THE GRAVITATIONAL FORCE AT MASS SEPARATIONS FROM 0.6 m TO 2.1 m AND THE PRECISE MEASUREMENT OF G

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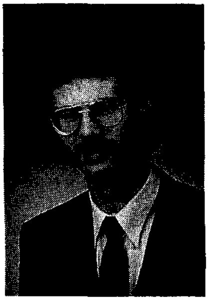

Abstract

We present the results of a gravitational experiment which is based on a microwave resonator. The gravitational force of a test mass acting on the resonator is measured as a function of distance. No deviation from Newton's law has been found and the gravitational constant G has been determined with a relative accuracy of 2.2×10^{-4} to be $G=(6.6724 \pm 0.0015)\times10^{-11}\,\mathrm{Nm^2kg^{-2}}$ in good agreement with the presently accepted value.

Introduction

Many aspects of Newton's law of gravitation have been investigated experimentally during the last three centuries and the precision has been improved steadily. Until today the most precise determination of the gravitational constant G is possible by means of a Cavendish torsion balance. The gravitational constant G is so far determined with a relative uncertainty of 1.3×10^{-4} only. This uncertainty is much larger than the uncertainty of all other fundamental constants [1,2], mainly due to systematic errors in the measurement of the weak gravitational force.

It is however remarkable, that almost all of the experiments which have been performed to obtain a precision value for G have not tested the inverse-square law at the same time [3] in order to detect and exclude systematic effects.

The fifth-force discussion starting in 1986 [4] has motivated us to develop a pendulum gravimeter based on a microwave Fabry-Perot resonator. The gravimeter was designed to measure the gravitational force of a laboratory test mass as a function of distance in order to determine the gravitational constant G with high precision [5,6].

The Basic Principle

The central part of the gravimeter consists of a Fabry-Perot microwave resonator (Fig. 1). Both mirrors are suspended as pendula of equal length l ($l \approx 2.6$ m) separated by a distance b ($b \approx 0.24$ m) from a suspension platform. The gravitational force of a laboratory test mass M ($M \approx 576$ kg) acting on this resonator changes the mirror separation which is measured by means of a shift in the resonator frequency.

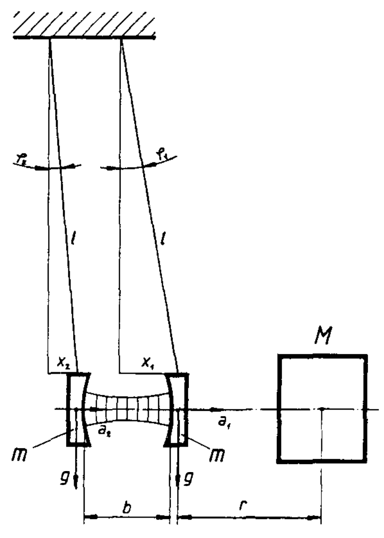

Fig. 1: The principle of the Fabry-Perot gravimeter

The deflection angles of the pendula (typically 10^{-8} rad) are very small, and therefore the displacement of each mirror can be approximated as a horizontal translation proportional to the gravitational field of the test mass.

The quasi static change in the mirror separation Δb is therefore directly proportional to the difference Δa of the acceleration of the two pendula (a_1, a_2):

$$\Delta f = \frac{df}{db}\Delta b = \frac{df}{db}\omega_o^{-2}\Delta a \qquad (1)$$

with ω_o the eigenfrequency of the pendula. The measured shift Δf of the resonance frequency can be converted into a shift Δb of the mirror separation using the conversion factor df/db based on standard theory of resonator performance [7].

The horizontal gravitational acceleration of each pendulum is calculated by a numerical integration of Newton's inverse square law over the mass distributions of the test mass (M) and the resonator (m_1, m_2) using a Gaussian integration formula. This leads to our basic relation:

$$\Delta f(r) = \frac{df}{db}\, \omega_o^{-2} GM \left[\left(\frac{1}{r^2} - \frac{1}{(r+b)^2} \right) K(r) - \left(\frac{1}{r_{ref}^2} - \frac{1}{(r_{ref}+b)^2} \right) K(r_{ref}) \right] \tag{2}$$

The terms in parentheses correspond to the difference of the gravitational force between the test mass and each pendulum. The function K takes the finite dimensions of the masses into account (K=1 corresponds to point masses). $\Delta f\ (r)$ is the measured frequency shift of the resonator obtained by moving the test mass periodically between a position r and a reference position r_{ref}. Different separations r between the test mass and the resonator from 0.6m to 2.1m have been chosen to measure the gravitational force as a function of distance. This measurement is used to test the inverse-square law and it is a powerful tool to detect otherwise hidden systematic errors in order to increase the precision of G.

Experimental Set-Up

A schematic sketch of the experimental set-up is shown in Fig. 2. Its main part, the Fabry-Perot resonator, consist of two spherical mirrors. They are fabricated from OFHC copper and the roughness of their diamond machined surfaces is about 50 nm. The diameter of the circular mirrors is 192 mm, their radius of curvature is 580 mm and they are separated by a distance b of 240 mm. In this case stable electro-magnetic modes exist in the Fabry-Perot resonator [9].

The resonator is suspended in loops of tungsten wire mounted to a special suspension platform. The mirror separation is held constant by means of a quarz plate in order to reduce thermal drift of the resonance frequency of the Fabry-Perot resonator.

To damp the pendulum oscillations caused by microseismic noise we use eddy-current brakes which consists of iron plates supporting an array of permanent magnets with a chess-board pattern. The brakes can be adjusted in situ in order to synchronize the oscillation of the two pendula and therefore to minimize the oscillation of their separation.

The resonator with the eddy-current brakes and the suspension platform are placed inside a vacuum tank. To avoid dielectric effects and convection in the residual gas which disturb the resonance frequency of the Fabry-Perot resonator the vacuum pressure is kept below 10^{-4} mbar.

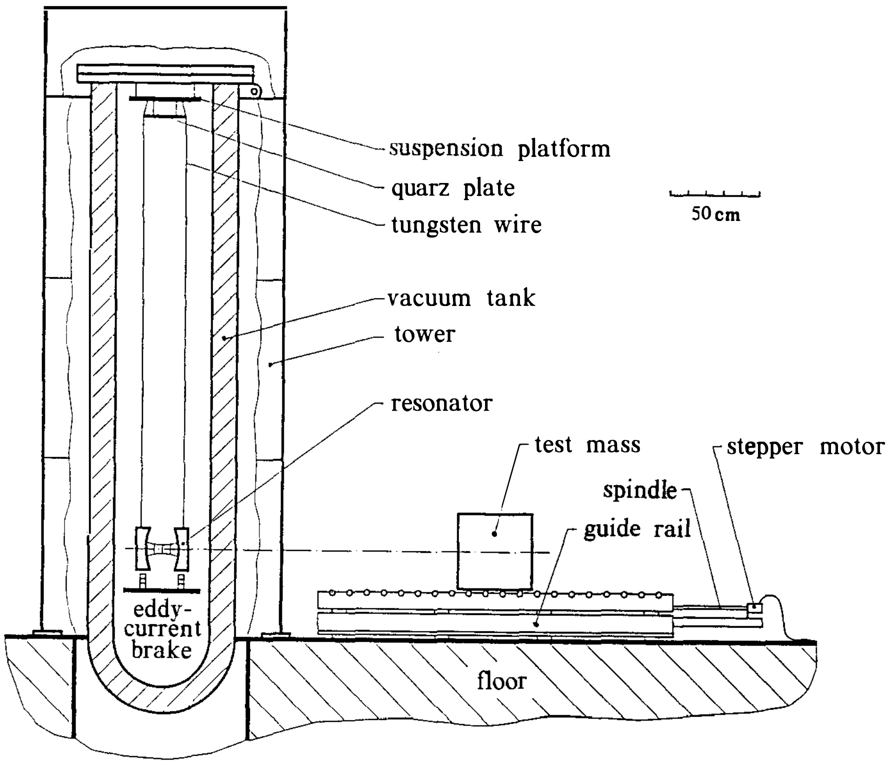

Fig. 2: The experimental set-up (schematic)

To keep thermal expansion effects small a good insulation of the vacuum tank is necessary. In our case we use a vacuum tank with an additional vacuum insulation and with additional layers of superinsulation inside.

The tank is mounted into a supporting steel trestle (called tower) to align the resonator on the same height as the test mass. The tower is build from strong steel girders to obtain such a rigidity that the lowest eigenfrequencies of the tower are much higher than the eigenfrequencies of the pendula.

The test mass outside the trestle rests on a special guide rail and glides on rollers which are mounted on ball bearings around axels fixed to this rail. The test mass is a cylinder with a diameter of 440mm and a length of 430mm. Its dimensions are chosen in a way that the gravitational force between test mass and resonator nearly behaves like point masses, e.g. the correction funktion K(r) which has been introduced in equation (2) takes values between 0.98 and 1.001. In order to avoid magnetic forces between the test mass and the resonator a special brass was choosen for the material of the test mass. The material is an alloy of 90% copper and 10% zinc with a magnetic susceptibility smaller than 10^{-5}. Thus the magnetic forces between the test mass and the resonator are well below the detection level.

The test mass is positioned precisely (~1 µm) by means of a spindle driven by a stepper motor. The stepper motor itself is controlled by a computer and allows an exact periodic motion of the test mass.

Experimental Results

The gravitational force between the test mass and the resonator is measured by moving the test mass periodically between a position r and a reference position r_{ref}.

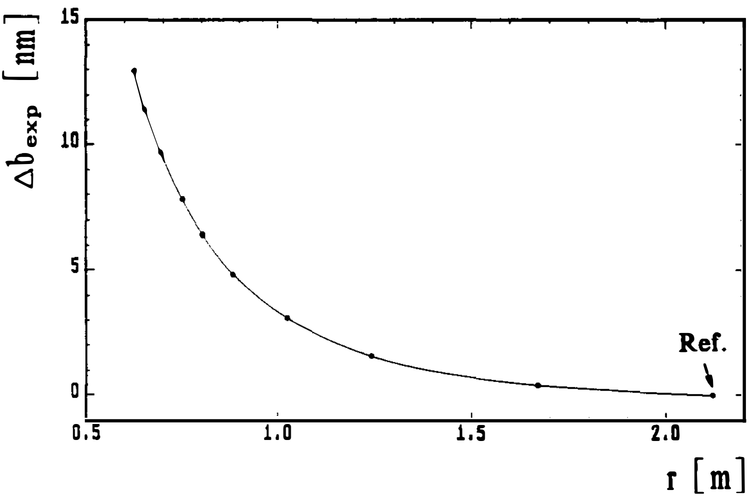

Fig. 3: The measured shift of mirror separation Δb due to the gravitational force versus the distance r.

This leads to a square wave modulation of the resonator frequency. The modulation amplitude Δf is determined from the data by means of a demodulation technique with high precision. Therefore disturbing thermal drift of the mirror separation and random noise is strongly suppressed and currently leads to a resolution of the change in mirror separation of about 1×10^{-12}m.

This procedure has been repeated for different positions of the test mass but always with the same reference position. The number of positions is usually chosen to be 9 where the number of cycles per position is about 12. In Fig. 3 the shift Δb of the mirror separation is plotted versus the distance r between the resonator and the test mass.

The residuals of Δb are shown in Fig.4 in more detail. The data points are normally distributed and no significant deviations from the inverse-square law are observed.

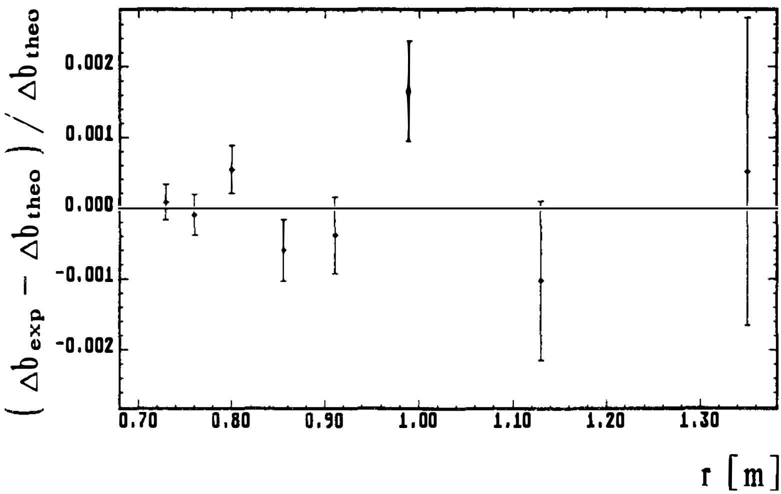

Fig. 4: The measured deviations from the inverse-square law.

The full line in Fig. 3 is a least-square fit to the data. From this fit we determined the gravitational constant with a relative accuracy of 2.2×10^{-4} to be:

$$G=(6.6724 \pm 0.0015)\times10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$$

The quoted error includes both statistical (100ppm) and systematical (200ppm) errors. The measured value of G is consistent with the CODATA value [2].

The precision achieved is not inherent to the chosen experimental method. Limits of the precision by which G can be determined caused by the finite Q ($\sim 2.4\times10^5$) of our normal conducting Fabry-Perot resonator are on a level of 5×10^{-5}. This limit can in principle be overcome by using a superconducting resonator instead of a normal conducting one. In an earlier experiment [9] it was demonstrated, that at the temperature of liquid helium (4.2K) a Q value of 1.8×10^7 can be obtained using niobium mirrors instead of copper mirrors.

The precision achieved so far is limited by systematic uncertainties and we try to get further improvements in our future work.

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