

# Flavored Dark Matter and its Laboratory Tests

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## Abstract.

In this talk, based on the paper 1109.3516, we consider theories where the dark matter particle carries lepton flavor quantum numbers, and has renormalizable contact interactions with the Standard Model fields. We find that the region of parameter space where dark matter has the right abundance to be a thermal relic is in general within reach of current direct detection experiments. In order to evaluate the collider prospects, we focus on a class of models where dark matter carries tau flavor, and show that the collider signals of these models include events with four or more isolated leptons and missing energy. A significant part of parameter space in these theories can be discovered above Standard Model backgrounds at the 14 TeV Large Hadron Collider. We also study the extent to which flavor and charge correlations among the final state leptons allows models of this type to be distinguished from theories where dark matter couples to leptons but does not carry flavor, similarly to a neutralino.

## 1. Introduction

It is now well established that about 80% of the matter in the universe is dark [1]. However, the masses and interactions of the particles of which dark matter is composed are not known. One simple and well-motivated possibility is that dark matter is made up of particles with masses close to the weak scale that have weak scale annihilation cross section to Standard Model (SM) particles. Dark matter candidates with these properties neatly fit into the ‘Weakly Interacting Massive Particle’ (WIMP) paradigm, and therefore naturally tend to have the right relic abundance to explain observations.

The matter fields ( $Q, U^c, D^c, L, E^c$ ) of the SM each come in three copies, or flavors, that differ only in their masses. This reflects the fact that the Lagrangian of the SM possesses an approximate  $U(3)^5$  flavor symmetry acting on the matter fields, which is explicitly broken by the Yukawa couplings that generate the quark and lepton masses. An interesting possibility is that the dark matter field, which we label by  $\chi$ , also carries flavor quantum numbers, with the physical dark matter particle being the lightest of three copies. Several specific dark matter candidates of this type have been studied extensively in the literature, including sneutrino dark matter [2, 3] in the Minimal Supersymmetric Standard Model (MSSM), and Kaluza-Klein (KK) neutrino dark matter [4] in extra dimensional models.

To incorporate three flavors of the dark matter field, the flavor symmetry of the SM is extended from  $U(3)^5$  to  $U(3)^5 \times U(3)_\chi$ , if  $\chi$  is a complex field such as a complex scalar, Dirac fermion or complex vector boson. If instead  $\chi$  is a real field, such as a real scalar, Majorana

fermion or real vector boson the flavor symmetry is extended from  $U(3)^5$  to  $U(3)^5 \times O(3)_\chi$ . The new flavor symmetry  $U(3)_\chi$  (or  $O(3)_\chi$ ) may be exact, or it may be explicitly broken as in the SM.

Our focus here will be on theories where dark matter has renormalizable contact interactions with the SM fields, which, by analogy with the SM Yukawa couplings, could represent an explicit breaking of the flavor symmetry. Then, if the SM matter field that  $\chi$  couples to is a lepton, there is an association between the different dark matter flavors and lepton flavors. Accordingly, we refer to this scenario as ‘lepton flavored dark matter’. In this scenario the fact that the SM flavor symmetries are not exact naturally results in a splitting of the states in the dark matter multiplet, the physical dark matter particle being identified with the lightest. A more extensive discussion including the case where dark matter carries quark flavor or an internal flavor can be found in [5].

## 2. Lepton Flavored Dark Matter

The lepton sector of the SM has a  $U(3)_L \times U(3)_E$  flavor symmetry, where  $U(3)_L$  acts on the  $SU(2)$  doublet leptons and  $U(3)_E$  on the singlets. This symmetry is explicitly broken down to  $U(1)^3$  by the Yukawa interactions that give the charged leptons their masses. (We neglect the tiny neutrino masses, which also break the symmetry).

The characteristic vertex of lepton flavored dark matter involves contact interactions between  $\chi$  and the SM leptons. For concreteness, in what follows we take  $\chi$  to be a Dirac fermion and  $\phi$  to be a complex scalar, and restrict our focus to the case where  $\chi$  couples to the  $SU(2)$  singlet lepton field  $E^c$ , such that the corresponding term in the Lagrangian is

$$\lambda_\alpha^i \chi^\alpha E_i^c \phi + \text{h.c.}, \quad (1)$$

where  $i$  is a  $U(3)_E$  flavor index and  $\alpha$  is a  $U(3)_\chi$  flavor index.

### 2.1. Flavor Structure

In general the matrix  $\lambda$  will contain both diagonal and off-diagonal elements, thereby giving rise to lepton flavor violation. The experimental bounds on such processes are satisfied if all the elements in  $\lambda < 10^{-3}$  for  $m_\phi \sim 200$  GeV, even in the absence of any special flavor structure. Unfortunately, however, if  $\chi$  is a SM singlet and has no sizable couplings beyond those in Eq. (1), couplings of this size are by themselves too small to generate the correct abundance for  $\chi$ , if it is to be a thermal relic, except if they are aligned with the lepton Yukawa couplings to avoid flavor bounds.

The matrix  $\lambda$  can naturally be aligned with the SM Yukawa couplings if this interaction is consistent with minimal flavor violation (MFV) [6]. In this scenario, the dark matter flavor symmetry  $U(3)_\chi$  is identified with either  $U(3)_E$  or  $U(3)_L$  of the SM, and the matrix  $\lambda$  respects these symmetries up to effects arising from the SM Yukawa couplings.

If we write the lepton Yukawa couplings of the SM as  $y_A^i L^A E_i^c H + \text{h.c.}$ , then the Yukawa matrix  $y_A^i$  can be thought of as a spurion transforming as  $(3, \bar{3})$  under the  $SU(3)_L \times SU(3)_E$  subgroup of  $U(3)_L \times U(3)_E$ . Consider the case where  $U(3)_\chi$  is identified with  $U(3)_L$ . Then

$$\lambda_\alpha^i \chi^\alpha E_i^c \phi + \text{h.c.} \rightarrow \lambda_A^i \chi^A E_i^c \phi + \text{h.c.} \quad (2)$$

MFV restricts the matrix  $\lambda$  to be of the form  $\lambda_A^i = \alpha y_A^i$ , where we are working only to the leading non-trivial order in an expansion in the SM Yukawa couplings. The dark matter mass term now takes the form

$$[m_\chi]_A^B = \left( m_0 \mathbf{1} + \Delta m y y^\dagger \right)_A^B. \quad (3)$$

We see that in this case the three dark matter flavors are close in mass, but their couplings to the SM fields, though still flavor-diagonal, are hierarchical. In particular, if the relic abundance is determined by  $\lambda$ , we expect that only the tau flavor can constitute dark matter, since the couplings of the other flavors are relatively small. The other case where  $U(3)_\chi$  is identified with  $U(3)_E$  can be worked out in a similar fashion [5].

### 2.2. Relic Abundance

If  $\chi$  is a thermal WIMP, its relic abundance is set by its annihilation rate to SM fields. If  $\chi$  is a SM singlet, and its only interactions are those of Eq. (1), then the primary annihilation mode is through  $t$ -channel  $\phi$  exchange to two leptons. In the relevant parameter space, the matrix  $\lambda$  is constrained by flavor bounds to be very nearly flavor diagonal, so that lightest state in the dark matter multiplet is associated with a specific lepton flavor. We assume that the splittings between the different states in this multiplet are large enough so that only the lightest state is stable on cosmological timescales, and constitutes all of dark matter.

Since the dark matter particle is non-relativistic at freeze-out, annihilation is dominated by the lowest partial wave. In this limit

$$\langle\sigma v\rangle = \frac{\lambda^4 m_\chi^2}{32\pi(m_\chi^2 + m_\phi^2)^2}, \quad (4)$$

where we have assumed that  $m_\chi \gg m_\ell$ , so that the masses of the final state leptons can be neglected.

### 2.3. Direct Detection

The direct detection signals of this class of theories depend on whether the dark matter particle  $\chi$  transforms non-trivially under the SM  $SU(2)$  gauge symmetry, or remains a SM singlet. If  $\chi$  is a SM singlet, the leading contribution to dark matter scattering off a nucleus arises from the (triangle) loop diagrams involving leptons and a photon exchange.

In the region of parameter space of interest to current direct detection experiments, bounds on lepton flavor violating processes constrain the coupling matrix  $\lambda$  to be flavor diagonal. Therefore the dark matter candidate carries the flavor of the lepton it couples to. This coupling gives rise to three distinct types of interactions between dark matter and the nucleus, specifically a charge-charge coupling, a dipole-charge coupling, and a dipole-dipole coupling.

The dipole-charge interaction is sub-dominant to the charge-charge interaction. The dipole-dipole coupling, being spin-dependent, is also sub-dominant. Consequently, we use the charge-charge cross section to derive limits. We find

$$\sigma_{ZZ}^0 = \frac{\mu^2 Z^2}{\pi} \left[ \frac{\lambda^2 e^2}{64\pi^2 m_\phi^2} \left[ 1 + \frac{2}{3} \log \left( \frac{m_\ell^2}{m_\phi^2} \right) \right] \right]^2. \quad (5)$$

Here  $Z$  is the total charge of the nucleus,  $\sigma_0$  is the cross section at zero-momentum transfer, and  $\mu$  is the reduced mass of the dark matter-nucleus system.

The ratio  $\lambda/m_\phi$  corresponding to a thermal WIMP is plotted in Fig. 1 as a function of the dark matter mass, for the tau flavored and electron flavored cases. The current limits from the Xenon100 experiment [7] are also shown. It is clear from the figure that the expected improvement in sensitivity of the experiment by an order of magnitude will bring a large part of the parameter space of these models within reach.

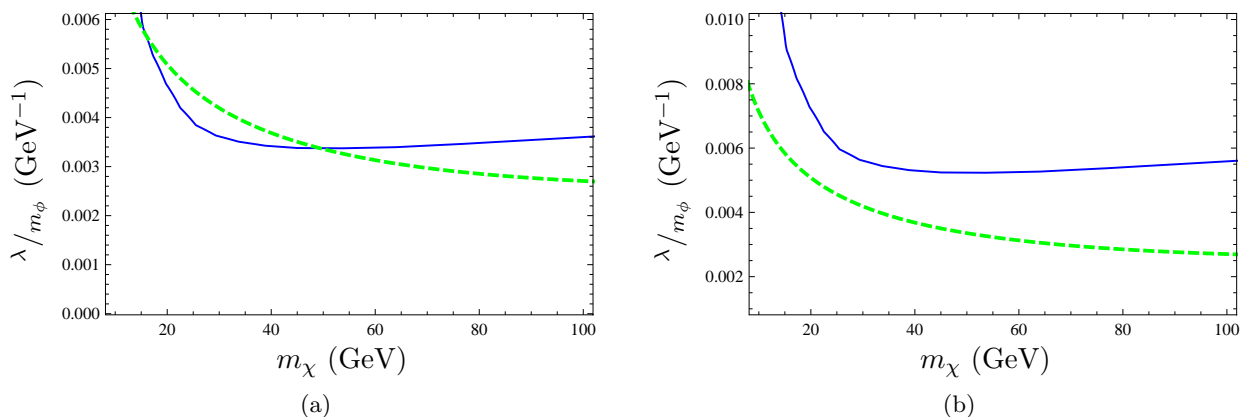


Figure 1: Direct detection and relic abundance constraints on lepton flavor dark matter for a)  $\chi_e$  and b)  $\chi_\tau$ , when  $m_\phi = 150$  GeV. The area above the solid blue curve is ruled out by the new Xenon100 [7] data. The green dashed curves signify the parameters for which we obtain correct relic abundance.

### 3. Collider Signals of Tau Flavored Dark Matter

In this section we discuss in detail the collider signals of a specific model where the dark matter particle carries tau flavor. The relevant part of the Lagrangian was given in Eq. (1). This interaction fixes the SM quantum numbers of  $\phi$ , which is charged under the photon and the  $Z$ , but does not couple to the  $W$ .

For concreteness, we consider two benchmark spectra that are consistent with MFV, with  $\chi$  transforming under  $U(3)_E$ . Then  $\chi_e$  and  $\chi_\mu$  are expected to be nearly degenerate since the corresponding SM Yukawa couplings are very small. We assume that  $\chi_\tau$  is lighter than  $\chi_e$  or  $\chi_\mu$ , and constitutes dark matter.

We label the first benchmark spectrum  $\tau$ FDM1, where  $m_{\chi,e} = m_{\chi,\mu} = 110$  GeV,  $m_{\chi,\tau} = 90$  GeV and  $m_\phi = 160$  GeV. The second benchmark spectrum we study has a lighter mediator, and therefore leads to a larger production cross section. We label this benchmark spectrum  $\tau$ FDM2, where  $m_{\chi,e} = m_{\chi,\mu} = 90$  GeV,  $m_{\chi,\tau} = 70$  GeV and  $m_\phi = 150$  GeV.

In these simple models, only the mediator  $\phi$  carries SM gauge quantum numbers, so dark matter events at colliders must arise from  $\phi^+\phi^-$  production. Since  $\phi^+\phi^-$  production proceeds through Drell-Yan and  $\phi$  is a scalar, the  $\phi$  pair comes out in a  $p$ -wave, leading to a small cross section, on the order of 10 fb at the 14 TeV LHC run. Therefore, we do not expect the early LHC data to be able to probe this model. In order to obtain reasonable signal over background discrimination, tens of inverse fb of data will be required.

#### 3.1. Signal topologies

Signal events come in three distinct topologies. Each  $\phi$  can decay directly into the dark matter particle and a  $\tau$ , corresponding to a short chain. Alternatively, it can decay to one of the heavier particles in the dark matter multiplet, which eventually cascades down to the dark matter particle, creating a long chain. Therefore each event can be categorized as comprising of short-short, short-long or long-long chains.

Since  $\tau$ 's are difficult to identify, we implicitly restrict ourselves to  $\ell = e, \mu$  final states in this section when we talk about leptons. The events with the long-long decay chain topology have four-lepton final states (not to mention a pair of  $\tau$ 's), which have small SM backgrounds, and therefore it is the most promising channel.

In order to simulate signal and background events we use the `usrmod` utility of

MadGraph/MadEvent [8, 9], and we use BRIDGE [10] for the  $\chi_{e,\mu}$  decays. Pythia [11] is used to simulate parton showers and hadronic physics, and PGS [12] with the default CMS parameter set is used to simulate detector effects.

### 3.2. Backgrounds

While four-lepton final states are rare in the SM, the signal cross section is also small so we carefully consider the three leading sources of backgrounds and devise cuts to reduce them as much as possible.

$(Z/\gamma)^*(Z/\gamma)^*$ : One of the dominant backgrounds is the production of two opposite-sign, same-flavor lepton pairs from either on-shell or off-shell  $Z$ 's and photons. Any missing energy in this background arises from mis-measurement of lepton momenta, which is small.

$t\bar{t}(Z/\gamma)^*$ : This background process, while it has a three-body final state, has a cross section comparable to the above process which is purely electroweak. When both tops decay leptonically and the  $(Z/\gamma)^*$  goes to leptons, the final state is  $4\ell$ +jets+MET.

$WW(Z/\gamma)^*$ : This process is qualitatively similar to the above process, but has a much smaller production cross section because it is purely electroweak.

*Backgrounds with fakes*: There are also backgrounds arising from jets that are misidentified as leptons. We find that provided the fake rates are of order  $10^{-3}$  or less, the irreducible backgrounds described above are the dominant ones.

### 3.3. Cuts

We use the following cut flow in order to maximize signal over background:

- Lepton cuts - We demand events with at least four leptons each with  $pT > 7$  GeV. At least two of these leptons are further required to have  $E > 50$  GeV.
- Dijet veto - We discard events with two or more jets of  $pT > 30$  GeV each.
- $Z$  veto - We veto events if the invariant mass of any  $Z$ -candidate (a pair of same-flavor and opposite-charge leptons) falls within 7 GeV of the  $Z$  mass. This is a tighter  $Z$ -veto than is usually used, but we find that the loss in signal efficiency is more than compensated for by the background reduction.
- Missing energy - We require at least 20 GeV of missing energy in each signal event. Since most backgrounds with high MET have already been eliminated by the previous cuts in the cut flow, we find that a mild threshold such as 20 GeV is sufficient.

### 3.4. Results

The signal and background events of each type that survive these cuts are listed in Table 1. These results show that it is possible to discover the  $\tau$ FDM2 benchmark above SM backgrounds at  $5\sigma$  significance with about  $20 \text{ fb}^{-1}$  of data at the 14 TeV LHC run. A higher luminosity ( $\sim 40 \text{ fb}^{-1}$ ) would be needed in order to distinguish the  $\tau$ FDM1 benchmark from the SM background.

While ATLAS and CMS have already performed searches in multilepton final states, considering the low cross section of the FDM benchmark model, they are not yet expected to have exclusion level sensitivity to this scenario.

## 4. Distinguishing $\tau$ FDM

Multi-lepton events with large missing energy are fairly common signals in theories with neutral stable particles and partners to the SM leptons, which include a variety of dark matter models. We would like to understand whether it is possible to distinguish  $\tau$ FDM from a specific 'strawman' model, where the dark matter does not carry flavor.

Dataset	Event rate after cuts at 100 fb <sup>-1</sup>			
	Lepton cuts	Jet cuts	Z veto	MET
$\tau$ FDM1	46.73	42.83	38.41	35.01
$\tau$ FDM2	75.39	69.30	63.26	57.04
$\ell^+\ell^-\ell^+\ell^-$	1617.94	1582.42	140.30	13.32
$t\bar{t}\ell^+\ell^-$	89.57	19.45	4.92	4.70
$WW\ell^+\ell^-$	14.70	13.98	2.51	2.51

Table 1: Signal and SM background event rates for processes yielding 4-lepton final states after each set of cuts is progressively applied (note that  $\ell = e, \mu$ ). All numbers are reported for the 14 TeV LHC run and include detector effects.

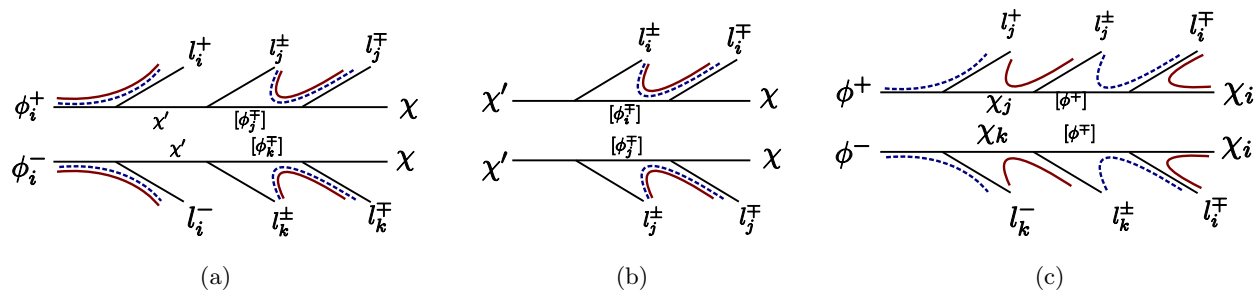


Figure 2: Flavor (red solid) and charge (blue dashed) correlations are shown for topologies in strawman models [(a) and (b)], and for  $\tau$ FDM (c).

The strawman model we choose is related to supersymmetric theories where the bino constitutes dark matter. The form of the lepton-slepton-bino vertex is very similar to the defining vertex of a theory of lepton FDM, except that in the supersymmetric case it is the slepton that carries flavor, not the bino. The strawman model we choose therefore consists of the bino, which we label by  $\chi$ , along with the three right-handed sleptons,  $\tilde{E}_i^c$ . The bino constitutes dark matter. To mimic the collider signals of  $\tau$ FDM, we add to the strawman model an additional ‘neutralino’  $\chi'$ , which is heavier than the slepton.  $\chi'$  is an admixture of a SM SU(2) doublet and singlet, so that it can be pair-produced through the Z, and is chosen to couple to leptons and sleptons in a flavor-blind way. This interaction takes the schematic form  $\lambda' E_i^c \chi' \tilde{E}_i^c + \text{h.c.}$

For simplicity, in what follows we assume that the three sleptons are degenerate in mass. In general,  $\chi'$  could either be lighter than or heavier than the sleptons, while the bino is the lightest of the new states. Both  $\chi'$  and  $\chi$  are taken to be Majorana fermions as is the case in the MSSM.

Signal events in the  $\tau$ FDM model involve four or more isolated leptons and missing energy. How does the strawman model generate similar events? The sleptons can be pair-produced in colliders. If they are heavier than  $\chi'$ , this leads to events of the form shown in Fig. 2(a), which involve six leptons, any or all of which could be taus. We label this possibility topology (a). Two  $\chi'$  particles can also be pair-produced, leading to events of the form shown in Fig. 2(b), which we label topology (b). These events involve four leptons, any or all of which could be taus.

How can we distinguish between signal events in the two classes of models? We focus here on charge and flavor correlations among the final state leptons in the event. In Fig. 2, we exhibit

the correlation of flavor and charge among the final state leptons in signal events for the  $\tau$ FDM model and the strawman model.

We generated fake signal events for the strawman model for three benchmark spectra.

### Spectrum 1

We assume the masses of the sleptons to be the same as that of the mediator  $\phi$  in  $\tau$ FDM. The masses of  $\chi'$  and  $\chi$  are also chosen equal to the  $\chi_{e,\mu}$  and  $\chi_\tau$  mass respectively. More specifically, we choose  $m_{\chi'} = 110$  GeV,  $m_\chi = 90$  GeV and  $m_{\tilde{e},\tilde{\mu},\tilde{\tau}} = 160$  GeV. Topology (a) in Fig. 2 dominates the phenomenology of this benchmark. In this topology, the two most upstream leptons are also the hardest, *and* are flavor-correlated. The  $\tau$ FDM leptons, as noted above, have no flavor correlation. Therefore, we expect that the flavor correlation of the two hardest leptons is a good discriminant in this case.

### Spectrum 2

If the mass of the sleptons is less than the mass of  $\chi'$ , then only the topology (b) is allowed. The decay of  $\chi'$  is on-shell in this case. The representative spectrum we study is  $m_{\chi'} = 160$  GeV,  $m_\chi = 90$  GeV and  $m_{\tilde{e},\tilde{\mu},\tilde{\tau}} = 110$  GeV. In this case, the hardest leptons should exhibit neither charge nor flavor correlations, allowing us to distinguish it from  $\tau$ FDM.

### Spectrum 3

In the intermediate case, however, the result is a mixture of the two topologies. In order to investigate this we study a third spectrum,  $m_{\chi'} = 140$  GeV,  $m_\chi = 90$  GeV and  $m_{\tilde{e},\tilde{\mu},\tilde{\tau}} = 160$  GeV. In the next section we study the extent to which each of these spectra can be distinguished from  $\tau$ FDM.

#### *4.1. Comparison*

The correlations we obtain are listed in the left panel of Fig. 3. The results are in agreement with our expectations. Events with topology (a) in Spectrum 1 clearly exhibit flavor correlation between the two hardest leptons, as expected for the upstream leptons created from (flavor-carrying) sleptons.  $\tau$ FDM, on the other hand exhibits no flavor correlation in the hardest two leptons.

In all the fake spectra with topology (b), the two hardest leptons show no preferential charge assignment beyond the ratio of 1 : 2 for same to opposite charge, as expected from random charge assignment. Consequently these cases have a weaker charge anti-correlation than the  $\tau$ FDM.

Events from topology (a) in Spectrum 3 fall in the middle, with somewhat significant charge anti-correlation, and a weak flavor correlation. While the correlation between charge and flavor is different from the  $\tau$ FDM case, higher statistics might be needed in this case to make a precise distinction.

In the right panel of Fig. 3, we plot the flavor and charge asymmetries of the two hardest leptons, along with statistical error contours for the  $\tau$ FDM1 model at the 14 TeV LHC run, with  $100 \text{ fb}^{-1}$  of integrated luminosity. The asymmetries for flavor and charge are defined as,

$$a_F, a_C = \frac{n_{\text{same}} - n_{\text{diff.}}}{n_{\text{same}} + n_{\text{diff.}}} \quad (6)$$

We see that with this amount of data,  $\tau$ FDM can be distinguished from the strawman models at  $2\sigma$ .

## **5. Conclusions**

In conclusion, we have studied the direct detection and collider prospects of theories where the dark matter particle carries lepton flavor quantum numbers, and has renormalizable contact interactions with the Standard Model fields. Assuming a coupling consistent with relic

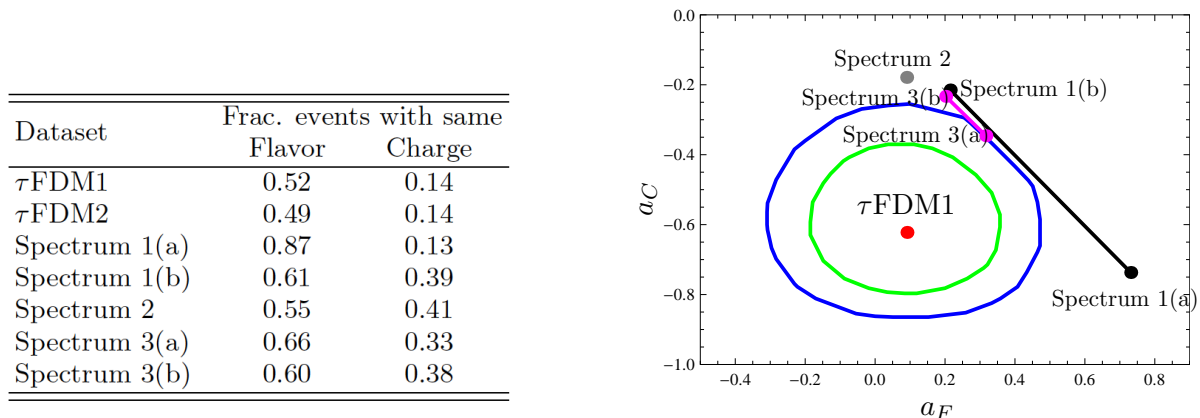


Figure 3: Left panel: Flavor and charge correlations for the two highest  $pT$  leptons in events passing cuts for different data samples. Right panel: Flavor and charge asymmetry for different models and event topologies. We show  $1\sigma$  (green) and  $2\sigma$  (blue) contours for the  $\tau$ FDM1 model at  $100 \text{ fb}^{-1}$ . The straight lines interpolate between points which correspond to different event topologies for each fake spectrum, in order to account for cases where both topologies contribute.

abundance considerations, we have shown that this scenario could be probed in the near future by upcoming direct detection experiments

We have then studied in detail a class of models where dark matter carries tau flavor, and the collider signals include events with four or more isolated leptons and missing energy. We have performed a full simulation of the signal and SM backgrounds, including detector effects, and shown that in a significant part of the parameter space favored by MFV, these theories can be discovered above SM backgrounds at the 14 TeV LHC run. We have also shown that flavor and charge correlations among the final state leptons may allow models of this type to be distinguished from simple theories where the dark matter particle couples to leptons but does not carry flavor.

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