pi,0,\mu) - F^{(n)}(m_K,0,\mu) - \frac{1}{2}F^{(n)}(m_\eta,0,\mu)\right], \\ F^{(n)}_{\mathbf{27}} &= \frac{1}{3}F^{(n)}(m_\pi,0,\mu) - \frac{4}{3}F^{(n)}(m_K,0,\mu) + F^{(n)}(m_\eta,0,\mu).\end{aligned}$$

Explicit expressions for the general function $F^{(n)}(m_\Pi,\Delta,\mu)$, defined by

$$F^{(n)}(m_\Pi,\Delta,\mu) \equiv \frac{\partial^n F(m_\Pi,\Delta,\mu)}{\partial\delta^n}, \tag{10}$$

can be found in Ref. [5]

4. Results and Conclusions

The analysis was performed at one-loop order, where the corrections to the baryon axial vector coupling arise at relative orders $1/N_c$, $1/N_c^2$, and so on, which is precisely the origin of the $1/N_c$ expansion. The predicted values for g_A are listed in Table 1. Our final results referring to the degeneracy limit have been analyzed in Ref. [1, 5].

Table 1. Relative orders $1/N_c$ to the coupling constants g_A.

			Figs. 1(a-d)					
			1		**8**		**27**	
Process	Total	Tree	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$
$n \to pe^-\bar{\nu}_e$	1.275	1.238	0.480	−0.549	−0.181	0.278	−0.001	0.009
$\Sigma^\pm \to \Lambda e^+\nu_e$	0.623	0.661	0.279	−0.319	−0.040	0.047	−0.004	0
$\Lambda \to pe^-\bar{\nu}_e$	−0.899	−0.855	−0.317	0.360	−0.007	−0.089	0.005	0.005
$\Sigma^- \to ne^-\bar{\nu}_e$	0.345	0.381	0.457	−0.488	−0.005	−0.002	−0.002	0.004
$\Xi^- \to \Lambda e^-\bar{\nu}_e$	0.225	0.194	0.062	−0.064	0.032	0.010	−0.007	−0.002
$\Xi^- \to \Sigma^0 e^-\bar{\nu}_e$	0.795	0.875	0.338	−0.387	0.064	−0.098	−0.006	0.008
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e$	1.124	1.238	0.480	−0.549	0.091	−0.139	0.013	0.020

Table 1 shows the numerical values of the g_A axial vector couplings for various semileptonic processes in the $1/N_c$ expansion, individually for the flavor singlet **1**, octet **8**, and **27** contributions. The singlet corrections are $1/N_c$ suppressed with respect to the tree-level value. Subsequent suppressions of the octet and **27** contributions are also noticeable. The results are perfectly consistent both with the expectations from the $1/N_c$ expansion and the experimental data.

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