Strange dibaryon and $\bar{K}NN$-$\pi\Sigma N$ coupled channel equation

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Abstract. $\bar{K}NN$ three body resonance has been studied by the $\bar{K}NN - \pi\Sigma N$ coupled channel Faddeev equation. The S-matrix pole has been investigated using the scattering amplitude on the unphysical Riemann sheet. As a result we found a three-body resonance of the strange dibaryon system with a binding energy $B \sim 76$MeV and a width $\Gamma \sim 54$MeV.

PACS. 11.30.Rd Chiral symmetries – 11.80.Jy Many-body scattering and Faddeev equation – 13.75.Jz Kaon-baryon interaction – 21.45.+v Few-body systems

1 Introduction

Meson-nucleus bound states offer an important tool to study the meson properties inside the nuclear medium and the interaction of the meson below the threshold energy. The $\bar{K}$-nuclear system is particularly interesting because of the $I = 0$ resonance $\Lambda(1405)$ below the $\bar{K}N$ threshold. The attractive kaon-nucleus interaction obtained from the analysis of the X-ray from the kanonic-atom[1,2] might be largely related to the $\Lambda(1405)$. In a few nucleon system, where one hopes to learn about the kaon-nucleon interaction with less ambiguity on nuclear many body dynamics, possible deeply bound states of the kaon in nuclei have been proposed by Akaishi and Yamazaki[3–5]. The kaon-nucleus optical potential is constructed from the kaon-nucleon g-matrix in the nuclear medium. The predicted binding energy B and width Γ of the smallest nuclear system K^-pp is $(B,\Gamma) = (48, 61)$MeV. FINUDA collaboration reported a signal of the K^-pp bound state from the analysis of the invariant mass distribution of $\Lambda - p$ in the K^- absorption reaction on nuclei[6]. The reported central value of the binding energy is $(B,\Gamma) = (115, 67)$MeV, which is twice larger than the theoretical prediction. Recently a question raised[7] on the $\bar{K}N - \pi\Sigma$ interaction used in Ref. [3,4], where the interaction in the $\pi\Sigma$ channel is absent, and on the interpretation of the invariant mass spectrum[8], which might be explained by the $K^-pp \to \Lambda p$ reaction with the final state interaction.

Intuitively, the K^-pp resonance may be regarded as the bound state of the $\Lambda(1405)$ and the nucleon interacting with the kaon exchange mechanism. The binding energy of this resonance will be strongly influenced by the dynamics of the $\Lambda(1405)$ resonance. For the resonance interaction in a few body system, it will be very important to take into account fully the kaon-nucleon dynamics in the $\bar{K}NN$ three-body system including the decay of the $\Lambda(1405)$ into the $\pi\Sigma$ state. The purpose of this work is to study the strange dibaryon system by taking into account the three-body dynamics using the $\bar{K}NN - \pi\Sigma N$ coupled channel Faddeev equation. The three-body resonance has been investigated on the three-neutron[10,12], πNN dibaryon[13] and ΣNN hypernuclei[14,15]. The resonance can be studied from the pole of the S-matrix or scattering amplitude. The pole position can be obtained by studying the eigenvalue of the kernel of scattering equation, which is analytically continued in the unphysical sheet. We briefly explain our $\bar{K}NN - \pi\Sigma N$ coupled channel equation and the procedure to search the three-body resonance in section 2.

The structure of the $\Lambda(1405)$ has been a long standing issue. The chiral Lagrangian[16,17] approach is able to describe well the low energy $\bar{K}N$ reaction. A genuine q^3 picture of the $\Lambda(1405)$ coupled with meson-baryon[18] may not be yet excluded. In this work we describe a $\bar{K}N - \pi\Sigma$ state using the s-wave meson-baryon potentials guided from the lowest order chiral Lagrangian. With this procedure, the strength of the potentials and the relative strength of the potentials among various meson-baryon channels are not parameters but determined from the chiral Lagrangian. In this model, the $\Lambda(1405)$ is 'unstable bound state', whose pole in the unphysical sheet will become the $\bar{K}N$ bound state when the coupling between $\bar{K}N$ and $\pi\Sigma$ is turned off. The model of the two-body meson-baryon interaction used in this work is explained in section 3. We then report our results on the $\bar{K}NN$ dibaryon resonance in section 4.

2 Coupled channel Faddeev equation and resonance pole

In this section we briefly explain our coupled channel equation and a method to find resonance pole from the coupled channel Faddeev equation. Our starting point is the

Alt-Grassberger-Sandhas equation[19] for the three-body scattering problem. The AGS equation for the three-body scattering amplitude $U_{i,j}$ is given as

$$U_{i,j} = (1-\delta_{i,j})G_0^{-1} + \sum_{n\neq i} t_n G_0 U_{n,j}. \quad (1)$$

We label the scattering amplitude U by the spectator particles $i,j = 1,2,3$. t_i is the two-body t-matrix with the spectator particle i and $G_0 = 1/(W - H_0 + i\epsilon)$ is the three particle Green's function.

With the separable two-body interaction given as

$$v_i = |g_i > \gamma_i < g_i|, \quad (2)$$

the AGS-equation in Eq. (1) takes the following form

$$X_{i,j} = (1-\delta_{i,j})Z_{i,j} + \sum_{n\neq i}\int d\boldsymbol{p}_n Z_{i,n}\tau_n X_{n,j}. \quad (3)$$

The amplitude $X_{i,j}$ is the matrix element of $U_{i,j}$ between states $G_0|\boldsymbol{p}_i, g_i>$ with the plane wave spectator $|\boldsymbol{p}_i>$ and the interacting pair $|g_i>$, as

$$X_{i,j} = < \boldsymbol{p}_i, g_i|G_0 U_{i,j} G_0|\boldsymbol{p}_j, g_j > . \quad (4)$$

The driving term $Z_{i,j}$ of Eq. (3) is the particle exchange interaction defined as

$$Z_{i,j} = < \boldsymbol{p}_i, g_i|G_0|\boldsymbol{p}_j, g_j > . \quad (5)$$

The 'isobar' propagator τ_i is given as

$$t_i = |g_i > \tau_i < g_i|, \quad (6)$$

with

$$\tau_i(W) = [1/\gamma_i - \int d\boldsymbol{q}_i \frac{< g_i|\boldsymbol{q}_i >< \boldsymbol{q}_i|g_i >}{W - E_i(\boldsymbol{p}_i) - E_{jk}(\boldsymbol{p}_i,\boldsymbol{q}_i)}]^{-1}, \quad (7)$$

where E_i and E_{jk} are the energies of the spectator and the interacting pair, respectively. $\boldsymbol{q}_i$ is the relative momentum of the pair j,k.

In our $\bar{K}NN$ resonance problem, we have included following $\bar{K}NN$ and $\pi\Sigma N$ Fock space components,

$$|a> = |N_1, N_2, \bar{K}_3>, \quad (8)$$
$$|b> = |N_1, \Sigma_2, \pi_3>, \quad (9)$$
$$|c> = |\Sigma_1, N_2, \pi_3> . \quad (10)$$

After symmetrizing the amplitude for N_1 and N_2[20] and the partial wave expansion of the amplitude[9] restricting s-wave, the AGS-equation reduces into the following coupled integral equation,

$$X_{l,m}(p_l,p_m) = Z_{l,m}(p_l,p_m) + \sum_n \int dp_n p_n^2 \times K_{l,n}(p_l,p_n)X_{n,m}(p_n,p_m). \quad (11)$$

Here we used simplified notation for the kernel $K = Z\tau$.

We follow the method for searching the three-body resonance used by Matsuyama and Yazaki[11–13]. The AGS-equation of Eq. (11) is the Fredholm type integral equation with the kernel $K = Z\tau$. Using the eigenvalue $\eta_a(W)$ and the eigenfunction $|\phi_a(W)>$ of the kernel for given energy W,

$$Z\tau|\phi_a(W)> = \eta_a(W)|\phi_a(W)>, \quad (12)$$

the scattering amplitude X can be written as

$$X = \sum_a \frac{|\phi_a(W)><\phi_a(W)|Z}{1-\eta_a(W)}. \quad (13)$$

At the energy $W = W_p$ where $\eta_a(W_p) = 1$, the amplitude has a pole and therefore W_p gives the bound state or resonance energy.

3 Model of the meson-baryon interaction

We investigate the strange $S = -1$ dibaryon state with the total angular momentum $J = 0$, parity $\pi = -1$ and isospin $I = 1/2$, which is expected to have a larger $I = 0$ $\bar{K}N$ component than the spin triplet state. The s-wave meson-baryon ($\bar{K}N - \pi\Sigma$, πN) interactions and baryon-baryon interactions are included. In the following we concentrate on the most important $\bar{K}N$ interaction.

The leading order chiral effective Lagrangian for the octet baryon ψ_B and the pseudoscalar meson ϕ fields is given as

$$L_{int} = \frac{i}{8F_\pi^2} tr(\bar{\psi}_B\gamma^\mu[[\phi,\partial_\mu\phi],\psi_B]). \quad (14)$$

The meson-baryon potential derived from the chiral Lagrangian can be written as

$$< \boldsymbol{p}', \alpha|V_{BM}|\boldsymbol{p}, \beta > = -C_{\alpha,\beta}\frac{1}{(2\pi)^3 8F_\pi^2}\frac{E_{M'}(\boldsymbol{p}') + E_M(\boldsymbol{p})}{\sqrt{4E_{M'}(\boldsymbol{p}')E_M(\boldsymbol{p})}} \times v_\alpha(\boldsymbol{p}')v_\beta(\boldsymbol{p}). \quad (15)$$

Here $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the momentum of the meson in the initial state β and the final state α. The strength of the potential at zero momentum is determined by the pion decay constant F_π. The relative strength among the meson-baryon states is given by the constants $C_{\alpha,\beta}$, which are $C_{\bar{K}N-\bar{K}N} = 6, C_{\bar{K}N-\pi\Sigma} = \sqrt{6}$ and $C_{\pi\Sigma-\pi\Sigma} = 8$. The only parameter of our model is cut off Λ of the phenomenologically introduced vertex function $v_\alpha(\boldsymbol{p}) = \Lambda_\alpha^4/(\boldsymbol{p}^2 + \Lambda_\alpha^2)^2$.

The cut off Λ is determined so as to reproduce the scattering length of $\bar{K}N$ by Martin[21], which is summarized in Table 1. We have two models for the non-relativistic and relativistic kinematic energies. The relativistic form of the kinetic energy may be necessary for the small pion mass in the $\pi\Sigma N$ channel. The $I = 0$ scattering length is close to the value $-1.70 + i0.68(fm)$ of Ref. [21] but the real part of our $I = 1$ scattering length is a little bit

Table 1. The relativistic and non-relativistic models of the $\bar{K}N$ interaction. The values of the cut off parameters (Λ) are shown. We also show the pole positions of the $\Lambda(1405)$ in our models.

		$\Lambda_{\bar{K}N}$(MeV)	$\Lambda_{\pi\Sigma}$(MeV)	$\Lambda_{\pi\Lambda}$(MeV)	Scattering Length(fm)	Resonance energy(MeV)
Relativistic Model	I=0	1125	1300		$-1.71 + i0.56$	$1413.6 - i29.0$
	I=1	1100	1100	1100	$0.66 + i0.64$	
Non-rela. Model	I=0	960	900		$-1.78 + i0.59$	$1414.3 - i26.4$
	I=1	850	950	900	$0.78 + i0.66$	

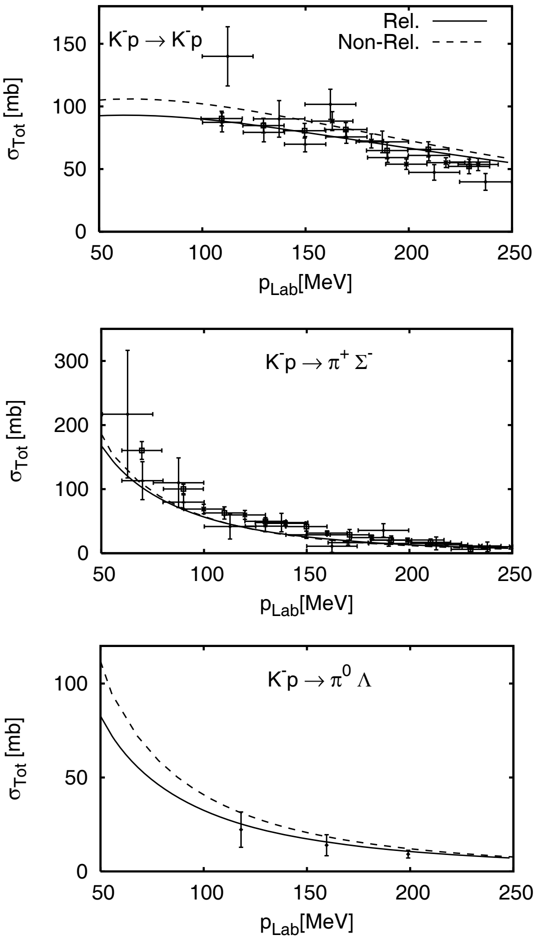

Fig. 1. The total cross section of $K^-p \to K^-p$ (Left), $K^-p \to \pi^+\Sigma^-$ (Center) and $K^-p \to \pi^0\Lambda$ (Right) reactions. The solid (dashed) curves show the results using relativistic (non-relativistic) model. Data are taken from Ref. [25–29].

larger than $0.37 + i0.60(fm)$ of Ref. [21]. They are consistent with the data of the kaonic hydrogen atom[22–24]. In $\bar{K}N - \pi\Sigma$ for $I = 0$ channel, the two models have a resonance in $\bar{K}N$ physical and $\pi\Sigma$ unphysical sheet. Both models give a satisfactory description of the total cross section of the K^-p reaction at low energies as can be seen in Fig. 1.

4 Results and discussion

We have searched for a resonance pole of the $\bar{K}NN-\pi\Sigma N$ coupled channel equation using the method described in section 2 and the $\bar{K}N$ interaction explained in section 3. In addition, the NN interaction for $^1S_0$ channel and the πN interaction are included in the AGS-equation. However the ΣN interaction is not included in this work.

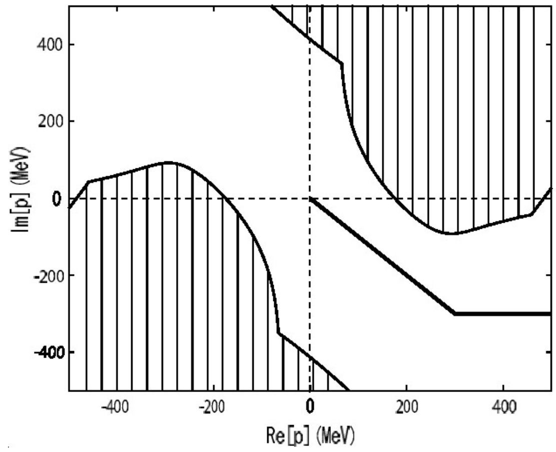

Fig. 2. Deformed contour of the momentum integration.

In order to investigate the resonance position, we have to analytically continue the equation into the unphysical energy sheet. For this purpose we deform the contour of the momentum integration so that we will not cross the singularity of the Z and τ of the kernel. The singularities are due to the $\pi\Sigma N, \bar{K}NN$ continuum in the Green's function and the $\bar{K}N$ two-body resonance corresponding to the $\Lambda(1405)$, and the singularity of the potential for the complex momentum. As an example the momentum contour is shown by the solid curve in Fig. 2 for $W = 2m_N + m_K - 70 - i32MeV$. We searched for a resonance energy below $\bar{K}NN$ and above the $\pi\Sigma N$ threshold in the $\bar{K}NN$-physical and $\pi\Sigma N$-unphysical Riemann-sheet. The shaded area is the 'forbidden region' due to the singularity of Z for the kaon exchange mechanism. We have studied all 'forbidden region' for π, N and K exchange mechanism and determined the integration contour.

At first, we take into account only the $\bar{K}N-\bar{K}N$ interaction. Therefore coupling with $\pi\Sigma N$ channel is switched off and the contour of the momentum integration is on the real axis. In this case, we find a bound state pole of the AGS-equation below $\bar{K}NN$ threshold on the physical sheet. The results are shown in Fig. 3 marked by a and a' for the 'relativistic' and 'non-relativistic' model. Then including the NN interaction, the binding energy is further increased to 29.1MeV (25.2MeV) at b (b') for 'relativistic' ('non-relativistic') model. The $\bar{K}N$ interaction included in τ and Z in this model is strong enough to bind the $\bar{K}NN$ system.

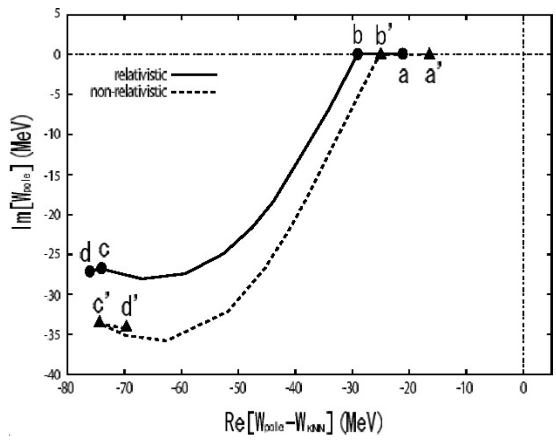

Fig. 3. The pole trajectories of the $\bar{K}NN - \pi\Sigma N$ scattering amplitude for the $J^\pi = 0^-$ and $T = 1/2$ state. Two trajectories correspond to the relativistic model (solid line and filled circles) and non-relativistic one (dashed line and filled triangles).

In the next step, we take into account the channel coupling between $\bar{K}N$ state and $\pi\Sigma$ state keeping the kaon and nucleon exchange mechanisms in Z. We trace the trajectory of the resonance pole by artificially modifying the strength of $\bar{K}N - \pi\Sigma$ transition potential from zero to the value of our model. The solid and dashed curve in Fig. 3 represent the pole trajectory corresponding to relativistic and non-relativistic model, respectively. The bound state pole moves into the $\bar{K}NN$ physical and $\pi\Sigma N$ unphysical energy sheet and reaches to c (c'). The width of the resonance is due to the decay of the $\bar{K}NN$ bound state to the $\pi\Sigma N$ and $\pi\Lambda N$ states through the imaginary part of the τ. Finally we include the π exchange mechanism in Z and the $\pi - N$ two-body scattering terms in τ, which plays a rather minor role in determining the pole position. The final result of the $\bar{K}NN - \pi\Sigma N$ resonance poles are denoted by d and d' in Fig. 3. The pole position of the three-body resonance is $W = M - i\Gamma/2 = 2m_N + m_K - 76.1 - 27.1i$MeV ($2m_N + m_K - 69.7 - 34.2i$) for the relativistic (non-relativistic) model. Our resonance has deeper binding energy and similar width compared with the prediction of Ref. [4]. Recently Shevchenko, Gal and Mares [30] studied K^-pp system using coupled channel Faddeev equation, which is a quite similar approach as our present study. They reported $B \sim 55 - 70$MeV and $\Gamma \sim 95 - 110$MeV. Their binding energy is similar to ours, while our relativistic model gives smaller width.

In summary we have studied strange dibaryon states using the $\bar{K}NN - \pi\Sigma N$ coupled channel Faddeev equation. We found a resonance pole of the strange dibaryon at $B \sim 76$MeV and $\Gamma \sim 54$MeV in a relativistic model. It is however noticed that the $\bar{K}N$ interaction is not well determined experimentally and further investigations to find a possible range of the resonance energy is necessary. The full content of our work will be reported elsewhere [31].

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