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Article

Searching for Extra Higgs Boson Effects in General Two-Higgs Doublet Model (2HDM)

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Abstract: Starting from our current *impasse* at the LHC, of observing an SM-like Higgs boson but nothing beyond, we focus on the General 2HDM (G2HDM), which possesses extra sets of Yukawa couplings as a likely *Next New Physics*. After expounding its merits, we explore our "*Decadal Mission of the New Higgs/Flavor era*", reporting on an Academic Summit Project (ASP) in Taiwan that conducts a four-pronged pursuit of G2HDM: CMS and Belle II searches, a lattice study of first-order electroweak phase transition, and phenomenology. The ASP Midterm report is based on ATLAS and CMS searches for $cg \to tH/tA \to tt\bar{c}$, where H and A are exotic neutral scalar bosons, and now progressing onto a post-Midterm $cg \to bH^+ \to bt\bar{b}$ search, where H^+ is the exotic charged Higgs boson, plus a few other searches at the LHC, all with discovery potential. We then discuss a plethora of flavor observables that can be explored by CMS and Belle II, as well as other dedicated experiments. Finally, we elucidate why G2HDM, providing myriad new dynamics, can remain well hidden so far. This brief report summarizes the progress of the ASP of the NSTC of Taiwan.

Keywords: LHC; Higgs boson; G2HDM; ASP; CMS; Belle II; flavor; lattice

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1. Our Current Impasse

The 125 GeV boson, h, was discovered [1,2] in 2012 at the Large Hadron Collider (LHC), but No New Physics ($\mathcal{NNP}$) beyond the Standard Model (BSM) has been found: not before 2012, not in 2012, and not in the dozen years since.

The alert *Science* reporter, Adrian Cho, gave the warning of this "*Nightmare Scenario: The Higgs and Nothing Else!*" beforehand, in an article published in *Science* magazine in March 2007 [3], which cited Jon Ellis concurring that "it would be the five-star disaster, because it would mean there would not need to be any new physics". And just before the ten-year anniversary celebration at CERN for the Higgs boson discovery, he published another news article [4], stating "Unless Europe's LHC *coughs* up a surprise, the field of particle physics may *wheeze* to its end". Such seems the lot for particle physics.

Indeed, people have descended on ALPs [5] (axion-like particles) and LLPs [6] (long-lived particles), a *sign* of our times, while direct and indirect searches for Dark Matter (DM) have come up empty-handed so far. We have no idea what DM is, while its "band-width" seems "infinite", as illustrated by our citation of all "White Papers" of Snowmass 2021 [7–21].

Another general direction that is now in vogue is EFT [10,22] (Effective Field Theory): since No New Particles ($\mathcal{NNP}$) are seen other than those of SM, one assumes that new states exist above some "cutoff" scale Λ, far above the known SM particles such as t, h, Z/W that are below the v.e.v. scale of 246 GeV. These latter particles give the dimension-4 terms of the SM Lagrangian, while we can only (nominally) *divine* minute deviations from SM with dimension-6 or higher operators as an expansion in $1/\Lambda$.

We, however, wish to explore a "Road Not Taken": we advocate the existence of an *extra* Higgs doublet (2HDM) that possesses *extra* Yukawa couplings, i.e., dropping the usual Z_2 symmetry, and *extra* Higgs quartic couplings grow from 5 to 7. We call this 2HDM without Z_2 the General 2HDM (G2HDM), i.e., 2HDM without any ad hoc assumptions [23].

We show that the exotic Higgs bosons, H, A, and H^+, are naturally sub-TeV in mass [24], which is precisely the niche for the LHC to explore.

This paper is organized as follows. In the next section, we turn to advocate the *5 Merits* of G2HDM. We then give our ASP Midterm Report of the "*Decadal Mission of the New Higgs/Flavor Era*", namely that both ATLAS and CMS have published their searches for $cg \to tH/tA \to tt\bar{c}$, with the signature of the same-sign top quark pair plus jet; we outline the 5-year Academic Summit Project of the same name to illustrate its scope. In Section 4, we give our Post-Midterm report on the $cg \to bH^+ \to bt\bar{b}$ and $cg \to tH/tA \to tt\bar{t}$ search program, where H^+, H, and A are from the exotic doublet. In Section 5, we illustrate G2HDM as the potential *Next New Physics* (*NNP*), and show how CKM enhancement was first revealed via the H^- mediation of $B^- \to \mu^-\nu$, i.e., a flavor physics process. After some discussion, we conclude in Section 6: from $\mathcal{NNP}$ to *NNP*.

2. General Two-Higgs Doublet Model

Having already observed one weak scalar doublet, and with no theorem forbidding a second, 2HDM should be a no-brainer. And dropping the Z_2 symmetry conforms with this author's maxim: "any added assumption should cost one $\mathcal{O}(\alpha)$ in terms of realizability". In G2HDM, there is no Z_2 symmetry, and hence, the two scalar doublets are identical and cannot be distinguished. The standard approach, then, is to choose the "Higgs basis" [24,25] and lump v.e.v., i.e., spontaneous symmetry breaking, to the Φ doublet, while the exotic doublet Φ' does not generate the v.e.v. Therefore, there should exist extra Yukawa matrices ρ^f for Φ', where $f = \ell, u, d$. Furthermore, without a Z_2 symmetry, the number of quartic self-couplings increases from 5 to 7.

With two identical scalar doublets, in the Higgs basis where only one doublet breaks the symmetry, the most general Higgs potential assuming CP conservation is [24,25]

$$
\begin{aligned}
V(\Phi,\Phi') &= \mu_{11}^2|\Phi|^2 + \mu_{22}^2|\Phi'|^2 - (\mu_{12}^2\Phi^\dagger\Phi' + \text{h.c.}) \\
&\quad + \frac{\eta_1}{2}|\Phi|^4 + \frac{\eta_2}{2}|\Phi'|^4 + \eta_3|\Phi|^2|\Phi'|^2 + \eta_4|\Phi^\dagger\Phi'|^2 \\
&\quad + \left[\frac{\eta_5}{2}(\Phi^\dagger\Phi')^2 + \left(\eta_6|\Phi|^2 + \eta_7|\Phi'|^2\right)\Phi^\dagger\Phi' + \text{h.c.}\right],
\end{aligned}
\tag{1}
$$

where η_i values are quartic couplings and taken as real. Φ generates v to break EW symmetry spontaneously via a first minimization condition, $\mu_{11}^2 = -\frac{1}{2}\eta_1 v^2$, while $\langle\Phi'\rangle = 0$ hence $\mu_{22}^2 > 0$. A second minimization condition [24], $\mu_{12}^2 = \frac{1}{2}\eta_6 v^2$, removes μ_{12}^2 as a parameter; this latter point seems more appealing than the usual 2HDMs with Z_2 symmetry.

The general Yukawa couplings are [25,26]

$$
\begin{aligned}
\mathcal{L}_Y = \frac{1}{\sqrt{2}} \sum_{f=u,d,\ell} \bar{f}_i \Big[(\lambda^f_{ij} s_\gamma - \rho^f_{ij} c_\gamma)\, h - (\lambda^f_{ij} c_\gamma + \rho^f_{ij} s_\gamma) H + i\,\text{sgn}(Q_f)\, \rho^f_{ij}\, A \Big] R\, f_j \\
- \bar{u}_i \big[(V\rho^d)_{ij} R - (\rho^{u\dagger} V)_{ij} L \big] d_j H^+ - \bar{\nu}_i\, \rho^\ell_{ij} R\, \ell_j H^+ + \text{h.c.},
\end{aligned}
\tag{2}
$$

where $i, j = 1, 2, 3$ are generation indices, $L, R = (1 \mp \gamma_5)/2$, $sgnQ_f = +1\ (-1)$ for $f = u$ $(f = d, \ell)$, $c_\gamma \equiv \cos\gamma$ is the h-H mixing angle between the two CP-even scalars, and $s_\gamma \equiv \sin\gamma$, while V is the CKM matrix. The elements $\lambda^f_{ij} = \delta_{ij}\sqrt{2} m^f_i / v \equiv \delta_{ij}\lambda_t$ are real as mass m^f_i is real and already measured, with $v \simeq 246$ GeV. The extra Yukawa couplings ρ^f_{ij} are non-diagonal and in general complex, befitting their Yukawa coupling nature. We will return to comment on these non-diagonal extra Yukawa couplings in Merit-**3** below.

Merit-**1** of G2HDM is the $\mathcal{O}(1)$ extra top Yukawa couplings, either ρ_{tt} or ρ_{tc}, and each could [27] drive the electroweak baryogenesis (EWBG). The leading driving formula is [27]

$$
\lambda_t\, \text{Im}\, \rho_{tt}, \tag{3}
$$

a beautiful result of co-operating doublets, with the exotic doublet providing the imaginary Yukawa coupling. Interestingly, Higgs quartic couplings η_i at $\mathcal{O}(1)$ can give [28] the first-order EW phase transition ($1^{\rm st}$EWPT $\longrightarrow$ *primordial* gravitational waves!), and hence, two Sakharov conditions [29] are satisfied; the baryon number violation at a high temperature is a given.

Billions and billions of stars, and all those protons burning to light up the Universe—but seeing to the end (i.e., beginning) of the Universe: no sign of antiprotons burning! That is, we see no violent matter–antimatter interfaces. The baryon asymmetry of the universe (BAU), or disappearance of antimatter from the very early universe, is indeed a problem as big as the universe itself, and at the very core of our own existence, hence a great motivator. Equation (3) shows the (already) measured $\lambda_t \simeq 1$, which is real, pairing with the imaginary part of ρ_{tt}, where a best guess would be $|\rho_{tt}| = \mathcal{O}(\lambda_t) \simeq 1$, which also holds for the imaginary part. But this brings about the next point: how to survive electron electric dipole moment (eEDM) bounds of ACME [30] and JILA [31], the current cutting edge of CP violation (CPV) search. This is of course a generic challenge to *any* attempt at EWBG.

Keeping the ρ_{ee} of the charged lepton extra Yukawa matrix ρ^{ℓ}, Merit-**2** of G2HDM is a spectacular diagrammatic cancellation [32] of two-loop Barr–Zee diagrams for eEDM, as illustrated in Figure 1. This is rather impressive, resulting in

$$|\rho_{ee}/\rho_{tt}| \sim \lambda_e/\lambda_t, \tag{4}$$

where a "phase lock" [32] of $\arg \rho_{ee} = -\arg \rho_{tt}$ is a prerequisite for Equation (4). As we argue later, we may have unraveled "the flavor code": did *Nature* set up the observed charged fermion mass and mixing hierarchies—the *flavor enigma* itself—to ensure this cancellation!?

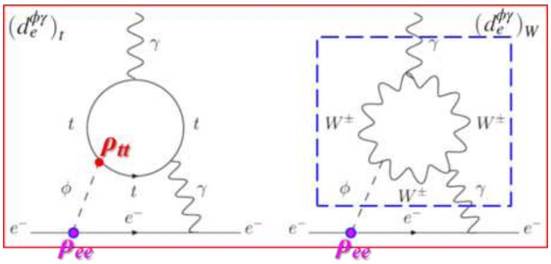

Figure 1. Two-loop Barr–Zee diagrams for eEDM, where the top loop and the W loop naturally tend to cancel, effectively a ϕ-γ-γ^* insertion, where ϕ runs over h, H, A, and even H^+.

Merit-**3** of G2HDM addresses the *Natural Flavor Conservation* (NFC) condition of Glashow and Weinberg [33]. The alert reader should have perceived that, when theorists—even Nobel laureates—blow the "*Natural*" trumpet, it *ain't natural*. Glashow had the legitimate worry of flavor changing neutral couplings; e.g., ρ_{tc} can induce $t \to ch$ [34] (Ref. [34] pointed out that $t \to ch$ would be *naturally* controlled by mass and mixing hierarchies observed in SM fermions.) But let us stress that, having discovered $h(125)$ being lighter than the top, it is "a PDG duty" [35] for us to search. Curiously, $t \to ch$ decay remains elusive to date, though both ATLAS and CMS have searched for it quite extensively. Noticing that h is rather close to the SM-Higgs, *Nature* threw in *alignment* (i.e., small h-H mixing, $c_\gamma \equiv \cos\gamma$, where H is the CP-even scalar from the exotic doublet), an *emergent* phenomenon from circa 2015–2016. *Who would have thought!?* After all, c_γ is a purely Higgs sector parameter that has nothing to do with *flavor*! Alignment also alleviates any $h \to \tau\mu, \tau e$ and μe constraints, as bounds are rather poor.

Merit-**4** of G2HDM is that a small c_γ does *not* [24] contradict $\mathcal{O}(1)$ quartics, and that

$$c_\gamma \simeq \frac{\eta_6 v^2}{m_H^2 - m_h^2}, \tag{5}$$

since $s_\gamma \equiv \sin\gamma \to 1$ with a small c_γ. In fact, one can turn the argument around [24] to show that exotic scalars H, A, and H^+ populate 300–600 GeV. However, $\mathcal{O}(1)$ quartics imply Landau pole behavior, which can only be properly studied on the lattice (see below).

With $t \to ch$ suppressed by alignment (small c_γ), Merit-**5** of G2HDM is that it is more natural to pursue [36]

$$cg \to tH/tA \to tt\bar{c}, \tag{6}$$

which is not alignment-suppressed, but controlled by $s_\gamma \to 1$. It was subsequently found that a better process to probe would be [37]

$$cg \to bH^+ \to bt\bar{b}. \tag{7}$$

Not only would the associated b-jet be less costly compared with an associated t in Equation (6), but it turns out to be CKM-enhanced compared with 2HDM-II, the SUSY-type 2HDM.

3. Decadal Mission of the New Higgs/Flavor Era

We are happy to report that both ATLAS and CMS have completed their search for $cg \to tH/tA \to tt\bar{c}$ of Equation (6), with both search results published, in *JHEP* [38] and *PLB* [39], respectively. Though the CMS effort already commenced in February 2020, it started with just one experienced researcher and a graduate student. We therefore express our sincere gratitude for the timely approval of a 5-year Academic Summit Project by the NSTC of Taiwan in August 2021, such that we could build-up sufficient resources in time to complete our CMS search, lagging ATLAS only by several months.

3.1. Academic Summit Project (ASP)

The ASP lends its name to the title of this section, and it has four subprojects:

1. CMS: H, A, H^+ search @ LHC (since 2020).
2. Flavor physics searches:
   Belle II ($B \to \mu\nu$, $\tau\nu$; $\tau \to \mu\gamma$);
   CMS ($B_{s,d} \to \mu\mu$; $t \to ch$).
3. Lattice: Higgs potential (1$^{\text{st}}$EWPT and Landau pole).
4. Steering: Pheno (since 2017).

The ASP provided a timely injection of funds to assemble the CERN-side CMS team especially. We recently passed the Midterm point of ASP execution, and hence, we turn to our "ASP Midterm Report" on $cg \to tH/tA \to tt\bar{c}$ search. But let us report some recent Pheno progress: (1) To improve H^+ reconstruction in Equation (7), which is hampered by having three b-jets in the final state, making the $t\bar{b}$ pairing difficult, we studied $bg \to cH^- \to c\bar{t}b$ instead, where $H^- \to \bar{t}b$ reconstruction is unambiguous [40], while the recoiling c provides both a tag, and as a discriminant to suppress the background; (2) A revisit of eEDM exposed a larger parameter range, indicating eEDM might be discovered soon, and possibly followed by neutron EDM (nEDM) in a decade or two [41]; (3) The work was further extended to direct CPV difference in B^+ vs $B^0 \to X_s\gamma$ [42] for further study via Belle II.

3.2. ASP Midterm Report: $cg \to tH/tA \to tt\bar{c}$ Search

As stated in Section 2, with $t \to ch$ alignment-suppressed, it is natural to pursue $cg \to tH/tA \to tt\bar{c}$, which is controlled by $s_\gamma \simeq 1$. We will not go into the details of the ATLAS paper, as we were not involved, but we show their Figure 10 in Figure 2 below, where the star indicates the highest observed significance of 2.8 σ at $m_H = 900$ GeV. See Ref. [38] for further details.

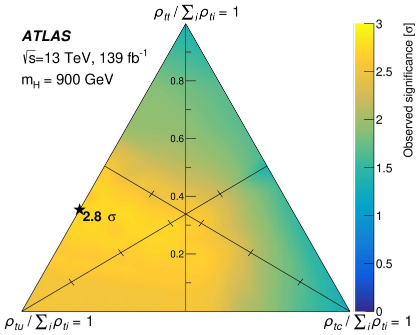

Figure 2. Figure 10 as taken from ATLAS paper [38].

A representative diagram for $cg \to tH/tA \to tt\bar{c}$, taken from Figure 1 of the CMS paper [39], is given in Figure 3 below, where $q' = q = u, c$ is assumed. The same-sign top pair leads to same-sign di-leptons, and the additional jet serves as a further discriminant.

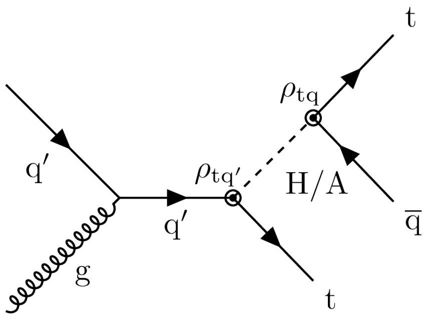

Figure 3. Figure 1 as taken from CMS paper [39]. Note that $H/A \to t\bar{t}$ is also possible.

For the CMS study, we again do not give details, but show some results. Let us begin with Table 3 of the published paper, which we display in Figure 4. The CMS study takes into account H–A interference [36]: since A couples to $t\bar{c}$ with an i, it can hence be destructive against H if the two are nearby. The results in Table 3 of CMS (our Figure 4) can be verified by inspection of Figures 4 and 5 of the published CMS paper [39], which we do not display.

The main result of the CMS paper is given in Figures 5 and 6 below, which correspond to Figures 6 and 7 of the CMS paper [39]. One can see that the constraint on ρ_{tu} is considerably more stringent than on ρ_{tc}, and more so for the A–H interference case.

	Observed (expected) mass limit [GeV]		
	without interference	with interference	with interference
	m_{A} or m_{H}	m_{A}	m_{H}
ρ_{tu}			
0.4	920 (920)	1000 (1000)	950 (950)
1.0	1000 (1000)	1000 (1000)	950 (950)
ρ_{tc}			
0.4	no limit	340 (370)	290 (320)
1.0	770 (680)	810 (670)	760 (620)

Figure 4. Table 3 as taken from CMS paper [39].

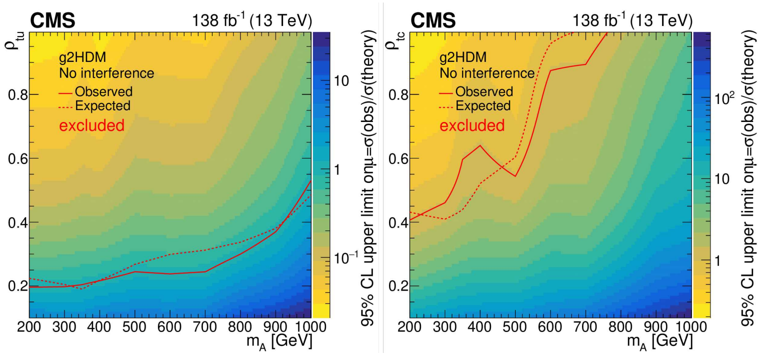

Figure 5. Observed 95% CL upper limit on signal strength vs m_A and ρ_{tu} (**left**) and ρ_{tc} (**right**) for G2HDM without A–H interference, for the combination of $e^\pm e^\pm$, $\mu^\pm\mu^\pm$, and $e^\pm\mu^\pm$ categories. The color axis represents the observed upper limit on the signal strength. Expected (dashed) and observed (solid) exclusion contours are also shown (taken from Figure 6 of Ref. [39]).

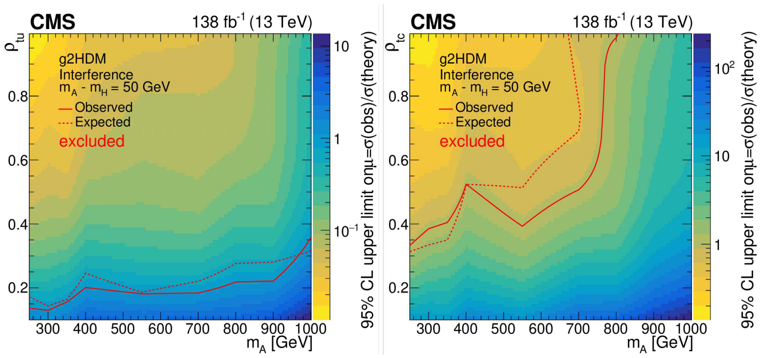

Figure 6. Same as Figure 5, but with A–H interference (taken from Figure 7 of Ref. [39]).

To conclude this section, we note that neither ATLAS [38] nor CMS [39] has seen any evidence of a signal so far, but that may not be too surprising.

4. Post-Midterm: $pp \to bH^+ \to bt\bar{b}$ and $pp \to tH/tA \to tt\bar{t}$ @ CMS

As discussed in Section 2, the $cg \to bH^+ \to bt\bar{b}$ process of Equation (7) is more promising than the $cg \to tH/tA \to tt\bar{c}$ process of Equation (6). Not only is the cross section several times larger [37] due to the CKM enhancement (as illustrated in Figure 7 below), but having an accompanying b-jet also means one can probe a broader, more reasonable m_{H^+} range, rather than suffering the higher threshold by an accompanying t quark in Equation (6).

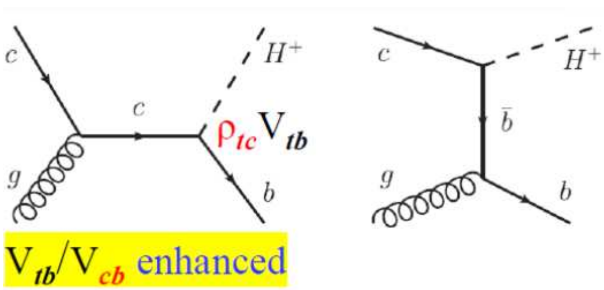

Figure 7. The $|V_{tb}/V_{cb}|$ enhancement of $cg \to bH^+$ process w.r.t. 2HDM-II. $H^+ \to t\bar{b}$ decay receives the same CKM factor V_{tb} multiplying ρ_{tt} [37].

However, aside from moving on to $cg \to bH^+ \to bt\bar{b}$ search, the ASP group is also considering $cg \to tH/tA \to tt\bar{t}$ (see caption of Figure 4), or a triple-top search simultaneously, which was not touched upon in our previous $tt\bar{c}$ search [39].

Furthermore, since Run 3 is now progressing well, as the amount of data accumulated is beyond twice that of Run 2, the ASP group would like to repeat the $t \to ch$ search, as well as the $tt\bar{c}$ search. The former is alignment-suppressed, but we do not know c_γ, the h-H mixing angle, while with the $tt\bar{c}$ search with double (or triple) Run 2 data, one could possibly see a hint. Thus, all four modes have a "discovery" prospect. The $t \to ch$ process could plainly "emerge" at any time.

We trust that the ATLAS team would do the same, for healthy competition.

5. G2HDM as Next NP!?

Now, we point out that the "magic" of the H^+ couplings in G2HDM for the process of Equation (7), i.e., CKM enhancement of $cg \to bH^+ \to bt\bar{b}$, was first noticed [43] through the study of flavor physics, namely $B^+ \to \mu^+\nu$, as illustrated in Figure 8 below.

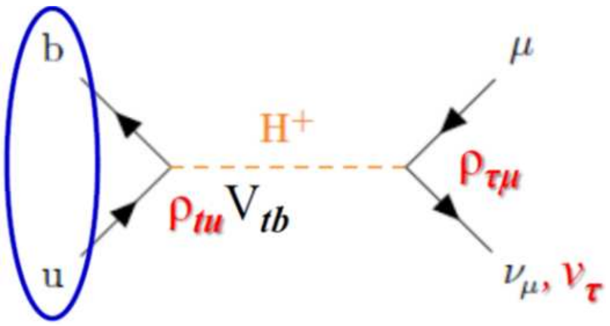

Figure 8. $B^+ \to \mu^+\nu$ decay, where the extra Yukawa coupling $\rho_{\tau\mu}$ enters the process.

Here, the $\bar{b}$ quark annihilates the u quark and the B^+ meson disappears into the purely leptonic μ^+ plus a neutrino. Since the flavor of the latter cannot be detected, the escaping neutrino could be a ν_τ, bringing in the extra Yukawa coupling $\rho_{\tau\mu}$ for charged leptons [43]. One notices that the $\bar{b}u$ annihilation receives an astounding V_{tb}/V_{ub} enhancement factor compared with 2HDM-II, as one would need V_{ub} to proceed for the latter; the process uniquely probes the extra Yukawa coupling product $\rho_{tu}\rho_{\tau\mu}$. The SM value for $\mathcal{B}(B^+ \to \mu\nu)/\mathcal{B}(B^+ \to \tau\nu)$ would be 0.0045, from kinematic factors of m_μ and m_τ, but 2HDM-II would give the *same* value [44]! Thus, a measurement of this ratio could not only facilitate the discovery of BSM physics, but also rule out 2HDM-II, which is realized with SUSY!

That $\mathcal{B}(B \to \mu\nu)/\mathcal{B}(B \to \tau\nu)$ in G2HDM could differ from SM (and even 2HDM-II) was first pointed out in an experimental FPCP review [45].

The measurement of the $\mathcal{B}(B \to \mu\nu)/\mathcal{B}(B \to \tau\nu)$ ratio, however, is quite nontrivial. The numerator is not yet measured at the evidence level [35], which would be dominated by statistical errors for some time to come. However, there is no well-defined methodology for measuring $B \to \tau\nu$ (though it appears consistent with SM), which would be dominated by not so well-defined systematic errors until a more definite method emerged.

To perform this important ratio measurement at Belle-II, a definite new method for $B \to \tau\nu$ with better systematic control is called for.

From the $B \to \mu\nu, \tau\nu$ prelude, we present a rather rich and pictorial "flavor-table" in Figure 9, where we display the expected flavor effects in G2HDM to guide the eye.

Perhaps the most visible are the "five gray boxes". These reflect the $B_s \to \tau\tau$, $B \to K\tau\tau$, $B_s \to \tau\mu$, $B \to K\tau\mu$ decays that were hotly pursued by LHCb, aiming for discovery, and $\tau \to \mu\gamma$ pursued by Belle and Belle II. Though these signatures are rather interesting in their own right, the backdrop was the various *B-anomalies* of the 2010s (a warning against the *B-anomalies* from an experimental perspective was sounded in Ref. [46]), that suffered from the December 2022 LHCb confession [47] that the R_K and R_{K^*} "*anomalies*" that dominated the flavor scene were in fact driven by hadrons faking electrons (hence driving up the numerator)—quite a known effect! With the disappearance of the R_K and R_{K^*} "*anomalies*", these modes are of less concern, though we stress again that they are certainly interesting in their own right and ought to be pursued.

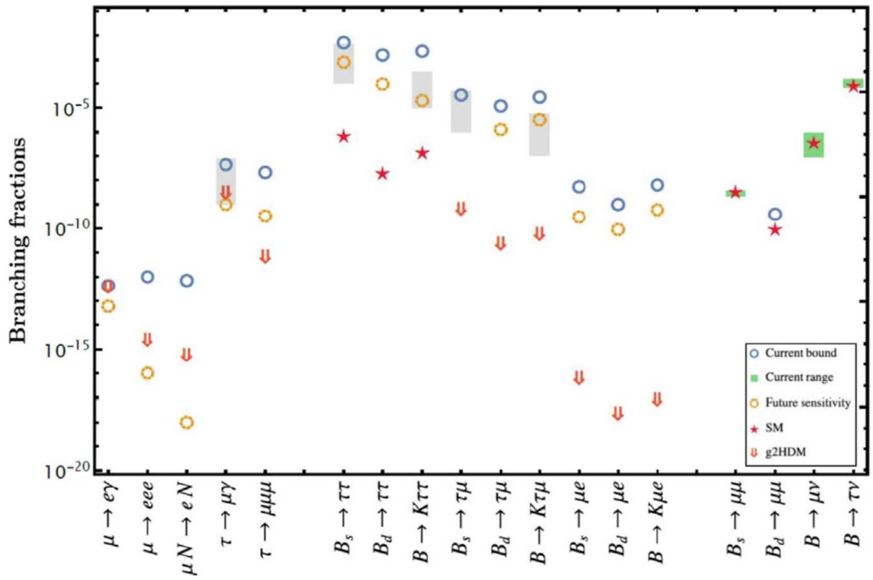

Figure 9. A picture table of the *new flavor era* of G2HDM in the coming decades [48]. The five gray boxes are remnants of *B-anomalies*, which evaporated with disappearance of R_K and R_{K^*} "*anomalies*" [47]. Blue circles are the experimental limits as of 8/2020, while orange circles are projected limits. The downward red arrow depicts the G2HDM expectation according to the "Rule of Thumb" of Equation (8) below, which explains why G2HDM effects are well-hidden when lighter generations are involved. The red stars reflect SM expectations, and the green boxes are SM-G2HDM interference ranges.

But now, we can draw our eye to the G2HDM perspective, the 2HDM without Z_2.

First, the $\mu \to e\gamma$, $\mu \to eee$, and $\mu N \to eN$ processes at the lower left are of great interest. The first process is currently pursued by MEG II, which just published the 90% CL bound of $\mathcal{B}(\mu \to e\gamma) < 7.5 \times 10^{-13}$ [49]. Combining with the final result of MEG [50] gives $\mathcal{B}(\mu \to e\gamma) < 3.1 \times 10^{-13}$ [49]. The $\mu \to e\gamma$ process still has discovery potential in G2HDM through two-loop diagrams that are not so different from the eEDM diagrams of Figure 1, i.e., changing the initial electron to a muon. All three $\mu \to e$ processes have dedicated experiments, with the eventual reach of $\mu N \to eN$ particularly impressive, and augmented further by healthy competition between KEK and Fermilab. The red arrows depicted in Figure 9 are relatively far away from current sensitivity. This is because G2HDM predicts the $\mu e\gamma$ vertex to be dipole-dominant, which can be tested by Mu3e by measuring the relatively co-linear e^+e^- pairs, while for PRISM/Mu2e experiments, more thought and study would be needed. Likewise, G2HDM also predicts $\tau\mu\gamma$ to be dipole-dominant, hence the "late" discovery of $\tau \to \mu\gamma$ if at all, while $\tau \to \mu\mu\mu$ lies beyond reach for Belle II.

Next, $B_{s,d} \to \tau\tau$ and $B \to K\tau\tau$ occur in SM, but are beyond the projected sensitivity. Likewise, $B_{s,d} \to \tau\mu$ decays also occur in G2HDM, but our "Rule of Thumb" for flavor control,

$$\rho_{ii} \lesssim \mathcal{O}(\lambda_i); \quad \rho_{1i} \lesssim \mathcal{O}(\lambda_1); \quad \rho_{3j} \lesssim \mathcal{O}(\lambda_3)\ (j \neq 1), \tag{8}$$

gives the position of red arrows in Figure 9, and illustrates why G2HDM, while rich in extra dynamics, is so far quite well hidden, as is visible from the greatly suppressed $B_{s,d} \to \mu e$ and $B \to K\mu e$ decays: flavor protection of Equation (8), our "Rule of Thumb". This "Rule of Thumb" is supported by the miraculous eEDM cancellation mechanism of Figure 1.

Finally, we draw our attention to the remaining four "red stars", the processes $B_{s,d} \to \mu\mu$ and the aforementioned $B \to \mu\nu, \tau\nu$, which can all occur within SM. After hinting at sub-SM strength between CMS and LHCb for some while [35], the measurement of $B_s \to \mu\mu$ by CMS [51], which turned out to be consistent with SM, served as a prelude to the evaporation [47] of the "*B-anomalies*" by LHCb. But $B_d \to \mu\mu$ remains unmeasured.

As previously discussed, $\mathcal{B}(B \to \mu\nu)/\mathcal{B}(\to \tau\nu) = 0.0045$ is expected in SM, as well as in 2HDM-II [44], and offers a very interesting test of G2HDM. We note that the illustrated range for $B \to \mu\nu$ is quite wide due to SM-G2HDM interference, inasmuch as the current $B \to \tau\nu$ value is consistent with SM. The measurement of this ratio would be a major target for Belle II.

6. Discussion and Conclusions

We have followed an *unconventionally* conventional "Road Not Taken", as G2HDM has not gained much traction so far! But we wish to emphasize that it may very well end up being our *Next New Physics* (*NNP*)! The definite observation of one Higgs doublet should make the 2HDM a no-brainer, while G2HDM has no added ad hoc assumptions, such as NFC. We have seen the plethora of modes open for search in Figure 9, as well as H, A, H^+ direct search modes of Equations (6) and (7).

It was with Merit-**4** of G2HDM, having an extra doublet but with extra Yukawa couplings, that we emphasized that the exotic Higgs bosons from Φ' would likely [24] populate 300–600 GeV, as depicted in Figure 10. This is based on Higgs quartics η_is being $\mathcal{O}(1)$ in strength, i.e., in the *naive naturalness* sense of $|\eta_i| < 3$, which could [28] give the 1$^{\rm st}$EWPT. This is why we brought in the lattice arm to the ASP, since one cannot trust perturbation theory any more. With 7 $\mathcal{O}(1)$ quartic couplings, one can easily run into the Landau pole problem, making the lattice simulation quite challenging. The Landau pole could be at 10 to 20 TeV [24]. If we find a hint for this at the LHC, it may in turn guarantee, or justify, the FCC and CEPC/SppC developments! It could well be that we got the SUSY scale wrong, and we get another shot at SUSY at a much higher scale.

As such, there may be a lot of work ahead for particle physicists.

We therefore conclude with our Decadal Mission:

"*Find* the **extra *H*, *A*, *H*$^+$ bosons**;
Crack the **Flavor Code**;
Solve the **Mysterious BAU!**"

As for the *Flavor Code*, we wonder whether the eEDM cancellation point, $|\rho_{ee}/\rho_{tt}| \simeq \lambda_e/\lambda_t$, implies that the extra ρ^ℓ and ρ^u matrices seem to *know* the observed mass and mixing hierarchies of the SM sector already. Does this reflect the "flavor design" of *Nature*?

It is up to *Nature* whether our "Wish for Discovery" is granted, or not.

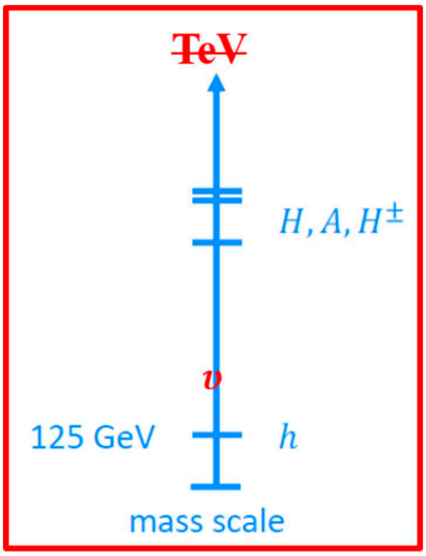

Figure 10. Illustration of the "Road Not Taken": sub-TeV H, A, and $H^\pm$ exotic Higgs bosons.

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Abbreviations

The following abbreviations are used in this manuscript:

G2HDM	General Two-Higgs Doublet Model
ASP	Academic Summit Project
NSTC	National Science and Technology Council (Taiwan)
LHC	Large Hadron Collider
$\mathcal{NNP}$	No New Physics, or No New Particles
CKM	Cabibbo–Kobayashi–Maskawa
NNP	Next New Physics
SM	Standard Model
BSM	Beyond SM
ALP	Axion-like particles
LLP	Long-lived particles
DM	Dark Matter
EFT	Effective Field Theory
CP	Charge–Parity
EWBG	Electroweak baryogenesis
1^{st}EWPT	First-order electroweak phase transition
BAU	Baryon asymmetry of the universe
eEDM	Electron electric dipole moment
CPV	CP violation
NFC	Natural Flavor Conservation
PDG	Particle Data Group
SUSY	Supersymmetry
nEDM	Neutron electric dipole moment
FPCP	Flavor physics and CP violation
PRD-L	Physical Review D Letter
FCC	Future Circular Collider
CEPC	Circular Electron–Positron Collider
SppC	Super proton–proton Collider

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