

Superbound Electrons

The binding energy of an electron in the field of a point-like nucleus with charge Z (for the ground state) is, according to Dirac,¹ equal to

$$\epsilon = m[1 - \sqrt{1 - Z^2 \alpha^2}],$$

where $\alpha = 1/137$ and m is the electron mass (we are using units in which $\hbar = c = 1$). This expression loses its meaning for $Z > 137$. In 1945 Pomeranchuk and Smorodinsky² noted that, taking into account the finite size of a nucleus, the electron may be bound with binding energy equal to $2m$. Recently Gerstein and Zeldovich,³ Pieper and Greiner,⁴ Popov⁵ and Migdal⁶ have returned to this problem, considering superbound electrons with binding energy larger than $2m$.

The solution of the Dirac equation for an electron in the field of a non-point-like nucleus with $Z > 137$, obtained analytically and by numerical methods,^{4,5} leads to the conclusion that binding energy $\epsilon = 2m$ may be achieved for the $1s$ -electron at $Z = Z_c = 170$.

Such a value of critical charge Z_c corresponds to the nuclear radius $R = r_0 A^{1/3}$ where $r_0 = 1.2 \times 10^{-13}$ cm.

With further increase of Z , binding energy continues to increase and the electron becomes superbound ($2p$ -electrons remain non-superbound up to $Z \simeq 185$, $2s$ -electrons up to $Z \simeq 230$; the electrons of higher shells become superbound at still larger Z). The atom with two superbound electrons in the $1s$ -shell does not differ drastically from those whose nuclei have $Z < Z_c$. In particular, the electron in a superbound state is localized in the region of its Compton wavelength $1/m \sim 10^{-11}$ cm.

However, unlike usual nuclei, a bare nucleus with $Z > Z_c$ spontaneously produces its own $1s$ -electrons. The theory predicts that the bare nucleus with $Z > Z_c$ is unstable; it must spontaneously produce two electron-positron pairs, the electrons of which will be captured in the $1s$ level while the positrons with kinetic energy $\epsilon - 2m$ go to infinity. The necessary energy is available because the binding energy of the electrons is greater than $2m$. Consequently there is enough energy not only to produce the electron and positron but to give the positron kinetic energy. A. B. Migdal⁶ has taken into account the

interaction between electrons and positrons when two positrons are emitted by a bare nucleus with simultaneous capture of two electrons into superbound states. It appears that inclusion of the electron-positron interaction effectively results in a small increase of Z_c . Besides, what is more interesting, there appears a region near Z_c in which the emission of one positron takes place while the emission of two positrons is energetically impossible.

If $\epsilon - 2m \sim m$, and consequently the positron is very energetic, the time necessary for this process is of order of 10^{-17} s. This time, however, may become very large if $\epsilon - 2m \ll m$, since the positron must tunnel through the Coulomb barrier before it goes to infinity.

Experimentally the observation of such positrons accompanying the production of superbound electrons may be achieved in principle, by colliding two bare nuclei such that $Z_1 + Z_2 > Z_c$. As was noted by S. S. Gerstein and V. S. Popov, this process occurs also when only one of the colliding nuclei is bare.

It would be possible to observe an interesting phenomenon if one could direct a positron beam onto atoms with $Z > Z_c$. At a positron energy equal to $\epsilon - 2m$ resonant scattering would take place. The intermediate resonant state in this case would be the atom with one $1s$ -electron. The second superbound electron would vanish for a moment, cancelling out with the positron, and then would again take its place in the $1s$ -shell resulting from the spontaneous decay of the nucleus just described.

The mathematical description of a superbound electron is a very interesting problem. The method of effective potential developed by Popov⁵ turns out to be very convenient for analysis of the solutions of the Dirac equation at $Z \sim Z_c$. The equation is here reduced to a Schrödinger-type equation with effective attraction at small distances, both for electrons and positrons.

However, the analysis of the one-particle equation does not totally settle the problem. The point is that the superbound electron, having quite definite binding energy and being localized in space, analogously to the usual $1s$ -electron, is nevertheless in a state not belonging to the discrete spectrum of the one-particle Dirac equation. The charge density distribution in this state is expressed by an integral over energy of charge densities of one-particle states.

Although the experiments described here seem today to be very difficult, it may be that as time goes on the superbound electrons will be studied experimentally in the laboratory. It is possible that conditions for the production of such electrons may be realized in astrophysical objects. But, even if this is not so, the superbound electrons are a very interesting subject for a theorist. The physics of this phenomenon is interesting, instructive and no doubt will find applications in the analysis of theoretical models of elementary particles for which strong fields are of great importance, and in the theory of nuclear matter.

Can mesons as well as electrons be superbound? This question has been analysed by Migdal.⁶ It appears that, according to Bose statistics, the number of mesons on the lowest level will be such that the screening field will not allow that level to achieve its critical value. As a result the superbound state cannot appear. Analogously, as shown by Migdal, the strong nuclear potential cannot effectively surpass a definite limit.

Details of the superbound electron theory may be found in a review by Zeldovich and Popov.⁷

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References

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