Arguments against using h^{-1} Mpc units in observational cosmology

Ariel G. Sánchez[*]
Max-Planck-Institut für extraterrestrische Physik, Postfach 1312, Giessenbachstr., 85741 Garching, Germany

(Received 26 February 2020; revised 28 August 2020; accepted 4 November 2020; published 2 December 2020)

It is common to express cosmological measurements in units of h^{-1} Mpc. Here, we review some of the complications that originate from this practice. A crucial problem caused by these units is related to the normalization of the matter power spectrum, which is commonly characterized in terms of the linear-theory rms mass fluctuation in spheres of radius 8 h^{-1} Mpc, σ_8. This parameter does not correctly capture the impact of h on the amplitude of density fluctuations. We show that the use of σ_8 has caused critical misconceptions for both the so-called σ_8 tension regarding the consistency between low-redshift probes and cosmic microwave background data and the way in which growth-rate estimates inferred from redshift-space distortions are commonly expressed. We propose to abandon the use of h^{-1} Mpc units in cosmology and to characterize the amplitude of the matter power spectrum in terms of σ_{12}, defined as the mass fluctuation in spheres of radius 12 Mpc, whose value is similar to the standard σ_8 for $h \sim 0.67$.

DOI: 10.1103/PhysRevD.102.123511

I. INTRODUCTION

Most statistics used to analyze the large-scale structure of the Universe require the assumption of a fiducial cosmology to relate observable quantities such as galaxy angular positions and redshifts to density fluctuations on a given physical scale. To avoid adopting a specific value of the Hubble parameter, it is common to express all scales in units of h^{-1} Mpc, where h determines the present-day value of the Hubble parameter as $H_0 = 100\ h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. At low redshift, where the comoving distance, $\chi(z)$, can be approximated as

$$\chi(z) \approx \frac{c}{H_0} z, \tag{1}$$

using h^{-1} Mpc units effectively yields a distance independent of the fiducial cosmology. This approach was applied to the analysis of the first galaxy redshift surveys [1,2], which probed only small volumes. However, this practice has continued until the analysis of present-day samples, such as the Baryon Oscillation Spectroscopic Survey (BOSS) [3], which covers a larger redshift range in which computing $\chi(z)$ requires the assumption of a full set of fiducial cosmological parameters.

As cosmological observations are expressed in h^{-1} Mpc units, theoretical predictions follow the same approach. These units obscure the dependence of the matter power spectrum, $P(k)$, on h. Moreover, the amplitude of $P(k)$ is often characterized in terms of the rms linear perturbation theory variance in spheres of radius $R = 8\ h^{-1}$ Mpc, commonly denoted as σ_8. In this paper, we discuss the misconceptions related with the use of h^{-1} Mpc units and the normalization of model predictions in terms of σ_8 and how they can be avoided.

II. IMPACT OF THE FIDUCIAL COSMOLOGY

Three-dimensional galaxy clustering measurements depend on the cosmology used to transform the observed redshifts into distances. Any difference between this fiducial cosmology and the true one gives rise to the so-called Alcock-Paczynski (AP) distortions [4]. This geometric effect distorts the inferred components parallel and perpendicular to the line of sight, $s_\parallel$ and $s_\perp$, of the separation vector $\mathbf{s}$ between any two galaxies as [5,6]

$$s_\parallel = q_\parallel s'_\parallel, \tag{2}$$

$$s_\perp = q_\perp s'_\perp, \tag{3}$$

where the primes denote the quantities in the fiducial cosmology, and the scaling factors are given by

[*]arielsan@mpe.mpg.de

$$q_{\parallel} = \frac{H'(z_m)}{H(z_m)}, \quad (4)$$

$$q_{\perp} = \frac{D_M(z_m)}{D'_M(z_m)}, \quad (5)$$

where $H(z)$ is the Hubble parameter, $D_M(z)$ is the comoving angular diameter distance, and z_m is the effective redshift of the galaxy sample. If the clustering measurements are expressed in h^{-1} Mpc, the quantities appearing in Eqs. (4) and (5) must also be computed in these units.

Using h^{-1} Mpc units or simply Mpc would lead to identical parameter constraints, as the factors of h in the model and fiducial cosmologies that enter in $q_{\perp,\parallel}$ would simply cancel out with those in $s_{\perp,\parallel}$ in Eqs. (2) and (3). This simply reflects that, when h is correctly taken into account in the scaling parameters $q_{\perp,\parallel}$, the constraints derived from clustering data are not sensitive to the units in which they are expressed. The fact that clustering measurements can be expressed in h^{-1} Mpc without the explicit assumption of a value of h has no impact on the information content of these data, and it does not imply that only quantities referred to scales in h^{-1} Mpc units can be derived from them.

III. THE NORMALIZATION OF THE POWER SPECTRUM

Model predictions are often expressed in h^{-1} Mpc units before AP distortions are taken into account. Using h^{-1} Mpc or Mpc units yields identical cosmological constraints. However, h^{-1} Mpc units obscure the response of $P(k)$ to changes in h.

Panel (a) of Fig. 1 shows the linear matter power spectra at $z = 0$ of three ΛCDM models expressed in h^{-1} Mpc units, computed using CAMB [7]. These models have identical baryon, cold dark matter, and neutrino physical density parameters, ω_b, ω_c, and ω_ν, as well as scalar mode amplitude and spectral index, A_s and n_s, and differ only in their values of h. Panel (b) of Fig. 1 shows the same $P(k)$ in units of Mpc, which have the same shape and differ only in their amplitude. Expressing these power spectra in h^{-1} Mpc units obscures the fact that h only affects the overall clustering amplitude.

For a ΛCDM universe, the amplitude of $P(k)$ is controlled by both h and A_s. The joint effect of these parameters is usually described in terms of σ_8. When h varies, σ_8 changes due to two effects:

(i) the change in the amplitude of $P(k)$ itself, and
(ii) the change in the reference scale $R = 8\ h^{-1}$ Mpc, which corresponds to a different scale in Mpc for different values of h.

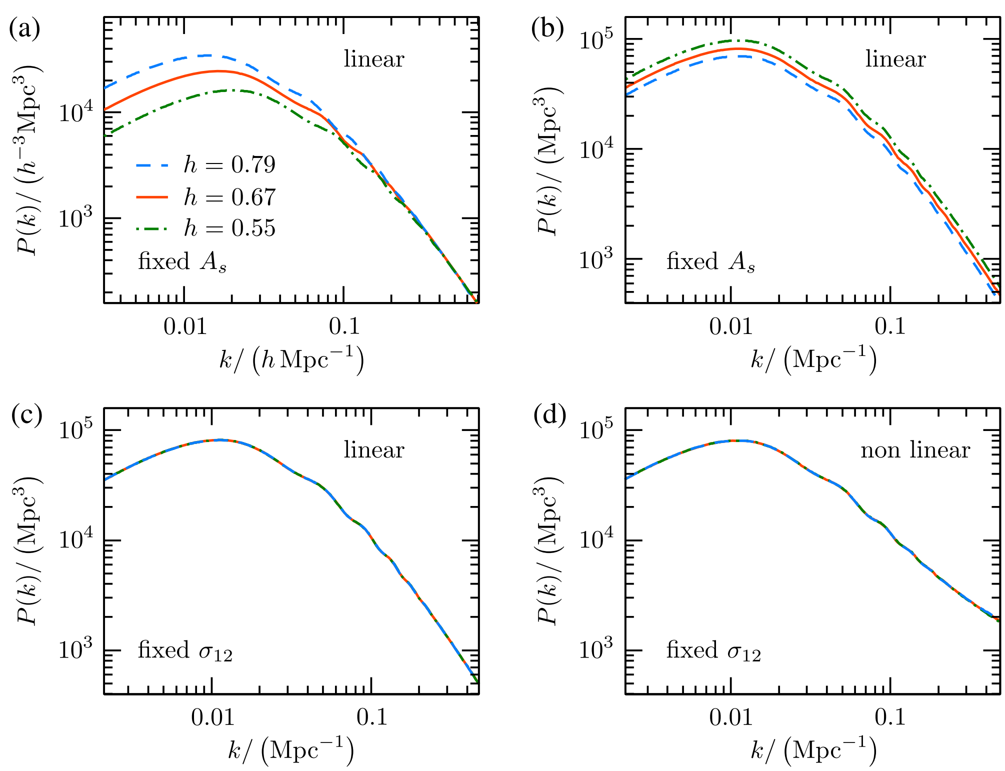

FIG. 1. Panel (a): Linear matter power spectra at $z = 0$ of three ΛCDM models defined by identical values of ω_b, ω_c, ω_ν, A_s, and n_s, and varying h, expressed in h^{-1} Mpc units. Panel (b): The same power spectra of panel a shown in Mpc units. Panel (c): The power spectra of the same models of panel (b) but with their values of A_s adapted to produce the same value of σ_{12}. Panel (d): Nonlinear matter power spectra corresponding to the same models of panel (c).

Point (ii) implies that σ_8 does not capture the impact of h on the amplitude of $P(k)$. For different values of h, σ_8 characterizes the amplitude of density fluctuations on different scales. Normalizing the power spectra of Fig. 1 to the same value of σ_8 increases their amplitude mismatch.

A better choice to describe the degenerate effect of h and A_s is to normalize $P(k)$ using a reference scale in Mpc. We propose to use σ_{12}, defined as the rms linear theory variance at $R = 12$ Mpc. For models with $h \simeq 0.67$ as suggested by current CMB data, $8\,h^{-1}\,\mathrm{Mpc} \simeq 12$ Mpc, and σ_{12} has a similar value to σ_8. However, these parameters differ for other values of h. Panel (c) of Fig. 1 shows $P(k)$ for the same models of panel (b) with their values of A_s modified to produce the same value of σ_{12}. These power spectra are identical, showing that the perfect degeneracy between h and A_s is better described in terms of σ_{12} than the standard σ_8.

Panel (d) of Fig. 1 shows the nonlinear $P(k)$ of the same models as panel (c), computed using HaloFit [8]. The observed agreement, with differences of only a few percent at high k, shows that σ_{12} is a more adequate parameter to characterize the nonlinear $P(k)$ than σ_8.

IV. REVISING THE σ_8 TENSION

The value of σ_8 preferred by Planck CMB data [9] under the assumption that a ΛCDM universe is higher than the estimates derived from all recent weak lensing (WL) datasets [10–13] and the clustering measurements from BOSS [14–17]. These discrepancies, dubbed the σ_8 tension, are illustrated in panel (a) of Fig. 2, which shows the constraints on Ω_m and σ_8 recovered from Planck [9], the auto- and cross-correlations between the cosmic shear and galaxy positions from the Dark Energy Survey (DES) [18], and clustering measurements from BOSS [14,19]. These results assume a ΛCDM cosmology with the same wide uniform priors as in [14]. Panel (b) of Fig. 2 shows these constraints expressed in terms of $S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$. For the values of Ω_m preferred by Planck, the low-redshift data prefer lower values of S_8 than the CMB.

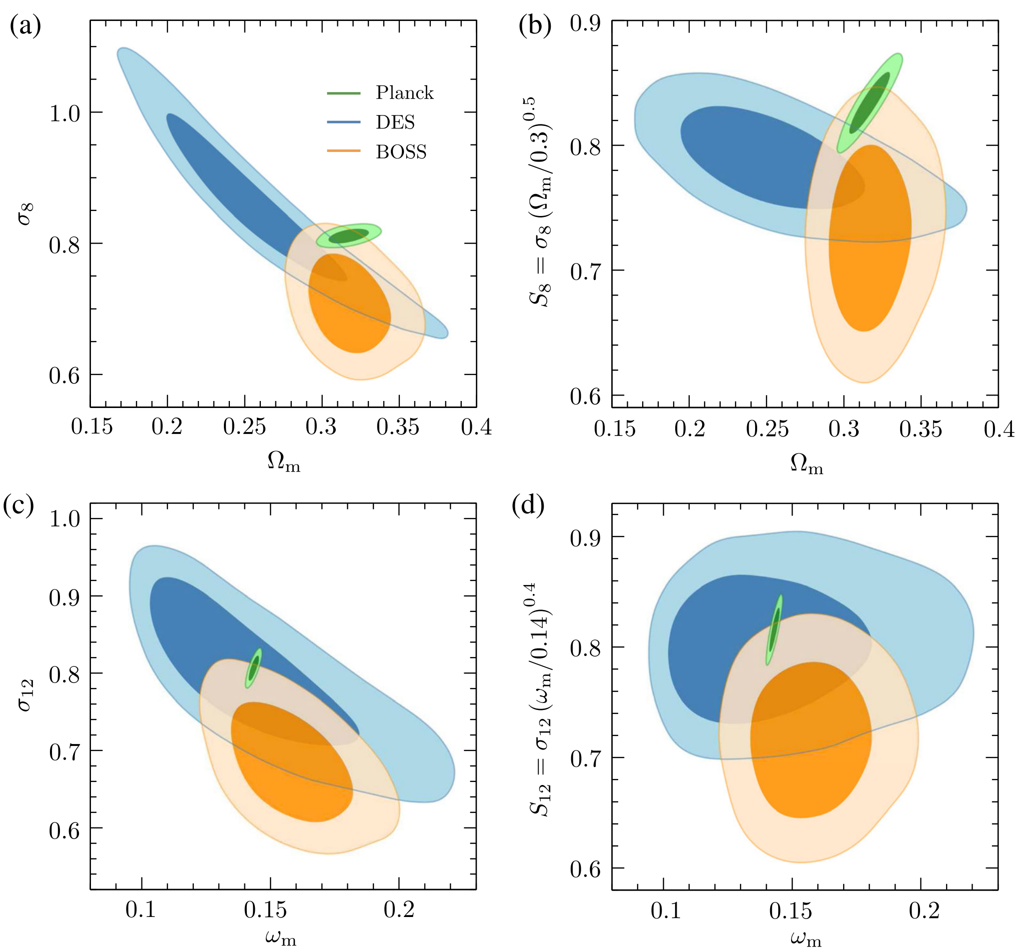

FIG. 2. Two-dimensional 68% and 95% constraints recovered from Planck (green), the 3×2pt analysis of DES (blue), and BOSS (orange) under the assumption of a ΛCDM cosmology on the parameters Ω_m–σ_8 [panel (a)], Ω_m–$S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$ [panel (b)], ω_m–σ_{12} [panel (c)], and ω_m–$S_{12} = \sigma_{12}(\omega_m/0.14)^{0.4}$ [panel (d)].

A drawback of using σ_8 to characterize the amplitude of $P(k)$ is that the reference scale $R = 8\ h^{-1}$ Mpc depends on h. Panel (a) of Fig. 3 shows the posterior distribution on h, $\mathcal{P}(h)$, inferred from DES, Planck, and BOSS. Although they are consistent, DES gives a wider posterior than Planck or BOSS. The posterior $\mathcal{P}(h)$ impacts the constraints on σ_8, which are given by

$$\sigma_8 = \int \sigma(R = (8/h)\ \mathrm{Mpc}|h)\mathcal{P}(h)\mathrm{d}h; \tag{6}$$

that is, they represent the average of $\sigma(R)$ over the range of scales defined by the posterior distribution of $R=(8/h)\,$Mpc, shown in panel (b) of Fig. 3. Averaging $\sigma(R)$ over different scales will give different, not necessarily consistent, results.

This issue can be avoided by using σ_{12}, which only depends on h through its impact on the amplitude of $P(k)$. Panel (c) of Fig. 2 shows the constraints in the ω_{m}—σ_{12} plane recovered from the same data. We use the physical density ω_{m} instead of Ω_{m}, as the former is the most relevant quantity to characterize the shape of $P(k)$. When expressed in terms of σ_{12}, the constraints inferred from DES and Planck are in excellent agreement. BOSS data prefer lower values of σ_{12}. Panel (d) of Fig. 2 shows these results in terms of the parameter $S_{12} = \sigma_{12}(\omega_{\mathrm{m}}/0.14)^{0.4}$, which matches the degeneracy between ω_{m} and σ_{12} recovered from DES data. Planck and DES imply $S_{12} = 0.815 \pm 0.013$ and $S_{12} = 0.798 \pm 0.043$, respectively, while BOSS gives $S_{12} = 0.716 \pm 0.047$. A detailed assessment of the consistency between Planck and low-redshift data is out of the scope of this paper. However, such studies should characterize the amplitude of density fluctuations in terms of σ_{12}.

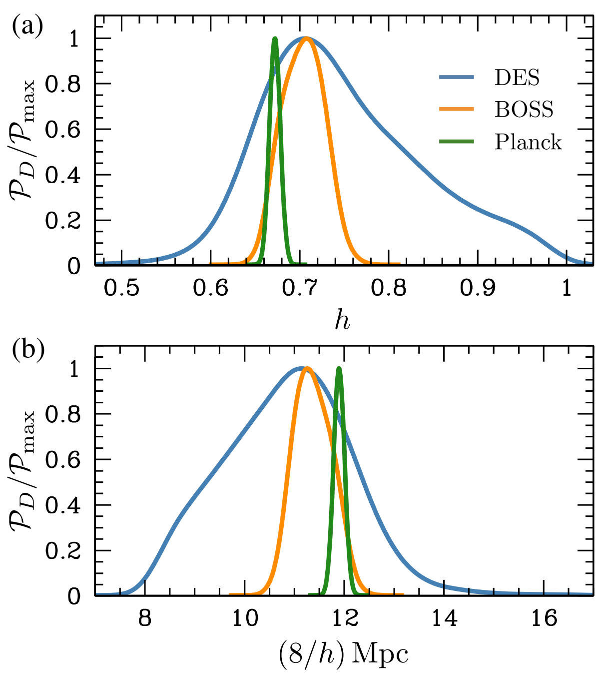

FIG. 3. Posterior distributions of the dimensionless Hubble parameter h [panel (a)] and the reference scale $(8/h)$ Mpc, where σ_8 is measured [panel (b)] recovered from DES, BOSS, and Planck under the assumption of a ΛCDM universe.

V. THE GROWTH RATE OF COSMIC STRUCTURES

The analysis of redshift-space distortions (RSD) on clustering measurements is considered as one of the most robust probes of the growth-rate of structures [20]. In linear perturbation theory, the relation between the two-dimensional galaxy power spectrum, $P_{\mathrm{g}}(k,\mu,z)$, and the real-space matter power spectrum can be written as [21]

$$P_{\mathrm{g}}(k,\mu,z) = (b\sigma_8(z) + f\sigma_8(z)\mu^2)^2 \frac{P(k,z)}{\sigma_8^2(z)}, \tag{7}$$

where μ represents the cosine of the angle between $\mathbf{k}$ and the line-of-sight direction, $b(z)$ is the galaxy bias factor, and $f(z)$ is the linear growth rate parameter. If $\sigma_8^2(z)$ described the amplitude of the power spectrum, the ratio $P(k,z)/\sigma_8^2(z)$ would only depend on the parameters that control its shape. In this case, the anisotropies in $P_{\mathrm{g}}(k,\mu,z)$ would depend on the combination $f\sigma_8(z)$. For this reason, the results of RSD analyses are usually expressed as measurements of $f\sigma_8(z)$. However, this argument is flawed, as the ratio $P(k,z)/\sigma_8^2(z)$ depends on h. Instead, the ratio $P(k)/\sigma_{12}^2(z)$ is truly constant, independently of the values of h or σ_{12}. Hence, the argument usually applied to justify the use of $f\sigma_8$ actually implies that $f\sigma_{12}$ is the most relevant quantity to describe RSD.

In most RSD studies, $f\sigma_8(z)$ is constrained together with the baryon acoustic oscillation (BAO) shift parameters, which describe the impact of AP distortions on the sound horizon scale, while the cosmological parameters that determine the shape and amplitude of the matter $P(k)$, including h, are kept fixed. We can then expect to obtain different results depending on the assumed value of h or when this parameter is marginalized over. To illustrate this point, we used linear theory to compute the Legendre multipoles $P_{\ell=0,2,4}(k)$ of a galaxy sample roughly matching the volume, bias, and number density of the BOSS CMASS sample [22] and used a Gaussian prediction for their covariance matrix [23]. We used these data to constrain $b\sigma_8(z)$, $f\sigma_8(z)$, and the BAO shift parameters. Panel (a) of Fig. 4 shows the constraints in the $b\sigma_8(z)$—$f\sigma_8(z)$ plane obtained when both A_{s} and h are kept fixed to their true values (orange), when A_{s} is varied while h is kept fixed (green), and when both A_{s} and h are varied (blue). The dashed lines indicate the true values of these parameters. When only A_{s} is varied, the constraints follow the degeneracies defined by constant values of $b\sigma_8(z)$ and $f\sigma_8(z)$, leading to identical results to the ones

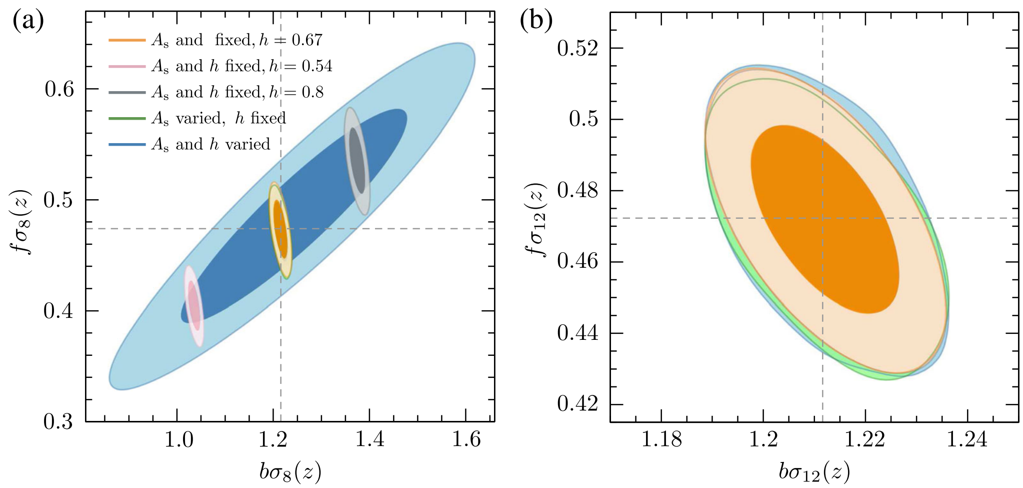

FIG. 4. Panel (a): Constraints on $b\sigma_8(z)$ and $f\sigma_8(z)$ derived from synthetic Legendre multipoles $P_{\ell=0,2,4}(k)$. The contours correspond to the cases in which A_s and h are fixed ($h = 0.67$ orange, $h = 0.54$ pink, and $h = 0.8$ gray), when A_s is varied and h is fixed (green) and when both are varied (blue). Panel (b): Same constraints as panel a but expressed in terms of $b\sigma_{12}(z)$ and $f\sigma_{12}(z)$.

obtained when it is fixed. However, when h is also varied, the constraints deviate significantly from those of the standard case. The uncertainties on $f\sigma_8(z)$ derived under a fixed h are significantly underestimated. Furthermore, the results obtained when fixing h depend on the particular value adopted. This is illustrated by the pink and gray contours in Fig. 4, which show the results obtained assuming values of h that differ by $\pm 20\%$ from the true value $h = 0.67$.

Panel (b) of Fig. 4 shows the same constraints as in panel (a) but expressed in terms of $b\sigma_{12}(z)$ and $f\sigma_{12}(z)$. The results are the same irrespective of whether A_s or h are kept fixed or marginalized over. This shows that $f\sigma_{12}(z)$ provides a more correct description of the information retrieved from the standard RSD analyses.

VI. CONCLUSIONS

Although the use of h^{-1} Mpc units has no impact on the information content of cosmological data, they have generated misconceptions related to the normalization of the matter power spectrum in terms of σ_8. This parameter does not correctly capture the impact of h on the amplitude of $P(k)$, which is better described in terms of a reference scale in Mpc. A convenient choice is 12 Mpc, which results in a mass variance σ_{12} with a similar value to the standard σ_8 for $h \sim 0.67$.

The amplitude of density fluctuations inferred from low- and high-redshift data should be characterized in terms of σ_{12}, eliminating the dependency of the reference scale $R = 8\ h^{-1}$ Mpc on the constraints on h. The results of standard RSD analyses are more correctly described in terms of $f\sigma_{12}(z)$, which changes the cosmological implications of most available growth-rate measurements. We propose to abandon the traditional h^{-1} Mpc units in the analysis of new surveys [24,25] and to replace σ_8 by σ_{12} to characterize the amplitude of density fluctuations.

ACKNOWLEDGMENTS

A. G. S. would like to thank Daniel Farrow, Daniel Grün, Catherine Heymans, Jiamin Hou, Martha Lippich, and Agne Semenaite for their help and useful discussions. Some of the figures in this work were created with GetDist, making use of the NumPy [26] and SciPy [27] software packages. This reasearch was supported by the Excellence Cluster ORIGINS, which is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strateg—EXC-2094—390783311.

[1] M. Davis and P. J. E. Peebles, Astrophys. J. **267**, 465 (1983).
[2] S. A. Shectman, S. D. Landy, A. Oemler, D. L. Tucker, H. Lin, R. P. Kirshner, and P. L. Schechter, Astrophys. J. **470**, 172 (1996).
[3] K. S. Dawson, D. J. Schlegel, C. P. Ahn, S. F. Anderson, É. Aubourg, S. Bailey, R. H. Barkhouser, J. E. Bautista, A. Beifiori, A. A. Berlind, V. Bhardwaj, D. Bizyaev, C. H. Blake, M. R. Blanton *et al.*, Astron. J. **145**, 10 (2013).

[4] C. Alcock and B. Paczynski, Nature (London) **281**, 358 (1979).
[5] N. Padmanabhan and M. White, Phys. Rev. D **77**, 123540 (2008).
[6] E. A. Kazin, A. G. Sánchez, and M. R. Blanton, Mon. Not. R. Astron. Soc. **419**, 3223 (2012).
[7] A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. **538**, 473 (2000).
[8] R. E. Smith, R. Scoccimarro, and R. K. Sheth, Phys. Rev. D **77**, 043525 (2008).
[9] N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. B. Barreiro, N. Bartolo, S. Basak, R. Battye, K. Benabed, J. P. Bernard, M. Bersanelli *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A6 (2020).
[10] C. Heymans, L. Van Waerbeke, L. Miller, T. Erben, H. Hildebrandt, H. Hoekstra, T. D. Kitching, Y. Mellier, P. Simon, C. Bonnett, J. Coupon, L. Fu, J. Harnois Déraps, M. J. Hudson *et al.*, Mon. Not. R. Astron. Soc. **427**, 146 (2012).
[11] H. Hildebrandt, M. Viola, C. Heymans, S. Joudaki, K. Kuijken, C. Blake, T. Erben, B. Joachimi, D. Klaes, L. Miller, C. B. Morrison, R. Nakajima, G. Verdoes Kleijn, A. Amon *et al.*, Mon. Not. R. Astron. Soc. **465**, 1454 (2017).
[12] M. A. Troxel, N. MacCrann, J. Zuntz, T. F. Eifler, E. Krause, S. Dodelson, D. Gruen, J. Blazek, O. Friedrich, S. Samuroff, J. Prat, L. F. Secco, C. Davis, A. Ferté, J. DeRose *et al.*, Phys. Rev. D **98**, 043528 (2018).
[13] C. Hikage, M. Oguri, T. Hamana, S. More, R. Mandelbaum, M. Takada, F. Köhlinger, H. Miyatake, A. J. Nishizawa, H. Aihara, R. Armstrong, J. Bosch *et al.*, arXiv:1809.09148.
[14] T. Tröster, A. G. Sánchez, M. Asgari, C. Blake, M. Crocce, C. Heymans, H. Hildebrandt, B. Joachimi, S. Joudaki, A. Kannawadi, C.-A. Lin, and A. Wright, Astron. Astrophys. **633**, L10 (2020).
[15] G. D'Amico, J. Gleyzes, N. Kokron, D. Markovic, L. Senatore, P. Zhang, F. Beutler, and H. Gil-Marín, J. Cosmol. Astropart. Phys. 05 (2020) 005.
[16] T. Colas, G. D'Amico, L. Senatore, P. Zhang, and F. Beutler, J. Cosmol. Astropart. Phys. 06 (2020) 001.
[17] M. M. Ivanov, M. Simonović, and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 05 (**2020**) 042.
[18] T. M. C. Abbott, F. B. Abdalla, A. Alarcon, J. Aleksić, S. Allam, S. Allen, A. Amara, J. Annis, J. Asorey, S. Avila, D. Bacon, E. Balbinot, M. Banerji, N. Banik, W. Barkhouse *et al.*, Phys. Rev. D **98**, 043526 (2018).
[19] A. G. Sánchez, R. Scoccimarro, M. Crocce, J. N. Grieb, S. Salazar-Albornoz, C. Dalla Vecchia, M. Lippich, F. Beutler, J. R. Brownstein, C.-H. Chuang, D. J. Eisenstein, F.-S. Kitaura, M. D. Olmstead, W. J. Percival, F. Prada *et al.*, Mon. Not. R. Astron. Soc. **464**, 1640 (2017).
[20] L. Guzzo, M. Pierleoni, B. Meneux, E. Branchini, O. Le Fèvre, C. Marinoni, B. Garilli, J. Blaizot, G. De Lucia, A. Pollo, H. J. McCracken, D. Bottini, V. Le Brun, D. Maccagni, J. P. Picat *et al.*, Nature (London) **451**, 541 (2008).
[21] N. Kaiser, Mon. Not. R. Astron. Soc. **227**, 1 (1987).
[22] B. Reid, S. Ho, N. Padmanabhan, W. J. Percival, J. Tinker, R. Tojeiro, M. White, D. J. Eisenstein, C. Maraston, A. J. Ross, A. G. Sánchez, D. Schlegel, E. Sheldon, M. A. Strauss, D. Thomas, D. Wake *et al.*, Mon. Not. R. Astron. Soc. **455**, 1553 (2016).
[23] J. N. Grieb, A. G. Sánchez, S. Salazar-Albornoz, and C. Dalla Vecchia, Mon. Not. R. Astron. Soc. **457**, 1577 (2016).
[24] R. Laureijs *et al.* (EUCLID Collaboration), arXiv:1110.3193.
[25] M. Levi *et al.* (DESI Collaboration), arXiv:1308.0847.
[26] T. E. Oliphant, *A guide to NumPy*, Vol. 1 (Trelgol Publishing, USA, 2006).
[27] E. Jones, T. Oliphant, P. Peterson *et al.*, SciPy: Open source scientific tools for PYTHON, http://www.scipy.org/ (2001).